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MATHEMATICAL MODELING OF SWIRLING FLOW IN A VORTEX DUST SEPARATOR

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Abstract. The problem of swirling flow motion in an axisymmetric channel is numerically investigated. The results of flow field calculation are obtained by solving the Navier-Stokes equations. Various flow regimes with formation of axial recirculation zones are presented. A convective diffusion model is used to determine the concentration of flow particles and the formation of characteristic sedimentation zones. The obtained numerical solutions are compared with the results of available experimental data.

Keywords: swirling flows, two-phase flow, finite-difference method, convection-diffusion model, particle sedimentation

МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ЗАКРУЧЕННОГО ТЕЧЕНИЯ В ВИХРЕВОМ ПЫЛЕОТДЕЛИТЕЛЕ

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Аннотация. Численно исследована задача о движении закрученного потока в осесимметричном канале. Результаты расчета поля течения получены решением системы уравнений Навье-Стокса. Представлены различные режимы течения с образованием осевых зон рециркуляции. Для определения концентрации частиц потока и формирования характерных зон осаждения использована модель конвективной диффузии. Проведено сравнение полученных численных решений с результатами имеющихся экспериментальных данных.

Ключевые слова: закрученные течения, двухфазный поток, метод конечных разностей, конвективно-диффузионная модель, осаждение частиц

1. INTRODUCTION

Swirling flows have a wide usage in various technical applications. They can be observed in natural phenomena and are the object of theoretical and experimental study. The most important properties of swirling flows are described in [1, 2]. The structure of swirling flows in channels limited by solid walls was investigated numerically in [3-5]. Free flows in an unlimited medium were studied numerically in [6, 7]. Experimental data on the flow structure were presented in [8-10]. The stability of swirling flows was studied in [11-14].

Many engineering applications utilize swirling flows that contain particulate admixtures. The properties of swirling flow are used in cyclone-type devices for gas cleaning against dust, separation of particles, separation of liquid mixtures, in particular for purification of produced oil, in swirl nozzles for atomization of liquids, in drying chambers for removal of moisture from polydisperse products, in chemical, food, construction industries and in agricultural production.

This study considers swirling two-phase flow in a straight-flow cyclone dust separator. Devices of this type are used in air cleaning plants. The principle of operation of the dust separator is as follows. Dusty gas through the tangential swirler 1 enters the pre-separation chamber 2 (figure 1). Here coarse dust is separated. It is

then ejected through tangentially located outlets 3. Fine dust together with the gas moves further in radial direction to the axis of the vortex chamber 5. Due to the action of centrifugal forces, the dust is separated and removed through a number of annular slits 6 in the side surface of the channel.

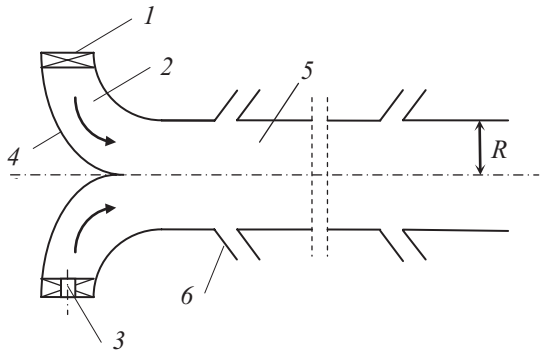


Figure 1. Flow diagram

Experimental studies of the considered device were carried out in [15]. The objective of this study is to develop a mathematical model and investigate the process of particle deposition under the action of swirling flow.

2. PROBLEM STATEMENT AND RESEARCH METHOD

A model problem of two-phase swirling flow in a cylindrical channel of radius R is examened. Finely dispersed particles are considered as a discrete system, which is carried by the flow without affecting it inversely. In cylindrical coordinate system r, φ, z, the Navier-Stokes equations can be represented with respect to the flow function ψ, vorticity Ω and azimuthal velocity V_φ as follows:

$$\frac{1}{r}\frac{\partial^2\psi}{\partial z^2}+\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\psi}{\partial r}\right)=-\Omega, \qquad (1)$$

$$\frac{\partial\Omega}{\partial t}+\frac{\partial}{\partial z}\left(V_z\Omega\right)+\frac{\partial}{\partial r}\left(V_r\Omega\right)=$$
$$=\frac{1}{\mathrm{Re}}\left[\frac{\partial^2\Omega}{\partial z^2}+\frac{\partial^2\Omega}{\partial r^2}+\frac{\partial}{\partial r}\left(\frac{\Omega}{r}\right)\right]+G^2\frac{1}{r}\frac{\partial\left(V_\varphi\right)^2}{\partial z}, \qquad (2)$$

$$\frac{\partial V_\varphi}{\partial t}+\frac{\partial}{\partial z}\left(V_zV_\varphi\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(rV_rV_\varphi\right)+\frac{V_rV_\varphi}{r}=$$
$$=\frac{1}{\mathrm{Re}}\left[\frac{\partial^2V_\varphi}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_\varphi}{\partial r}\right)-\frac{V_\varphi}{r^2}\right], \qquad (3)$$

$$V_r=-\frac{1}{r}\frac{\partial\psi}{\partial z},\ V_z=\frac{1}{r}\frac{\partial\psi}{\partial r},\ \Omega=\frac{\partial V_r}{\partial z}-\frac{\partial V_z}{\partial r}\ . \qquad (4)$$

The system of equations (1)-(4) is written in the conservative dimensionless form. Axial V_z and radial V_r velocities are related with the characteristic value U, azimuthal velocity V_φ is related with the value W_0, coordinates are related with radius R, time is related with the ratio R/U. The equation system (1)-(4) contains two dimensionless parameters: Reynolds number $\mathrm{Re}=UR/\nu$, where ν is the kinematic viscosity, and swirl parameter $G=W_0/U$.

The solution of the problem under consideration was defined in the domain $D(0\le z\le z_k,\ 0\le r\le 1)$. The velocity distribution in the initial cross section was assumed to be as follows:

$$V_{z0}=1,\quad V_{r0}=0,$$
$$V_{\varphi0}=\frac{b_1}{r}(1-\exp(-b_2r^2)),\quad 0\le r\le 1,\quad z=0,$$

where $b_1=0.554$, $b_2=8$.

At the lateral surface $r=1$, the adhesion conditions were specified. At the axis $r=0$, the symmetry conditions were specified. At the exit from the channel for $z=z_k$, soft boundary conditions were assigned. The set of boundary conditions in the variables flow function ψ and vorticity Ω is represented in dimensionless form as follows:

$$\psi = \frac{r^2}{2}, V_\varphi = V_{\varphi 0}(r), \quad \frac{\partial \psi}{\partial z} = 0, 0 \le r \le 1, z = 0,$$
$$\psi = \frac{1}{2}, \; V_\varphi = 0, \; \frac{\partial \psi}{\partial r} = 0, \; 0 \le z \le z_k, \; r = 1, \quad (5)$$
$$\psi = 0, \quad V_\varphi = 0, \quad \Omega = 0, \quad 0 \le z \le z_k, \quad r = 0,$$
$$\frac{\partial \psi}{\partial z} = \frac{\partial \Omega}{\partial z} = \frac{\partial V_\varphi}{\partial z} = 0, \quad 0 \le r \le 1, \quad z = z_k .$$

To solve the boundary value problem (1)-(4) at boundary conditions (5), the block iteration method was employed, which was successfully used, for example, in [16, 17] to analyze swirling flows of various types. A uniform 81x256 grid was utilized. The time step was chosen in the range from 0.05 to 0.2.

3. CONVECTION-DIFFUSION MODEL

Among all the forces acting on a particle in a vortex dust separator, the most significant is the Stokes viscous resistance force. In cylindrical coordinates, the equations of particle motion under the action of this force have a dimensionless form:

$$\frac{dV_{zs}}{dt} = \frac{1}{2\,\mathrm{St}}(V_z - V_{zs}), \quad (6)$$

$$\frac{dV_{rs}}{dt} = G^2 \frac{V_{\varphi s}^2}{r} + \frac{1}{2\,\mathrm{St}}(V_r - V_{rs}), \quad (7)$$

$$\frac{dV_{\varphi s}}{dt} = -\frac{V_{rs} V_{\varphi s}}{r} + \frac{1}{2\,\mathrm{St}}(V_\varphi - V_{\varphi s}). \quad (8)$$

Where $\mathrm{St} = (\rho_s r_s^2 U)/(9\nu\rho R)$ is the Stokes number, and the index s is associated with particles. According the equations (6), (8) axial V_{zs} and azimuthal $V_{\varphi s}$ particle velocities coincide with the corresponding velocities of the main flow for $\mathrm{St} << 1$. Радиальная скорость частиц V_{rs} определяется уравнением (7), из которого при условиях $\mathrm{St} << 1$, $G^2\,\mathrm{St} = O(1)$ получаем

If the conditions $\mathrm{St} << 1$, $G^2\,\mathrm{St} = O(1)$ are met, the radial velocity of particles V_{rs} can be determined using equation (7) as follows:

$$V_{rs} - V_r = 2\,\mathrm{St}\,G^2 \frac{V_{\varphi s}^2}{r}. \quad (9)$$

Since the considered flow includes a recirculation zone and significantly inhomogeneous velocity and concentration fields, two approaches were also investigated using averaging procedures with respect to r for the calculation of the V_{rs}. In the first one, the deposition velocity was determined as a function by averaging (9) with respect to r as follows:

$$V_{rs}(z) = 2StG^2 \left\langle V_{\varphi s}^2 / r \right\rangle + \left\langle V_{rs} \right\rangle. \quad (10)$$

In the second approach, the effective deposition velocity V_{rs}^* was introduced by the following relation:

$$\frac{1}{r}\int_0^r V_{rs}\, c r\, dr = V_{rs}^* \int_0^r c\, dr. \quad (11)$$

where the left part is the averaged flow $V_{rs}cr$ at the range $[0, r]$, and the right part is the effective flow calculated for velocity $V_{rs}^* = V_{rs}^*(z,t)$ and total partial concentration at the range $[0, r]$.

Consider the process of particle transportation for the calculated flows. Following the method based on the convection-diffusion model for mixtures of gas with small low-inertia particles, we can neglect the inverse effect of particles on the fluid flow (passive admixture approximation). The velocity field is found using equations (6)-(8). The equation of conservation of mass of particles is transformed to the diffusion equation for a passive scalar

$$\frac{\partial c}{\partial t}+\frac{\partial}{\partial z}(V_z c)+\frac{1}{r}\frac{\partial}{\partial r}(rV_{rs} c)= \\ =\frac{1}{\mathrm{Re}\,\mathrm{Sc}}\left[\frac{\partial^2 c}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c}{\partial r}\right)\right], \quad (12)$$

where c is the particle concentration, $\mathrm{Sc} = \nu/D$ is the Schmidt number, D is the diffusion factor. The convection-diffusion equation (12) has been used to simulate flows of mixtures with particle mass loss by deposition on walls, e.g., in [18, 19].

To determine V_{rs}, we shall consider three models in which the deposition velocity is defined by (12) similarly to $V_{rs}(z)$ by (10), or similarly to $V_{rs}^*(z,t)$ by (11), or by solving equation (7) with the condition $V_{rs}(z,0,t)=0$.

The boundary conditions for equation (12) can be written as follows:

$$z=0: c=1,\ r_1\le r\le r_2;\ c=0,\ r<r_1, r>r_2, \quad (13)$$

$$z=z_k:\ \frac{\partial c}{\partial z}=0,\quad 0\le r\le 1, \quad (14)$$

$$r=0,\ r=1:\ \frac{\partial c}{\partial r}=0,\quad 0\le z\le z_k. \quad (15)$$

In the calculations, $r_1=0.1$, $r_2=0.9$, $z_k=10$ were assumed according to [15].

By integrating equation (12) over the initial volume, taking into account boundary conditions (13)-(15), we obtain

$$2\pi\frac{\partial}{\partial t}\int_0^1\int_0^{z_k} r\,c\,dr\,dz+2\pi\int_0^1 (rV_z c)\big|_{z=z_k}\,dr+ \\ +2\pi\int_0^{z_k}(rV_{rs} c)\big|_{r=1}dz=0. \quad (16)$$

In accordance with (16), the following integral characteristics have been introduced: the particle flow rates Q_k and Q_b through the outlet section of the tube at $z=z_k$ and the side surface at $r=1$, respectively, as well as the particle mass m_f located in the volume under consideration:

$$Q_k(t)=2\pi\int_0^1 r(V_z c)\big|_{z=z_k}\,dr,$$

$$Q_b(t)=2\pi\int_0^{z_k} r(V_{rs} c)\big|_{r=1}\,dz,$$

$$m_f(t)=2\pi\int_0^{z_k}\int_0^1 cr\,dr\,dz.$$

According to these notations, equation (16) will take the form:

$$\frac{\partial}{\partial t}(m_f(t))+Q_k(t)+Q_b(t)=0. \quad (17)$$

By integrating equation (17) over time, we can determine the mass of particles m_k, which came out through the exit section, and the mass of particles m_b, which remained on the side surface:

$$m_k=\int_0^T Q_k(t)\,dt,\quad m_b=\int_0^T Q_b(t)\,dt.$$

The effects of particle reflection during deposition on the lateral surface $r=1$ are not considered. It is assumed that the particle flow $j=D\,\partial c/\partial r+V_{rs} c$ is determined only by the convective component, and the influence of diffusion is not taken into account, i.e., $(\partial c/\partial r=0)$.

The boundary value problem (12)-(15) was solved using the difference scheme applied to the transportation equations (2), (3). The main calculations were carried out using the relation (10).

4. RESULTS

In [15], a straight-flow vortex dust separator with the following parameters was investigated.

The length of the channel working part L was 1 m, the Reynolds number calculated from the average flow velocity and channel radius of 0.05 m was Re ~ 10^5, and the swirl parameter G was varied from 0 to 4.

In the present study, the flow fields corresponding to the boundary value problem (1)-(5) were calculated for values of Reynolds numbers Re = 1000, 2000, 2500 and swirl parameter G = 3.6. In this case, a region of return flow was formed in the channel, which started in the vicinity of the inlet section and extended to the channel exit. The axial velocity profiles along the cross-sections practically did not change and weakly depended on the Reynolds number.

The typical profile $V_z(r)$ obtained by solving the boundary value problem (1)-(5) at Re = 1000 in section z = 2.5 is shown in figure 2 (curve *4*). Symbols 5-7 in this figure correspond to the measurement data [15] for flow rates Q = 0.3, 0.25, 0.2 kg/s and show the auto modeling of the profiles by flow rate.

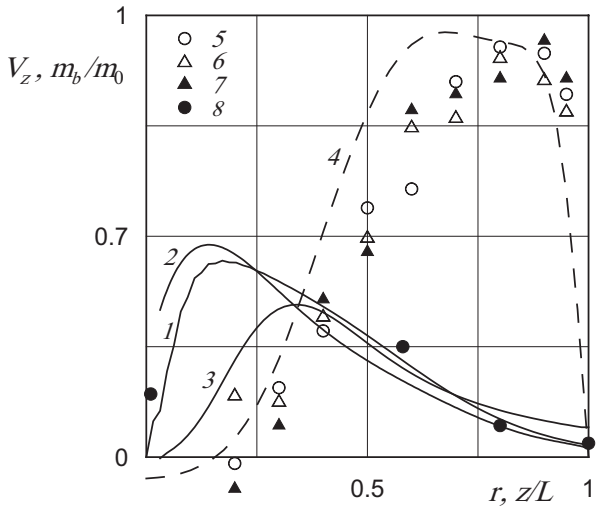


Figure 2. Comparison of computational models with experiments. Mass mb /m0 at r = 1 (models 1-3 for Vrs), data [15] (8), axial velocity $V_z(r)$ (4), flow rate [15] Q = 0.3; 0.25; 0.2 kg/s (5-7)

Generally, the obtained numerical results qualitatively correctly describe the structure of the investigated flow, including the diameter of the near-axis recirculation zone.

To investigate the dust separation process we will use the found velocity field at Re = 1000, G = 3.6.

In experiments [15], quartz dust with a particle diameter of $d'_s = 5.5$ μm and density of $\rho'_s = 2000$ kg/m3 (respectively $\rho_s = \rho'_s / \rho = 1550$) were used. The dimensionless particle radius for the calculations is determined as follows. If $\mathrm{Re}_s < 1$, where $\mathrm{Re}_s = V_{rs}\, r_s / \nu$ is the Reynolds number for the particles, the resistance coefficient is expressed by Stokes' law $c_d = 12 / \mathrm{Re}_s$. Hence, the calculated particle radius is related to the particle radius in the experiment by the expression

$$r_s = r_s^* \sqrt{\mathrm{Re}^* / \mathrm{Re}}\,, \qquad (18)$$

where the values with asterisks refer to the experiment. In [15], the value Re_s was in the transition from laminar to turbulent regime $1 < \mathrm{Re}_s < 500$. In this case, the resistance coefficient is $c_d = 12 / \mathrm{Re}_s^{0,6}$, and the relation (18) for r_s takes the form:

$$r_s = r_s^* (\mathrm{Re}^* / \mathrm{Re})^{3/8}. \qquad (19)$$

Substituting the above parameter values into (19), we obtain $r_s = (0.2 - 0.3) \cdot 10^{-4}$.

Figure 2 (curves 1-3) presents results for mass m_b/m_0, where $m_0 = m_f(0)$, of particles deposited on the lateral surface as a function of length distribution z/L which have been obtained by solving the boundary value problem (12)-(15) at Re =1000, G = 3.6, ρ_s = 1550, r_s = 0.2 104 for all three approaches of finding V_{rs}. Maximum particle deposition is observed at z/L = 0.22-0.35 (z/L = 0.3 in the experiment). At $z/L \geq 0.3$, relations 1-3 practically coincide. Comparison of the obtained data with the experiment [15] (symbols 8) confirms the suitability of the proposed approach using the convective diffusion equation (12) for modeling the transport and deposition of particles in swirlers.

The total mass of particles m_k/m_0, passed through the exit section $z = z_k$, as well as in [15], is about 0.04.

5. CONCLUSIONS

These calculations give a sufficiently detailed view of the dust separation process using the properties of swirled flow. The used mathematical model allows to describe accurately enough the main properties of the flow, such as: the formation of recirculation zones, the emergence of rarefaction in the vicinity of the flow axis under the action of the swirler, the drift of fine dispersed particles by the flow through the side surface of the straight-flow dust separator. There is a general agreement of the calculations with the experimental data.

REFERENCES

1. **Gupta A.K, Lilley D.G, Syred N.** Swirl Flows 1984 (Abacus Press)
2. **Alekseenko S.V., Kuibin P.A., Okulov V.L.** Introduction to theory of concentrated vortices. Moskow–Izhevsk: Institute of Computer Science. 2005. 504 p.
3. **Wang Y., Xingjian Wang X., Yang V.** Evolution and transition mechanisms of internal swirling flows with tangential entry // Physics of Fluids. 2018. Vol. 30. 013601.
4. **Ogus G., Baelmans M., Vanierschot M.** On the flow structures and hysteresis of laminar swirling jets // Physics of Fluids. 2016. Vol. 28. 123604.
5. **Akhmetov V.K., Shkadov V.Ya., Shkadova V.P.** Mixing of hot gases in an axisymmetric channel with a pre-swirled flow // Fluid Dynamics. 2006. Vol. 41. Pp. 504-513.
6. **Orlandi P.** Two-dimensional and three-dimensional direct numerical simulation of co-rotating vortices // Physics of Fluids. 2007. Vol. 19. 013101.
7. **Akhmetov V.K., Shkadov V.Ya.** Ustoychivost svobodnykh i ogranichennykh zakruchennykh techeniy s zonami retsirkulyatsii // Inzhenernaya fizika. 2008. № 6. S. 6-13 (in Russian).
8. **Oberleithner K., Paschereit C., Seele R., Wygnanski T.** Formation of turbulent vortex breakdown: intermittency, criticality, and global instability // AIAA Journal. 2012. Vol. 50. № 7**.** Pp. 1437-1452.
9. **Escudier M.P., Nickson A.K., Poole R.J.** Influence of outlet geometry on strongly swirling turbulent flow through a circular tube // Physics of Fluids. 2006. Vol. 18. 125103.
10. **Mergheni M.A., Riahi Z., Sautet J.C., Nasrallah S.B.** Swirl effects on dynamics characteristics of a coaxial jet // Thermal Science. 2017. Vol. 21. № 6. Pp. 2543-2552.
11. **Blanco-Rodríguez F.J., Rodríguez-García J.O., Parras L., del Pino C.** Optimal response of Batchelor vortex // Physics of Fluids. 2017. Vol. 29. 064108.
12. **Yadav N.K., Samanta A.** The stability of compressible swirling pipe flows with density stratification // Journal of Fluid Mechanics. 2017. Vol. 823. Pp. 689-715.
13. **Delbende I., Rossi M.** Nonlinear evolution of a swirling jet instability // Physics of Fluids. 2005. Vol. 17. 044103.
14. **Akhmetov V.K.** Gidrodinamicheskaya ustojchivost' kontrvihrevyh techenij // Gidrotekhnicheskoe stroitel'stvo. 2018. № 2. S. 13-18 (in Russian).
15. **Kutateladze S.S., Volchkov E.P., Terekhov V.I.** Aerodynamics and heat and mass exchange in bounded vortex flows. Novosibirsk: Institute of thermophysics, 1987. 283 p. (in Russian).
16. **Akhmetov V.K., Shkadov V.Ya.** Chislennoe issledovanie recirkulyacionnyh zon v vihrevoj kamere // Aeromekhanika i gazovaya dinamika. 2003. № 3. S. 39-45 (in Russian).
17. **Akhmetov V.K., Volshanik V.V.** Investigation of the propagation an aerated submerged jet // Hydrotechnical

Construction. 1994. Vol. 28. № 10. C. 605-609.
18. **Mang J., Minkov E., Schaflinger U., Ungarish M.** Particle entrainment in a bounded rotating flow with a drain // Trans. ASME. J. Fluids Eng. 1998. Vol. 120. Pp. 676–679.
19. **Tsai R., Chang Y.P., Lin T.Y.** Combined effects of thermophoresis and electrophoresis on particle deposition onto a wafer // J. Aerosol Sci. 1998. V. 29. № 7. Pp. 811-825.

СПИСОК ЛИТЕРАТУРЫ

1. **Гупта А., Лилли Д., Сайред Н.** Закрученные потоки. М.: Мир, 1987. 588 с.
2. **Алексеенко С.В., Куйбин П.А., Окулов В.Л.** Введение в теорию концентрированных вихрей. Москва–Ижевск: Институт компьютерных исследований, 2005. 504 с.
3. **Wang Y., Xingjian Wang X., Yang V.** Evolution and transition mechanisms of internal swirling flows with tangential entry // Physics of Fluids. 2018. Vol. 30. 013601.
4. **Ogus G., Baelmans M., Vanierschot M.** On the flow structures and hysteresis of laminar swirling jets // Physics of Fluids. 2016. Vol. 28. 123604.
5. **Ахметов В.К., Шкадов В.Я., Шкадова В.П.** Смешение нагретых газов в осесимметричном канале с предварительной закруткой потока // Известия РАН. Механика жидкости и газа. 2006. № 3. С. 19-29.
6. **Orlandi P.** Two-dimensional and three-dimensional direct numerical simulation of co-rotating vortices // Physics of Fluids. 2007. Vol. 19. 013101.
7. **Ахметов В.К., Шкадов В.Я.** Устойчивость свободных и ограниченных закрученных течений с зонами рециркуляции // Инженерная физика. 2008. № 6. С. 6-13.
8. **Oberleithner K., Paschereit C., Seele R., Wygnanski T.** Formation of turbulent vortex breakdown: intermittency, criticality, and global instability // AIAA Journal. 2012. Vol. 50. № 7**.** Pp. 1437-1452.
9. **Escudier M.P., Nickson A.K., Poole R.J.** Influence of outlet geometry on strongly swirling turbulent flow through a circular tube // Physics of Fluids. 2006. Vol. 18. 125103.
10. **Mergheni M.A., Riahi Z., Sautet J.C., Nasrallah S.B.** Swirl effects on dynamics characteristics of a coaxial jet // Thermal Science. 2017. Vol. 21. № 6. Pp. 2543-2552.
11. **Blanco-Rodríguez F.J., Rodríguez-García J.O., Parras L., del Pino C.** Optimal response of Batchelor vortex // Physics of Fluids. 2017. Vol. 29. 064108.
12. **Yadav N.K., Samanta A.** The stability of compressible swirling pipe flows with density stratification // Journal of Fluid Mechanics. 2017. Vol. 823. Pp. 689-715.
13. **Delbende I., Rossi M.** Nonlinear evolution of a swirling jet instability // Physics of Fluids. 2005. Vol. 17. 044103.
14. **Ахметов В.К.** Гидродинамическая устойчивость контрвихревых течений // Гидротехническое строительство. 2018. № 2. С. 13-18.
15. **Кутателадзе С.С., Волчков Э.П., Терехов В.И.** Аэродинамика и тепломассообмен в ограниченных вихревых потоках. Новосибирск: Институт теплофизики, 1987. 283 с.
16. **Ахметов В.К., Шкадов В.Я.** Численное исследование рециркуляционных зон в вихревой камере // Аэромеханика и газовая динамика. 2003. № 3. С. 39-45.
17. **Ахметов В.К., Волшаник В.В.** Исследование распространения аэрированной затопленной струи // Гидротехническое строительство. 1994. № 10. С. 24-26.
18. **Mang J., Minkov E., Schaflinger U., Ungarish M.** Particle entrainment in a bounded rotating flow with a drain // Trans. ASME. J. Fluids Eng. 1998. Vol. 120. Pp. 676-679.

19. **Tsai R., Chang Y.P., Lin T.Y.** Combined effects of thermophoresis and electrophoresis on particle deposition onto a wafer // J. Aerosol Sci. 1998. V. 29. № 7. Pp. 811-825.

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