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NUMERICAL SIMULATION OF SWIRLING FLOWS WITH SEPARATION ZONES

Vadim K. Akhmetov

Moscow State University of Civil Engineering, Moscow, RUSSIA

Abstact. A numerical study of axisymmetric viscous swirling flows of fluid was carried out at moderate Reynolds numbers. The mathematical model is based on the use of the complete Navier-Stokes system equations, written in terms of the stream function, vorticity and azimuthal velocity variables. Boundary value problems for flows with different configurations are formulated. The numerical solution is carried out by the finite-difference relaxation method using the Leonard scheme of the third order of accuracy for approximating the convective terms in the transport equations and the method of incomplete reduction for solving the Poisson equation. The flow streamlines and velocity distribution patterns were obtained. The results are compared with available experimental data. The possibility of using the obtained results for the analysis of turbulent flows by introducing effective parameters is considered.

Keywords: swirling flows, coaxial flows, finite difference method, recirculation zones, vortex breakdown, effective parameters

ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ ЗАКРУЧЕННЫХ ТЕЧЕНИЙ С ОТРЫВНЫМИ ЗОНАМИ

В.К. Ахметов

Национальный исследовательский Московский государственный строительный университет, Москва, РОССИЯ

Аннотация. Проведено численное исследование осесимметричных закрученных течений вязкой жидкости при умеренных числах Рейнольдса. Математическая модель основана на использовании полной системы уравнений Навье-Стокса, записанной в переменных функции тока, завихренности и азимутальной скорости. Сформулированы краевые задачи для течений с различной конфигурацией. Численное решение проводится конечно-разностным методом установления с применением схемы Леонарда третьего порядка точности для аппроксимации конвективных членов в уравнениях переноса и метода неполной редукции при решении уравнения Пуассона. Получены картины линий тока и распределения скоростей в потоке. Результаты сравниваются с имеющимися экспериментальными данными. Рассмотрена возможность использования полученных результатов для анализа турбулентных течений за счет введения эффективных параметров.

Ключевые слова: закрученные течения, коаксиальные потоки, метод конечных разностей, рециркуляционные зоны, распад вихря, эффективные параметры

1. INTRODUCTION

Swirling flows are widely used in various technical devices [1, 2] and hydraulic engineering structures [3-5]. The properties of swirling flows are utilized for combustion stabilization, intensification of mixing processes, gas cleaning from dust, particle separation, thermal separation of flows, energy dissipation in vortex drop shafts, in counter-vortex devices for reservoir aeration, as well as in many other fields, particularly in the petrochemical, microbiological, and food industries. A distinctive feature of swirling flows is the formation of separated recirculation zones upon reaching a certain critical value of the initial flow swirl. These zones can be beneficial or may cause undesirable effects. The aim of this work is to perform numerical simulation of swirling flows with separation

zones in relation to various technical applications and problems in hydraulic engineering construction.

2. PROBLEM STATEMENT AND RESEARCH METHODS

Studies of the hydrodynamics of swirling flows of a viscous incompressible fluid at moderate Reynolds numbers are based on the complete system of Navier-Stokes equations. Assuming axisymmetry of the flow in a cylindrical coordinate system r, φ, z. Here, z-axis is directed along the axis of the flow under study, in terms of the stream function ψ, vorticity Ω, and azimuthal velocity V_φ, these equations can be represented as follows:

$$\frac{1}{r}\frac{\partial^2\psi}{\partial z^2}+\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\psi}{\partial r}\right)=-\Omega, \qquad (1)$$

$$\frac{\partial\Omega}{\partial t}+\frac{\partial}{\partial z}\left(V_z\Omega\right)+\frac{\partial}{\partial r}\left(V_r\Omega\right)=$$
$$=\frac{1}{\mathrm{Re}}\left[\frac{\partial^2\Omega}{\partial z^2}+\frac{\partial^2\Omega}{\partial r^2}+\frac{\partial}{\partial r}\left(\frac{\Omega}{r}\right)\right]+G^2\frac{1}{r}\frac{\partial\left(V_\varphi\right)^2}{\partial z}, \qquad (2)$$

$$\frac{\partial V_\varphi}{\partial t}+\frac{\partial}{\partial z}\left(V_zV_\varphi\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(rV_rV_\varphi\right)+\frac{V_rV_\varphi}{r}=$$
$$=\frac{1}{\mathrm{Re}}\left[\frac{\partial^2V_\varphi}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_\varphi}{\partial r}\right)-\frac{V_\varphi}{r^2}\right], \qquad (3)$$

$$V_r=-\frac{1}{r}\frac{\partial\psi}{\partial z},\quad V_z=\frac{1}{r}\frac{\partial\psi}{\partial r},\quad \Omega=\frac{\partial V_r}{\partial z}-\frac{\partial V_z}{\partial r}. \qquad (4)$$

The system of equations (1)-(4) is written in a conservative dimensionless form. The axial V_z and radial V_r velocities are referred to the characteristic value U_0. The azimuthal velocity V_φ refers to the characteristic value W_0. The coordinates refer to the characteristic size R, time refers to the value R/U_0. The parameters of the system are the Reynolds number $\mathrm{Re}=U_0R/\nu$, where ν is the kinematic viscosity, and the swirl parameter $G=W_0/U_0$. These equations are essentially nonlinear. Analytical solutions for specific problems can only be obtained in a number of special cases.

The boundary value problem is formulated by setting boundary conditions, which include a given velocity distribution of the flow at the inlet to the considered flow region, no-slip conditions on solid surfaces, symmetry conditions on the flow axis, and soft boundary conditions at the outlet.

For an axisymmetric channel with solid walls, at the inlet of which a swirling flow arrives with a velocity distribution:

$$\begin{aligned}
&V_{z0}(r)=D_0+D_1\exp(-B_1r^2),\ 0\le r\le r_0,\\
&V_{z0}(r)=a_0+a_1r+a_2r^2,\qquad r_0\le r\le 1,\\
&V_{\varphi0}(r)=\frac{A}{r}(1-\exp(-B_2r^2)),\ 0\le r\le r_0,\\
&V_{\varphi0}(r)=b_0+b_1r+b_2r^2,\qquad r_0\le r\le 1
\end{aligned} \qquad (5)$$

the set of boundary conditions is written as follows:

$$\psi=\psi_0(r),\quad V_\varphi=V_{\varphi0}(r),$$
$$\frac{\partial\psi}{\partial z}=0,\quad 0\le r\le 1,\quad z=0, \qquad (6)$$

$$\psi=\psi_0(1)=\psi_1,\quad V_\varphi=0,$$
$$\frac{\partial\psi}{\partial r}=0,\quad 0\le z\le z_k,\quad r=1, \qquad (7)$$

$$\psi=0,\ V_\varphi=0,\ \Omega=0,\ 0\le z\le z_k,\ r=0, \qquad (8)$$

$$\frac{\partial\psi}{\partial z}=\frac{\partial\Omega}{\partial z}=\frac{\partial V_\varphi}{\partial z}=0,\ 0\le r\le 1,\ z=z_k. \qquad (9)$$

To solve the boundary value problem (1)-(4), (6)-(9) with the initial distribution (5), the finite difference method was used, employing the incomplete reduction method [6] for equation (1) and approximation of the convective terms in equations (2)-(3) using Leonard's scheme [7]. The calculations assumed the following values: $D_0=1$, $D_1=0$, $B_1=0$, $B_2=14$, $A=0.419$, $r_0=0.75$, $z_k=10$. The characteristic flow pattern with two separation zones is presented in Fig. 1.

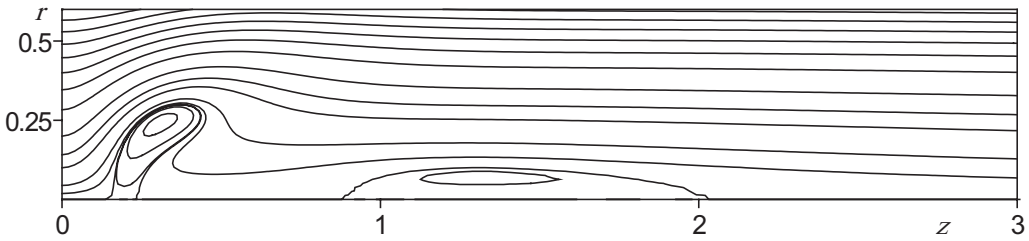


Figure 1. Streamlines for Re=150, G=2.6

If the flow is in a channel with permeable walls, one can assume that additional inflow occurs at a constant velocity $-V_0$ on a part of the lateral surface $0 \le z \le z_0$, $r = 1$. Then the boundary condition (7) for the stream function at $r = 1$ must be replaced by the following:

$$\begin{aligned} \psi &= V_0 z + \psi_1, \quad 0 \le z \le z_0, \\ \psi &= V_0 z_0 + \psi_1, \quad z_0 \le z \le z_k . \end{aligned} \tag{10}$$

The main effect in this case is that by setting the appropriate inflow velocity, it is possible to influence the boundaries of the reverse flow zones, up to their complete elimination. Examples of the calculated flows are shown in Fig. 2.

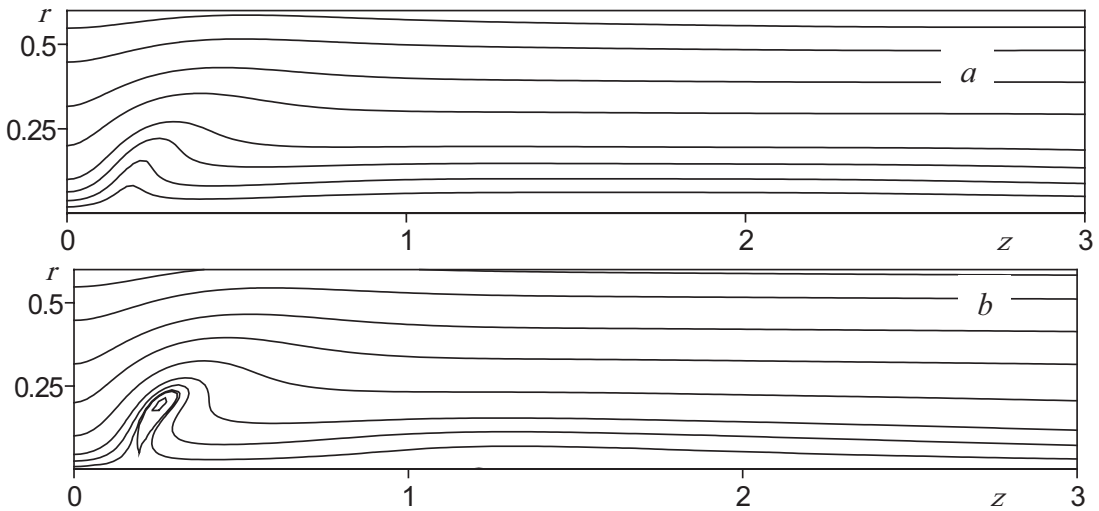


Figure 2. Streamlines for Re=150, G=2.6, $z_0 = 2.5$, V_0=0.01, 0.03 (a, b)

To study the swirling flow in an unbounded medium (without solid walls), a coordinate transformation was used in system (1)-(4), refining the mesh in the near-axis region of the flow, and instead of r, a variable y was introduced by the following transformation:

$$y = \frac{1}{a}\ln(1 + r/b), \quad r \in [0, r_k], \quad y \in [0, 1], \tag{11}$$

where a and b are constants. A characteristic pattern of separated flows is shown in Fig. 3. In this case, the front part of the recirculation zone becomes more pointed, its diameter relative to its length decreases, and overall, its shape corresponds more closely to the bubble-type vortex breakdown observed in experiments [8].

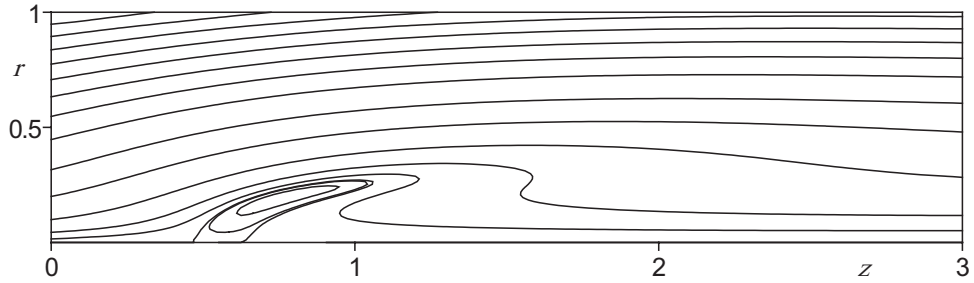


Figure 3. Streamlines for Re=750, G=0.95

The problem of the interaction of two axisymmetric swirling flows is relevant in connection with the modeling of flows in combustion chambers and gas turbines [2], vortex chambers, and gas curtains [9]. We will consider the case where the flow is organized as follows. Two coaxial flows, separated by a thin annular partition — an inner non-swirling flow and an outer swirling flow — enter a cylindrical channel and interact intensively within it.

For the calculations of the interaction between the axial and annular swirling flows at the inlet $z = 0$, velocity profiles with a uniform distribution and the following values were specified:

$$\begin{aligned} &U_0 = 1,\ V_{\varphi 0}(r) = 0,\ 0 \le r \le r_0, \\ &U_1 = 2.25, V_{\varphi 0}(r) = W_1 = 1, r_0 \le r \le r_1, \end{aligned} \tag{12}$$

in which it was adopted that $r_0 = 0.25,\ r_1 = 0.5$ for $R = 1$.

The most important properties of the flows discovered during the study are the emergence of a near-axis recirculation zone upon reaching a critical swirl rate, the formation downstream of this zone of a secondary recirculation region, and the development of an annular recirculation zone due to the splitting of the first zone by the inner jet flow. Characteristic streamline patterns are presented in Fig. 4.

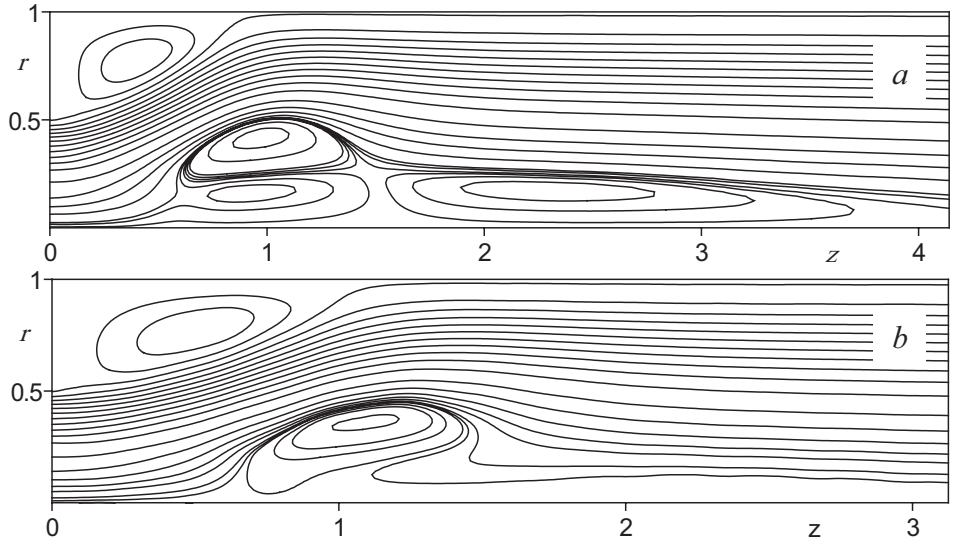


Figure 4. Streamlines for Re=150, G=2.25 (a), 2.75 (b)

Coaxial swirling flows are used in vortex combustion chambers [1]. The resulting near-axis recirculation zone serves as a kind of flame holder and is an example of vortex breakdown [8]. Consider the flow in an axisymmetric channel, at the inlet of which two coaxially swirling flows are supplied: an inner flow ($0 \le r \le r_0$) and an outer flow ($r_0 \le r \le r_1$). The

rotation can occur in either the same or opposite directions. The velocity distribution at $z = 0$ is specified as follows:

$$V_{z0}(r)=\begin{cases}1, & 0\le r\le r_{c1},\\ 1-k_1(r-r_{c1})^2, & r_{c1}\le r\le 1,\\ V_z^*-k_2(r-r_{c2})^2, & 1\le r\le r_{c2},\\ V_z^*, & 1\le r\le r_{c2},\\ V_z^*-k_3(r-r_{c3})^2, & r_{c3}\le r\le r_k,\end{cases} \quad (13)$$

$$V_{\varphi 0}(r)=\begin{cases}s_1 r, & 0\le r\le r_{a1},\\ 1-s_2(r-r_{a1})^2, & r_{a1}\le r\le r_{a2},\\ 1-s_3(r-r_{a2})^2, & r_{a2}\le r\le 1,\\ V_\varphi^*-s_4(r-r_{a3})^2, & 1\le r\le r_{a3},\\ \dfrac{Ar_k}{r}[1-\exp(-B(r/r_k)^2)], & r_{a3}\le r\le r_{a4},\\ b_0+b_1 r+b_2 r^2, & r_{a4}\le r\le r_k.\end{cases}$$

where $r_k = r_1/r_0 = 4$, $A = 0.554$, $B = 8$, $r_{c1} = 0.94$, $r_{c2} = 1.06$, $r_{c3} = 3.94$, $r_{a1} = 0.9$, $r_{a2} = 0.94$, $r_{a3} = 1.6$, $r_{a4} = 3.94$.

The solution of the boundary value problem (1)-(4) with the given distribution (13) depends on four parameters: the Reynolds number Re, the swirl parameter G, and the initial velocity values of the outer flow V_z^* and V_φ^*. Flow calculations were performed in the parameter range: $30 \le \mathrm{Re} \le 500$, $0 \le G \le 3$, $0.1 \le V_z^* \le 0.8$, $-1.6 \le V_\varphi^* \le 1.4$. An example of the flow calculation for the case of initial swirl in opposite directions is shown in Fig. 5. The two-cell structure of the recirculation zone (Fig. 5b) has been noted for bubble-type vortex breakdown in experiments [11] and numerical studies [12]. The calculations established that moderate swirl of the external flow in both the same and opposite directions can lead to both a reduction in the size of the near-axis recirculation zone (up to its complete disappearance at $V_\varphi^* > 0$) and an increase in its size. Strong swirl of the external flow $|V_\varphi^*| > 1$ always leads to an increase in the size of the recirculation zone.

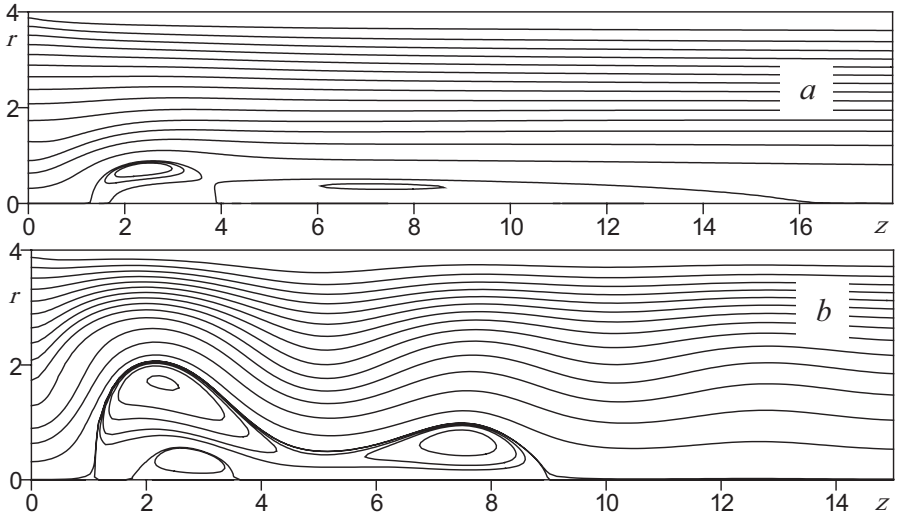


Figure 5. Streamlines for $\mathrm{Re} = 250$, $G = 1.8$, $V_z^* = 0.3$, $V_\varphi^* = -0.1, -0.8$ *(a, b)*

Let us consider the possibility of describing experimentally studied turbulent flows using the above-proposed model of swirling flows based on the concept of effective parameters. The experiments [13] were conducted on an aerodynamic stand, the working section of which was a plexiglass tube with a diameter of D = 14.5 cm. The inner jet was supplied into the measured test section from a copper tube with an outer diameter of 3.86 cm and an inner

diameter of 3.43 cm. The Reynolds number, based on the average flow velocity U_{cp} and the tube diameter D, was $\mathrm{Re} = 2.5 \cdot 10^5$.

We introduce into consideration the effective values of the turbulent viscosity coefficients ν_{tz} and $\nu_{t\varphi}$ in the longitudinal and azimuthal directions. Experiments [2] have shown that strong anisotropy of turbulent viscosity is observed in the flow, and $\nu_{t,z} / \nu_{t,\varphi} \approx 4$.

We define the turbulent value of the Reynolds number Re_{tz} as follows:

$$\mathrm{Re}_{tz} = \frac{U_{cp}\, R}{\nu_{t,z}} = \frac{1}{\chi}\sqrt{\frac{8}{\lambda}}, \qquad (14)$$

where U_{cp} is the average flow velocity, λ is the hydraulic resistance coefficient, and χ is a universal constant.

Based on the available experimental data, the hydraulic resistance coefficient along the pipe length is calculated using the formula of A.D. Altshul:

$$\lambda = 0.11\left(\frac{k_э}{D} + \frac{68}{\mathrm{Re}}\right)^{0.25} = $$
$$= 0.11\left(\frac{0.02}{145} + \frac{68}{2.5 \cdot 10^5}\right)^{0.25} = 0.015,$$

where $k_э = 0.02$ mm – коэффициент шероховатости полированного оргстекла. Используя значение универсальной постоянной для воздуха $\chi = 0.07$, из (14) находим $\mathrm{Re}_{tz} = 330$. Пересчитывая полученное число Рейнольдса в соответствии с характерными величинами – максимальной скорости внутренней струи U_0 в начальном сечении $z = 0$ и радиусу r_0, получим:

where $k_э = 0.02$ mm is the roughness coefficient of polished plexiglass. Using the value of the universal constant for air $\chi =$ 0.07, from (14) we find $\mathrm{Re}_{tz} = 330$. Recalculating the obtained Reynolds number in accordance with the characteristic values – the maximum velocity of the inner jet U_0 at the initial section z = 0 and the radius r_0 – we obtain:

$$\mathrm{Re}_{tz} = 330 \frac{U_0}{U_{cp}} \frac{r_0}{r_1} = 117.9.$$

In the experiments [13], integral swirl parameters for the inner and outer flows (S_0 and S_1, respectively) were used, defined by the relations:

$$S_j = \frac{\int_{r'}^{r''} V_z V_\varphi r^2 dr}{r'' \int_{r'}^{r''} V_z^2\, r\, dr}, \qquad j = 0; 1, \quad (15)$$

$$j = 0: \; r_j' = 0, \, r_j'' = r_0; \quad j = 1: \; r_j' = r_0, \, r_j'' = r_1.$$

In the series of experiments, the ratio $V_z^* = U_1 / U_0$ of the average axial velocities at the chamber inlet and the swirl parameter S_0 of the inner flow varied insignificantly: $V_z^* = 0.69 - 0.74$; $S_0 = 0.59 - 0.71$. Recirculation zones were observed for five cases under consideration, in which the values of the initial swirl of the outer flow were:

$$S_1 = -0.38, -0.19, 0, 0.21, 0.42. \qquad (16)$$

To conduct a comparison with the experiments, we take the average values of the inner jet parameters $V_z^* = 0.7$, $S_0 = 0.66$. Substituting the specified initial velocity distribution (13) into (15) considering (16), we find the effective swirl parameters for the calculation $G = 0.763$ and:

$$V_\varphi^* = -0.408, -0.25, 0, 0.27, 0.43. \qquad (17)$$

We determine, by analogy with [14], the effective values of the flow parameters using the expressions:

$$G_* = G\frac{\nu_{tz}}{\nu_{t\varphi}} = 0.763 \cdot 4 \approx 3, \qquad (18)$$

$$\mathrm{Re}_* = \mathrm{Re}_{tz}\frac{\nu_{t\varphi}}{\nu_{tz}} = \frac{117.9}{4} \approx 30. \qquad (19)$$

Taking this into account, the numerical solution of the boundary value problem (1)-(4), (13) for comparison with the experiment [13] was carried out for $\mathrm{Re}_* = 30$, $G_* = 3$, $V_z^* = 0.7$, and the values of V_φ^* from (17).

The calculated and experimental distributions of the axial velocity $V_z(0,z)$ along the channel length for the cases studied in [13] are presented in Fig. 6. The good agreement confirms the possibility of using the proposed model for calculating turbulent flows.

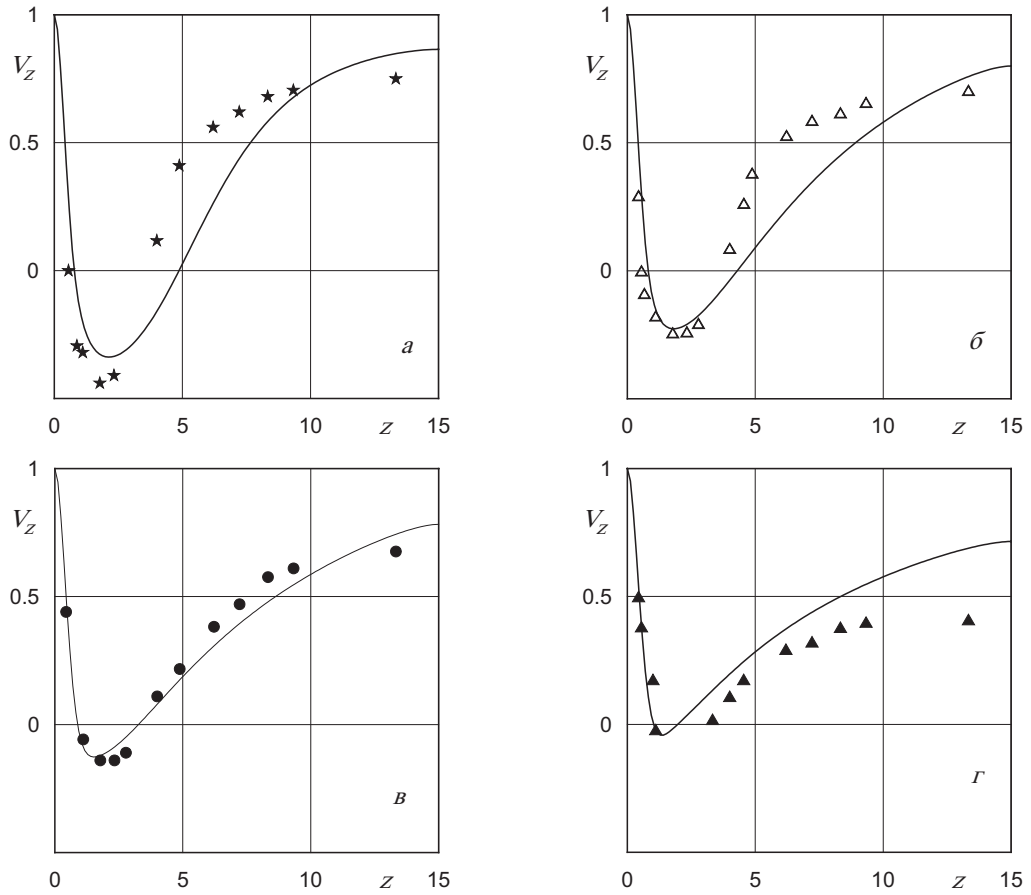


Figure 6. Comparison of the calculated (solid curves) and experimental [13] distributions of the velocities V_z at the channel axis; $S_1 = -0.38, -0.19, 0, 0.21$ (a–d)

The influence of the outer flow swirl V_φ^* on the diameter d_0 of the near-axis recirculation zone and the value of the maximum reverse flow velocity V_b is shown in Fig. 7 (calculated curves 1 and 2). An increase in the swirl of the outer flow in the opposite direction relative to the inner flow leads to an increase in the diameter d_0 of the recirculation zone and the reverse flow velocity V_b. In the case of the flows rotating in the same direction, an increase in the swirl of the outer flow leads to a decrease in the values of d_0 and V_b. The obtained calculated results are in good agreement with the experimental data [13] for all five investigated cases of varying outer flow swirl.

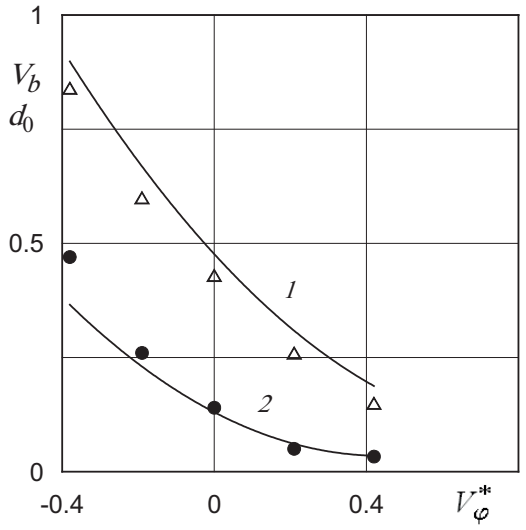


Figure 7. Comparison of calculated and experimental [13] dependencies of the recirculation zone diameter (curve 1 and experimental points) and the reverse flow velocity (curve 2 and experimental points) on the outer flow swirl

5. CONCLUSIONS

Based on the complete system of Navier-Stokes equations, mathematical modeling of swirling flows of various configurations has been carried out. For the numerical solution of the boundary value problems, a software package has been developed that allows for the efficient calculation of the structure of separated flows. Detailed streamline patterns were obtained, the range of parameters within which recirculation zones form in the flow was determined, and the internal structure of the separation zones was investigated. A comparison of the presented calculations with experimental data using the method of effective parameters demonstrates the applicability of the obtained results for the qualitative description of turbulent separated flows.

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Vadim K. Akhmetov, Doctor of Science, Professor, Department of Computer Science and Applied Mathematics, Moscow State University of Civil Engineering, 26, Yaroslavskoe Shosse, Moscow, 129337, Russia, e-mail: vadim.akhmetov@gmail.com

Ахметов Вадим Каюмович, доктор технических наук, профессор кафедры информатики и прикладной математики Национального исследовательского Московского государственного строительного университета, 129337, г. Москва, Ярославское шоссе, д. 26, e-mail: vadim.akhmetov@gmail.com