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- привлечение внимания к наиболее актуальным, перспективным, прорывным и интересным направлениям развития и приложений численных и численно-аналитических методов решения фундаментальных и прикладных технических задач, совершенствования технологий математического, компьютерного моделирования, разработки и верификации реализующего программно-алгоритмического обеспечения;
- обеспечение обмена мнениями между исследователями из разных регионов и государств.

Тематика журнала. К рассмотрению и публикации в журнале принимаются аналитические материалы, научные статьи, обзоры, рецензии и отзывы на научные публикации по фундаментальным и прикладным вопросам технических наук, прежде всего в области строительства. В журнале также публикуются информационные материалы, освещающие научные мероприятия и передовые достижения Российской академии архитектуры и строительных наук, научно-образовательных и проектно-конструкторских организаций.

Тематика статей, принимаемых к публикации в журнале, соответствует его названию и охватывает направления научных исследований в области разработки, исследования и приложений численных и численно-аналитических методов, программного обеспечения, технологий компьютерного моделирования в решении прикладных задач в области строительства, а также соответствующие профильные специальности, представленные в диссертационных советах профильных образовательных организациях высшего образования.

Редакционная политика. Политика редакционной коллегии журнала базируется на современных юридических требованиях в отношении авторского права, законности, плагиата и клеветы, изложенных в законодательстве Российской Федерации, и этических принципах, поддерживаемых сообществом ведущих издателей научной периодики.

За публикацию статей плата с авторов не взымается. Публикация статей в журнале бесплатная. На платной основе в журнале могут быть опубликованы материалы рекламного характера, имеющие прямое отношение к тематике журнала.

Журнал предоставляет непосредственный открытый доступ к своему контенту, исходя из следующего принципа: свободный открытый доступ к результатам исследований способствует увеличению глобального обмена знаниями.

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По вопросам подписки на международный научный журнал “International Journal for Computational Civil and Structural Engineering (Международный журнал по расчету гражданских и строительных конструкций)” обращайтесь в Агентство «Роспечать» (Официальный сайт в сети Интернет: http://www.rosp.ru/) или в издательство Ассоциации строительных вузов (АСВ) в соответствии со следующими контактными данными:

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С октября 2016 года стал возможен прием статей на рассмотрение и рецензирование через онлайн систему приема статей Open Journal Systems на сайте журнала (электронная редакция): http://ijccse.iasv.ru/index.php/IJCCSE.

Автор имеет возможность следить за продвижением статьи в редакции журнала в личном кабинете Open Journal Systems и получать соответствующие уведомления по электронной почте.

В феврале 2018 года журнал был зарегистрирован в Directory of open access journals (DOAJ) (это один из самых известных поисковых сервисов в мире, который предоставляет открытый доступ к материалам и индексирует не только заголовки журналов, но и научные статьи), в сентябре 2018 года включен в продукты EBSCO Publishing.

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LONG-PERIOD SEISMIC EFFECTS AND THEIR INFLUENCE ON THE STRUCTURAL STRENGTH OF HIGH-RISE BUILDINGS

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Abstract: Currently, in all building codes, the diagrams of dynamics coefficient are limited to a maximum natural oscillation period of 1.8 s. However, this range is clearly not enough for the calculation of constructions of high-rise structures with characteristic basic periods of about 4-5 s and more. This article analyzes the available seismological data presented in the Center for Engineering Strong Motion Data (CESMD) database. The spectra of Tohoku earthquakes (Tohoku earthquake, Japan, March 11, 2011) and Emberley (New Zealand Earthquake, New Zealand, November 13, 2016) were studied, and dynamic factors for periods of natural oscillations of structures 4–5 s are calculated. The results of the study allow to establish reasonable values of dynamic coefficients in the field of high periods.

Keywords: dynamic analysis, seismic analysis, long-period seismic action, amplification factor, accelerograms, tall buildings

ДЛИННОПЕРИОДНЫЕ СЕЙСМИЧЕСКИЕ ВОЗДЕЙСТВИЯ И ИХ ВЛИЯНИЕ НА ПРОЧНОСТЬ КОНСТРУКЦИЙ ВЫСОТНЫХ ЗДАНИЙ

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Аннотация: В настоящее время во всех строительных нормах диаграммы коэффициентов динамичности ограничены максимальным периодом собственных колебаний сооружений в 1.8 с. Однако такого диапазона явно недостаточно для расчета конструкций высотных сооружений с характерными основными собственными периодами порядка 4-5 с и более. В настоящей статье проанализированы доступные сейсмологические данные, представленные в базе данных Center for Engineering Strong Motion Data (CESMD). Исследованы спектры землетрясений Тохоку (Tohoku earthquake, Япония, 11 марта 2011 г.) и Эмберли (Amberley New Zealand Earthquake, Новая Зеландия, 13 ноября 2016), рассчитаны коэффициенты динамичности для периодов собственных колебаний сооружений 4-5 с. Результаты исследования помогут установить обоснованные значения динамических коэффициентов в области высоких периодов.

Ключевые слова: динамический анализ, сейсмический анализ, длиннопериодное сейсмическое воздействие, коэффициент динамичности, акселерограммы, высотные здания

The authors of the National Building Code of Russian Federation SP 267.1325800.2016 [1] faced the following problem: in all the actual building standards governing seismic calculations [1-3], the diagrams of dynamic coefficients are limited to a maximum period of natural oscillations of structures of 1.8 s. This restriction made us seriously think about extending the diagram of the values of the dynamic coefficient to the region of periods that are more characteristic for high-rise buildings - about 4-5 s [4]. In this article authors attempts to analyze the available seismological data and find out what values the dynamic coefficient takes in the field of high periods of natural oscillations of structures.

Long-period seismic vibrations of the soil usually occur in the far zone of impact, remote from the epicenter for hundreds kilometers. For the long-period impacts, the dominant periods of the order about 2–5 s and more (0.2–0.5 Hz) are characteristic. It may cause resonances in such structures as high-rise buildings, telecommunication towers, long-span suspended and cable-stayed bridges, seismic-insulated structures with artificially reduced natural frequencies. Long-period movements of high-rise buildings were observed repeatedly, during large-scale earthquakes in the far zone; it examples can be found in papers [5-7]. Among the most significant it was the following: the July 7, 1952 earthquake in Los Angeles, USA (Kern County earthquake, hypocentral distance about 100-150 km (by the authors of the source material definition), M 7.3); March 28, 1970 in western Turkey (Gediz earthquake, hypocentral distance 135 km, M 7.1); September 19, 1985 Michoacan, Mexico (Michoacan, Mexico earthquake, hypocentral distance about 400 km, M 8.0); March 11, 2011 in Tokyo, Japan (Tohoku earthquake, hypocentral distance about 370 km, M9).

Using the example of the Tohoku earthquake, it is possible to analyze how the intensity and impacts' spectrum change with increasing of distance from the epicenter. In figures 1 and 2, the records from seismic stations of the Japanese network Kyoshin Net (KNET) MYG011 in Oshika at a hypocentral distance of 81.3 km and TKY017 in Tokyo at a hypocentral distance of 373 km (records of seismic stations can be found on the Center for Engineering Strong Motion Data website, CESMD, http : //cesmd.org) are shown. Parameters of seismic impact intensity are presented in table 1, that includes the maximum values of accelerations in each direction and the maximum values of the magnitude of the acceleration vector of seismic impact. In figures 3 and 4, the graphs of the dynamic coefficients for both impacts at 5% damping are constructed using the Odyssey software (developed by Company "Eurosoft") that allows process accelerograms of earthquakes and obtain the design parameters of seismic impacts [9-11]. In figures 5 and 6 the modules of corresponding acceleration vectors are shown. The direction of NS in table 1 and in figures 1-4 correspond to the original direction "0", EW – "90", Z – "Up". Obviously, that with an increase of the hypocentral distance, the intensity of the impact decreases, and the spectrum shifts noticeably towards longer periods — in the zone of 4–5 seconds, the dynamic coefficient increases from 0.2 to 0.65–0.5. According to the table 1 and figures 3 and 4, the following estimation can be provided: let us consider, that the fundamental natural period of the construction T1 = 5 s, the following dynamic coefficient β and intensity coefficient I (maximum values of acceleration vector modulus of a soil) for earthquakes in Oshika and Tokyo are obtained:

$$\beta_{Osh}(T_1) = 0.15, \quad I_{Osh} = 9.39 \text{ м/с}^2,$$
$$\beta_{Tky}(T_1) = 0.5, \quad I_{Tky} = 2.24 \text{ м/с}^2.$$

Then, at ratio of the intensities

$$I_{Osh} / I_{Tky} = 9.39/2.24 = 4.20,$$

the internal forces for the first mode of oscillations for the two impacts are related as

$$I_{Osh}\beta_{Osh}(T_1) / \left(I_{Tky}\beta_{Tky}(T_1)\right) = \\ = 4.20 \cdot 0.15/0.50 = 1.26.$$

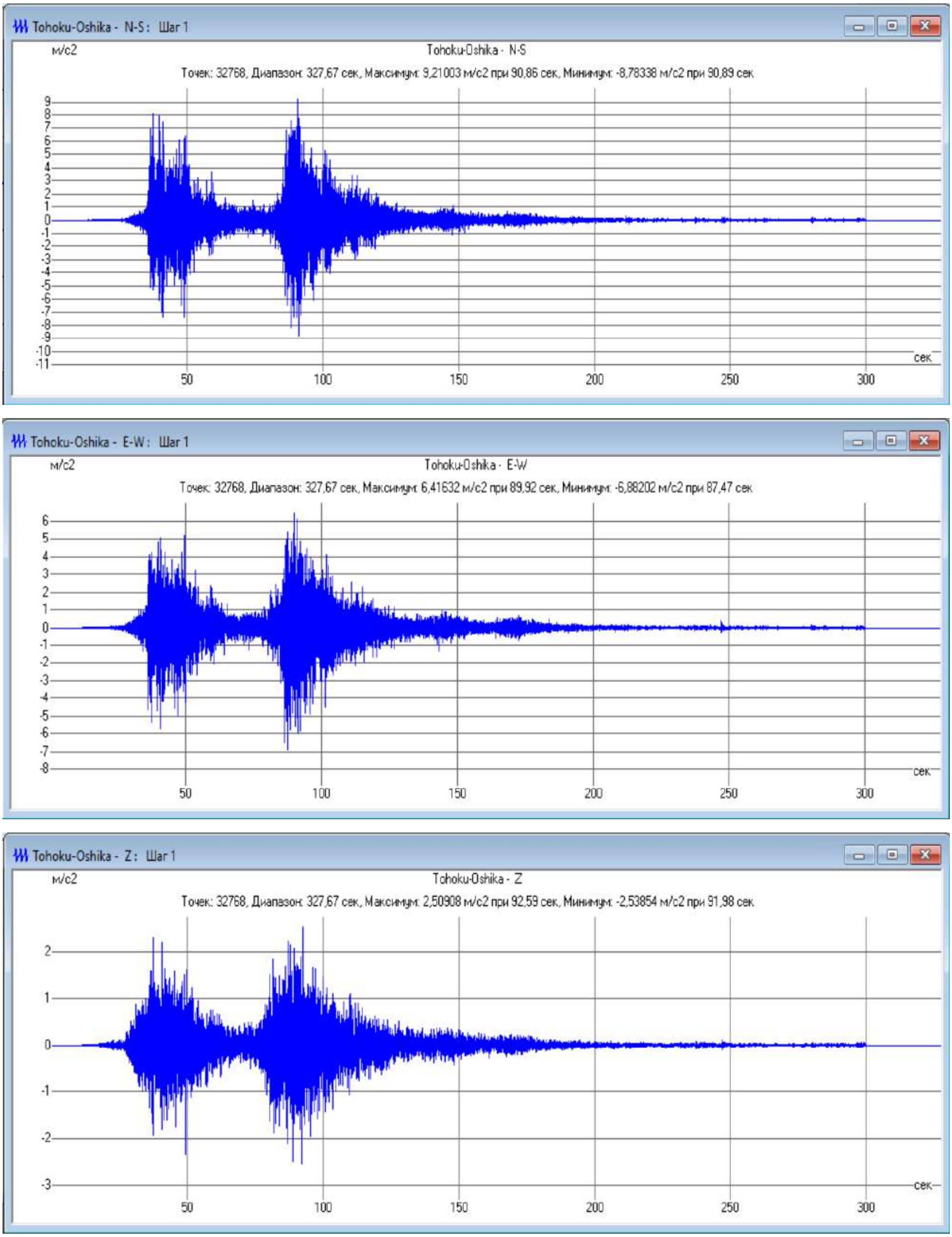


Figure 1. Accelerograms of Tohoku earthquake
(station MYG011, Oshika, hypocentral distance 81.3 km).

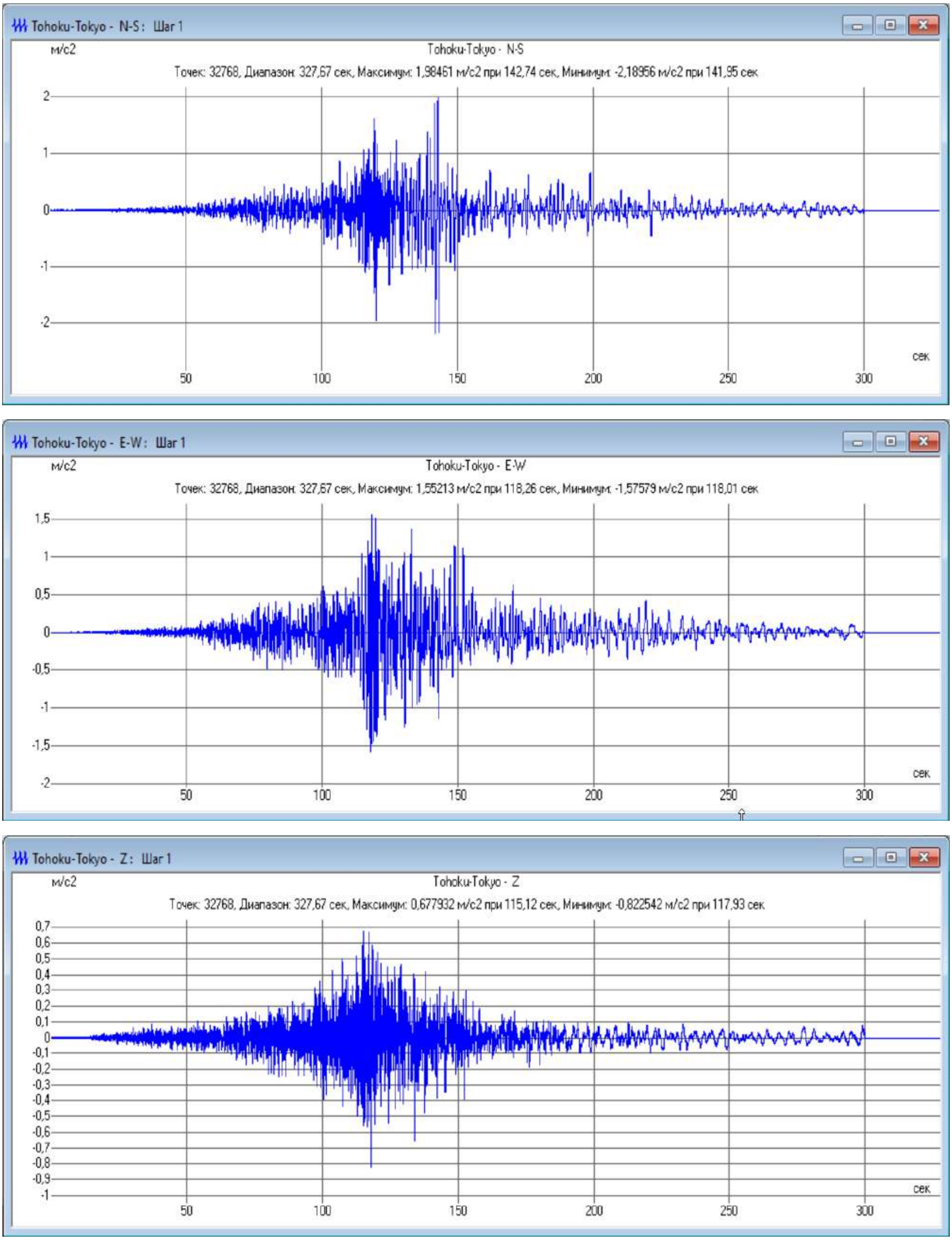
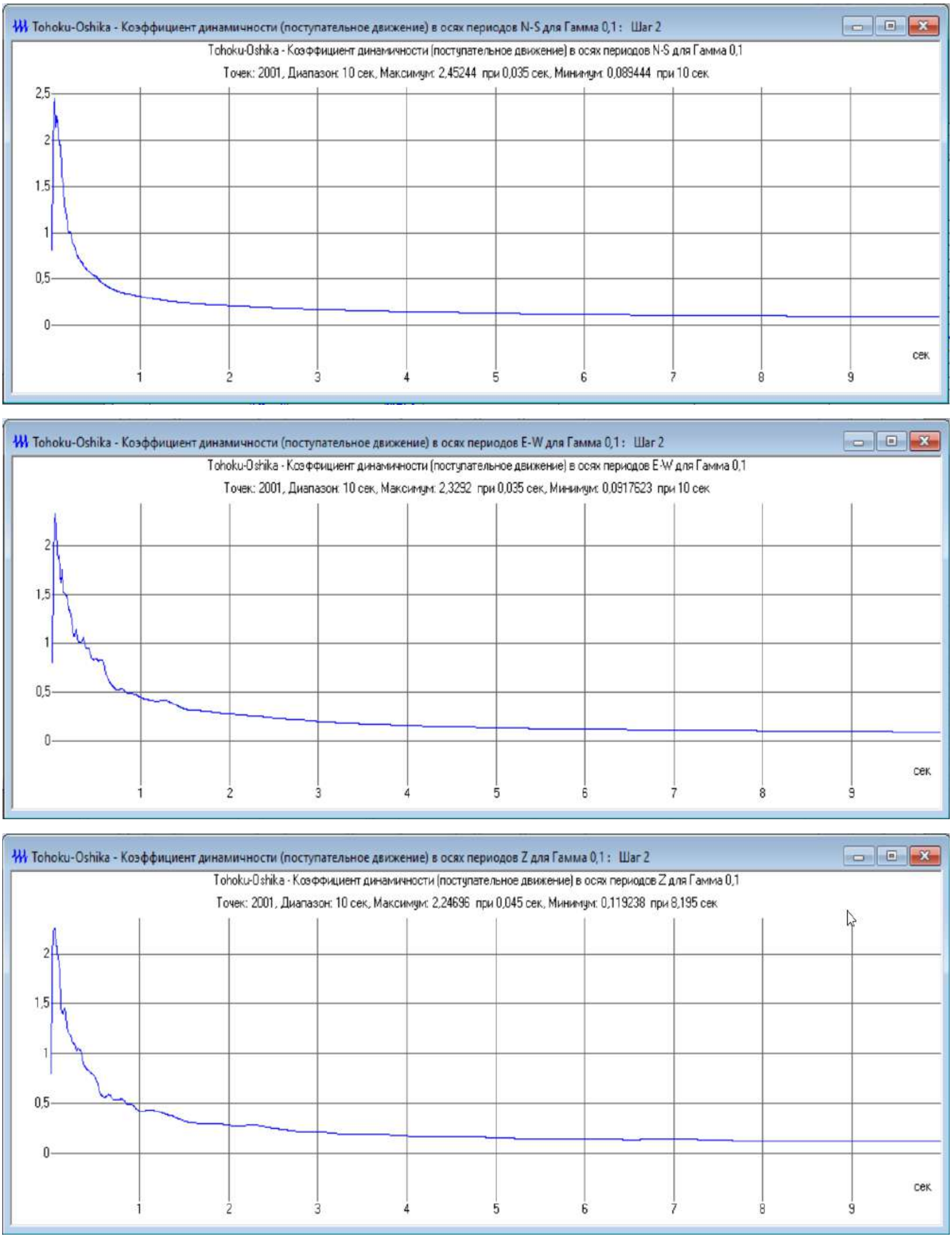


Figure 2. Accelerograms of Tohoku Earthquake
(TKY017 station, Tokyo, hypocentral distance 373 km).



<u>Figure 3.</u> Dynamic coefficients of the Tohoku earthquake (station MYG011, Oshika, hypocentral distance 81.3 km).

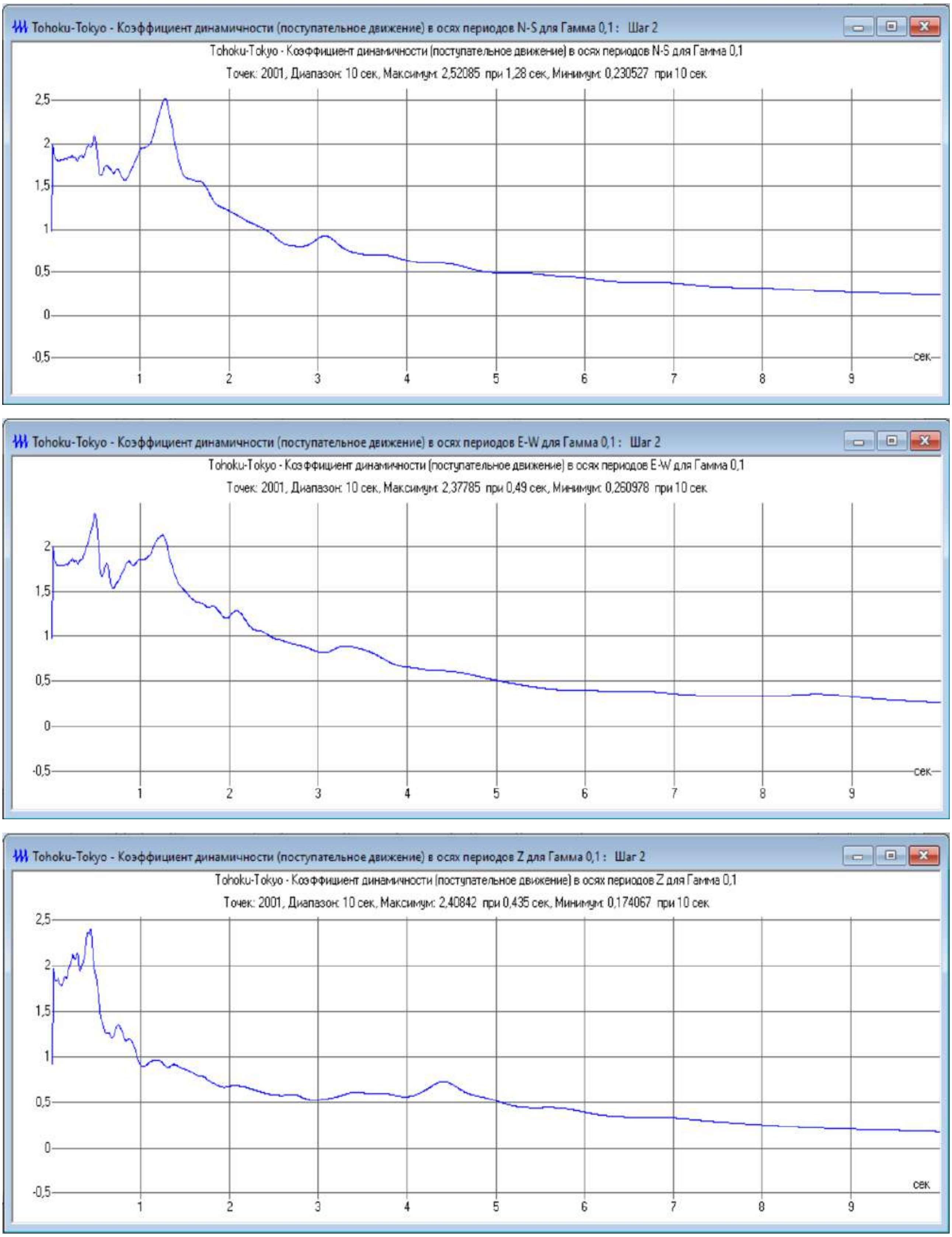


Figure 4. Dynamic factors of the Tohoku earthquake (TKY017 station, Tokyo, hypocentral distance 373 km).

Table 1. Tohoku Earthquake intensity at stations MYG011 – Oshika and TKY017 – Tokyo.

Station	Maximum acceleration, m/s²			
	NS	*EW*	*Z*	*Modulus*
MYG011	9.21	6.88	2.54	9.39
TKY017	2.19	1.58	0.82	2.24

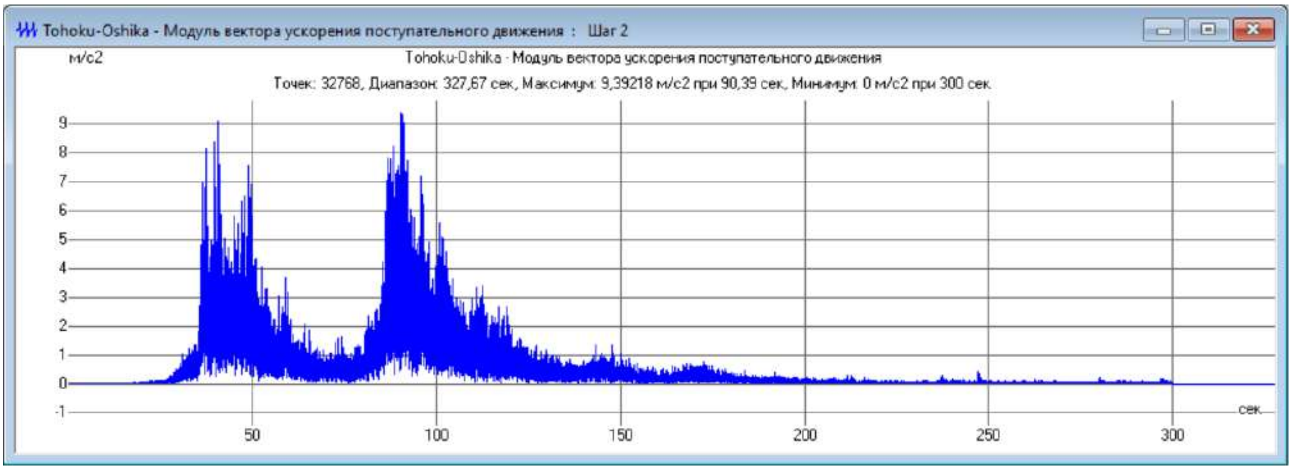


Figure 5. Seismic impact modulus (station MYG011, Oshika).

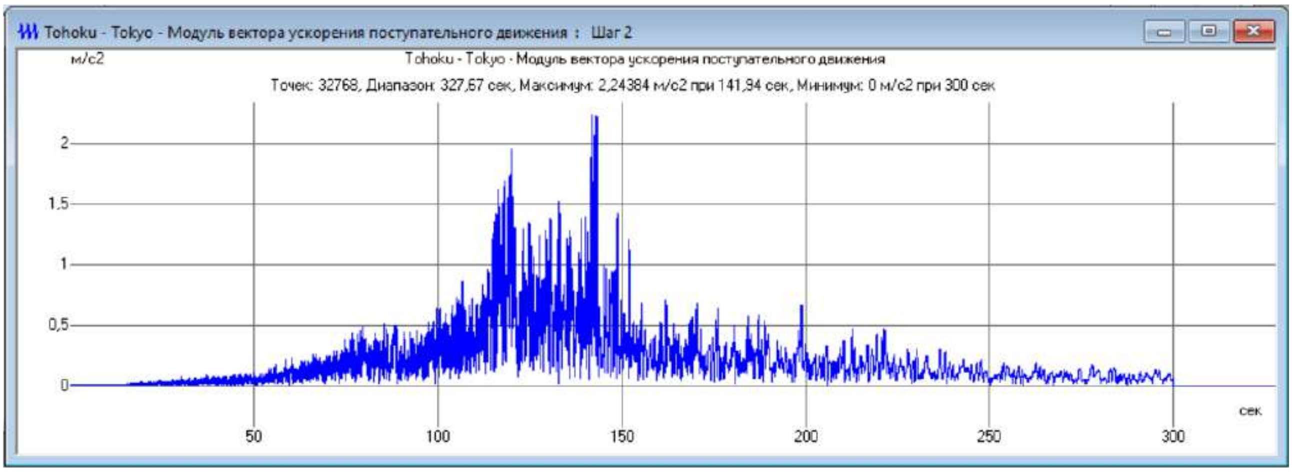


Figure 6. Seismic impact modulus (station TKY017, Tokyo).

Thus, although the intensity in the far zone falls more than 4 times, the internal forces (calculated by the linear spectral method) corresponding to the near zone are higher than the forces for the far zone about only 26%.

To obtain an averaged reasonable estimation of the dynamic cofficient for long-term impacts, it is required to analyze the spectrums of real earthquakes in the region of large periods for tens and hundreds of similar records. As a first step to this work, we consider the Emberley earthquake (New Zealand, November 13, 2016, M7.8 Emberley New Zealand Earthquake of 13 Nov 2016), presented in the CESMD database with instrumental records from 83 seismic stations. On the CESMD website, it can be found ready-made Emberley acceleration spectrums with a range of natural periods of up to 4 s and 5% damping (Fig. 7).

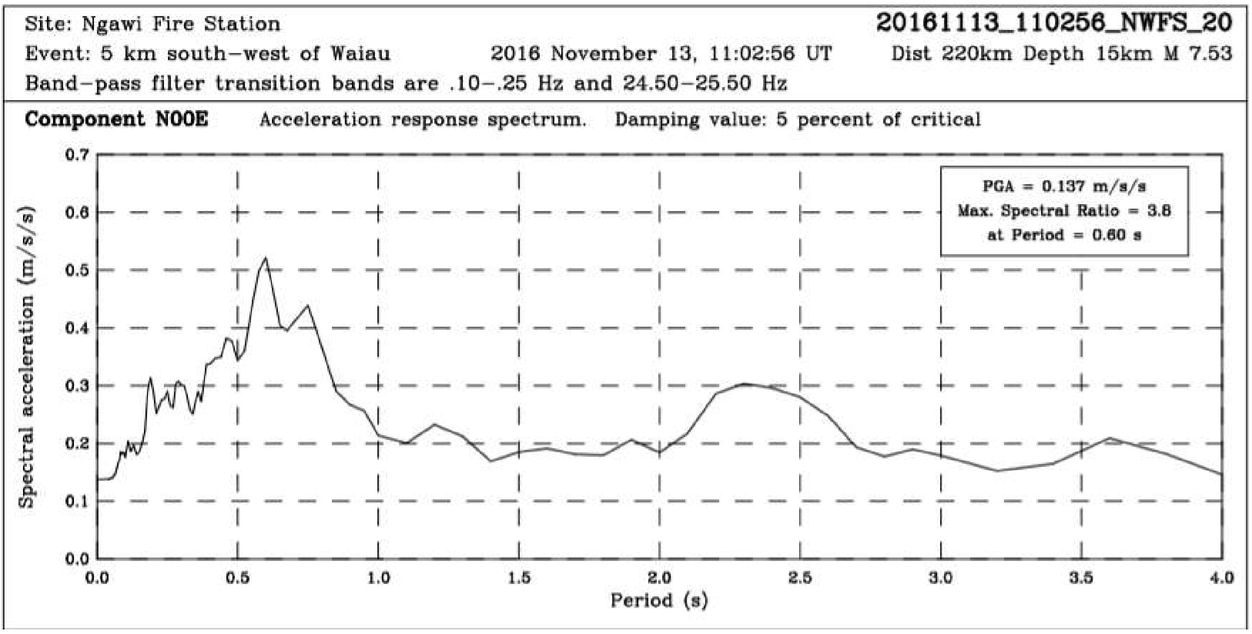
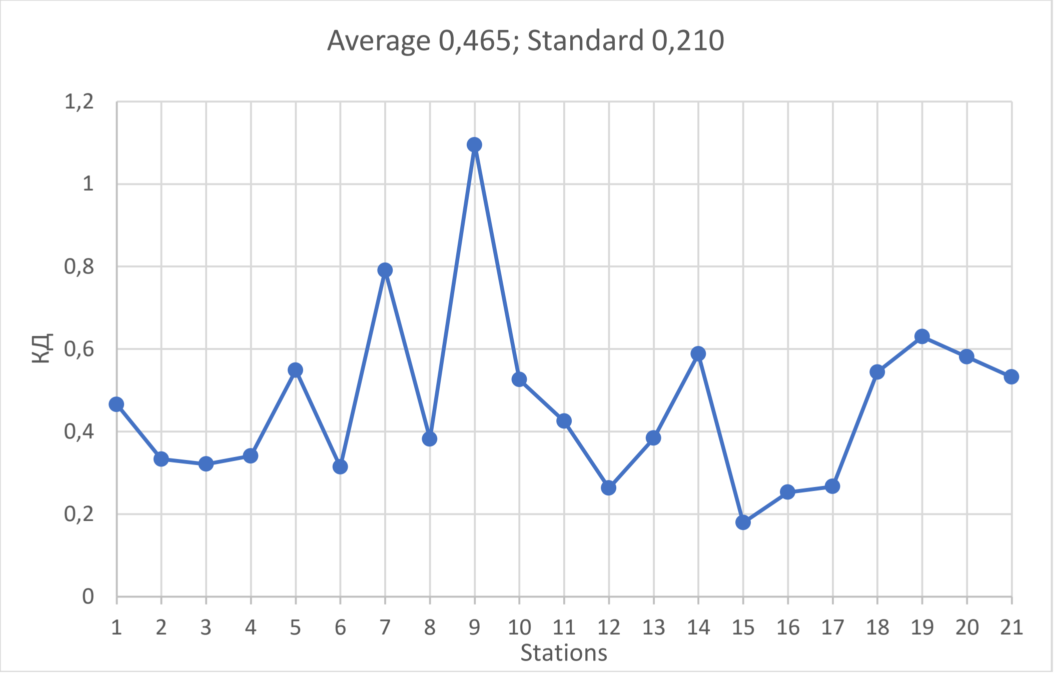


Figure 7. Spectrum of horizontal acceleration of Emberley earthquake in the CESMD database.

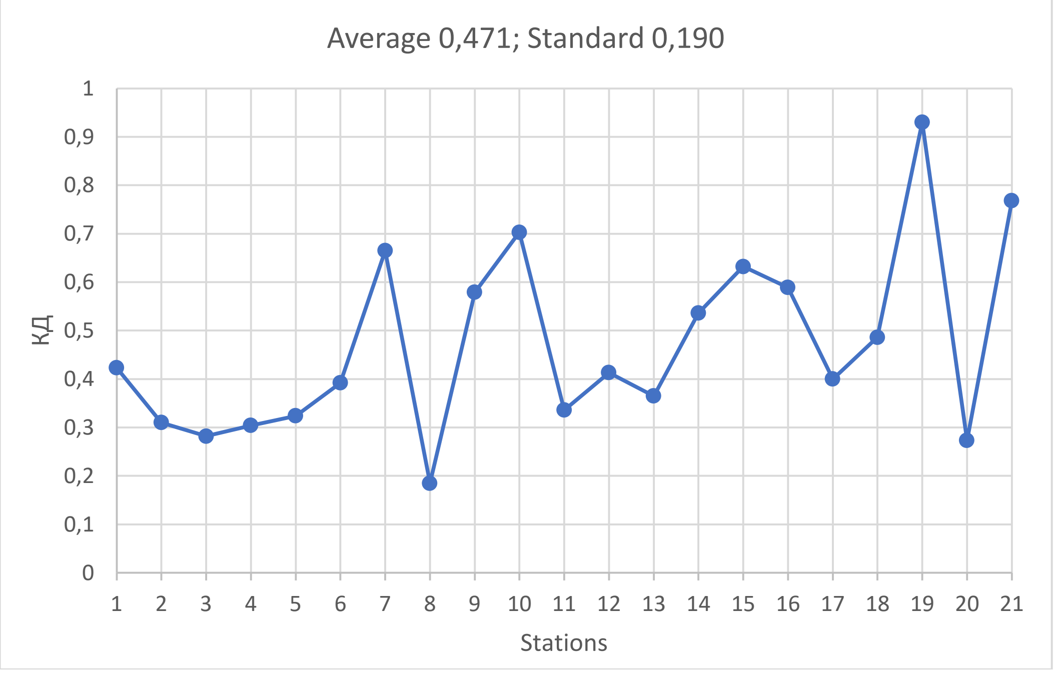
Table 2. Emberley Seismic Impact Parameters in the zone of 200-300 km from the epicenter (http://cesmd.org).

No	Station, code	Epicentral distance (km) and magnitude	Direction	DCmax (Max. Spectral Ratio)	PGA, m/s^2	S_a at T=4 s, m/s^2	**DC at T=4 s**
1	2	3	4	5	6	7	8
1.	Harihari Fire Station, HAFS	209 km M7.53	Hor1 Hor2 UP	3.5 4.1 4.3	0.161 0.213 0.062	0.075 0.090 0.075	**0.466** **0.423** **1.210**
2.	Wellington, WEL	214.8 M7.53	Hor1 Hor2 UP	3.3 3.7 3.6	1.202 1.455 0.669	0.400 0.450 0.400	**0.333** **0.310** **0.600**
3.	Wellington Te Papa Museum, TEPS	214.8 M7.53	Hor1 Hor2 UP	5.3 5.1 3.1	0.936 1.593 0.446	0.300 0.450 0.300	**0.321** **0.282** **0.673**
4.	Wellington Emergency Management Office, WEMS	216.2 M7.53	Hor1 Hor2 UP	3.5 3.2 3.2	1.321 1.478 0.547	0.450 0.45 0.30	**0.341** **0.304** **0.548**
5.	Makara Bunker, MKBS	216.2 M7.53	Hor1 Hor2 UP	3.1 3.7 3.1	0.911 0.618 0.395	0.50 0.20 0.20	**0.549** **0.324** **0.506**
6.	Wellington Pottery Association, POTS	216.4 M7.53	Hor1 Hor2 UP	3.9 3.6 2.5	0.795 0.638 0.371	0.25 0.25 0.25	**0.314** **0.392** **0.674**

1	2	3	4	5	6	7	8
7.	Quartz Range, QRZ	216.6 M7.53	Hor1 Hor2 UP	3.4 3.6 3.2	0.158 0.188 0.127	0.125 0.125 0.100	**0.791** **0.665** **0.787**
8.	Aotea Quay Pipitea, PIPS	217.4 M7.82	Hor1 Hor2 UP	4.0 3.0 3.0	2.094 2.710 0.845	0.8 0.5 0.3	**0.382** **0.185** **0.355**
9.	Ngawi Fire Station, NWFS	220.7 M7.53	Hor1 Hor2 UP	3.8 3.3 2.8	0.137 0.190 0.079	0.15 0.11 0.10	**1.095** **Figure 7** **0.579** **1.266**
10.	Newlands, NEWS	222.3 M7.82	Hor1 Hor2 UP	2.6 3.6 2.9	0.950 0.569 0.538	0.505 0.400 0.125	**0.526** **0.703** **0.233**
11.	Petone Overbridge/ PTOS	225.0 M7.53	Hor1 Hor2 UP	3.3 3.4 2.6	0.706 0.743 0.359	0.300 0.250 0.125	**0.425** **0.336** **0.348**
12.	Petone Victoria Street, PVCS	225.7 M7.53	Hor1 Hor2 UP	2.9 3.5 3.2	1.900 1.211 0.506	0.500 0.500 0.200	**0.263** **0.413** **0.395**
13.	Petone Municipal Building, PGMS	225.9 M7.82	Hor1 Hor2 UP	4.0 4.2 3.7	1.301 1.370 0.404	0.500 0.500 0.200	**0.384** **0.365** **0.495**
14.	Whataroa Fire Station, WHFS	227.3 M7.53	Hor1 Hor2 UP	4.5 3.8 4.3	0.136 0.112 0.066	0.080 0.060 0.030	**0.588** **0.536** **0.455**
15.	Lower Hutt Normandale, LHRS	228.5 M7.82	Hor1 Hor2 UP	3.0 3.9 2.5	0.699 0.633 0.361	0.125 0.400 0.125	**0.179** **0.632** **0.346**
16.	Lower Hutt Normandale, LHBS	229.1 M7.82	Hor1 Hor2 UP	3.3 3.3 3.6	0.987 0.637 0.348	0.250 0.375 0.200	**0.253** **0.589** **0.575**
17.	Lower Hutt St Orans College, SOCS	229.7 M7.82	Hor1 Hor2 UP	3.9 4.5 3.4	1.685 1.252 0.617	0.450 0.500 0.100	**0.267** **0.400** **0.162**
18.	Fairfield, FAIS	230.7 M7.53	Hor1 Hor2 UP	3.7 3.3 3.1	0.827 0.823 0.349	0.450 0.400 0.200	**0.544** **0.486** **0.573**
19.	Foxton Beach School, FXBS	311.0 M7.82	Hor1 Hor2 UP	3.2 3.2 4.5	0.952 0.860 0.285	0.600 0.800 0.400	**0.630** **0.930** **1.404**
20.	Paraparaumu Primary School, PAPS	259.4 M7.53	Hor1 Hor2 UP	3.6 4.4 4.1	0.860 0.915 0.393	0.500 0.250 0.125	**0.581** **0.273** **0.318**
21.	Te Horo House, THOB	274.9 M7.53	Hor1 Hor2 UP	4.6 3.6 4.4	0.939 1.042 0.257	0.500 0.800 0.100	**0.532** **0.768** **0.389**



<u>Figure 8.</u> Dynamic coefficients of the Emberley earthquake when moving propagates in a horizontal direction 1.



<u>Figure 9.</u> Dynamic coefficients of the Emberley earthquake when moving propagates in the horizontal direction 2.

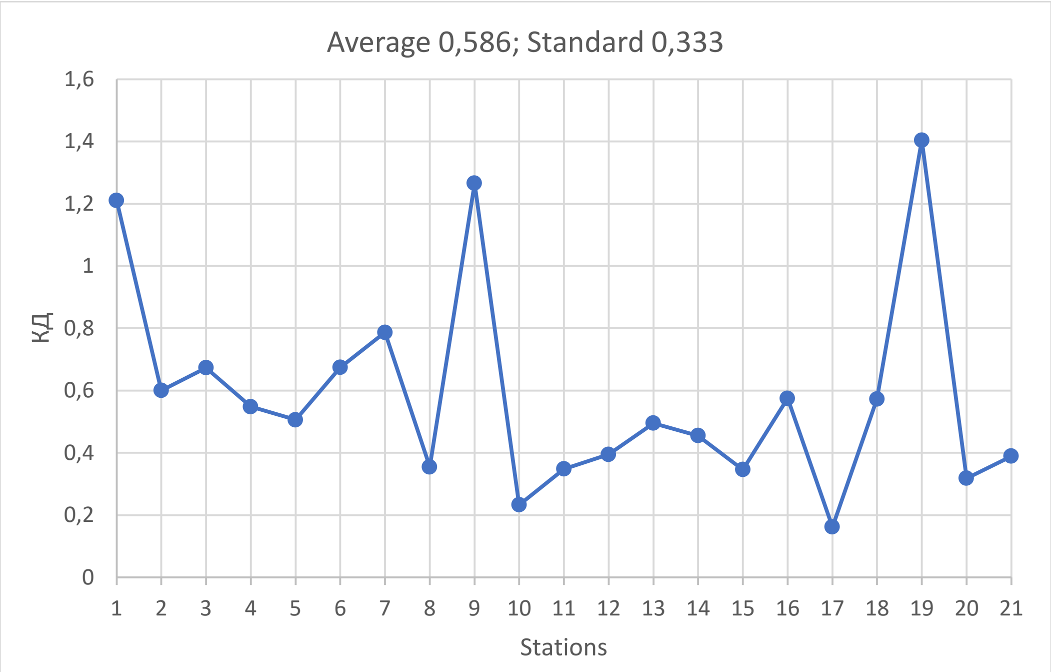


Figure 10. Dynamic coefficients of the Emberley earthquake when moving propagates vertically.

Let us consider the recordings from stations with epicentral distances (these definitions are taken from the initial data) from 200 to 300 km (41 stations in total) to record precisely the long-period seismic movement. Using the records from 41 stations, we have selected only those, the dynamic coefficient for horizontal-motion of which at least for one direction exceeds 0.3 for the period T = 4 s. It is turned out that there are only 21 seismic stations with such recording parameters. Table 2 contains the following data from 21 seismic stations: the name of the seismic station with its code, epicenter distance and magnitude; for each station, the values of maximum dynamic coefficients (Max Spectral Ratio) for two horizontal and vertical directions, peak ground accelerations (PGA), spectral acceleration Sa and dynamic coefficient at T = 4 s, that equals to the ratio of Sa and PGA are presented. The obtained dynamic coefficient values are shown in Figures 8-10, there the average values of dynamic coefficients and it standards are given too.

Analysis of the records of the Emberley earthquake using data from 41 far-zone seismic stations showed that at the half of the all cases, the values of dynamic coefficients of horizontal movement at a period of natural oscillations of structures equal to 4 s exceed 0.3, while the average DC values are about 0.465-0.471 (figures 8, 9), and the maximum values of DC can reach 1 or more. Investigations in the evaluation of dynamic coefficients in the field of high periods should be continued, and the obtained values should be taken into account when developing construction standards governing calculations of high-rise structures for the seismic resistance.

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DYNAMIC EFFECTS IN THE BEAM ON AN ELASTIC FOUNDATION CAUSED BY THE SUDDEN TRANSFORMATION OF SUPPORTING CONDITIONS

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Abstract: A mathematical model of a dynamic process in a loaded beam on an elastic Winkler base with the sudden formation of a defect in the form of a change in the boundary conditions is constructed. The solution of the static problem of bending of the beam fixed at the ends serves as the initial condition for the process of forced vibrations hinged supported at the ends of the beam, which arose after a sudden break in the bonds that prevented the rotation of the end sections. Dynamic increments of stresses in the beam for various combinations of beam and foundation parameters are determined. determined.

Keywords: beam on an elastic foundation, sudden transformation of boundary conditions, deflections, bending moments, natural oscillation frequencies, forced oscillations, stress increments

ДИНАМИЧЕСКИЕ ЭФФЕКТЫ В БАЛКЕ НА УПРУГОМ ОСНОВАНИИ, ВЫЗВАННЫЕ ВНЕЗАПНЫМ ПРЕОБРАЗОВАНИЕМ УСЛОВИЙ ОПИРАНИЯ

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Аннотация: Построена математическая модель динамического процесса в нагруженной балке на упругом основании Винклера при внезапном образовании дефекта в виде изменения граничных условий. Решение статической задачи изгиба защемленной по концам балки служит начальным условием процесса вынужденных колебаний шарнирно опертой по концам балки, возникшего после внезапного обрыва связей, препятствующих повороту концевых сечений. Определены динамические приращения напряжений в балке при различных сочетаниях параметров балки и основания.

Ключевые слова: балка на упругом основании, внезапное преобразование граничных условий, прогибы, изгибающие моменты, собственные частоты колебаний, вынужденные колебания, приращения напряжений

An important problem of Structural Mechanics is the analysis of the sensitivity of load-bearing structures to structural transformations under load, such as sudden connections' removal, cracks formation, partial destruction, etc. Obtaining such information for real structures requires the development of special methods, since this problem cannot be solved by ordinary methods. In accordance with structural mechanic's provisions to solve these tasks, it becomes necessary to calculate such systems as structurally non-linear, changing the design scheme under loads, including dynamic loadings caused by sudden overlimit impacts [1-5].

And if the ordinary design situations are well analyzed and exactly are regulated in the relevant documents, the overlimit situations are not classified and the response of structural elements to such impacts is not sufficiently investigated.

There is only a few number of engineering methods to design and calculate structures at such impacts, which take into account sudden transformation and damage of structural systems, and it is not perfect. The dearth of knowledge about the deformation and stress state of structural elements during dynamic processes, initiated by sudden damage, restrains the development of the theory and design methods that take into account the possibility and potential consequences of overlimit impacts and ensure a high level of structural safety at its operating. As an example of investigations, performed in this direction, it should be noted a number of scientific publications [6–11] containing the simulation results of transient dynamic processes performing in loaded beams at sudden damage of supports, delamination, transverse or longitudinal cracks formation, partial destruction, change of connection conditions of structural elements, etc. All these investigations were performed for beams supported only at the edges that is, there is solid ground under the beams. There is theoretical and practical interest to extend similar approaches to beams on an elastic ground.

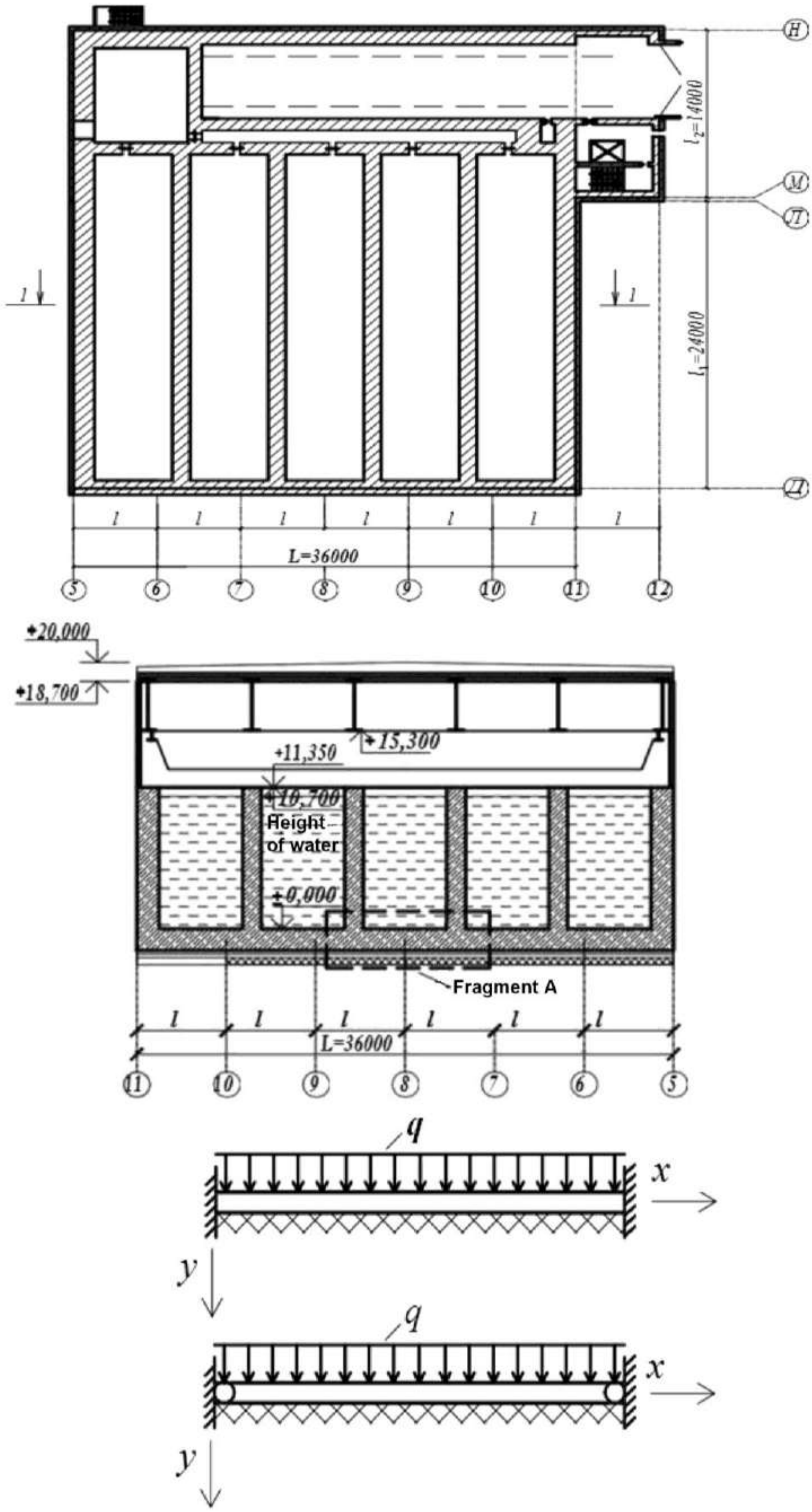
In the present paper, the problem of constructing a mathematical model of transient dynamic processes in a beam on an elastic foundation at the sudden changing of boundary conditions is described. Before the formation of a defect, the reaction of the structure is determined by a static impact. The sudden formation of a defect leads to a decrease in the overall rigidity of the structure, which does not ensure the static balance of the system. The arising inertial forces cause a dynamic reaction, redistribution and growth of deformations and stresses. As a result, there may be a violation of the normal operating of the structure or loss of bearing capacity and destruction.

Currently, in the scientific literature related with the problem of the dynamics of the "beam-ground" systems, there are many solved problems. The majority of researches on the dynamics of interaction between a beam and a foundation are devoted to the analysis of natural oscillations. At the same time, in these researches, natural and forced oscillations of beams on elastic ground are considered only for cases when the design scheme of the beam-ground system in the loaded state does not change. Appearance of constructive nonlinearity, i.e. changing in the design scheme of a loaded beam on an elastic foundation and it consequences are described only in a few papers [12–18], in which the sudden partial or complete destruction of the foundation was considered.

The paper [19] presents the results of the computational analysis of the long-term deformation of the building-foundation system of one of the nuclear power facilities — the complex of nuclear waste storage buildings, including the nuclear waste storage facility of nuclear power plants (Fig. 1 a, b). To study the dynamic effects in buildings and structures of this type, a second level design scheme - beams on an elastic foundation as an element of such a structural system [1] is considered (Fig. 1 c).

The following formulation of the problem is formulated.

An elastic beam with flexural stiffness EI, rigidly clamped at the ends rests along the entire length l on an elastic Winkler foundation with stiffness coefficient k (Figure 1c).



<u>Figure 1.</u> The design scheme of static bending of the beam on an elastic foundation.

On the outer layers of the beam acts even distributed load of q intensity and the reaction of the foundation. It is assumed that at some point in time t = 0 of the statically deforming of the beam, the connections, which impede the rotation of the end sections in the supports, suddenly collapses, forming hinges at the before clamped points. The static equilibrium of the loaded beam is interrupted and the beam begin motion $\upsilon(x,t)$, during of which the deformation and stress in the beam receive dynamic increments.

The solution to the problem is carried out in Cartesian coordinates x, y. Linear dimensions and transverse displacements are related with the length of the beam. The sought parameters are static and dynamic deflections, the frequencies and modes of the natural flexural vibrations of a hinged beam, internal bending moments. Taking into account the practical significance of the problem of ensuring the strength and survivability of structures on elastic grounds and the dearth of well-known papers by this problem, the described problems seems to be actual.

The solution to the problem is constructed in the following sequence:

1) determination of a static deflection of the beam clamped at the ends (“intact”) on an elastic foundation. These parameters will be used later as the initial condition of a dynamic process initiated in the system by a sudden transformation of boundary conditions;

2) determination of the frequencies and forms of natural flexural oscillations of hinged at the ends of the (“damaged”) beam on an elastic base;

3) investigation of forced bending vibrations of a loaded beam. In this case, the load, the static deflection of the "intact" beam and the unknown dynamic deflection are arranged in rows according to the natural vibration forms of the "damaged" beam.

PROBLEM SOLUTION

1. Static bending of clamped at the edges beam on an elastic Winkler foundation is described by equation with dimensionless parameters [20, 21]

$$\frac{d^4 w_{cm}}{d\xi^4} + 4\alpha^4 w_{cm} = \bar{q}, \qquad (1)$$

where

$$\xi = \frac{x}{l}, \quad w_{cm} = \frac{\upsilon}{l}, \quad \bar{q} = \frac{ql^3}{EI}, \quad \alpha = \sqrt[4]{\frac{kl^4}{4EI}}.$$

General solution to the equation (1) for the case, when edges are clamped [20, 21]

$$w_{cm} = \frac{\bar{q}}{4\alpha^4}\left(1 - k_n(\alpha\xi)\right) + w_0'' k_2(\alpha\xi) + w_0''' k_1(\alpha\xi), \qquad (2)$$

where $k_i(\alpha\xi)$ $(i = 1 \div 4)$ – Krylov function, that takes the following form

$$k_1(\alpha\xi) = \frac{\sin\alpha\xi ch\alpha\xi - \cos\alpha\xi sh\alpha\xi}{4\alpha^3}$$

$$k_2(\alpha\xi) = \frac{\sin\alpha\xi sh\alpha\xi}{2\alpha^2}$$

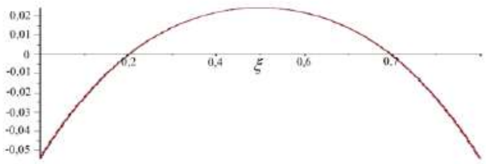
$$k_3(\alpha\xi) = \frac{\sin\alpha\xi ch\alpha\xi + \cos\alpha\xi sh\alpha\xi}{2\alpha}$$

$$k_4(\alpha\xi) = \cos\alpha\xi ch\alpha\xi$$

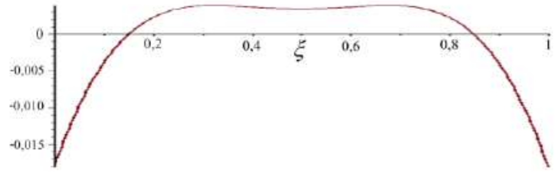
w_0'', w_0''' – initial parameters, dimensionless bending moment and shear force in a cross section at the point $\xi = 0$

$$w_0'' = \frac{\bar{q}}{k_2^2(\alpha) - k_1(\alpha)k_3(\alpha)} \times \left(\frac{k_4(\alpha) - 1}{4\alpha^4} k_2(\alpha) + k_1^2(\alpha)\right),$$

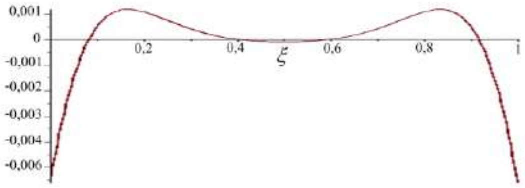
$$w_0''' = \frac{\bar{q}}{k_1(\alpha)k_3(\alpha) - k_2^2(\alpha)} \times \left(\frac{k_4(\alpha) - 1}{4\alpha^4} k_3(\alpha) + k_1(\alpha)k_2(\alpha)\right).$$



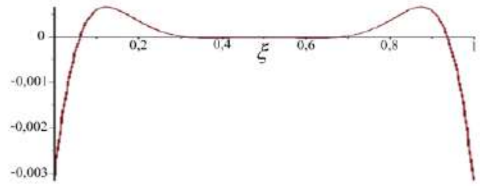
λ=10^{2,5}



λ=10^{3,5}



λ=10^{4,5}



λ=10^5

<u>Figure 2.</u> Diagrams of bending moments at the initial static state, depending on the stiffness indicator of the beam-foundation system λ.

Dimensionless bending moment at static state is determined by function

$$w_{cm} = \bar{q}k_2\left(\alpha\xi\right) + w_0''k_4\left(\alpha\xi\right) + w_0'''k_3\left(\alpha\xi\right). \quad (3)$$

In figure 2, diagrams of bending moments in the beam with clamped edges at different values of generalized stiffness $\lambda = 4\alpha^4$. of "beam-foundation" system are presented. It should be noted the "extraordinary" [22] view of diagrams of bending moments at growing of stiffness of the system. Bending moments in the middle of the beam is significantly lower, than in the quarters of the span. This is result of complex action of external load and reaction of an elasti foundation to the beam.

2. Appeared motion $\upsilon_{\partial u н} = \upsilon\left(x, t\right)$ after sudden transformation of clamped supports of the beam to hinges is described by equation [10]

$$\frac{\partial^4 w_{\partial u н}}{\partial \xi^4} + 4\alpha^4\left(w_{\partial u н} + \frac{\partial^2 w_{\partial u н}}{\partial \tau^2}\right) = \bar{q}, \quad (4)$$

where

$$w_{\partial u н} = \frac{\upsilon\left(x, t\right)}{l}, \ \tau = w_0 t.$$

$$w_0 = \sqrt{\frac{k}{\rho A}}$$

is parameter, that has frequency dimension, and called as "conventional" frequency.

Equation (4) describes forced vibrations of the loaded beam. Winkler model does not suppose dynamic effects in an elastic foundation. Necessary for the further calculation eigen functions can be obtained from the equation (4) with zero padded right part. After parameters division by performance

$$w_{\partial u н} = W\left(\xi\right)\sin\bar{\omega}\tau, \quad (5)$$

takes the form

$$\frac{d^4 W}{d\xi^4} + 4\alpha^4\left(1 - \bar{\omega}^2\right)W = 0, \quad (6)$$

where

$$\bar{\omega} = \frac{\omega}{\omega_0}$$

is dimensionless natural frequency of vibration of the beam on an elastic foundation.

Using "conventional" frequency ω_0, characterizing stiffness and insertional properties of the system "beam – foundation", and known the basic frequency of flexural oscillations of the beam with the same support at the edges, but without an elastic foundation

$$\omega_{1св} = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}},$$

We transform equation (6) to the form

$$\frac{d^4W}{d\xi^4} = \pi^4\left(-\bar{\omega}_0^2 + \tilde{\omega}^2\right)W, \qquad (7)$$

where

$$\bar{\omega}_0 = \frac{\omega_0}{\omega_{1св}}$$

is relative "conventional" frequency;

$$\tilde{\omega} = \frac{\omega}{\omega_{1св}}$$

is relative finding frequency.

Using Euler substitution

$$W = Ae^{r\xi}, \qquad (8)$$

we obtain characteristic equation for differential equation (7),

$$r^4 + \pi^4\left(\bar{\omega}_0^2 - \tilde{\omega}^2\right) = 0, \qquad (9)$$

The roots of which can be represented in both forms in accordance with frequencies ratio $\bar{\omega}_0$ and $\tilde{\omega}$:

– if $\tilde{\omega} > \bar{\omega}_0$, then roots of the equation (9) are real and imaginary

$$r_{1,2} = \pm\beta_1, \quad r_{3,4} = \pm i\beta_1, \quad \beta_1 = \pi\sqrt[4]{\tilde{\omega}^2 - \bar{\omega}_0^2}, \qquad (10)$$

In this case a deflection function (8) takes the form

$$\begin{aligned} W = {} & A_1 ch\beta_1\xi + A_2 sh\beta_1\xi + \\ & + A_3\cos\beta_1\xi + A_4\sin\beta_1\xi; \end{aligned} \qquad (11)$$

– if $\tilde{\omega} < \bar{\omega}_0$, then roots of the equation (9) are complex

$$r_{1,2,3,4} = (\pm i \pm 1)\beta_2, \quad \beta_2 = \frac{\pi}{\sqrt{2}}\sqrt[4]{\bar{\omega}^2 - \tilde{\omega}^2} \qquad (12)$$

and a deflection function takes the form

$$\begin{aligned} W = {} & A_1 sh\beta_2\xi\sin\beta_2\xi + \\ & + A_2 sh\beta_2\xi\cos\beta_2\xi + \\ & + A_3 ch\beta_2\xi\sin\beta_2\xi + \\ & + A_4 ch\beta_2\xi\cos\beta_2\xi. \end{aligned} \qquad (13)$$

In the papers [23, 24] it is shown that for a beam fully supported on the elastic Winkler foundation, in the case of canonical symmetric boundary conditions: clamping-clamping, hinge-hinge, free ends, only option (10) is realized.

The principally possible case of fourfold root

$$r_{1,2,3,4} = 0, \qquad (14)$$

when $\bar{\omega}_0 = \tilde{\omega}$ and deflection function takes the form

$$W = A_1 + A_2\xi + A_3\frac{\xi^2}{2} + A_4\sin\frac{\xi^3}{6} \qquad (15)$$

in the case of limiting the deflections of the ends of the beam, as it is in our case with the hinged supports, also is not implemented. Let us note that with partial support of the beam on an elastic foundation and at various boundary

conditions, all three modes of oscillations (11), (13) and (15) are possible.
Using initial parameters

$$W_0 = W(0),\quad W_0' = W'(0),\quad W_0'' = W''(0),\quad W_0''' = W'''(0)$$

instead constants of integration $A_i\,(i = 1 \div 4)$, let us write relationships, characterizing state of arbitrary section ξ of the beam, using variant (10), (11). The deflection function in this case takes the form

$$W(\xi) = W_0 R_4(\beta_1\xi) + W_0' R_3(\beta_1\xi) + W_0'' R_2(\beta_1\xi) + W_0''' R_1(\beta_1\xi), \tag{16}$$

where $R_i\,(i = 1 \div 4)$ is Krylov function, that takes the form

$$R_1(\beta_1\xi) = \frac{sh\beta_1\xi - \sin\beta_1\xi}{2\beta_1\xi}$$
$$R_2(\beta_1\xi) = \frac{ch\beta_1\xi - \cos\beta_1\xi}{2\beta_1^2}$$
$$R_3(\beta_1\xi) = \frac{sh\beta_1\xi + \sin\beta_1\xi}{2\beta_1}$$
$$R_4(\beta_1\xi) = \frac{ch\beta_1\xi + \cos\beta_1\xi}{2}$$

The state of an arbitrary section of the beam is described by equation in the matrix form

$$\bar{W}(\xi) = V_1(\xi)\bar{W}_0, \tag{17}$$

where $\bar{W}(\xi)$ is vector of the state of an arbitrary section ξ

$$\bar{W}(\xi) = \{W(\xi)\;\; W'(\xi)\;\; W''(\xi)\;\; W'''(\xi)\};$$

$\bar{W}_0(\xi)$ is vector of initial parametrs

$$\bar{W}_0 = \{W_0\;\; W_0'\;\; W_0''\;\; W_0'''\};$$

$V_1(\xi) = \{\upsilon_{ij}\}$ – functional matrix, characterizes affecting of initial parameters to state of a section ξ

$$V_1(\xi) = \begin{pmatrix} R_4(\beta_1\xi) & R_3(\beta_1\xi) & R_2(\beta_1\xi) & R_1(\beta_1\xi) \\ \beta_1^4 R_1(\beta_1\xi) & R_4(\beta_1\xi) & R_3(\beta_1\xi) & R_2(\beta_1\xi) \\ \beta_1^4 R_2(\beta_1\xi) & \beta_1^4 R_1(\beta_1\xi) & R_4(\beta_1\xi) & R_3(\beta_1\xi) \\ \beta_1^4 R_3(\beta_1\xi) & \beta_1^4 R_2(\beta_1\xi) & \beta_1^4 R_1(\beta_1\xi) & R_4(\beta_1\xi) \end{pmatrix}.$$

3. Let us perform an analysis of the natural frequencies and modes of bending oscillations of the beam on an elastic foundation with hinged ends. In this case, the boundary conditions and the deflection function are

$$W_0 = W_0'' = 0$$
$$W(1) = W''(1) = 0. \tag{18}$$

$$W(\xi) = W_0' R_3(\beta_1\xi) + W_0''' R_1(\beta_1\xi). \tag{19}$$

Satisfying the second pair of boundary conditions (18), from function (19) and its second derivative, we obtain a system of algebraic equations for the unknown initial parameters W_0' and W_0'''

$$\begin{cases} W_0' R_3(\beta_1) + W_0''' R_1(\beta_1) = 0, \\ W_0' \beta_1^4 R_1(\beta_1) + W_0''' R_3(\beta_1) = 0. \end{cases} \tag{20}$$

The condition for the existence of nonzero solutions of a given homogeneous system is the equality to zero of the determinant of the coefficient matrix of this system

$$\begin{vmatrix} R_3(\beta_1) & R_1(\beta_1) \\ \beta_1^4 R_1(\beta_1) & R_3(\beta_1) \end{vmatrix} = 0. \tag{21}$$

Expanding the determinant, we obtain the frequency equation

$$4\sin\beta_1 sh\beta_1 = 0,$$

From where follows

$$\sin\beta_1 = 0$$

$$\beta_{1n} = n\pi \quad (n = 1, 2, 3, ...).$$

Taking in account the formula (10), we obtain frequencies spectrum

$$\tilde{\omega}_n = \sqrt{\bar{\omega}_0^2 + n^4}. \qquad (22)$$

From any equation of the system (20) when $\beta_n = n\pi$ follows

$$\frac{W_0'''}{W_0'} = -(n\pi)^2,$$

then, in accordance with (19), *n*-th mode with frequency ω_n, takes the form

$$W_n(\xi) = A_n \sin n\pi\xi, \qquad (23)$$

where n is a number of half-waves of sinusoid along the beam length l; A_n is unknown amplitude of oscillations for *n*-th mode.

Thus, the forms of natural oscillations of a beam on an elastic foundation remain the same as that of a free beam, but with frequencies ω_n, greater than the corresponding frequencies of a free beam $\omega_{св_n}$ in $\sqrt{\bar{\omega}_0^2 + n^4}$ times, i.e. in accordance with (22)

$$\omega_n = \sqrt{\bar{\omega}_0^2 + n^4}\,\omega_{св_n}.$$

4. The solution of the differential equation of forced oscillations (4) can be obtained by expanding the function $w_{дин}(\xi, \tau)$ in a series in eigenfunctions $W_n(\xi)$ (23) with coefficients in the form of unknown functions of time $Q_n(\tau)$

$$w_{дин} = \sum_{n=1}^{\infty} Q_n(\tau) W_n(\xi). \qquad (24)$$

The functions $Q_n(\tau)$ can be found, applying the following procedures: substituting series (24) and expression (7) into equation (4), multiplying both sides of this equation by $W_n(\xi)$, integrating both sides by ξ from 0 to 1 and, using the orthogonality property of the natural vibration forms $W_n(\xi)$, we obtain the differential equation for function $Q_n(\tau)$

$$\frac{d^2 Q_n}{d\tau^2} + \bar{\omega}_n^2 Q_n = S_n, \qquad (25)$$

where

$$S_n = \frac{2\bar{q}}{\pi^4 \bar{\omega}_0^2} \frac{\sin^2 \frac{n\pi}{2}}{\frac{n\pi}{2}}.$$

General solution of inhomogeneous equation (25)

$$Q_n = D_{1n} \cos\frac{\tilde{\omega}_n}{\bar{\omega}_0}\tau + D_{2n} \sin\frac{\tilde{\omega}_n}{\bar{\omega}_0}\tau + \frac{S_n}{\bar{\omega}_n^2} \qquad (26)$$

is a sum of relative homogeneous solution (first and second additives) and partial solution, corresponding to the right part of (25) (third additive).

Now, according to (24), the dynamic deflection function takes the form

$$w_{дин} = \\ = \sum_{n=1}^{\infty} \left(D_{1n} \cos\frac{\tilde{\omega}_n}{\bar{\omega}_0}\tau + D_{2n} \sin\frac{\tilde{\omega}_n}{\bar{\omega}_0}\tau + \frac{S_n}{\bar{\omega}_n^2} \right) \sin n\pi\xi. \qquad (27)$$

Integration constants D_{1n} and D_{2n} is determined from initial condition

$$w_{дин}(\xi,0)=w_{cm}(\xi)$$
$$\left.\frac{\partial w_{дин}}{\partial \tau}\right|_{\xi,0}=0. \tag{28}$$

From the second condition (28) we define one constant

$$D_{2n}=0. \tag{29}$$

Multiplying both sides of the first condition (28) in accordance with (27) and (29) to $\sin n\pi\xi$ and integrating by ξ from 0 to 1, we obtain another constant

$$D_{1n}=B_n-\frac{S_n}{\bar{\omega}_n^2}, \tag{30}$$

where

$$B_n=2\int_0^1 w_{cm}(\xi)\sin n\pi\xi d\xi.$$

Substituting (29) and (30) into (27) and taking into account the trigonometric identity

$$1-\cos\frac{\tilde{\omega}_n}{\bar{\omega}_0}\tau=2\sin^2\frac{\tilde{\omega}_n}{2\bar{\omega}_0}\tau,$$

we obtain

$$w_{дин}=$$
$$=\sum_{n=1}^{\infty}\left(B_n\cos\frac{\tilde{\omega}_n}{\bar{\omega}_0}\tau+C_n\sin^2\frac{\tilde{\omega}_n}{2\bar{\omega}_0}\tau\right)\sin n\pi\xi, \tag{31}$$

where

$$C_n=\frac{4\bar{q}}{\pi^4\tilde{\omega}_n^2}\frac{\sin^2\frac{n\pi}{2}}{\frac{n\pi}{2}}.$$

The dimensionless bending moment is obtained by twice differentiating the series (31)

$$w''_{дин}=$$
$$=-\pi^2\sum_{n=1}^{\infty}n^2\left(B_n\cos\frac{\tilde{\omega}_n}{\bar{\omega}_0}\tau+C_n\sin^2\frac{\tilde{\omega}_n}{2\bar{\omega}_0}\tau\right)\sin n\pi\xi \tag{32}$$

5. Numerical example.

Using the Maple software package, we calculated the dimensionless deflections $w(\xi)$ and bending moments $w''(\xi)$ in the beam loaded with a even distributed load of $\bar{q}=1$ intensity on an elastic Winkler foundation

– in the initial static state with clamping of its ends: $w_{cm}(\xi), w''_{cm}(\xi)$;

– in the static state, formed after quasi-static transformation of clamping to hinges $w_{кв}(\xi)$, $w''_{кв}(\xi)$;

– in the dynamic process that occurs at a sudden transformation of clamping to hinges: $w_{дин}(\xi,\tau)$, $w''_{дин}(\xi)$.

In practical calculations, 20 members of the series (31) and (32) were taken into account. In this case, we obtain a practical coincidence of the diagrams of dynamic deflections $w_{дин}(\xi, 0)$ and static deflection $w_{cm}(\xi)$, that is

$$\sum_{n=1}^{20}B_n\sin n\pi\xi\approx w_{cm}(\xi).$$

The calculation results are shown in figures 3 and 4, as well as in table 1. In figures 3 and 4 are shown respectively: diagrams of bending moments $w''_{кв}(\xi)$ in the beam after quasi-static transformation of clamped points into hinges and during oscillations $w''_{дин}(\xi,\tau_0)$ after sudden transformation of clamped points into hinges at the moment τ_0 of reaching the highest values.

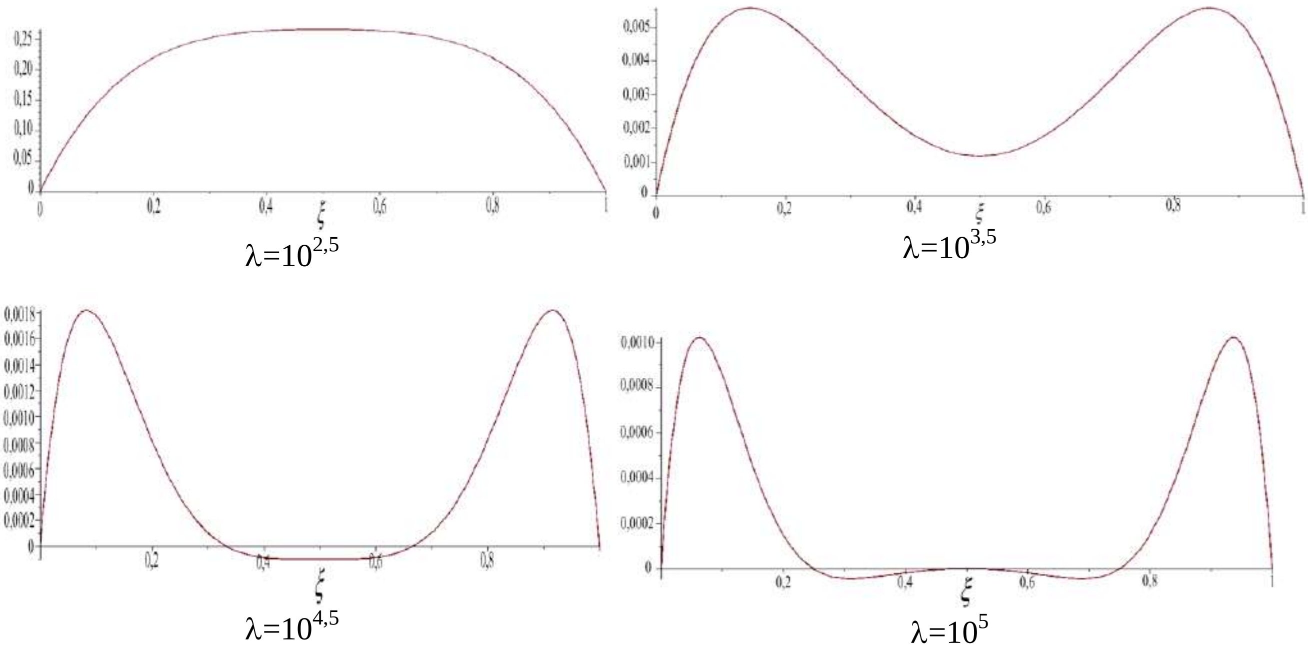


Figure 3. Diagrams of bending moments after quasistatic transformation of the boundary conditions depending on the stiffness indicator of the "beam-foundation" system λ.

Table 1. Affecting of stiffness of the "beam – foundation" system to the increment of bending moments.

λ	$\left\lvert w''_{\underset{\max}{ст.}} \right\rvert$	$w''_{\underset{\max}{кв.}}$	$K_{ст}$	$w''_{\underset{\max}{дин.}}$	$K_{дин}$
0	0.083	0.125	1.506	0.3	3.614
10	0.082	0.112	1.366	0.269	3.28
$10^{1.5}$	0.079	0.093	1.177	0.225	2.848
10^{2}	0.071	0.06	0.845	1.156	2.197
$10^{2.5}$	0.0544	0.0265	0.487	0.07	1.287
10^{3}	0.0334	0.0104	0.311	0.0346	1.036
$10^{3.5}$	0.018	0.0055	0.305	0.0186	1.033
10^{4}	0.01	0.003	0.3	0.01	1

Diagrams are constructed for different values of the stiffness parameter of the "beam-foundation" system λ.

Table 1 contains the values of the largest bending moments (dimensionless stresses) in three states $w''_{\underset{\max}{ст.}}$, $w''_{\underset{\max}{кв.}}$, $w''_{\underset{\max}{дин.}}$ with different parameters of the stiffness of the "beam-foundation" system λ, as well as the coefficients

$$K_{ст} = \frac{w''_{\underset{\max}{кв.}}}{w''_{\underset{\max}{ст.}}}; \quad K_{дин} = \frac{w''_{\underset{\max}{дин.}}}{w''_{\underset{\max}{ст.}}},$$

characterizing increasing (decreasing) order of the maximum bending moment at quasi-static ($K_{ст}$) and dynamic ($K_{дин}$) changing of boundary conditions.

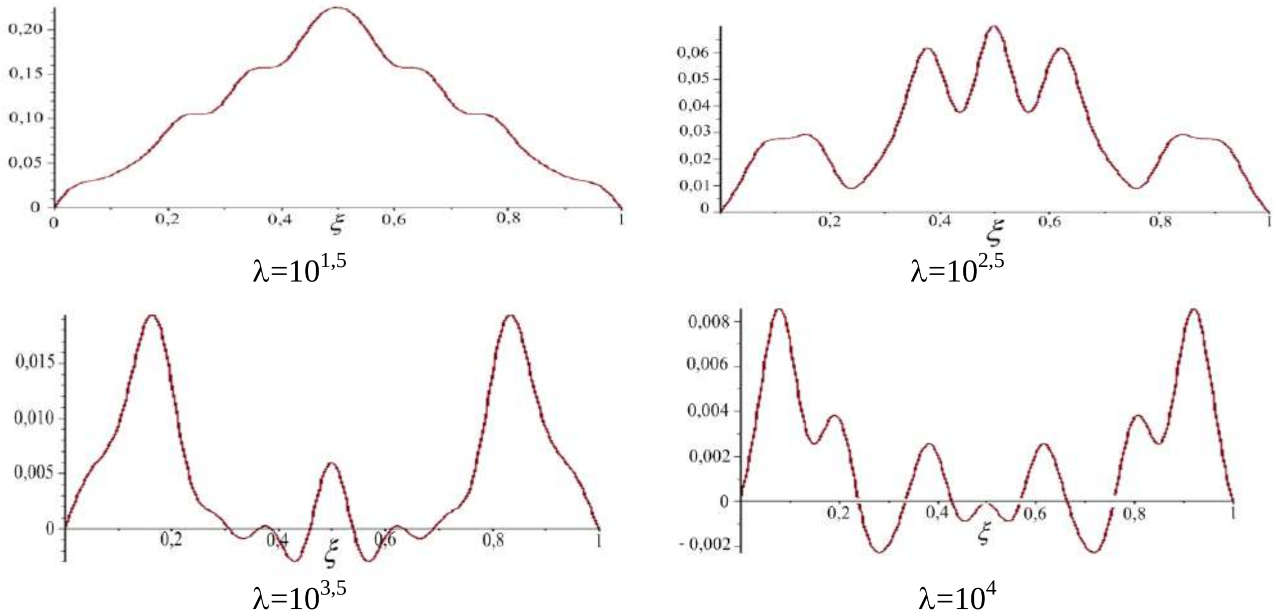


Figure 4. Bending moment diagrams after a sudden transformation of the boundary conditions at the time of reaching the highest values depending on the rigidity indicator of "beam – foundation" systems λ.

CONCLUSION

If we consider the transformation of the boundary conditions in this "beam-foundation" system under load caused by a damage, then the provided study shows that quasi-static defect formation, that is, a decrease in the stiffness of the end supports, leads to an insignificant increase in the stresses in the beam ($K_{cm} > 1$) when there is not foundation ($\lambda = 0$) and low indicator values of the "beam-foundation" system ($0< \lambda \leq 10^{1,79}$). For beams based on more rigid bases ($\lambda > 10^{1,79}$), the formation of the same defect, on the contrary, leads to a decrease in the greatest stresses ($K_{cm} < 1$).

The sudden formation of a defect gives a more than threefold ($K_{дин}$ = 3,614) increase in the greatest stress in the free beam ($\lambda = 0$). For systems with higher stiffness, the effect of transforming the boundary conditions is reduced. There is a redistribution of stresses along the span, but the greatest stress at $\lambda > 10^4$ does not exceed the value of the initial static one($K_{дин}$= 1). In addition, regardless of the speed of formation of a defect, with the increase in the rigidity of the system, the greatest stresses move from the center of the beam to the periphery of the span.

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NUMERICAL ANALYSIS OF NON-LINEAR VIBRATIONS OF A FRACTIONALLY DAMPED CYLINDRICAL SHELL UNDER THE ADDITIVE COMBINATIONAL INTERNAL RESONANCE

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Abstract: Non-linear damped vibrations of a cylindrical shell subjected to the additive type combinational internal resonance are investigated numerically using two different numerical methods. The damping features of the surrounding medium are described by the fractional derivative Kelvin-Voigt model involving the Riemann-Liouville fractional derivatives. Within the first method, the generalized displacements of a coupled set of nonlinear ordinary differential are estimated using numerical solution of nonlinear multi-term fractional differential equations by the procedure based on the reduction of the problem to a system of fractional differential equations. According to the second method, the amplitudes and phases of nonlinear vibrations are estimated from the governing nonlinear differential equations describing amplitude-and-phase modulations for the case of the additive combinational internal resonance. A good agreement in results is declared.

Keywords: cylindrical shell, free nonlinear damped vibrations, additive combinational internal resonance, method of multiple time scales, multi-term fractional differential equations

ЧИСЛЕННЫЙ АНАЛИЗ НЕЛИНЕЙНЫХ КОЛЕБАНИЙ ЦИЛИНДРИЧЕСКОЙ ОБОЛОЧКИ С ДРОБНЫМ ДЕМПФИРОВАНИЕМ ПРИ АДДИТИВНОМ КОМБИНАЦИОННОМ ВНУТРЕННЕМ РЕЗОНАНСЕ

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Аннотация: Рассматриваются нелинейные затухающие колебания цилиндрической оболочки при аддитивном комбинационном внутреннем резонансе. Для решения соответствующих задач применяются два различных численных метода. Демпфирующие особенности окружающей среды описываются с помощью дробной производной модели Кельвина-Фойгта, включающей дробные производные Римана-Лиувилля. В рамках первого метода обобщенные смещения связного набора нелинейных обыкновенных дифференциалов оцениваются на основе численного решения нелинейных многочленных уравнений с дробными производными по методике, предусматривающей сведение исходной задачи к системе уравнений с дробными производными. Согласно второму методу, амплитуды и фазы нелинейных колебаний оцениваются из определяющих нелинейных дифференциальных уравнений, описывающих амплитудно-фазовые модуляции для случая аддитивного комбинационного внутреннего резонанса. Отмечена хорошая согласованность полученных результатов.

Ключевые слова: цилиндрическая оболочка, свободные нелинейно-затухающие колебания, аддитивный комбинационный внутренний резонанс, метод кратных временных шкал, многочленные уравнения с дробными производными

1. INTRODUCTION

In mechanical nonlinear vibrations, the phenomena of internal resonance and energy exchange are quite often what requires the thorough studies, since in the case of low damping it could result in long-time vibrations accompanied by the two-sided or one-sided energy interchange between coupled modes [1]. It will suffice to mention the state-of-the-art articles [1,2] and the monograph [3] involving the extensive review of literature in the field of internal resonances in different mechanical systems. Different types of the internal resonance: one-to-one, two-to-one, three-to-one, as well as a variety of combinational resonances, when three and more natural modes interact, have been discussed. The enumerated internal resonances were investigated in various mechanical systems with multiple degree-of-freedom, as well as in strings, beams, plates, and shells.

It has been emphasized by many researchers [4-13] that the phenomenon of internal resonances can be very critical especially for circular cylindrical shells. Thus, the nonlinear vibrations of infinitely long circular cylindrical shells under the conditions of the two-to-one internal resonance were studied in [6] via the method of multiple time scales using the simple plane strain theory of shells. Parametrically excited vibrations of infinitely long cylindrical shells and nonlinear forced vibrations of a simply supported, circular cylindrical shell filled with an incompressible, inviscid, quiescent and dense fluid were investigated in [4,5,7] using Donnell’s nonlinear shallow-shell theory. The flexural deformation is usually expanded by using the linear shell eigenmodes, in so doing the flexural response involves several nodal diameters and one or two longitudinal half-waves. Internal resonances of different types have been analyzed in [8-13].

The extensive review of studies on shallow shells nonlinear vibrations could be found in the state-of-the-art articles [14-16]. In spite of the fact that many studies have been carried out on large amplitude vibrations of circular cylindrical shells and many different approaches to the problem have been used, we agree with Breslavsky and Amabili [10] that this research area is still far from being well understood.

In recent years much attention is given to damping features of mechanical systems subjected to the conditions of different internal resonances. Damping properties of nonlinear systems are described mainly by the first-order time-derivative of a generalized displacement [3]. However, as it has been shown by Rossikhin and Shitikova [17], who analyzed free damped vibrations of suspension combined system under the conditions of the one-to-one internal resonance, for good fit of the theoretical investigations with the experimental results it is better to describe the damping features of nonlinear mechanical systems in terms of fractional time-derivatives of the generalized displacements [18].

During the last decade, fractional calculus entered the mainstream of engineering analysis. And it has been widely applied to structural dynamics problems both in discrete and continuous equations. The history of the fractional calculus applications in mechanics could be found in the retrospective paper by Rossikhin [19], while a comprehensive review of the fractional calculus models in different dynamic problems of solids and structures is presented in the state-of-the-art article [18], wherein the results obtained in the field critically estimated in the light of the present view of the place and role of the fractional calculus in engineering problems and practice.

It has been suggested in 2011 to examine the nonlinear dynamic response of a thin cylindrical shell vibrating in a fractionally damped medium [20], when the dynamic behavior of the shell is described by a set of three coupled nonlinear differential equations with due account for the fact that the shell is being under the conditions of the internal resonance resulting in the interaction of modes corresponding to the mutually orthogonal displacements. The displacement functions are determined in terms of eigenfunctions of linear vibrations.

A new procedure resulting in decoupling linear parts of equations has been proposed in Rossikhin and Shitikova [21] with the further utilization of the method of multiple scales for solving nonlinear governing equations of motion, in so doing the amplitude functions are expanded into power series in terms of the small parameter and depend on different time scales. It is shown that the phenomenon of the internal resonance between vibrational subsystems of the cylindrical shell under consideration can be very critical, since in the circular cylindrical shell of such a type the two-to-one [21], one-to-one, three-to-one [22] internal resonances, as well as combinational internal resonances [23] could occur, which are governed by the order of smallness of viscosity. All possible cases of the internal resonance have been recently revealed in [22], which belong to the resonances of the constructive type, since all of them depend on the geometrical dimensions of the shell under consideration and its mechanical characteristics, that is why such resonances could not be ignored and eliminated for a particularly designed shell. It has been shown that the energy exchange could occur between two or three subsystems at a time: normal vibrations of the shell, its torsional vibrations and shear vibrations along the shell axis. Such an energy exchange, if it takes place for a rather long time, could result in crack formation in the shell, and finally to its failure. The energy exchange has been illustrated pictorially by the phase portraits, wherein the phase trajectories of the phase fluid motion are depicted.

In the present paper, we are going to verify parameter values of the cylindrical shell model [20-23], resulting in the nonlinear vibrations of a fractionally damped cylindrical shell under the conditions of combinational internal resonance, and to study such phenomenon using two different numerical methods [24]. In the first method, the generalized displacements of a coupled set of nonlinear ordinary differential equations of the second order are estimated using numerical solution of nonlinear multi-term fractional differential equations by the procedure based on the reducing of the problem to a system of fractional differential equations [25-28]. According to the second method, the amplitudes and phases of nonlinear vibrations are estimated from the governing nonlinear differential equations describing amplitude-and-phase modulations for the case of the combinational internal resonance [23] using the Runge-Kutta fourth order method.

2. PROBLEM FORMULATION AND GOVERNING EQUATIONS

Let us examine the dynamic response of a free supported non-linear elastic circular cylindrical shell of radius R and length l, vibrations of which in the cylindrical system of coordinates described by the Donnell–Mushtari–Vlasov equations with respect to the three displacements [12] considering that damping features of the surrounding medium are described by the time-differential operator of the fractional order [20]:

$$\begin{aligned}
&\frac{\partial^2 u}{\partial x^2}+\frac{1-\sigma}{2}\frac{1}{R^2}\frac{\partial^2 u}{\partial \varphi^2}+ \\
&+\frac{1+\sigma}{2}\frac{1}{R}\frac{\partial^2 v}{\partial x\partial\varphi}-\sigma\frac{1}{R}\frac{\partial w}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}+ \\
&+\frac{1+\sigma}{2}\frac{1}{R^2}\frac{\partial w}{\partial \varphi}\frac{\partial^2 w}{\partial x\partial\varphi}+\frac{1-\sigma}{2}\frac{1}{R^2}\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial \varphi^2}= \\
&=\frac{\rho(1-\sigma^2)}{E}\frac{\partial^2 u}{\partial t^2}+\text{æ}_1\left(\frac{d}{dt}\right)^{\gamma}u,
\end{aligned}\tag{1}$$

$$\begin{aligned}
&\frac{1}{R^2}\frac{\partial^2 v}{\partial \varphi^2}+\frac{1-\sigma}{2}\frac{\partial^2 v}{\partial x^2}+\frac{1+\sigma}{2}\frac{1}{R}\frac{\partial^2 u}{\partial x\partial\varphi}- \\
&-\frac{1}{R^2}\frac{\partial w}{\partial \varphi}+\frac{1}{R^3}\frac{\partial w}{\partial \varphi}\frac{\partial^2 w}{\partial \varphi^2}+ \\
&+\frac{1+\sigma}{2}\frac{1}{R}\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x\partial\varphi}+\frac{1-\sigma}{2}\frac{1}{R}\frac{\partial w}{\partial \varphi}\frac{\partial^2 w}{\partial x^2}= \\
&=\frac{\rho(1-\sigma^2)}{E}\frac{\partial^2 v}{\partial t^2}+\text{æ}_2\left(\frac{d}{dt}\right)^{\gamma}v,
\end{aligned}\tag{2}$$

$$\frac{h^2}{12}\nabla^4 w+\frac{1}{R^2}\,w-\sigma\,\frac{1}{R}\frac{\partial u}{\partial x}-\frac{1}{R^2}\frac{\partial v}{\partial \varphi}-$$
$$-\frac{1}{2}\frac{\sigma}{R}\left(\frac{\partial w}{\partial x}\right)^2-\frac{1}{2}\frac{1}{R^3}\left(\frac{\partial w}{\partial \varphi}\right)^2-$$
$$-\frac{\partial}{\partial x}\left[\frac{\partial w}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\sigma}{R}\frac{\partial v}{\partial \varphi}-\frac{\sigma}{R}\,w\right)+\right.$$
$$\left.+\frac{1-\sigma}{2}\frac{1}{R}\frac{\partial w}{\partial \varphi}\left(\frac{1}{R}\frac{\partial u}{\partial \varphi}+\frac{\partial v}{\partial x}\right)\right]-$$
$$-\frac{1}{R}\frac{\partial}{\partial \varphi}\left[\frac{1}{R}\frac{\partial w}{\partial \varphi}\left(\sigma\frac{\partial u}{\partial x}+\frac{1}{R}\frac{\partial v}{\partial \varphi}-\frac{1}{R}\,w\right)+\right.$$
$$\left.+\frac{1-\sigma}{2}\frac{\partial w}{\partial x}\left(\frac{1}{R}\frac{\partial u}{\partial \varphi}+\frac{\partial v}{\partial x}\right)\right]-$$
$$=-\frac{\rho(1-\sigma^2)}{E}\frac{\partial^2 w}{\partial t^2}-æ_3\left(\frac{d}{dt}\right)^{\gamma} w, \qquad (3)$$

where x-axis is directed along the axis of the cylinder, φ is the polar angle in the plane perpendicular to the x-axis,

$u=u(x,\varphi,t)$, $v=v(x,\varphi,t)$, and $w=w(x,\varphi,t)$

are the displacements of points located in the shell's middle surface in three mutually orthogonal directions x,φ,r with r as the polar radius, h is the thickness, ρ is the density, E and σ are the elastic modulus and Poisson's ratio, respectively, t is the time, $æ_1$, $æ_2$, $æ_3$ are the damping coefficients, and

$$\nabla^4=\nabla^2\nabla^2=\frac{\partial^4}{\partial x^4}+2\frac{1}{R^2}\frac{\partial^4}{\partial x^2\partial\varphi^2}+\frac{1}{R^4}\frac{\partial^4}{\partial\varphi^4}$$

The initial conditions

$$u\,|_{t=0}=v\,|_{t=0}=w\,|_{t=0}; \qquad (4)$$
$$\dot u\,|_{t=0}=\varepsilon V_1^0(x,\varphi),\quad \dot v\,|_{t=0}=\varepsilon V_2^0(x,\varphi),\quad \dot w\,|_{t=0}=\varepsilon V_3^0(x,\varphi) \qquad (5)$$

where $V_i^0(x,\varphi)$ (i=1,2,3) are the corresponding initial velocities, and ε is a small value, should be added to Eqs. (1)-(3). Hereafter over dots denote time-derivatives.

The boundary conditions for the simply supported shell (the Navier-type conditions for the edges free supported in the x-direction) have the form [12]:

$$w\,|_{x=0}=w\,|_{x=l}=0,\quad v\,|_{x=0}=v\,|_{x=l}=0,$$
$$\frac{\partial^2 w}{\partial x^2}\,|_{x=0}=\frac{\partial^2 w}{\partial x^2}\,|_{x=l}=0,$$
$$\frac{\partial u}{\partial x}\,|_{x=0}=\frac{\partial u}{\partial x}\,|_{x=l}=0. \qquad (6)$$

From relationships (5) it follows that free vibrations are excited by the weak disturbance from the equilibrium position.

It has been proposed in [20] to rewrite Eqs. (1)-(5) in the nondimensioned form in terms of the following dimensionless parameters:

$$u^*=\frac{u}{l},\quad v^*=\frac{v}{l},\quad w^*=\frac{w}{l},$$
$$x^*=\frac{x}{l},\quad t^*=\frac{t}{l}\sqrt{\frac{E}{\rho(1-\sigma^2)}}.$$

Dropping hereafter the asterisks for the ease of presentation, let us admit the solution of the Navier type for Eqs. (1)-(3) in the form

$$u(x,\varphi,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}x_{1mn}(t)\eta_{1mn}(x,\varphi);$$
$$v(x,\varphi,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}x_{2mn}(t)\eta_{2mn}(x,\varphi); \qquad (7)$$
$$w(x,\varphi,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}x_{3mn}(t)\eta_{3mn}(x,\varphi),$$

where $x_{jmn}(t)$ and $\eta_{jmn}(x,\varphi)$ (j=1,2,3) are, respectively, the generalized displacements and eigenfunctions satisfying the boundary conditions (6), and m and n are integers.

Distinct to the traditional modeling the viscous resistance forces via first order time-derivatives [16], in the present research we adopt the fractional order time-derivative

$$(d/dt)^{\gamma},$$

what, as it had been shown in [17,18], allows one to obtain the damping coefficients dependent on the natural frequency of vibrations. It has been demonstrated in [29] on the example of the Golden Gate suspension bridge that such an approach for modeling the damped non-linear vibrations provides the good agreement between the theoretical results and the experimental data through the appropriate choice of the fractional parameter (the order of the fractional derivative) and the viscosity coefficient.

It was shown in Samko et al. [30] (see Chapter 2, Paragraph 5, point 7^0) that the fractional order of the operator of differentiation

$$(d/dt)^{\gamma}$$

is equal to the Marsho fractional derivative, which, in its turn, equal to the Riemann–Liouville derivative D_+^{γ}.

It has been noted in [17,18] that a fractional derivative is the immediate extension of an ordinary derivative. In fact, when $\gamma \to 1$ the fractional derivative goes over into the ordinary time-derivative of the first order, and the mathematical model of the viscoelastic shell under consideration transforms into the conventional Kelvin–Voigt model, wherein the elastic element behaves non-lineally, but the viscous element behaves linearly. When

$$\gamma \to 0,$$

the fractional derivative

$$D_+^{\gamma} f \text{ tends to } f(t).$$

To put it otherwise, the introduction of the new fractional parameter along with the parameters $æ_i$ allows one to change not only the magnitude of viscosity at the cost of an increase or decrease in the parameters $æ_i$, but also the character of viscosity at the sacrifice of variations in the fractional parameter γ.

Now substituting the proposed solution (7) in nondimensioned Eqs. (1)-(3), multiplying then each equation by the corresponding function $\eta_{jmn}(x,\varphi)$, integrating over x and φ, and using the orthogonality conditions for linear modes within the domains of

$$0 \le x \le 1 \text{ and } 0 \le \varphi \le 2\pi,$$

we are led to a coupled set of nonlinear ordinary differential equations of the second order in $x_{imn}(t)$. However, a new procedure has been proposed in [21] for decoupling the linear parts of nonlinear differential equations.

Thus, the system is reduced to the following form:

$$\begin{aligned}&\ddot{X}_{1mn} + æ_1 D^{\gamma} X_{1mn} + \Omega_{1mn}^2 X_{1mn} = \\ &= -\sum_{i=1}^{3} F_{imn} L_{imn}^{I};\end{aligned} \tag{8}$$

$$\begin{aligned}&\ddot{X}_{2mn} + æ_2 D^{\gamma} X_{2mn} + \Omega_{2mn}^2 X_{2mn} = \\ &= -\sum_{i=1}^{3} F_{imn} L_{imn}^{II};\end{aligned} \tag{9}$$

$$\begin{aligned}&\ddot{X}_{3mn} + æ_3 D^{\gamma} X_{3mn} + \Omega_{3mn}^2 X_{2mn} = \\ &= -\sum_{i=1}^{3} F_{imn} L_{imn}^{III},\end{aligned} \tag{10}$$

where $D^{\gamma} = (d/dt)^{\gamma}$, and X_i (i=1,2,3) are new generalized displacements which are connected with $x_{imn}(t)$ via eigenvectors L_{imn}^{I}, L_{imn}^{II}, L_{imn}^{III}

$$x_{imn}(t) = X_{1mn} L_{imn}^{I} + X_{2mn} L_{imn}^{II} + X_{3mn} L_{imn}^{III}$$

of the matrix S_{ij}^{mn} with the corresponding eigenvalues Ω_{1mn}, Ω_{2mn}, and Ω_{3mn}, the elements of which are the following:

$$S_{ij}^{mn} = \begin{bmatrix} S_{11}^{mn} & S_{12}^{mn} & S_{13}^{mn} \\ S_{21}^{mn} & S_{22}^{mn} & S_{23}^{mn} \\ S_{31}^{mn} & S_{32}^{mn} & S_{33}^{mn} \end{bmatrix} = \begin{bmatrix} \left(\pi^2 m^2 + \frac{1-\sigma}{2}\beta_1^2 n^2\right) & \frac{1+\sigma}{2}\beta_1 \pi mn & \sigma\beta_1\pi m \\ \frac{1+\sigma}{2}\beta_1\pi mn & \left(\frac{1-\sigma}{2}\pi^2 m^2 + \beta_1^2 n^2\right) & \beta_1^2 n \\ \sigma\beta_1\pi m & \beta_1^2 n & \frac{\beta_2^2}{12}(\pi^2 m^2 + \beta_1^2 n^2)^2 + \beta_1 \end{bmatrix}, \qquad (11)$$

where

$$\beta_1 = l / R \text{ and } \beta_2 = h / l$$

are the parameters defining the dimensions of the shell.

From Eqs. (8)-(10) it is seen that their left-hand side parts are linear and independent of each other, while they are coupled only by non-linear terms F_{imn} in their right-hand sides.

It is known [3, 31] that during nonstationary excitation of thin bodies not all possible modes of vibration would be excited. Moreover, the modes which are strongly coupled by any of the so-called internal resonance conditions are initiated and dominate in the process of vibration, resulting in the energy transfer from one subsystem to another between the coupled modes, in so doing the types of modes to be excited are dependent of the character of the external excitation. It was emphasized in [31] that in the presence of damping, all modes that are not directly or indirectly excited by an internal resonance decay with time.

Assume hereafter that the vibration process occurs in such a way that only three natural modes corresponding to the complex generalized displacements

$$X_{1s_1s_2},\ X_{2l_1l_2},\text{ and } X_{3k_1k_2}$$

are excited and dominate over other natural modes. In this case, the right parts of Eqs. (8)-(10) are significantly simplified.

According to [20], the approximate solution of these three nonlinear equations (wherein the low indices s_1s_2, l_1l_2 and k_1k_2 are omitted for the ease of presentation) for small but finite amplitudes weakly varying with time could be represented by a uniform expansion in terms of different time scales:

$$X_i = \varepsilon X_{i1}(T_0, T_1, ...) + \varepsilon^2 X_{i2}(T_0, T_1, ...) + ... \qquad (12)$$

where $i = 1, 2, 3$, ε is a small dimensionless parameter of the same order of magnitude as the amplitudes,

$$T_n = \varepsilon^2 t \quad (n = 0, 1, 2, \dots)$$

are new independent variables, among them:

$$T_0 = t$$

is a fast scale characterizing motions with the natural frequencies, and

$$T_1 = \varepsilon t$$

is a slow scale characterizing the modulation of the amplitudes and phases of the modes with nonlinearity.

Applying the method of multiple scales directly to the governing partial-differential equations by substituting (12) in them and considering that the first and second time-derivatives, as well as the fractional order time-derivative are defined in terms of new time scales, respectively, as follows:

$$\frac{d}{dt} = D_0 + \varepsilon D_1 + ..., \quad \frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 ... \qquad (13)$$

$$\left(\frac{d}{dt}\right)^{\gamma} = D^{\gamma} = (D_0 + \varepsilon D_1 + ...)^{\gamma} =$$

$$= D_0^{\gamma} + \varepsilon\gamma D_0^{\gamma-1} D_1 + \frac{1}{2}\varepsilon^2 \gamma(\gamma-1) D_0^{\gamma-2} D_1^2 + ... \tag{14}$$

where $D_n = \partial / \partial T_n$, and D_0^{γ}, $D_0^{\gamma-1}$, $D_0^{\gamma-2}\dots$ are the Riemann-Liouville fractional derivatives in time t [17,18], after separating the terms at the same powers of ε, we could obtain the equations corresponding to different orders of ε [21].

It should be noted that the expansion of the fractional order operator of differentiation (14) for the first time was suggested in 1998 by Rossikhin and Shitikova [17], and nowadays it is used by the researchers worldwide when solving the nonlinear dynamic problems with fractional order damping.

The case of the order of ε has been considered in detail in [21], wherein all types of the internal resonance, which could occur on this step, have been detected and classified: (1) the two-to-one internal resonance, when one natural frequency is twice the other natural frequency, (2) the one-to-one-to-two or one-to-two-to-two internal resonance, and (3) the combinational resonances of the additive-difference type of the first order, among them, the case of $\Omega_2 = \Omega_1 + \Omega_3$, which we are going to study below numerically.

Utilizing the procedure described in [22] and considering that the fractional order damping coefficients have the form of

$$æ_i = \varepsilon \mu_i \tau_i^{\gamma},$$

where τ_i is the relaxation time of the i-th generalized displacement and μ_i is a finite value, the following six first-order nonlinear ordinary-differential equations governing the modulation of the amplitudes and phases of the three interacting modes in case of combinational additive internal resonance $\Omega_2 = \Omega_1 + \Omega_3$ have been obtained:

$$(a_1^2)^{\cdot} + s_1 a_1^2 = -\Omega_1^{-1} a_{23}^{I}\, a_1 a_2 a_3 \sin\delta\,; \tag{15}$$

$$(a_2^2)^{\cdot} + s_2 a_2^2 = \Omega_2^{-1} a_{13}^{II}\, a_1 a_2 a_3 \sin\delta\,; \tag{16}$$

$$(a_3^2)^{\cdot} + s_3 a_3^2 = -\Omega_3^{-1} a_{12}^{III}\, a_1 a_2 a_3 \sin\delta\,; \tag{17}$$

$$\dot{\varphi}_1 - \frac{1}{2}\sigma_1 - \frac{1}{2}\frac{a_{23}^{I}}{\Omega_1}\frac{a_2 a_3}{a_1}\cos\delta = 0\,; \tag{18}$$

$$\dot{\varphi}_2 - \frac{1}{2}\sigma_2 - \frac{1}{2}\frac{a_{13}^{II}}{\Omega_2}\frac{a_1 a_3}{a_2}\cos\delta = 0\,; \tag{19}$$

$$\dot{\varphi}_3 - \frac{1}{2}\sigma_3 - \frac{1}{2}\frac{a_{12}^{III}}{\Omega_3}\frac{a_1 a_2}{a_3}\cos\delta = 0\,; \tag{20}$$

where a_i and φ_i (i=1,2,3) are the amplitudes and phases, respectively,

$$\delta = \varphi_2 - (\varphi_1 + \varphi_3)$$

is the phase difference, an over dot denotes the differentiation with respect to T_1,

$$s_i = \mu_i \tau_i^{\gamma} \Omega_i^{\gamma-1} \sin\psi\,, \quad \sigma_i = \mu_i \tau_i^{\gamma} \Omega_i^{\gamma-1} \cos\psi \;\left(i = 1,2,3\right),$$

$$\psi = \frac{1}{2}\pi\gamma\,,$$

and a_{23}^{I}, a_{13}^{II}, a_{12}^{III} are constant coefficients defined by the coupled modes of vibrations [22].

3. NUMERICAL METHOD OF SOLUTION

3.1. Defining the shell parameters that satisfy the condition of the combinational internal resonance $\Omega_2 = \Omega_1 + \Omega_3$

Before proceeding to numerical investigations, let us find the shell parameters which could satisfy the condition of the additive combinational internal resonance

$$\Omega_2 = \Omega_1 + \Omega_3 .$$

Table 1. Part of shell parameters which satisfy the resonance condition $\Omega_2 = \Omega_1 + \Omega_3$.

Ω_1	m_1	n_1	Ω_2	m_2	n_2	Ω_3	m_3	n_3	σ	β_1	β_2
30.4137	5	3	44.412	4	5	13.9983	3	1	0.33	8.37	0.004
27.0251	5	2	43.1784	3	4	16.1531	3	1	0.33	10.23	0.004
19.8875	5	1	44.6532	4	4	24.7656	3	2	0.33	10.42	$5\ 10^{-5}$
18.9932	5	1	48.9931	4	5	29.9997	2	3	0.33	9.30	0.005
17.1999	5	1	33.883	3	5	16.6832	3	2	0.33	6.40	0.002
16.4713	3	3	23.0467	2	5	6.57529	1	1	0.33	4.36	0.004
15.7683	4	1	41.7693	1	5	26.0007	1	3	0.33	8.17	0.005

For this purpose we should use the properties of the symmetric matrix s_{ij}^{mn} (11) possessing three real eigenvalues $\Omega_{i\,mn}$ (i =1,2,3) which are in the correspondence with three mutually orthogonal eigenvectors $L_{i\,mn}$.

We search for values $\Omega_{1m_1n_1}$, $\Omega_{2m_2n_2}$, and $\Omega_{3m_3n_3}$ corresponding to the fixed shell's parameters σ, β_1 and β_2, which could satisfy the additive combinational resonance $\Omega_2 = \Omega_1 + \Omega_3$ (here subindices $m_i n_i$ are omitted for the ease of presentation), resulting in coupling of these particular three modes of vibration. Some results are shown in Table 1, from which it is evident that the situation of such a combinational resonance could be realized rather often in real shells used as parts of different civil engineering structures.

3.2. Numerical solution of general multi-term linear equations

Using the numerical method proposed in [25]-[28], the procedure based on the reduction of the problem to a set of fractional differential equations to estimate numerically the solution of Eqs. (8-10) is as follows:

let

$$Y_1 = X_1, \tag{21}$$
$$Y_2 = D^{\gamma} X_1 = D^{\gamma} Y_1,$$
$$Y_3 = DX_1 = DY_1,$$
$$\ddot{X}_1 = DDX_1 = DY_3.$$

First substitute these equalities in equation (8), resulting in

$$DY_3 = -\sum_{i=1}^{3} F_{1\,mn} L_{i\,mn}^{I} - æ_1 Y_2 - \Omega_1^2 Y_1, \tag{22}$$
$$Y_4 = X_2,$$
$$Y_5 = D^{\gamma} X_2 = D^{\gamma} Y_4,$$
$$Y_6 = DX_2 = DY_4,$$
$$\ddot{X}_2 = DDX_2 = DY_6;$$

then in equation (9), resulting in

$$DY_6 = -\sum_{i=1}^{3} F_{2\,mn} L_{i\,mn}^{II} - æ_2 Y_5 - \Omega_2^2 Y_4, \tag{23}$$
$$Y_7 = X_3,$$
$$Y_8 = D^{\gamma} X_3 = D^{\gamma} Y_7,$$
$$Y_9 = DX_3 = DY_7,$$

$$\ddot{X}_3 = DDX_3 = DY_9,$$

and finally in equation (10), resulting in

$$DY_9 = -\sum_{i=1}^{3} F_{3\,mn} L_{i\,mn}^{II} - æ_3 Y_8 - \Omega_3^2 Y_7. \quad (24)$$

Thus, the governing set of nine equations in nine unknown values Y_i in the matrix form could be written as

$$\begin{bmatrix} D^\gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & D & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & D^\gamma & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & D & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & D & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & D^\gamma & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & D & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & D \end{bmatrix} * \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \\ Y_6 \\ Y_7 \\ Y_8 \\ Y_9 \end{bmatrix} =$$

$$= \begin{bmatrix} Y_2 \\ Y_3 \\ -\sum_{i=1}^{3} F_{1\,mn} L_{i\,mn}^{I} - æ_1 Y_2 - \Omega_1^2 Y_1 \\ Y_5 \\ Y_6 \\ -\sum_{i=1}^{3} F_{2\,mn} L_{i\,mn}^{II} - æ_2 Y_5 - \Omega_2^2 Y_4 \\ Y_8 \\ Y_9 \\ -\sum_{i=1}^{3} F_{3\,mn} L_{i\,mn}^{III} - æ_3 Y_8 - \Omega_3^2 Y_7 \end{bmatrix} \quad (25)$$

Two different types of discretization of derivatives in (25) could be utilized [25-28]. For the first order of differentiation, the trapezoidal rule is usually used:

$$DY = f \xrightarrow{yields} Y_i = Y_{i-1} + \frac{1}{2} h(f_i + f_{i-1}). \quad (26)$$

So in our problem, the discrete derivatives (D) will take the form

$$Y_i - \frac{1}{2} hf_i = Y_{i-1} + \frac{1}{2} h(f_{i-1}). \quad (27)$$

To discretize the fractional derivative, the Diethelm's method could be used [25]:

$$D^\gamma Y = \frac{1}{{}^\gamma\chi_i} \left(\sum_{k=0}^{i} {}^\gamma\omega_{k,i} Y_{i-k} + \frac{Y_0}{\alpha} \right) \quad (28)$$

where

$${}^\gamma\chi_i = (ih)^\alpha \Gamma(-\gamma)$$

and ${}^\gamma\omega_{k,0}, \ldots, {}^\gamma\omega_{k,i}$ [25, 26] are the convolution weights derived from the fact that the fractional operator defined in terms of a convolution integral. We will use the weights of the quadrature formula [25]

$$f(\gamma) = \gamma(1-\gamma)\, j^{-\gamma}\, {}^\gamma\omega_{k,0} =$$
$$= \begin{cases} -1 & \text{for } k = 0 \\ 2k^{1-\gamma} - (k-1)^{1-\gamma} - (k+1)^{1-\gamma} & \text{for } k = 1, 2, ., j-1 \\ (\gamma-1)k^{-\gamma} - (k-1)^{1-\gamma} + k^{1-\gamma} & \text{for } k = j \end{cases} \quad (29)$$

Discretization of the equation

$$D^\gamma Y_1 = Y_2 \quad (30)$$

results in the following relationships (note $\alpha = \gamma$):

$$\frac{1}{{}^\gamma\chi_i} \left(\sum_{k=0}^{i} {}^\gamma\omega_{k,i} Y_{1\,i-k} + \frac{Y_{10}}{\gamma} \right) = Y_2; \quad (31)$$

$$\left({}^\gamma\omega_{0,i} Y_{1\,i} + \sum_{k=1}^{i} {}^\gamma\omega_{k,i} Y_{1\,i-k} + \frac{Y_{10}}{\alpha} \right) = {}^\gamma\chi_i Y_{2\,i}; \quad (32)$$

Let

$$\sum_{k=1}^{i} {}^\gamma\omega_{k,i} Y_{1\,i-k} + \frac{Y_{10}}{\gamma} = s_{1\,i-1},$$

so we have

$$({}^{\gamma}\omega_{0,i} Y_{1i} + s_{1i-1}) = {}^{\gamma}\chi_i Y_{2i}\,, \tag{33}$$

$$s_{1i} = -{}^{\gamma}\omega_{0,i} Y_{1i} + {}^{\gamma}\chi_i Y_{2i}\,. \tag{34}$$

By the trapezoidal rule we can represent

$$Y_3 = DY_1$$

in a discrete form as

$$Y_{1i} = Y_{1i-1} + \frac{h}{2}(Y_{3i} + Y_{3i-1})\,. \tag{35}$$

So rearranging the terms

$$Y_{1i} - \frac{h}{2}Y_{3i} = Y_{1i-1} + \frac{h}{2}Y_{3i-1} = s_{2i-1}\,, \tag{36}$$

and utilizing the trapezoidal rule, we can discretize

$$DY_3 = -\sum_{j=1}^{3} F_{1mn} L^I_{j\,mn} - æ_1 Y_2 - \Omega_1^2 Y_1\,; \tag{37}$$

$$Y_{3i} = Y_{3i-1} + \frac{h}{2}\Big[-\sum_{j=1}^{3}\left(F_{1\,mn\,i} + F_{1\,mn\,i-1}\right)L^I_{j\,mn} - æ_1(Y_{2i} + Y_{2i-1}) - \Omega_1^2(Y_{1i} + Y_{1i-1})\Big]. \tag{38}$$

Rearranging the terms, we have

$$\left(Y_{3i} + \frac{h}{2}(æ_1 Y_{2i} + \Omega_1^2)Y_{1i}\right) = Y_{3i-1} + \frac{h}{2}\left(-\sum_{j=1}^{3}(F_{1\,mn\,i} + F_{1\,mn\,i-1})L^I_{j\,mn} - æ_1 Y_{2i-1} - \Omega_1^2 Y_{1i-1}\right) = s_{3i-1} \tag{39}$$

Repeating these steps (as we have done in Eqs. (30)-(39)) for all other values (Y_4-Y_9), and arranging them in a matrix form, we obtain (40). Then it is quite straightforward to solve (40) (Figure 1).

$$\begin{bmatrix} -{}^{\gamma}\omega_{0,i} & {}^{\gamma}\chi_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -\frac{h}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{h}{2}\Omega_1^2 & \frac{h}{2}æ_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -{}^{\gamma}\omega_{0,i} & {}^{\gamma}\chi_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -\frac{h}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{h}{2}\Omega_2^2 & \frac{h}{2}æ_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -{}^{\gamma}\omega_{0,i} & {}^{\gamma}\chi_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\frac{h}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{h}{2}\Omega_3^2 & \frac{h}{2}æ_3 & 1 \end{bmatrix} * \begin{bmatrix} Y_{1i} \\ Y_{2i} \\ Y_{3i} \\ Y_{4i} \\ Y_{5i} \\ Y_{6i} \\ Y_{7i} \\ Y_{8i} \\ Y_{9i} \end{bmatrix} = \begin{bmatrix} s_{1i-1} \\ s_{2i-1} \\ s_{3i-1} \\ s_{4i-1} \\ s_{5i-1} \\ s_{6i-1} \\ s_{7i-1} \\ s_{8i-1} \\ s_{9i-1} \end{bmatrix} \tag{40}$$

Figure 1. Formula (40).

3.3. Numerical solution of the governing equations for the combinational additive internal resonance using the Runge-Kutta fourth order method

To utilize the Runge-Kutta fourth order method to estimate numerically the solution of equations (15)-(20), we first rewrite these equations as follows:

$$\dot{a}_1 = \frac{1}{2}(-\Omega_1^{-1} a_{23}^{I}\, a_2 a_3 \sin\delta - s_1 a_1)\,; \qquad (41)$$

$$\dot{a}_2 = \frac{1}{2}(\Omega_2^{-1} a_{13}^{II}\, a_1 a_3 \sin\delta - s_2 a_2)\,; \qquad (42)$$

$$\dot{a}_3 = \frac{1}{2}(-\Omega_3^{-1} a_{12}^{III}\, a_1 a_2 \sin\delta - s_3 a_3)\,; \qquad (43)$$

$$\dot{\varphi}_1 = \frac{1}{2}\sigma_1 + \frac{1}{2}\frac{a_{23}^{I}}{\Omega_1}\frac{a_2 a_3}{a_1}\cos\delta\,; \qquad (44)$$

$$\dot{\varphi}_2 = \frac{1}{2}\sigma_2 + \frac{1}{2}\frac{a_{13}^{II}}{\Omega_2}\frac{a_1 a_3}{a_2}\cos\delta\,; \qquad (45)$$

$$\dot{\varphi}_3 = \frac{1}{2}\sigma_3 + \frac{1}{2}\frac{a_{12}^{III}}{\Omega_3}\frac{a_1 a_2}{a_3}\cos\delta\,. \qquad (46)$$

4. NUMERICAL RESULTS

4.1. Method 1: multi-step fractional differential equations.

The numerical solution using the multi step method of equation (40) has been carried out at the dimensionless parameters presented in Table 1 (for the case presented in the first line), and the results are presented in Fig. 1 for different magnitudes of the fractional parameter.

4.2. Method 2: the analysis of the amplitudes and phases using multiple time scales

Variation of the fractional parameter γ from 0 to 1 allows one to investigate vibrations of cylindrical shells in surrounding media with different viscous properties, including the pure elastic case at $\gamma = 0$ and conventional Kelvin-Voigt model at $\gamma \to 1$.

The dynamic behavior of a cylindrical shell in a viscous medium at γ = 0.02, 025, 0.5, and 0.98, which is found by using the first and second methods, is shown in Figures 2 and 3, respectively, for the parameters taken from Table 1, which correspond to the combinational internal resonance at $\Omega_2 = \Omega_1 + \Omega_3$.

The behavior of amplitudes of vibrations reveals the exchange of energy between the generalized displacements of the system under the considered case of the combinational internal resonance.

CONCLUSION

Free damped vibrations of a shallow nonlinear thin cylindrical shell in a fractional derivative viscoelastic medium are investigated numerically by two different methods based on the new approach proposed in [20-23].

The numerical solutions of the damped vibrations of the nonlinear cylindrical shell subjected to the conditions of the internal resonance have been estimated, and good agreement between the two methods has been achieved. Within the first method, the generalized displacements of a coupled set of nonlinear ordinary differential equations of the second order are calculated using the numerical solution of nonlinear multi-term fractional differential equations by the procedure based on the reducing the problem to a system of fractional differential equations. According to the second method, the amplitudes and phases of nonlinear vibrations are estimated from the governing nonlinear differential equations describing amplitude-and-phase modulations for the case of the combinational internal resonance.

It has been shown that, as in [22], the phenomenon of the internal resonance could be very critical, since in a circular cylindrical shell the internal additive and difference combinational resonances are always present. The effect of viscosity on the energy exchange mechanism is analyzed.

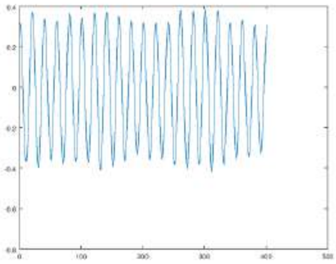
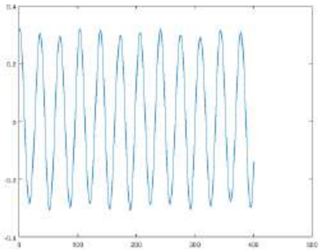
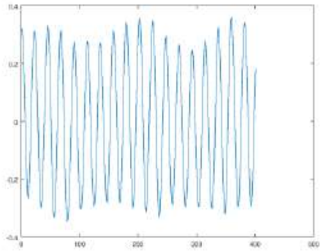
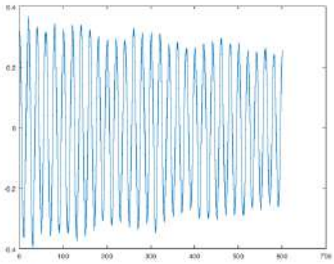
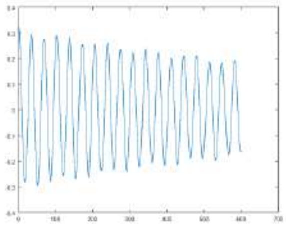
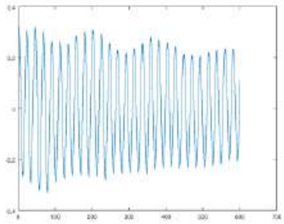
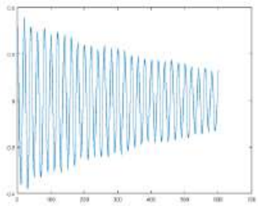
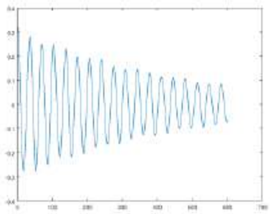
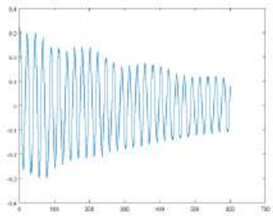
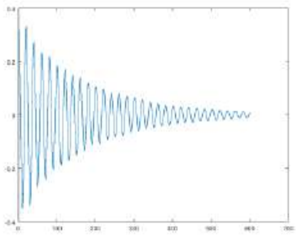
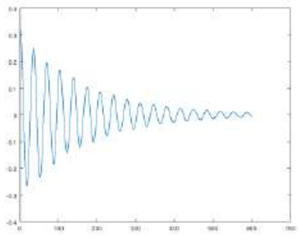
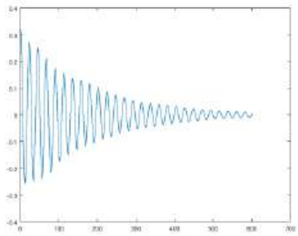
γ =0.02		
X_1	X_2	X_3
		
γ =0.25		
X_1	X_2	X_3
		
γ =0.5		
X_1	X_2	X_3
		
γ =0.98 ≈ 1		
X_1	X_2	X_3
		

Figure 2. The time-dependence of the generalized displacements at different magnitudes of the fractional parameter.

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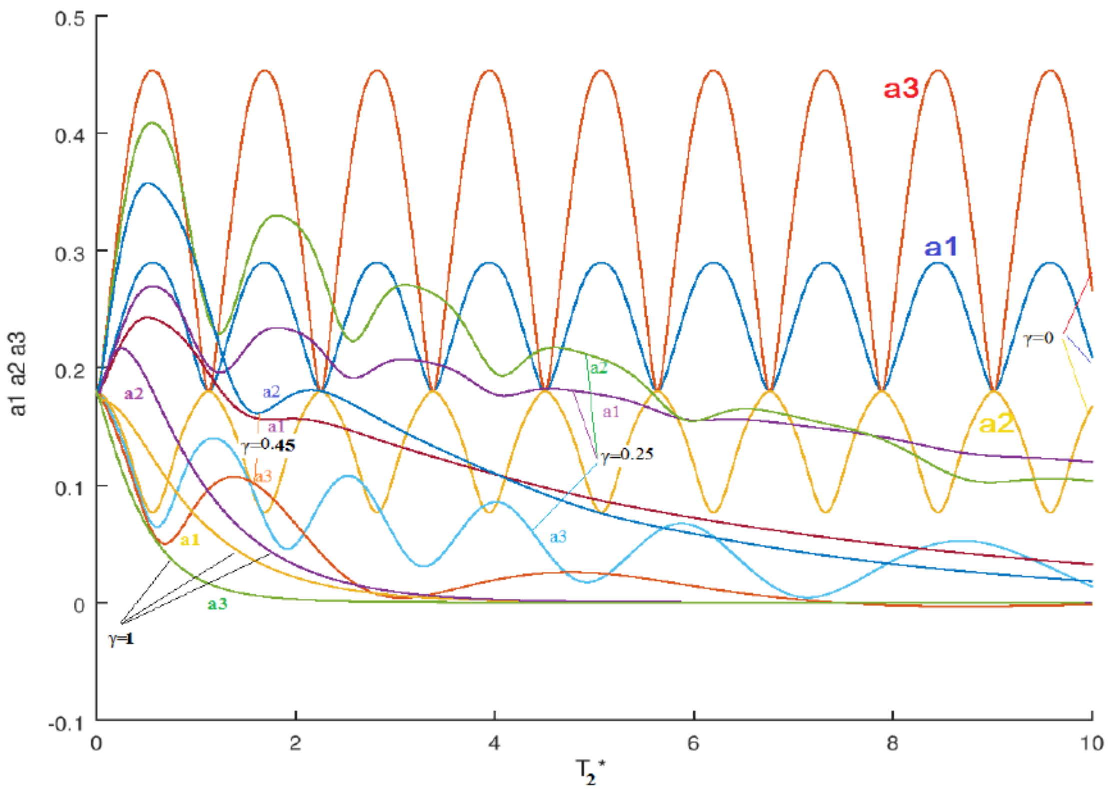

Figure 3. Dimensionless amplitude vs. dimensionless time as the solution of equations (41)-(46).

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ABOUT SEVERAL NUMERICAL AND SEMIANALYTICAL METHODS OF LOCAL STRUCTURAL ANALYSIS

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Abstract: Numerical or semianalytical solution of problems of structural mechanics with immense number of unknowns is time-consuming process. High-accuracy solution at all points of the model is not required normally, it is necessary to find only the most accurate solution in some pre-known domains. The choice of these domains is a priori data with respect to the structure being modelled. Designers usually choose domains with the so-called edge effect (with the risk of significant stresses that could lead to destruction of structures) and regions which are subject to specific operational requirements. Stress-strain state in such domains is important. Wavelets provide effective and popular tool for local structural analysis. Operational and variational formulations of problems of structural mechanics with the use of method of extended domain are presented. After discretization and obtaining of governing equations, problems are transformed to a multilevel space by multilevel wavelet transform. Discrete wavelet basis is used and corresponding direct and inverse algorithms of transformations are performed. Due to special algorithms of averaging, reduction of the problems is provided. Wavelet-based methods allows reducing the size of the problems and obtaining accurate results in selected domains simultaneously. These are rather efficient methods for evaluation of local phenomenon in structures.

Keywords: numerical methods, semianalytical methods, local structural analysis, structural mechanics, wavelet-based methods, reduction, operational formulations, variational formulation, boundary problem

О НЕКОТОРЫХ ЧИСЛЕННЫХ И ПОЛУАНАЛИТИЧЕСКИХ МЕТОДАХ ЛОКАЛЬНОГО РАСЧЕТА СТРОИТЕЛЬНЫХ КОНСТРУКЦИЙ

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Аннотация: Численное и полуаналитическое (численно-аналитическое) решение краевых задач строительной механики, нередко характеризующих огромным количеством неизвестных, сопряжено с большим объемом вычислительной работы и значительными временными затратами. Как правило, отсутствует необходимость в обеспечении высокой точности решения во всех точках соответствующей вычислительной модели, зачастую имеется потребность в нахождении высокоточного решения в некотором наборе областей (зон) конструкции, расположение которых, как правило, заранее известно (это своего рода априорная оценка). Расчетчики в этом отношении традиционно выбирают зоны так называемого краевого эффекта (потенциально опасные с точки зрения уровня возникающих напряжений, способных повлечь разрушение конструкций), а также зоны, внимание к которым обусловлено, например, разного рода технологической спецификой и т.д. Для указанных областей важнейшим вопросом является достоверное определение напряженно-деформированного состояния. Вейвлет-анализ является исключительно эффективным инструментарием для построения локальных решений соответствующих краевых задач строительной механики. Постановка последних в статье приводится в операторном и вариационном видах на основе использования метода расширенной (стандартной) области. После введения соответствующей аппроксимации реализуется переход от указанных континуальных постановок к дискретным и дискретно-континуальным. Далее реализуется прямое вейвлет-преобразование с привлечением дискретного вейвлет-базиса (заметим, что предложены соответствующие эффективные алгоритмы прямого и обратного вейвлет-преобразования). Разработанные вейвлет-версии методов локального расчета строительных конструкций позволяют с одной стороны значительно сократить вычислительную размерность решаемых задач, а с другой стороны обеспечить высокую точность получаемых результатов в выбранных областях (зонах) строительных конструкций.

Ключевые слова: численные методы, полуаналитические методы, локальный расчет строительных конструкций, локальные решения краевых задач, строительная механика, вейвлет-реализации, редукция, операторные постановки задач, вариационные постановки задач, краевая задача

1. BASIC FORMULAS OF FAST DIRECT AND INVERSE DISCRETE HAAR TRANSFORMS AND AVERAGING

1.1. One-dimensional problems

1.1.1. Algorithms of fast direct and inverse discrete Haar transforms. Let us consider the one-dimensional region

$$\omega = \{ x : \; a \le x \le b \} ,$$

where x is coordinate, a, b are lower and upper limits of interval. Let us divide ω into $(n-1)$ equal parts, where $n = 2^M$, M is the number of levels in the Haar basis [1-5]. Coordinates of mesh nodes are

$$x_i = a + (i-1)h, \; i = 1, 2, ..., n; \quad h = (b-a)/(n-1) . \quad (1)$$

Haar mesh functions are defined by formulas (N_p is the number of Haar functions at level p):

$$\psi_j^p(i) = \\ = \alpha_p^{-1} \begin{cases} 1, & 2^{p+1}(j-1) < i \le 2^p(2j-1) \\ -1, & 2^p(2j-1) < i \le 2^{p+1} j \\ 0, & i \le 2^{p+1}(j-1) \;\cup\; i > 2^{p+1} j, \end{cases} \\ i = 1, 2, ..., n, \;\; 0 \le p < M; \\ \psi_1^M(i) = \alpha_M^{-1}, \;\; i = 1, 2, ..., n; \quad (2)$$

$$N_p = \begin{cases} n/2^{p+1}, & 0 \le p < M \\ 1, & p = M; \end{cases}$$

$$\alpha_p = \begin{cases} \sqrt{2^{p+1}}, & 0 \le p < M \\ \sqrt{2^M} = \sqrt{n}, & p = M. \end{cases} \quad (3)$$

Let $f(i)$ be arbitrary mesh function. Then we have

$$f(i)=\sum_{p=0}^{M}\sum_{j=1}^{N_p} v_j^p \psi_j^p(i),$$
$$v_j^p=\sum_{i=1}^{n} f(i)\psi_j^p(i),\ \ j=1,2,\dots,N_p, \quad p=0,1,\dots,M, \tag{4}$$

where v_j^p, $j=1,2,\dots,N_p$, $p=0,1,\dots,M$ are Haar expansion coefficients.

Algorithm of fast direct discrete Haar transform is described below.

$$u_j^0=f(j),\ \ j=1,2,\dots,n;\quad \alpha_0=\sqrt{2}. \tag{5}$$

We have (for all $p=0,1,\dots,M-1$, $j=1,2,\dots,N_p$):

$$v_j^p=\alpha_p^{-1}(u_{2j-1}^p-u_{2j}^p);$$
$$u_j^{p+1}=u_{2j-1}^p+u_{2j}^p;\ \ \alpha_{p+1}=\sqrt{2}\alpha_p; \tag{6}$$
$$\alpha_M=\sqrt{n};\quad v_1^M=\alpha_M^{-1}u_1^M, \tag{7}$$

where u_j^p, $j=1,2,\dots,N_p$, $p=0,1,\dots,M$ are auxiliary quantities.

Algorithm of fast inverse Haar transform is described below.

$$\alpha_M=\sqrt{n};\quad \alpha_{M-1}=\sqrt{n};\quad u_1^M=\alpha_M^{-1}v_1^M. \tag{8}$$

We have ($p=M-1,M-2,\dots,0$, $i=1,2,\dots,N_p$):

$$j=[(i+1)/2];\quad z=(-1)^{i+1};$$
$$u_j^p=\alpha_p^{-1}zv_j^p+u_j^{p+1};\quad \alpha_{p-1}=\alpha_p/\sqrt{2}. \tag{9}$$

Thus,

$$f(i)=u_i^0,\ \ i=1,2,\dots,n. \tag{10}$$

1.1.2. Algorithm of averaging. In many cases it is not necessary to obtain global solution in the domain. Local solution for several prescribed subdomains is normally required. If we don’t need to find a complete solution we can reduce the number of unknowns without significant loss of accuracy or with a small error in local solutions. It is reasonable to eliminate unknown expansion coefficients of the basis functions with supports substantially distant from the considering area. Algorithm of averaging in one-dimensional case is described below.

Let us assume that it is necessary to make averaging at some level number q. For all $p=0,1,\dots,q$ and $j=1,2,\dots,N_p$ we suppose

$$(Du^p)_{2j-1}\approx(Du^p)_{2j}\approx(D\tilde{u}^p)_{2j-1},$$
$$v_{2j-1}^p=v_{2j}^p,\quad j=1,2,\dots,N_{p+1}, \tag{11}$$
$$\tilde{u}_{2j-1}^p=(u_{2j-1}^p+u_{2j}^p)/2;$$
$$(D\tilde{u}^p)_{2j-1}=(\tilde{u}_{2j}^p-\tilde{u}_{2j-1}^p)/(2^{p+1}h); \tag{12}$$

Then formulas of averaging have the form

$$v_{2j-1}^p=v_{2j}^p=\beta v_j^{p+1},\quad j=1,2,\dots,N_{p+1};$$
$$\beta=1/(2\sqrt{2}). \tag{13}$$

1.2. Two-dimensional problems.

1.2.1. Algorithms of fast direct and inverse discrete Haar transforms. Let us consider the two-dimensional rectangular domain

$$\omega=\{(x_1,x_2):\ 0\le x_1\le l_1,\ 0\le x_2\le l_2\},$$

where x_1, x_2 are coordinates; l_1, l_2 are dimensions along x_1, x_2. Let us divide ω into $(n-1)$ equal parts along x_1 and into $(n-1)$ equal parts along x_2, where $n=2^M$, M is the number of levels in the Haar basis. We have the following formulas for coordinates of mesh nodes:

$$x_{1,i}=(i_1-1)h_1,\ \ i_1=1,2,\dots,n;$$
$$x_{2,i}=(i_2-1)h_2,\ \ i_2=1,2,\dots,n,$$
$$h_1=l_1/(n-1);\quad h_2=l_2/(n-1). \tag{14}$$

Haar mesh functions

$$\psi_{s_1,s_2,j_1,j_2}^p(i_1,i_2),\ \ p=1,2,\dots,M,$$
$$j_1,\ j_2=1,2,\dots,N_p,\ \ s_1,s_2=0,1$$

(except $s_1 = s_2 = 0$) can be defined by formulas:

$$\psi^p_{s_1,s_2,j_1,j_2}(i_1,i_2) = \\ = \alpha_p^{-1}\begin{cases} (-1)^{k_1 s_1 + k_2 s_2}, \\ \bigcap_{q=1}^{2}\bigcup_{k_q=0}^{1}\left(\begin{array}{l} 2^{p+1}(j_q - 1 + \frac{k_q}{2}) < i_q \wedge \\ i_q \le 2^{p+1}(j_q - \frac{1}{2} + \frac{k_q}{2}) \end{array} \right), \\ 0, \quad \text{in other cases;} \end{cases}$$

$$\psi^M_{0,0,1,1}(i_1,i_2) = \alpha_M^{-1}; \quad (15)$$

$$N_p = \begin{cases} n/2^{p+1}, 0 \le p < M \\ 1, \ p = M; \end{cases}$$

$$\alpha_p = \begin{cases} 2^{p+1}, \ 0 \le p < M \\ 2^M = n, \ p = M. \end{cases} \quad (16)$$

Let $f(i_1,i_2)$ be an arbitrary mesh function. Consequently we have

$$f(i_1,i_2) = v^M_{0,0,1,1}\psi^M_{0,0,1,1} + \\ + \sum_{p=0}^{M-1}\sum_{j_1=1}^{N_p}\sum_{j_2=1}^{N_p}(v^p_{1,0,j_1,j_2}\psi^p_{1,0,j_1,j_2}(i_1,i_2) + \\ + v^p_{0,1,j_1,j_2}\psi^p_{0,1,j_1,j_2}(i_1,i_2) + v^p_{1,1,j_1,j_2}\psi^p_{1,1,j_1,j_2}(i_1,i_2)), \quad (17)$$

where $v^p_{1,0,j_1,j_2}, v^p_{0,1,j_1,j_2}, v^p_{1,1,j_1,j_2}$, $j_1, j_2 = 1,2,..., N_p$, $p = 1, 2, ..., M$ are Haar expansion coefficients,

$$v^p_{s_1,s_2,j_1,j_2} = \sum_{i_1=1}^{N}\sum_{i_2=1}^{N} f(i_1,i_2)\psi^p_{s_1,s_2,j_1,j_2}(i_1,i_2). \quad (18)$$

Algorithm of fast direct discrete Haar transform is described below.

$$u^0_{j_1,j_2} = f(j_1,j_2), \ j_1 = 1,2,...,n, \ j_2 = 1,2,...,n; \\ \alpha_0 = 2. \quad (19)$$

We have (for all $p = 0,1,..., M-1$, $j_1, j_2 = 0,1,..., N_p$, $s_1, s_2 = 0,1$ (except $s_1 = s_2 = 0$)):

$$z_1 = (-1)^{s_1}, \quad z_2 = (-1)^{s_2}; \quad \alpha_{p+1} = 2\cdot\alpha_p; \quad (20)$$

$$v^p_{s_1,s_2,j_1,j_2} = \alpha_p^{-1}(u^p_{2j_1-1,2j_2-1} + z_1 u^p_{2j_1,2j_2-1} + \\ + z_2 u^p_{2j_1-1,2j_2} + z_1 z_2 u^p_{2j_1,2j_2}); \quad (21)$$

$$u^{p+1}_{j_1,j_2} = u^p_{2j_1-1,2j_2-1} + u^p_{2j_1,2j_2-1} + \\ + u^p_{2j_1-1,2j_2} + u^p_{2j_1,2j_2}; \quad (22)$$

$$\alpha_M = n; \quad v^M_{0,0,1,1} = \alpha_M^{-1}u^M_{1,1}, \quad (23)$$

where $u^p_{j_1,j_2}$, $j_1, j_2 = 1, 2, ..., N_p$, $p = 1, 2, ..., M$ are auxiliary quantities.

Algorithm of fast inverse Haar transform is described below.

$$\alpha_M = n; \quad \alpha_{M-1} = n; \quad u^M_{1,1} = \alpha_M^{-1}v^M_{0,0,1,1}. \quad (24)$$

We have ($p = M-1, M-2, ..., 0, i_1, i_2 = 1, 2, ..., N_p$):

$$j_1 = [(i_1+1)/2]; \quad j_2 = [(i_2+1)/2]; \\ z_1 = (-1)^{i_1+1}; \quad z_2 = (-1)^{i_2+1}; \quad \alpha_{p-1} = \alpha_p/2; \quad (25)$$

$$u^p_{i_1,i_2} = \\ = \alpha_p^{-1}(z_1 v^p_{1,0,j_1,j_2} + z_2 v^p_{0,1,j_1,j_2} + z_1 z_2 v^p_{1,1,j_1,j_2}) + \\ + u^{p+1}_{j_1,j_2}. \quad (26)$$

Thus,

$$f(i_1,i_2) = u^0_{i_1,i_2}, \quad i_1 = 0,1,...,n, \quad i_2 = 0,1,...,n. \quad (27)$$

1.2.2. Algorithm of averaging. Let us assume that it is necessary to make averaging at level q. For all $p = 1, 2, ..., q$, $j_1, j_2 = 1, 2, ..., N_p$, $s_1, s_2 = 0,1$ (except $s_1 = s_2 = 0$) we suppose

$$(D_1 u^p)_{2j_1-1,2j_2-1} = (D_1 u^p)_{2j_1-1,2j_2} = \\ = (D_1 u^p)_{2j_1,2j_2-1} = (D_1 u^p)_{2j_1,2j_2} \approx \\ \approx (D_1 \tilde{u}^p)_{2j_1-1,2j_2-1}; \quad (28)$$

$$(D_2 u^p)_{2j_1-1,2j_2-1} = (D_2 u^p)_{2j_1-1,2j_2} = \\ = (D_2 u^p)_{2j_1,2j_2-1} = (D_2 u^p)_{2j_1,2j_2} \approx \\ \approx (D_2 \tilde{u}^p)_{2j_1-1,2j_2-1}; \quad (29)$$

$$(D_2^+ D_1^+ u^p)_{2j_1-1,2j_2-1} = (D_2^+ D_1^+ u^p)_{2j_1-1,2j_2} = \\ = (D_2^+ D_1^+ u^p)_{2j_1,2j_2-1} = (D_2^+ D_1^+ u^p)_{2j_1,2j_2} \approx \\ \approx (D_2^+ D_1^+ \tilde{u}^p)_{2j_1-1,2j_2-1}; \quad (30)$$

$$v^p_{s_1,s_2,2j_1-1,2j_2-1} = v^p_{s_1,s_2,2j_1,2j_2-1} = \\ = v^p_{s_1,s_2,2j_1-1,2j_2} = v^p_{s_1,s_2,2j_1,2j_2}; \quad (31)$$

$$\tilde{u}^p_{j_1,j_2} = (u^p_{j_1,j_2} + u^p_{j_1+1,j_2} + u^p_{j_1,j_2+1} + u^p_{j_1+1,j_2+1})/4; \quad (32)$$

$$(D_1^+ u^p)_{j_1,j_2} = (u^p_{j_1+1,j_2} - u^p_{j_1,j_2})/(2^p h); \\ (D_2^+ u^p)_{j_1,j_2} = (u^p_{j_1,j_2+1} - u^p_{j_1,j_2})/(2^p h); \quad (33)$$

$$(T_1^+ u^p)_{j_1,j_2} = u^p_{j_1+1,j_2} + u^p_{j_1,j_2}; \\ (T_2^+ u^p)_{j_1,j_2} = u^p_{j_1,j_2+1} + u^p_{j_1,j_2}; \quad (34)$$

$$D_1 = 0.5 \cdot T_2^+ D_1^+; \quad D_2 = 0.5 \cdot T_1^+ D_2^+. \quad (35)$$

Final formulas of averaging have the form

$$v^p_{1,0,2j_1-1,2j_2-1} = v^p_{1,0,2j_1,2j_2-1} = v^p_{1,0,2j_1-1,2j_2} = \\ = v^p_{1,0,2j_1,2j_2} = \beta_{1,0} v^{p+1}_{1,0,j_1,j_2}, \ j_1, j_2 = 1, 2, ..., N_{p+1}; \quad (36)$$

$$v^p_{0,1,2j_1-1,2j_2-1} = v^p_{0,1,2j_1,2j_2-1} = v^p_{0,1,2j_1-1,2j_2} = \\ = v^p_{0,1,2j_1,2j_2} = \beta_{0,1} v^{p+1}_{0,1,j_1,j_2}, \ j_1, j_2 = 1, 2, ..., N_{p+1}; \quad (37)$$

$$v^p_{1,1,2j_1-1,2j_2-1} = v^p_{1,1,2j_1,2j_2-1} = v^p_{1,1,2j_1-1,2j_2} = \\ = v^p_{1,1,2j_1,2j_2} = \beta_{1,1} v^{p+1}_{1,1,j_1,j_2}, \ j_1, j_2 = 1, 2, ..., N_{p+1}; \quad (38)$$

$$\beta_{1,0} = 0.25; \quad \beta_{0,1} = 0.25; \quad \beta_{1,1} = 0.125. \quad (39)$$

1.3. Three-dimensional problems.

This most cumbersome case is described in [6].

2. MULTILEVEL WAVELET-BASED NUMERICAL METHOD OF LOCAL STRUCTURAL ANALYSIS

2.1. Formulation of the problem

Effective qualitative multilevel analysis of local and global stress-strain states of the structure is normally required in various technical problems. As is known, defects and failures are mostly local in nature. However total load-carrying ability of the structure, associated with the condition of limit equilibrium, is determined by the global behavior of the considering project. Therefore corresponding multilevel approach is peculiarly relevant and apparently preferable in all aspects for qualitative and quantitative analysis of calculation data. Wavelet analysis provides effective and popular tool for such researches. After expansion of the solution with the use of local wavelet basis corresponding components are considered at each level of the basis.

In accordance with the method of extended domain [7], the domain Ω, occupied by considering structure, is embordered by extended one ω of arbitrary shape, particularly elementary. Operational formulation of the problem in domain ω normally has the form

$$Lu = F, \quad (40)$$

where L is the operator of boundary problem, which takes into account the boundary conditions; u is the unknown function; F is the given right-side function.

Directly from operational formulation we have variational formulation of the problem:

$$\Phi(u) = 0.5 \cdot (Lu, u) - (F, u), \quad (41)$$

Solution of (41) is the critical point of (40). (f, g) denotes dot product of functions f and g.

Discrete formulation of the problem has the form:

$$A\bar{u} = \bar{f}, \quad (42)$$

where $A = \{a_{i,j}\}_{i,\,j=1,2,...,n_{gl}}$ is the difference approximation of operator L; $\bar{u} = [\,u_1 \ \ u_2 \ \ ... \ \ u_{n_{gl}}\,]^T$ is the unknown mesh function; $\bar{f} = [\,f_1 \ \ f_2 \ \ ... \ \ f_{n_{gl}}\,]^T$ is the given right-side mesh function; n_{gl} is dimension of problem.

Various methods can be used to form the matrix of the discrete operator. We recommend method of basis (local) variations. Its major peculiarities include universality and computer orientation. We can use the following formulas for linear problems:

$$a_{i,j} = \Phi(\bar{e}^{(i)} + \bar{e}^{(j)}) - \Phi(\bar{e}^{(i)}) - \Phi(\bar{e}^{(j)}) + \Phi(\bar{0})];$$

$$f_i = 0.5 \cdot [\Phi(\bar{e}^{(i)}) - \Phi(-\bar{e}^{(i)})], \qquad (43)$$

$$\begin{aligned} &\bar{e}^{(i)} = [\, e_1^{(i)} \;\; e_2^{(i)} \;\; \ldots \;\; e_{n_{gl}}^{(i)} \,]^T, \;\; i = 1, 2, \ldots, n_{gl}; \\ &e_j^{(i)} = \delta_{i,j}, \;\; j = 1, 2, \ldots n_{gl}; \end{aligned} \qquad (44)$$

$\bar{e}^{(i)}$, $i = 1, 2, \ldots, n_{gl}$ are basis mesh vectors; $\bar{0}$ is the null function; $\delta_{i,j}$ is the Kronecker delta.

2.2. Haar-based formulation of the problem

Let us consider Haar-based formulation of the problem:

$$\begin{aligned} \Phi(\bar{u}) &= 0.5 \cdot (A\bar{u}, \bar{u}) - (f, \bar{u}) = \\ &= 0.5 \cdot (LQ\bar{v}, Q\bar{v}) - (\bar{f}, Q\bar{v}) = \\ &= 0.5 \cdot (Q^* LQ\bar{v}, \bar{v}) - (Q^* \bar{f}, \bar{v}), \end{aligned} \qquad (45)$$

where Q is transition matrix consisting from Haar basis vectors, located in rows. Thus,

$$\tilde{\Phi}(\bar{v}) = 0.5 \cdot (Q^* LQ\bar{v}, \bar{v}) - (Q^* \bar{f}, \bar{v}), \qquad (46)$$

where $\bar{v}$ is vector of Haar expansion coefficients of the vector $\bar{u}$. Corresponding operational formulation of the problem has the form

$$\tilde{L}\bar{v} = \tilde{\bar{f}}, \quad \tilde{L} = Q^* LQ; \quad \tilde{f} = Q^* \bar{f}. \qquad (47)$$

Further reduction of the problem is based on the averaging algorithm specified above.

3. MULTILEVEL WAVELET-BASED SEMIANALYTICAL METHOD OF LOCAL STRUCTURAL ANALYSIS

The objects of the multilevel wavelet-based semianalytical (discrete-continual) method are structures with piecewise constancy of physical and geometrical parameters in one dimension (it is so-called "basic direction"). Special discrete-continual design model is introduced. It presupposes wavelet approximation of extended domain along non-basic directions, while along the basic direction problem remains continual. Analytical solution is apparently preferable in all aspects for qualitative analysis of calculation data. It allows investigator to consider boundary effects when some components of solution are rapidly varying functions. Due to the abrupt decrease inside of mesh elements in many cases their rate of change can't be adequately considered by conventional numerical methods while analytics enables study. Another feature of the proposing method is the absence of limitations on lengths of structures. Semianalytical formulation are contemporary mathematical models which currently becoming available for computer realization. Resultant multipoint boundary problem after reduction has the form [8-10]

$$\begin{cases} \bar{y}' = A_k \bar{y} + \bar{f}_k, \quad x \in (x_k^b, x_{k+1}^b), \\ \qquad\qquad k = 1, \ldots, n_k - 1 \\ B_k^- \bar{y}(x_k^b - 0) + B_k^+ \bar{y}(x_k^b + 0) = \bar{g}_k^- + \bar{g}_k^+, \\ \qquad\qquad k = 2, \ldots, n_k - 1 \\ B_1^+ \bar{y}(x_1^b + 0) + B_{n_k}^- \bar{y}(x_{n_k}^b - 0) = \bar{g}_1^+ + \bar{g}_{n_k}^-, \end{cases} \qquad (48)$$

where $x_k^b = x_{3,k}^b$, $k = 1, \ldots, n_k$ are coordinates of boundary points; A_k, $k = 1, 2, \ldots, n_k - 1$ are matrices of constant coefficients of order n; B_k^-, B_k^+, $k = 2, \ldots, n_k - 1$ and $B_1^+, B_{n_k}^-$ are matrices of boundary conditions of order n at point x_k^b; $\bar{g}_k^-, \bar{g}_k^+$, $k = 2, \ldots, n_k - 1$ and $\bar{g}_1^+, \bar{g}_{n_k}^-$ are right-side vectors of boundary conditions at point x_k^b; $\bar{y} = \bar{y}(x) = [\, y_1(x) \;\; y_2(x) \;\; \ldots \;\; y_n(x) \,]^T$ is the unknown vector function;

$$\bar{y}^{(1)} = \bar{y}^{(1)}(x) = d\bar{y}/dx;$$

$$\bar{f}_k = \bar{f}_k(x) = [\, f_{k,1}(x) \;\; f_{k,2}(x) \ldots f_{k,n}(x) \,]^T, \quad k = 1, 2, \ldots, n_k - 1$$

are right-side vector functions.

Solution of considering multipoint boundary problem of structural analysis is accentuated by numerous factors. They include boundary effects (stiff systems) and considerable number of differential equations (several thousands). Matrices of coefficients of a system normally have eigenvalues of opposite signs and corresponding Jordan matrices are not diagonal. Method of solution of multipoint boundary problems for systems of ordinary differential equations with piecewise constant coefficients in structural analysis has been developed. Not only does it overcome all difficulties, but its peculiarities also include universality, computer-oriented algorithm, computational stability, optimal conditionality of resultant systems and partial Jordan decomposition of matrix of coefficient, eliminating necessity of calculation of root vectors.

CONCLUSION

Currently, high-tech work is underway to integrate the developed numerical and semianalytical methods and corresponding algorithms of local structural analysis into the STADYO software package [19,20].

It should be noted that STADYO is the universal software package, which rovides temperature fields, static, stability and dynamic analysis (including response spectra and accelerations definition) as well as fracture mechanics and strength analysis and optimization of arbitrary combined 2-D and 3-D solid, shell, plate and beam mechanical systems by the finite elements, superelement and other modern numerical methods:

- STADYO-FIELD – stationary field (thermoconduction, filtration, fluid flow, etc) problems;
- STADYO-STAT – linear-elastic static stress-strain analysis;
- STADYO-EIG – solving the eigenvalue problems (natural frequencies and modes, loads and forms of buckling);
- STADYO-SEISM – “normative” spectral analysis of seismic responce under exitations, defined by acceleration spectra;
- STADYO-VIBR – evaluation of system stationary vibration parameters;
- STADYO-SPEC – linear spectral (modern superposition) dynamic analysis;
- STADYO-DYN – direct step-by-step integration of dynamic equations;
- STADYO-NFIELD – solving the non-stationary field problems;
- STADYO-FRAC – solving the linear problems of fracture mechanics, including intensivity ratio coefficients and J-integral definitions;
- STADYO-NLIN – solving the nonlinear static and dynamic problems of motion equations (large displacement, plasticity and viscoplasticity of metals, concrete and ground, opening cracks and joints etc.);
- STADYO-WIND – object-oriented code for 3D static and dynamic analysis of typical wind units;
- STADYO-ASTRA – object-oriented cod for 3D static analysis of typical pipe elements (elbows, tees, weld connections, etc);
- STADYO-INTER – object-oriented code for 3D static and dynamic analysis of combined “soilstructure” systems.

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LIRA-SAPR PROGRAM FOR GENERATING DESIGN MODELS OF RECONSTRUCTED BUILDINGS

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Abstract: The paper deals with technique of simulation for buildings at maintenance stage with account of changes in structural model during reconstruction. The authors suggest algorithm for linear and nonlinear analysis of structures in LIRA-SAPR program with account of erection process. Generation of design models for reconstructed buildings are illustrated with real examples from design practice (reconstruction of 3-storey office building with overstorey; reconstruction of 5-storey hostel with built-in nonresidential premises when floor slabs are changed; reconstruction of building with account of defects that were detected and strengthening that was made; reconstruction of 9-storey residential building where gas was exploded, with account of defects that were detected and strengthening that was made).

Keywords: reconstruction, stress-strain state, strengthening, design model, erection, assemblage, disassemblage

ИСПОЛЬЗОВАНИЕ ПРОГРАММНОГО КОМПЛЕКСА ЛИРА-САПР ПРИ СОСТАВЛЕНИИ РАСЧЕТНЫХ СХЕМ РЕКОНСТРУИРУЕМЫХ ЗДАНИЙ

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Аннотация: Рассматривается методика моделирования зданий в эксплуатационной стадии с учетом изменяющейся конструктивной схемы в процессе реконструкции. Предлагается алгоритм расчета конструкций в линейной и нелинейной постановке с учетом процесса возведения в программном комплексе ЛИРА-САПР. Создание расчетных схем реконструируемых зданий проиллюстрированы на примерах реальных задач из проектной практики (реконструкция 3-х этажного административного здания с надстройкой этажа; 5-ти этажного здания общежития со встроенными нежилыми помещениями при замене плит перекрытия; реконструкция здания с учетом выявленных дефектов и выполненного усиления; реконструкция жилого девятиэтажного здания, в котором произошел взрыв газа, с учетом выявленных дефектов и выполненного усиления).

Ключевые слова: реконструкция, напряженно-деформированное состояние, усиление, расчетная модель, возведение, монтаж, демонтаж

INTRODUCTION

Reconstruction is significantly different from new construction and has its own peculiarities in the design. Reconstruction, redevelopment and modernization of buildings and structures require strengthening of bearing elements in order to keep the ability of buildings and structures as a whole to bear additional loads.

It should be noted that in recent years the number of emergency cases during reconstruction has significantly increased [1]. One of the main reasons of accidents is the following: there is no unified and 'adequate' method for computing stress-strain state of the structure when the design model is modified as a result of assemblage/disassemblage of the load-bearing elements in building during reconstruction.

To reduce the risk of emergency during reconstruction, it is necessary to investigate and predict the stress-strain state of the structure [2-4]. Numerical modelling will enable the user to evaluate the consequences that may affect bearing capacity of the building as a whole.

MAIN BODY

Methods for computing buildings and structures during reconstruction are constantly improved. Modelling of behaviour of the building structures and generation of correct design models were studied by several scientists as V. Banakh [5, 6], A. Gorodetsky [4], E. Gorokhov, A. Dykhovychny, N. Zotsenko, S. Klovanich, V. Kulyabko, A. Perelmuter [7], V. Slivker, R.L. Taylor, O.C. Zienkiewicz, etc.

However, up to now correct modelling of behaviour of existing buildings and structures under reconstruction has not been sufficiently studied. For reconstructed objects, there are many features that should be considered in analysis of stress strain state: damage to structural elements of the building before reconstruction (cracks, reduction of the cross section of elements); deformations of the base that are present at the time of reconstruction; change in service conditions of the building, change in engineering and geological conditions; changes in the initial design model of the building in the process of reconstruction, etc.

Therefore, the urgent task is to create methods for generation of adequate design models for buildings under reconstruction. Modern software packages that simulate the behaviour of structures at different stages of their life cycle, and take into account the changes in stress strain state during reconstruction enable the user to correctly evaluate the bearing capacity of building structures.

The real stress-strain state of the elements of bearing structural systems may be determined when a number of numerical experiments are conducted:

- to model erection process, when the stress strain state is determined for all design models corresponding to the stages of construction, and the model of the ready structure ‘keeps memory’ about the history of its construction;
- to model loading process, for example, for RC structures it is possible to trace initial stages of linearly elastic behaviour of structure, the stages of successive formation and propagation of cracks in concrete, the stages of development of plastic strain in compressed concrete and tensioned reinforcement, the stages prior to destruction, and full or partial unloading at any of these stages and further loading [8-10].

Peculiarities of modelling such processes and the capabilities of modern software used in this case will be illustrated with examples of solving real problems from design practice based on LIRA-SAPR program.

GENERATION OF DESIGN MODELS FOR RECONSTRUCTED BUILDINGS

In LIRA-SAPR 2018, the bearing capacity of the cross sections of bar and plate elements is analysed with regard to real reinforcement pattern, according to the current regulatory and normative documents valid at the time of design process.

For the above-mentioned purpose, it is possible to define the pattern of reinforcing items in the bar and plate elements either within the whole design model or for individual elements (Fig. 1).

When generating design model of the building where the loads from the superstructure storeys will be transferred to the existing building, the actual location of the reinforcement items is defined in the existing building elements. After checking the bearing capacity of sections with a specified reinforcement, according to building codes, the safety factor of the bearing capacity in each element of the plates and in each section

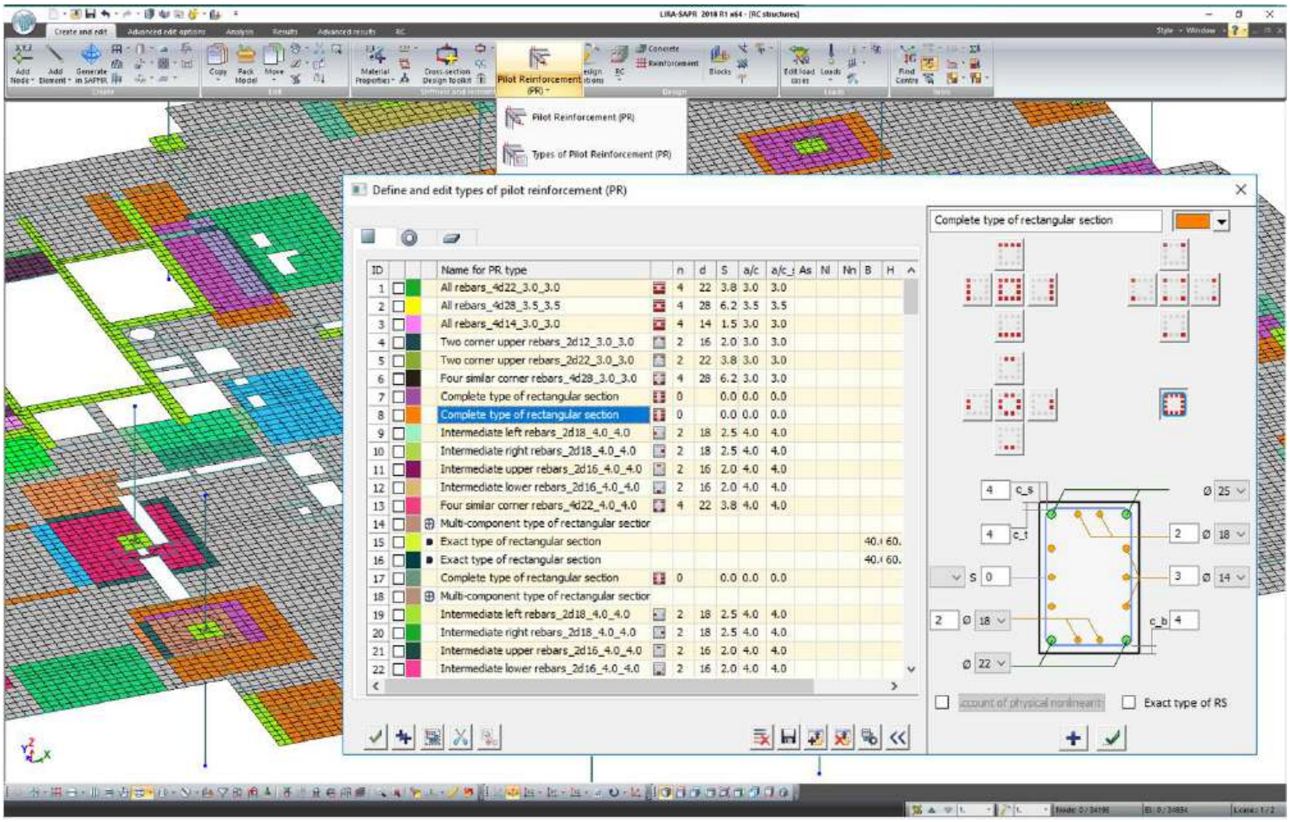


Figure 1. Generation of parametric types of specified reinforcement for elements of the model in LIRA-SAPR.

Figure 2. Building before reconstruction.

of the bars is determined. The required reinforcement area is selected for the superstructure part.

This method of analysis was used to estimate whether it is possible to reconstruct a 3-storey administrative building (Fig. 2).

The spatial rigidity of the building is provided by combined behaviour of the longitudinal bearing, transverse self-supporting walls and rigid bodies of floor slabs. During reconstruction of a non-residential building, it is proposed to remove existing brick partitions and install new ones. According to a survey of the building, conclusion is made that it could be reconstructed with a superstructure as the fourth storey. In design of this project, it was considered that the building is located in the existing residential area with its own basic facilities [11].

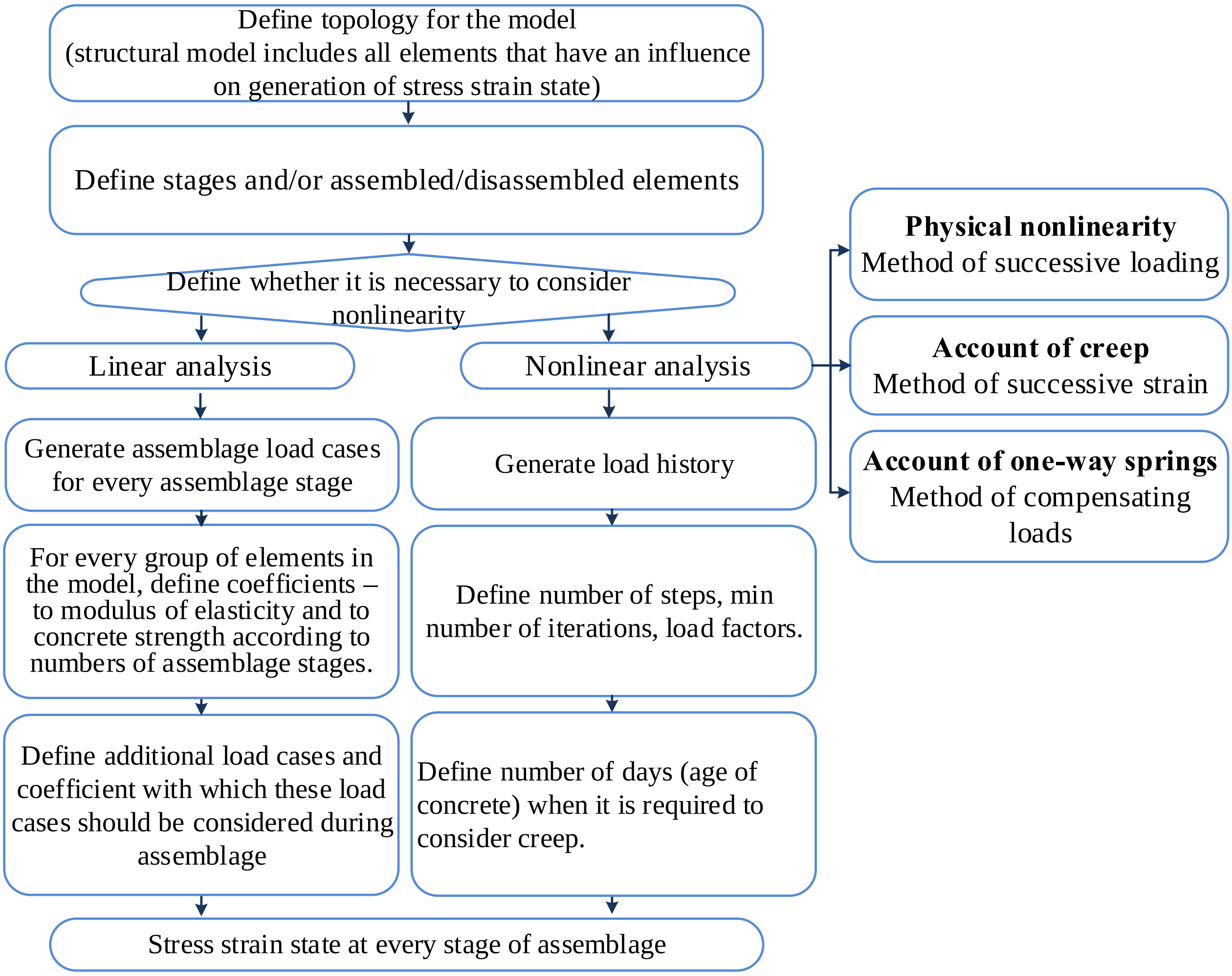


Figure 3. Algorithm of analysis with account of erection process.

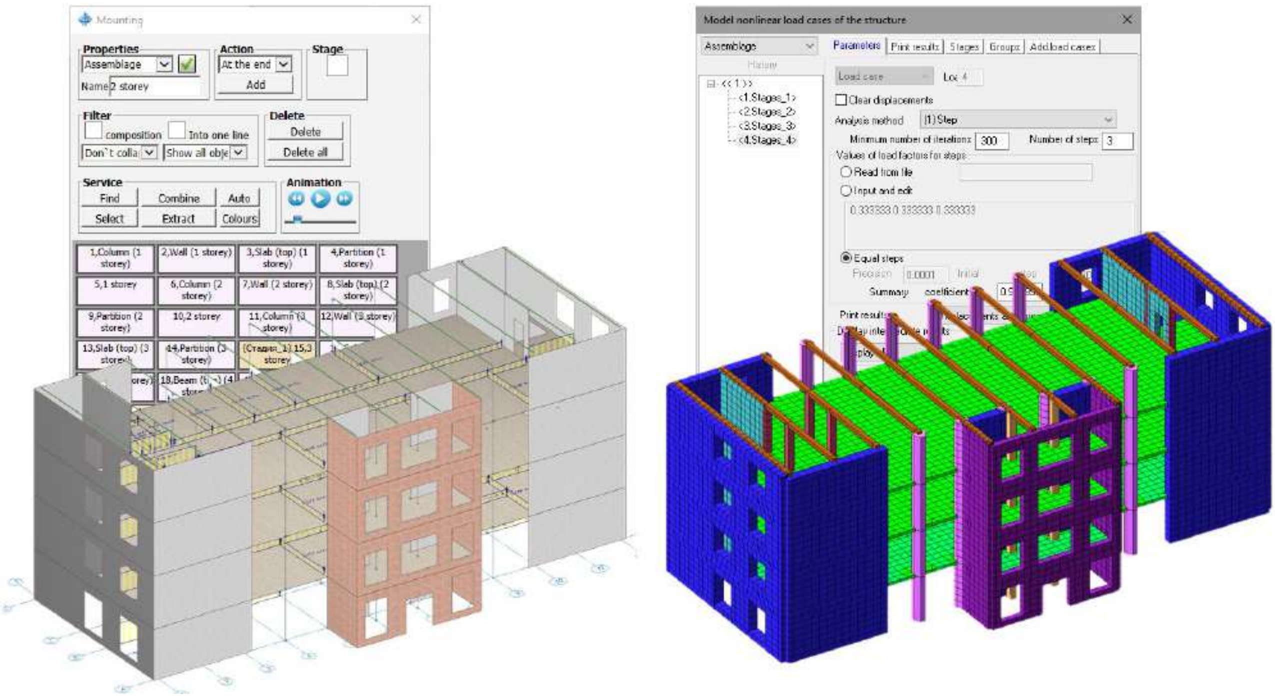
Analysis was carried out in LIRA-SAPR with account of erection process. The general algorithm for analysis with account of erection process for linear and physically nonlinear problems is presented in Fig. 3.

First of all, design model of the whole object is generated; it includes all elements that influence the stress strain state: the main load-bearing elements of the object - columns, beams, slabs, diaphragms, and temporary elements - formwork elements, struts, etc.

Then, for each stage of construction, all the structural elements that were erected or removed at the stage are indicated. Elements may be disassembled only once. Empty stages are allowed. The empty stage has the same elements as the previous stage. It is used only to define the load.

For each stage of assemblage you define loads (dead weight, assemblage loads) applied at this stage. Certain assemblage load should correspond to each stage of assemblage. Thus, the number of stages and the number of assemblage loads must be the same. Empty assemblage load cases are allowed. For example, a stage in which the elements of design model are only disassembled and no load is applied is considered as empty load case.

If necessary, correction factors are defined for each group of elements in the model — to the modulus of elasticity and to concrete strength in accordance with the numbers of assemblage stages. If information about the groups is not specified, then material properties remain the same at all stages.



a) *b)*

Figure 4. Design model of the administrative building with account of assemblage stages: a) in SAPFIR; b) in LIRA-SAPR.

For each stage you could also specify the numbers of additional load cases and the coefficients (including zero and negative ones) that should be considered during erection. Additional load cases are taken to mean load cases that contain loads from the storage of construction materials, from their movement within a storey or building, etc.

In addition to assemblage tables, you define parameters such as analysis method, number of stages, coefficients to load, etc. The load history is always considered.

After analysis by solver, the program computes the forces and stresses accumulated in the elements during erection process. By default, displacements of nodes are not accumulated in analysis; they are computed once more for each stage.

For this reconstructed administrative building, the following stages of assemblage are considered: the first stage is to assemble elements of the 1-3rd storeys; the second stage is to apply service loads; the third stage is to assemble elements of the fourth storey; fourth stage – to apply service loads (Fig. 4).

The following example demonstrates reconstruction of a hostel building. Wooden floor slabs are replaced with monolithic RC floor slabs, the opening is made for the whole height of the building.

The structural model of the building is a mixed frame: longitudinal and transverse frames of brick pillars and prefabricated reinforced concrete beams rigidly fixed in them; and load-bearing longitudinal and transverse walls that prefabricated reinforced concrete beams are supported with. Fig. 5 depicts general view of the hostel building before and after reconstruction.

To determine whether reconstruction and redevelopment are possible, a survey of the load-bearing structures of the building was carried out as well as numerical experiments to determine bearing capacity of the building structures. With the help of ASSEMBLAGE module in LIRA-SAPR program, wooden floor slabs were disassembled and reinforced concrete floor slabs were installed; rigid body of floor slab is replaced within entire area of the building (Fig. 6).

a) b)

Figure 5. General view of the hostel building:
a) before reconstruction; b) after reconstruction.

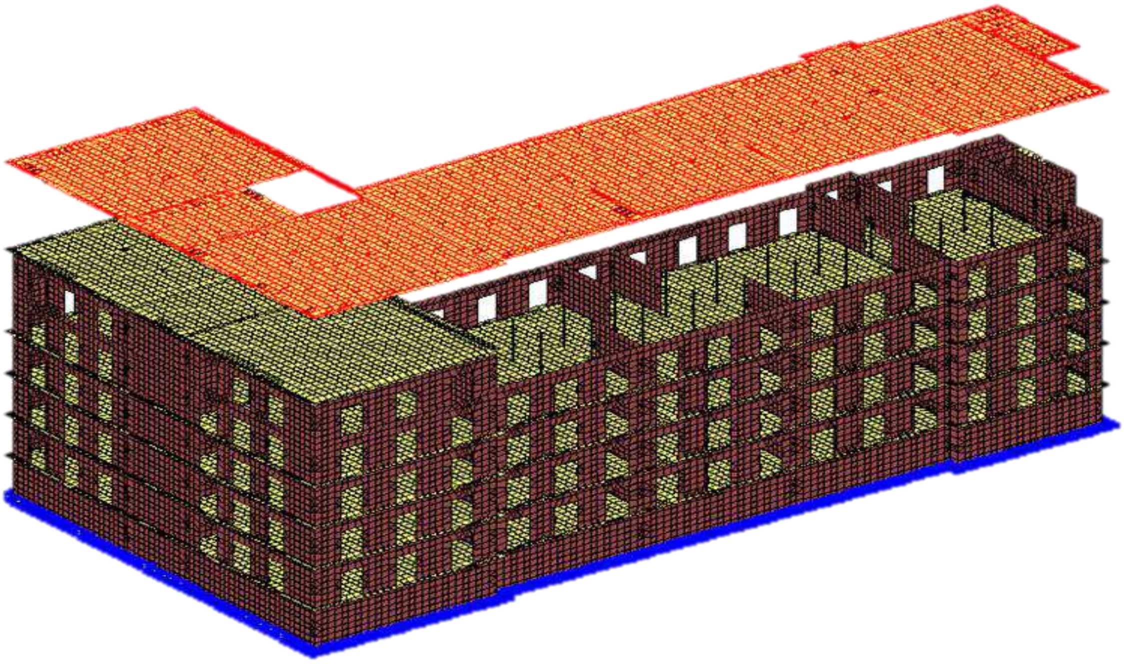
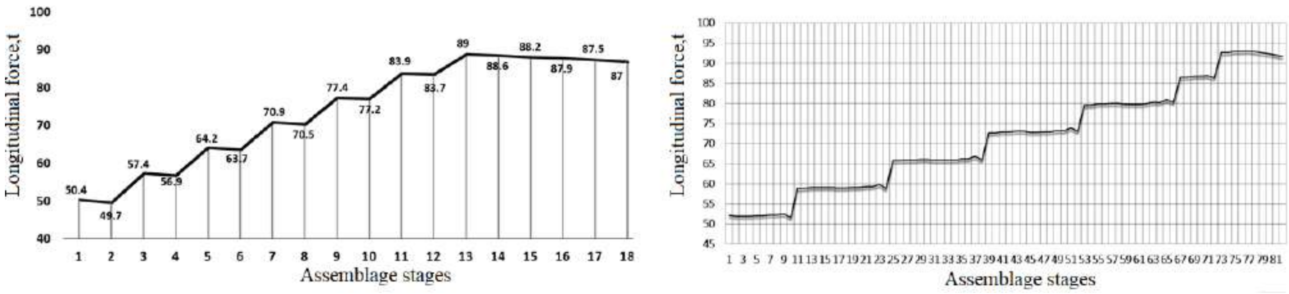


Figure 6. Design model of hostel (general view).



a) b)

Figure 7. Changes of longitudinal force in column of design model of the hostel:
a) 18 assemblage-disassemblage stages;
b) 81 assemblage-disassemblage stages.

a)

b)

Figure 8. Defects detected during evaluation of technical condition of the building structures: a) floor slabs got wet;b) load-bearing wall in the basement of the building is destroyed.

After analysis with account of assemblage / disassemblage process, the program evaluates changes of forces in the load-bearing structures during the reconstruction. For example, in one of the columns at level 0.0, the longitudinal force is changed from 49.7t (after disassemblage of the wooden floor slab at level 3.2m) to 89t (after installing the reinforced concrete floor slab at level 19.2m), it represents the difference 1.8 times.

Design model of the building was also studied. Every floor slab in the model was divided into 7 parts (according to the developed process list), it increased the number of assemblage / disassemblage stages up to 81 stages (Fig. 7). But obtained results did not significantly differ from the design model where floor slabs were assembled in enlarged details.

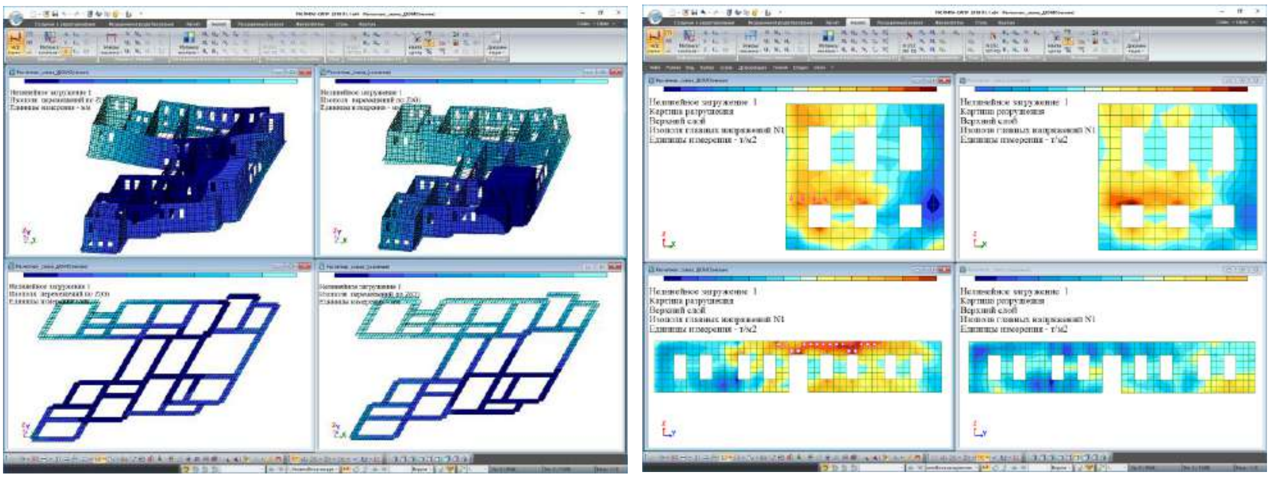
With such stage-by-stage simulation, it is convenient to evaluate the changes of the stress strain state in design model. So, it will be possible to provide detailed recommendations on the rational strengthening of structures in the reconstructed building and, if necessary, correct accepted design solutions during construction and assemblage.

For reconstructed buildings, it is very important to consider the damages and wear of the load-bearing structures identified during the technical checkup. If there are cracks and defects in the structures, it is necessary to simulate the structures and structural systems with account of accumulation of damages and structural destruction, reduction in stiffness properties of load-bearing elements. In such cases, it is important to determine the ability of damaged structural elements to take external load.

In the survey and evaluation of technical condition of building structures (that were carried out in order to determine its serviceability criterion and develop construction documents for strengthening of building structures and obtain certificate of a piece of architecture), serious defects were found: the load-bearing wall in the basement of the building was destroyed; floor slabs of the building got wet; cracks in the bearing walls of the building; destruction of the concrete cover and corrosion of reinforcement in monolithic gallery bulkheads, destruction of the plaster layer and corrosion of steel vaulted beams in the basement (Fig. 8).

Analysis of generated FE model of the building was carried out in LIRA-SAPR program with account of physical nonlinearity, based on the deformations and defects identified in the technical survey.

To consider reducing the strength properties of building materials, stiffness properties were modified in the design model. To be exact, modulus of elasticity is reduced, geometry of the cross-section is modified, several finite elements were ignore in analysis of design model.



a) b)

Figure 9. Fragment of design model before reconstruction and after reconstruction
a) contour plots of displacements;
b) contour plots of of principal stresses and the pattern of destruction.

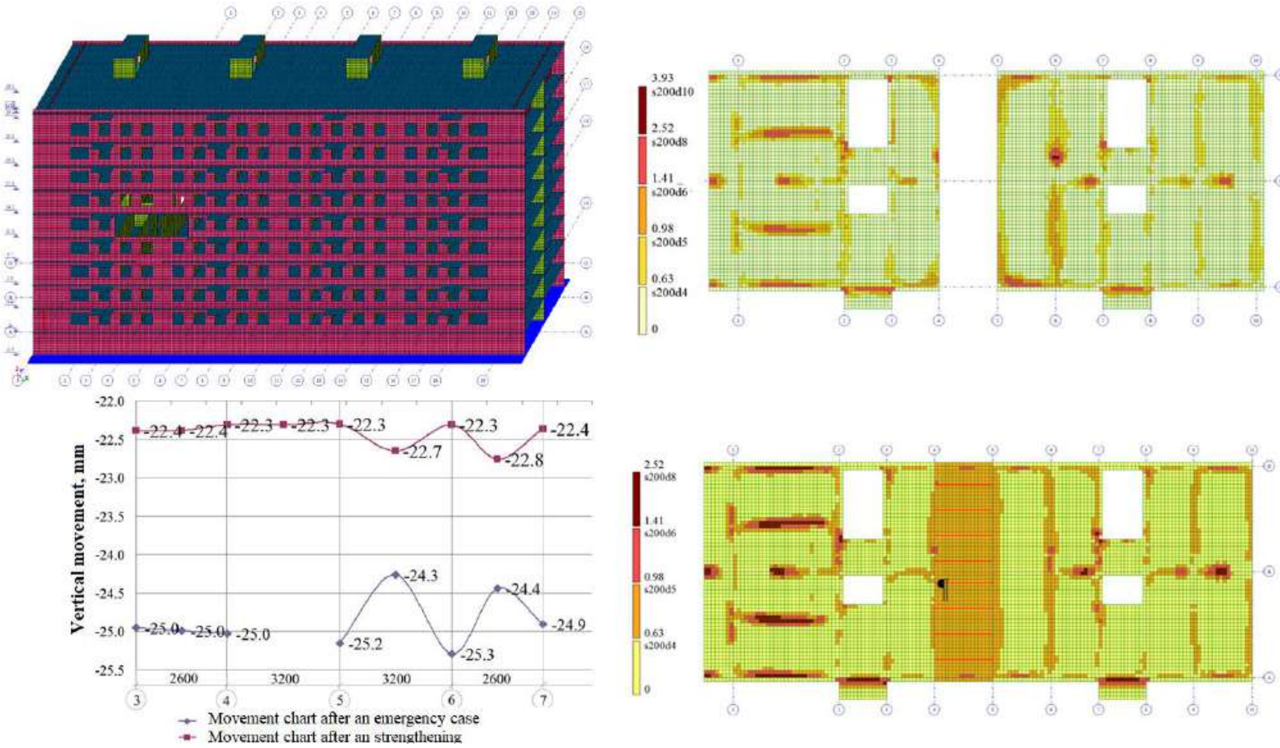


Figure 10. Results of numerical experiment and technical survey after explosion,
a fragment of the reinforcement in the floor slab after the explosion and after strengthening.

In elements of design model of the building, there are stresses that exceed the design strength of materials of the structures. It causes crack generation and other defects that were also found in the technical survey of the building. According to analysis of design model before the reconstruction, measures to strengthen the structural elements were developed. Comparison of the stress strain state of the building before reconstruction and after strengthening is presented in Fig. 9.

It is even more difficult to simulate such effects as an explosion, a fire inside a building, and other emergency situations. For example of such design model, there may be the model of a 9-storey residential building in which a gas explosion occurred. Analysis of the stress-strain state of the frame in explosion was carried out; so adaptability of structures to loads beyond design basis is determined. It causes less material consumption during the strengthening and reconstruction of a residential building in emergency case (Fig.10).

CONCLUSION

Thus, in reconstruction of buildings, it is necessary to carry out several numerical experiments to determine the most useful technique to strengthen the load-bearing elements, especially if it concerns the superstructure of storeys, replacement of floor slabs, and reconstruction after accidents. To carry out numerical experiments in LIRA-SAPR program, the following procedure is proposed:

1. Design model of the building is generated according to results of the survey on technical condition of the building, available documentation or/and technical check with account of damage and wear of load-bearing structures. Bearing capacity of structures is determined.
2. Design model of the building is generated with account of proposed reorganization, redevelopment, modernization, erected superstructures, additions to building. Analysis of stress strain state is carries out after several numerical experiments with account of restoration, strengthening and replacement of building structures. The program identifies dangerous elements of structures or their parts. Recommendations are provided whether reconstruction is possible. Measures to strengthen the bearing structures are suggested.

Proposed algorithm allows the user to obtain quite accurate stress strain state of the elements of bearing structures of the object with account of all changes and to predict its behaviour in the future as well. Analysis of the stress strain state of design model enables the user to draw conclusions about effectiveness of certain variant of structures' strengthening during their reconstruction.

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THE GENERALIZED BIFRACTIONAL BROWNIAN MOTION

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Abstract: To extend several known centered Gaussian processes, we introduce a new centered Gaussian process, named the generalized bifractional Brownian motion. This process depends on several parameters, namely $\alpha > 0$, $\beta > 0$, $0 < H < 1$ and $0 < K \le 1$. When $K = 1$, we investigate its convexity properties. Then, when $2HK \le 1$, we prove that this process is an element of the QHASI class, a class of centered Gaussian processes, which was introduced in 2015.

Keywords: convexity, quasi-helix, approximately stationary increments

ОБОБЩЕННОЕ БИФРАКТАЛЬНОЕ БРОУНОВСКОЕ ДВИЖЕНИЕ

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Аннотация: Расширение нескольких центрированных гауссовский процессов требует введения нового процесса, названного бифрактальным броуновским движением. Этот процесс зависит от нескольких параметров, а именно: $\alpha > 0$, $\beta > 0$, $0 < H < 1$ и $0 < K \le 1$. Для случая, когда параметр $K = 1$, исследуется свойство выпуклости. Для случая, когда $2HK \le 1$, доказывается принадлежность этого процесса к квази-классу (обладанием квази-канонической кривой постоянного склона), и к классу центральных гауссовских процессов.

Ключевые слова: выпуклость, квази-каноническая кривая постоянного склона, приближенно стационарные прирощении

1. INTRODUCTION

Let $\{B_{H,K}(t), t \in \mathrm{R}\}$ be a bifractional Brownian motion (bBm) with indices $0 < H < 1$ and $0 < K \le 1$, i.e. a centered Gaussian process such that $B_{H,K}(0) = 0$, with probability 1, and

$$\mathrm{E}\left(B_{H,K}(t)B_{H,K}(s)\right) = \frac{1}{2^K}\left(\left(|t|^{2H} + |s|^{2H}\right)^K - |t-s|^{2HK}\right). \tag{1.1}$$

We can verify that

$$\mathrm{Var}B_{H,K}(t) = |t|^{2HK}$$

and that the bBm is self-similar with index HK. Note also that the process $B_{H,1}$ is the fractional Brownian motion (fBm) and therefore the process $B_{1/2,1}$ is the ordinary Wiener process. Straightforward computations show $B_{H,K}$ has no stationary increment. However, the bBm is a HK-quasi-helix in the sense of Kahane ([1], p. 137) and its increments are approximately stationary for small increments. Houdré and Villa [2] introduced the bBm and established the previous results.

Consider the following centered Gaussian process $Y := Y_{\alpha,\beta,H,K}$ defined as follows:

$$\begin{aligned} Y(t) &:= Y_{\alpha,\beta,H,K}(t) \\ &= \alpha\, B_{H,K}(t) + \beta\, B_{H,K}(-t) \end{aligned} \quad (1.2)$$

with $t \geq 0, \alpha > 0, \beta > 0$. Set

$$\alpha(K) = 1/(2^{(2-K)/2}),\ 0 < K \leq 1.$$

The introduction of the process Y is motivated by the fact that this process was already introduced for specific values of α, β and K. Indeed, the process $Y_{\alpha(1),\alpha(1),H,1}$ was introduced in [3] and was named the sub-fractional Brownian motion. El-Nouty and Journé [4] extended the former process by introducing the process $Y_{\alpha(K),\alpha(K),H,K}$, which was named the sub-bifractional Brownian process (sbBm). Finally, Zili [5] introduced the process $Y_{\alpha,\beta,H,1}$, which was named the generalized fractional Brownian motion (gfBm). This is why we will name Y the generalized fractional Brownian motion (gbBm). Set for $s,t \geq 0$

$$\begin{aligned} \sigma^2(s,t) &:= \sigma^2_{\alpha,\beta,H,K}(s,t) \\ &= \mathrm{E}\left(\left(Y_{\alpha,\beta,H,K}(t) - Y_{\alpha,\beta,H,K}(s)\right)^2\right). \end{aligned} \quad (1.3)$$

Let us study the convexity properties of

$$\sigma^2(s,t) := \sigma_{\alpha,\beta,H,1}{}^2(s,t)$$

on the set

$$T = \{(s,t) \in [0,1]^2 \ :\ s \leq t\}.$$

Our first result is stated in the following proposition.

Proposition 1. *I. If* $H \geq 1/2$ *, then the function* $\sigma^2_{\alpha,\beta,H,1}(s,t)$, $(s,t) \in T$ *is convex and has a unique maximum at the point* $(0,1)$.
II. If $H < 1/2$ *, then the function* $\sigma^2_{\alpha,\beta,H,1}(s,t)$, $(s,t) \in T$ *is concave and has a unique maximum at the point* $(0,1)$.

Note the difference between the case $0 < H < 1/2$ and the case $1/2 < H < 1$, i.e. between short-range dependence and long-range dependence. This phenomenon was already observed by several authors in the fBm case (Beran ([6], p52), Samorodnitsky and Taqqu ([7], p. 123)). Proposition 1 establishes that the fBm and the gfBm are similar from the convexity point of view. However, when one compares Proposition 1 with Proposition 1.1 in [8], he can observe the difference between the gfBm and the bBm. This implies that there is a significant difference between the processes $Y_{\alpha,\beta,H,1}$ and $Y_{\alpha,\beta,H,K}$, with $K < 1$.

The quasi-helix with approximately stationary increments (QHASI) class of centered Gaussian processes was introduced by El-Nouty [9] and was defined as follows. A centered Gaussian process $\{X(t), t \in I \subset R\}$ belongs to the QHASI class if it fulfills the five following assumptions:

- A1: $X(0) = 0$ with probability 1,
- A2: there exists $\lambda > 0$ such that X is self-similar with index λ,
- A3: there exists $0 < C_1 \leq C_2 < +\infty$ such that $\forall (s,t) \in I^2$

$$C_1 |t-s|^{2\lambda} \leq \mathrm{E}\left(X(t) - X(s)\right)^2 \leq C_2 |t-s|^{2\lambda},$$

- A4: there exists $C_3 \in [C_1, C_2]$ such that

$$\forall (s,t) \in I^2,\ t \geq s,\ st \neq 0, \text{ when } t-s \to 0,$$
$$\mathrm{E}\left(X(t) - X(s)\right)^2 \sim C_3 (t-s)^{2\lambda}$$

- A5: there exists $C_4 \in [C_1, C_2]$ such that

$$\forall t \in I,\ \mathrm{E}X(t)^2 = C_4 |t|^{2\lambda}.$$

Let us state three known results. The first one is due Houdré and Villa [2], the second one to [10] and the last one to El-Nouty [9].

Theorem 2. *The bBM is an element of the QHASI class.*

Theorem 3. *The sfBm is an element of the QHASI class.*

Theorem 4. *The sbBm is an element of the QHASI class.*

We insist on the fact that the values of λ, C_1, C_2, C_3 and C_4 for the bBm, the sfBm and the sbBm can be found in El-Nouty [9]. Using some results of Zili [5] and introducing some additional computations, we get the following result.

Theorem 5. *The gfBm is an element of the QHASI class, with*

- $\lambda = H$,
- $C_1 = \min\left(\alpha^2+\beta^2, (\alpha+\beta)^2 - 2^{2H}\alpha\beta\right)$,
- $C_2 = \max\left(\alpha^2+\beta^2, \left((\alpha+\beta)^2 - 2^{2H}\alpha\beta\right)\right)$,
- $C_3 = \alpha^2+\beta^2$,
- $C_4 = \alpha^2 + 2\left(1-2^{2H-1}\right)\alpha\beta + \beta^2$.

Our main result is stated in the following theorem.

Theorem 6. *Assume that* $2HK \le 1$. *Then the gbBm is an element of the QHASI class, with*

- $\lambda = HK$,
- $C_1 = (\alpha+\beta)^2 - 2^{2-K}\alpha\beta$,
- $C_2 = 2^{1-K}\left((\alpha+\beta)^2 - 2^{2HK}\alpha\beta\right)$,
- $C_3 = 2^{1-K}\left(\alpha^2+\beta^2\right)$,
- $C_4 = \alpha^2 + 2\left(1-2^{2HK-K}\right)\alpha\beta + \beta^2$.

Let us make some comments on the above theorem. When

$$H \le 1/2 \text{ and } K = 1,$$

theorem TH is similar to theorem tutu. When

$$2HK \le 1 \text{ and } \alpha = \beta = \alpha(K),$$

theorem TH is similar to theorem JLJ. Finally, as expected, note the importance of the hyperbola

$$2HK = 1.$$

This phenomenon was already observed in El-Nouty [11], El-Nouty and Journé [12] and Russo and Tudor [13].

Let us investigate the case

$$1 < 2HK < 2 .$$

There is no difficulty to determine λ, C_3 and C_4. Indeed they have the same values as those found in the case

$$2HK \le 1.$$

A careful reading of the proof of theorem TH enables to state the following lemma.

Lemma 7. *Assume that* $2HK > 1$. *Then* $\forall\,(s,t) \in I^2$

$$\sigma^2(s,t) \le 2^{1-K}\left(\alpha^2+\beta^2\right)|t-s|^{2HK} .$$

The question of the existence of a constant C_1 is still an open one.

In section 2, we prove Proposition 1. The proof of Theorem 6is postponed to section 3. In the sequel, there is no loss of generality in assuming $K < 1$.

2. PROOF OF PROPOSITION 1

Recall that $K=1$ in this section. We have for any $t \geq s$

$$\begin{aligned}\sigma^2(s,t) &:= \sigma_{\alpha,\beta,H,1}{}^2(s,t)\\ &= s^{2H}\Big((\alpha^2+\beta^2)(u-1)^{2H}\\ &\quad + 2^{2H}\alpha\beta\left(2^{1-2H}(u+1)^{2H}-u^{2H}-1\right)\Big)\\ &:= s^{2H}\ \lambda(u),\end{aligned} \tag{2.1}$$

where

$$u = t/s \geq 1.$$

There is no difficulty to deal with the case $H=1/2$. When $H \neq 1/2$, the derivative of order 2 of the function λ is

$$\lambda^{(2)}(u) = 2H(2H-1)s^{2H}\ g(u), \tag{2.2}$$

where

$$\begin{aligned}g(u) &= (\alpha^2+\beta^2)(u-1)^{2H-2}\\ &\quad + 2^{2H}\alpha\beta\left(2^{1-2H}(u+1)^{2H-2}-u^{2H-2}\right).\end{aligned}$$

Let us study the sign of the function g. We have

$$g(u) \geq 2\ \alpha\ \beta\ h(u)\,, \tag{2.3}$$

where

$$h(u) = (u-1)^{2H-2} + (u+1)^{2H-2} - 2^{2H-1}u^{2H-2}.$$

Since

$$2H-2<0\,,$$

the function $u \to u^{2H-2}$, $u \geq 1$ is convex, and therefore

$$(u-1)^{2H-2} + (u+1)^{2H-2} \geq 2u^{2H-2}.$$

Hence,

$$h(u) = (2-2^{2H-1})u^{2H-2} > 0\,. \tag{2.4}$$

Combining (2.4) and (2.3) with (2.1), we establish that, if $H > 1/2$, then the function $\sigma_{\alpha,\beta,H,1}{}^2(s,t)$, $(s,t) \in T$ is convex, else the function is concave.
By using (2.1), we have for any real $s,\ t, s \neq t$, and $a>0$

$$\sigma^2(s,t) > 0,\ \sigma^2(s,s) = 0$$

and

$$\sigma^2(as,at) = a^{2H}\sigma^2(s,t)\,.$$

Thus, we get

$$s\frac{\partial\sigma^2(s,t)}{\partial s} + t\frac{\partial\sigma^2(s,t)}{\partial t} = 2H\sigma^2(s,t)\,. \tag{2.5}$$

If

$$\frac{\partial\sigma^2(s,t)}{\partial s} = \frac{\partial\sigma^2(s,t)}{\partial t} = 0\,,$$

then (2.5) yields that

$$\sigma^2(s,t) = 0$$

and consequently

$$s = t\,.$$

Thus, there is no maximum of $\sigma^2(s,t)$ in the interior of T.
Let us investigate the existence of a maximum of $\sigma^2(s,t)$ on the border of T. Note that

$$\sigma^2(0,t) = \left(\alpha^2 + (2-2^{2H})\alpha\ \beta + \beta^2\right)\ t^{2H}$$

has a unique maximum at the point $t=1$. Thus, we have to study the function

$$\sigma^2(s,1) = (\alpha^2+\beta^2)(1-s)^{2H} + 2^{2H}\alpha\beta\left(2^{1-2H}(1+s)^{2H} - s^{2H} - 1\right). \quad (2.6)$$

We have by differentiation

$$\frac{d\sigma^2(s,1)}{ds} = 2H\left(-\left(\alpha^2+\beta^2\right)(1-s)^{2H-1} + 2^{2H}\alpha\beta\left(2^{1-2H}(1+s)^{2H-1} - s^{2H-1}\right)\right).$$

We must consider the following three cases:

Case 1. $H = 1/2$.

Since

$$\frac{d\sigma^2(s,1)}{ds} = -\left(\alpha^2+\beta^2\right) < 0,$$

the function $\sigma^2(s,t), (s,t) \in T$ has a unique maximum at the point $(0,1)$. Using (2.6), we have

$$\sigma^2(0,1) = \alpha^2 + \beta^2.$$

Case 2. $2H > 1$.

We have

$$\frac{d\sigma^2(s,1)}{ds} \le 0 \Leftrightarrow 2\alpha\beta(1+s)^{2H-1} \le \left(\alpha^2+\beta^2\right)(1-s)^{2H-1} + 2\alpha\beta(2s)^{2H-1}. \quad (2.7)$$

Recall that

$$2\alpha\beta \le \alpha^2 + \beta^2.$$

To prove (2.7), it suffices to verify

$$(1+s)^{2H-1} \le (1-s)^{2H-1} + (2s)^{2H-1}. \quad (2.8)$$

Inequality (2.8) is true at the points 0 and 1. Set

$$u = 1/s \ge 1.$$

Thus, inequality (2.8) can be rewritten as follows:

$$(u+1)^{2H-1} \le (u-1)^{2H-1} + 2^{2H-1}. \quad (2.9)$$

Set

$$g(u) = (u+1)^{2H-1} - (u-1)^{2H-1} - 2^{2H-1}.$$

We have

$$g'(u) = (2H-1)\left((u+1)^{2H-2} - (u-1)^{2H-2}\right) \le 0.$$

Since

$$g(1) = 0,$$

we prove that $g \le 0$ and consequently inequality (2.9).

The function $\sigma^2(s,t), (s,t) \in T$ has a unique maximum at the point $(0,1)$. Using (2.6), we have

$$\sigma^2(0,1) = \alpha^2 + \left(2 - 2^{2H}\right)\alpha\beta + \beta^2.$$

Case 3. $2H < 1$.

To show that

$$\frac{d\sigma^2(s,1)}{ds} \le 0$$

it suffices to establish that

$$2^{1-2H}(1+s)^{2H-1} - s^{2H-1} \le 0$$
$$\Leftrightarrow (1+s)^{2H-1} \le (2s)^{2H-1}.$$

Since

$$2H - 1 < 0 \text{ and } s \le 1,$$

the result is true. Therefore, the function $\sigma^2(s,t),(s,t)\in T$ has a unique maximum at the point $(0,1)$. Using (2.6), we have

$$\sigma^2(0,1) = \alpha^2 + (2 - 2^{2H})\alpha\beta + \beta^2 .$$

The proof of Proposition 1 is complete. ■

3. PROOF OF THEOREM 6

We can easily remark that the process Y is a centered Gaussian process such that $Y(0)=0$ with probability 1 and Y is self-similar with index HK. The covariance function of the process Y is given in the following lemma.

Lemma 8. We have for $t \ge s \ge 0$

$$\mathrm{E}(Y(t)Y(s)) = \frac{1}{2^K}\Big((\alpha+\beta)^2(t^{2H}+s^{2H})^K - (\alpha^2+\beta^2)(t-s)^{2HK} - 2\alpha\beta(t+s)^{2HK}\Big)$$

and therefore

$$\mathrm{E}\left(Y(t)^2\right) = \left(\alpha^2 + 2\left(1-2^{2HK-K}\right)\alpha\beta + \beta^2\right)t^{2HK}.$$

Proof. It suffices to combine (1.1) with (1.2).

Remark 9. Lemma 8 gives the value of the constant C_4 .

Set for $t \ge s \ge 0$

$$F_{H,K}(s,t) = 2\left(\frac{t^{2H}+s^{2H}}{2}\right)^K - t^{2HK} - s^{2HK} \ge 0, \tag{3.1}$$

$$F_{\frac{1}{2},2HK}(s,t) = 2\left(\frac{t+s}{2}\right)^{2HK} - t^{2HK} - s^{2HK}. \tag{3.2}$$

The functions given in (3.1) and (3.2) will play a key role in the proofs of our results. Let us recall the following basic proposition.

Proposition 10. *When* $0 < 2HK < 1$, $F_{1/2,2HK} \ge 0$. *When* $2HK = 1$, $F_{1/2,1} = 0$. *When* $1 < 2HK < 2$, $F_{1/2,2HK} \le 0$.

Remark 9. *When* $2HK \le 1$, *the function* $F_{\frac{1}{2},2HK}$ *can be viewed as* $F_{H,K}$ *with* $H = \frac{1}{2}$.

We can state the second technical lemma.

Lemma 12. *We have for* $t \ge s \ge 0$

$$\sigma^2(s,t) = 2^{1-K}(\alpha^2+\beta^2)(t-s)^{2HK} - (\alpha+\beta)^2 F_{H,K}(s,t) + 2^{1-K+2HK}\alpha\,\beta\,F_{\frac{1}{2},2HK}(s,t).$$

Proof. It suffices to combine (1.3), (3.1) and (3.2) with lemma 8. ■

The next step consists in determining the value of the constant C_2.

Combining Lemma 12 with Proposition 10, we have

if $0 < 2HK < 1$, then

$$\sigma^2(s,t) \le 2^{1-K}(\alpha^2+\beta^2)(t-s)^{2HK} + 2^{1-K+2HK}\alpha\beta F_{\frac{1}{2},2HK}(s,t),$$

if $2HK = 1$, then

$$\sigma^2(s,t) \le 2^{1-K}\left(\alpha^2+\beta^2\right)(t-s),$$

and

if $1 < 2HK < 2$, then

$$\sigma^2(s,t) \le 2^{1-K}\left(\alpha^2+\beta^2\right)(t-s)^{2HK}.$$

El-Nouty and Journé [12] showed that we have for $0 < 2HK < 1$,

$$\begin{aligned}(t-s)^{2HK} &+ 2^{2HK-1}F_{\frac{1}{2},2HK}(s,t)\\ &\le \left(2-2^{2HK-1}\right)(t-s)^{2HK}.\end{aligned}$$

Then,

$$\begin{aligned}\sigma^2(s,t) &\le 2^{1-K}\left(\alpha^2+\beta^2\right)(t-s)^{2HK}\\ &+2^{1-K+2HK}\alpha\beta 2^{1-2HK}\left(1-2^{2HK-1}\right)(t-s)^{2HK}\\ &= 2^{1-K}\left((\alpha+\beta)^2-2^{2HK}\alpha\beta\right)(t-s)^{2HK}\\ &:= C_2\,(t-s)^{2HK}.\end{aligned}$$

Let us determine the value of the constant C_1 .

Combining Lemma 12 with Proposition 10, we get

$$\begin{aligned}\sigma^2(s,t) \ge\ & 2^{1-K}\left(\alpha^2+\beta^2\right)(t-s)^{2HK}\\ &-(\alpha+\beta)^2 F_{H,K}(s,t).\end{aligned}$$

It was proved by El-Nouty and Journé [12] that, when $2HK \le 1$,

$$\begin{aligned}(t-s)^{2HK} + 2^K\left(t^{2HK}+s^{2HK}\right) &- 2\left(t^{2H}+s^{2H}\right)\\ &\ge \left(2^K-1\right)(t-s)^{2HK},\end{aligned}$$

that is

$$\left(2-2^K\right)(t-s)^{2HK} \ge 2^K F_{H,K}(s,t).$$

Then,

$$\begin{aligned}\sigma^2(s,t) &\ge \left((\alpha+\beta)^2-2^{2-K}\,\alpha\,\beta\right)(t-s)^{2HK}\\ &:= C_1(t-s)^{2HK}.\end{aligned}$$

Finally, we determine the value of the constant C_3. Recall that $s>0$. Set

$$t-s=h.$$

When $h \to 0$, the Taylor expansions of the functions $F_{H,K}$ and $F_{1/2,2HK}$, given in (3.1) and (3.2), are

$$\begin{aligned}F_{H,K}(s,t) = s^{2HK}\Bigg(& H^2K(1-K)\frac{h^2}{s^2}\\ &+o\left(\frac{h^2}{s^2}\right)\Bigg)\end{aligned} \tag{3.3}$$

and for $2HK \ne 1$

$$\begin{aligned}F_{\frac{1}{2},2HK}(s,t) = -s^{2HK}\Bigg(& \frac{HK(2HK-1)}{2}\frac{h^2}{s^2}\\ &+o\left(\frac{h^2}{s^2}\right)\Bigg).\end{aligned} \tag{3.4}$$

Combining Lemma 12 with (3.3) and (3.4) (or Proposition 10 if $2HK = 1$), we obtain the Taylor expansion of $\sigma(s,t)$. Hence, since $2HK < 2$, we get the value of C_3.

To complete the proof of Theorem 6, we have to verify that

$$C_1 \le C_3 \le C_2$$

and

$$C_1 \le C_4 \le C_2 .$$

Assume

$$\alpha \ge \beta > 0$$

and set

$$x = \frac{\alpha}{\beta},\ x \ge 1.$$

The inequalities $C_1 \le C_3 \le C_2$ are equivalent to

$$\begin{aligned} x^2 + \left(2 - 2^{2-K}\right)x + 1 \\ &\le 2^{1-K}\left(x^2+1\right) \\ &\le 2^{1-K}\left(x^2 + \left(2-2^{2HK}\right)x + 1\right), \end{aligned}$$

that is

$$\left(x+1\right)^2 \ge 0$$

and

$$\left(2-2^{2HK}\right)x \ge 0 .$$

Since $2HK \le 1$, it proves the result. Similarly, we can establish that

$$C_1 \le C_4 \le C_2 .$$

The proof of Theorem 6 is complete. ■

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ONE FEATURE OF THE CONSTRUCTIVE SOLUTIONS OF THE LATTICE GIRDER

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Abstract: The problem of the deflection of a planar symmetric statically determinable truss with a double lattice depending on the number of panels was solved in an analytical form. The angle of inclination of the ascending and descending rods of the truss is different. A load is applied to the truss, evenly distributed over the nodes of the lower chord. Special operators of the Maple computer math system and the induction method were used to generalize individual particular solutions to an arbitrary case. Formulas are obtained for the forces the most compressed and stretched truss rods. Cases of kinematic variability of the structure are revealed. A picture of the possible speeds of truss nodes in these cases is constructed. The asymptotic behavior of the deflection is found with a large number of panels and a fixed span length. The deflection was determined by the formula of Maxwell – Mohr.

Keywords: truss, deflection, induction, Mohr' integral, Maple

ОСОБЕННОСТЬ ОДНОГО КОНСТРУКТИВНОГО РЕШЕНИЯ РЕШЕТКИ БАЛОЧНОЙ ФЕРМЫ

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Аннотация: В аналитической форме решена задача о прогибе плоской симметричной статически определимой фермы с двойной решеткой в зависимости от числа панелей. Угол наклона восходящих и нисходящих раскосов фермы разный. К ферме приложена нагрузка, равномерно распределенная по узлам нижнего пояса. Использованы специальные операторы системы компьютерной математики Maple и метод индукции для обобщения отдельных частных решений на произвольный случай. Получены формулы для усилий в наиболее сжатых и растянутых стержнях фермы. Выявлены случаи кинематической изменяемости конструкции. Построена картина возможных скоростей узлов фермы в этих случаях. Найдена асимптотика прогиба при большом числе панелей и фиксированной длине пролета. Прогиб определялся по формуле Максвелла – Мора.

Ключевые слова: ферма, прогиб, индукция, формула Максвелла – Мора, Maple

1. FORMULATION OF THE PROBLEM

The calculation of building structures is traditionally carried out by numerical methods, based mainly on the finite element method, incorporated in most standard computational packages. Along with this, recently the analytical line of research has become widespread. In particular, for planar [1–8] and spatial [9–11] statically definable trusses with the property of regularity, exact solutions were obtained for the deflection problem under the action of various loads. The main feature of these solutions is the inclusion of a number of panels in the truss parameters. This became possible with the advent of methods of symbolic mathematics (Reduce, Maple, Mathematica, Maxima, Derive, and others), allowing not only to solve problems of forces in the bars of a structure in symbolic form, but also to find generalizations of these solutions to an arbitrary number of rods or cells in truss. The latter can be done by induction, for the implementation of which (the compilation and solution of recurrent equations) in modern systems of computer mathematics there are corresponding special operators. It should be noted that together with the solution of the main problem — the derivation of a compact formula for the expression of the deflection, using the methods of computer mathematics, it became possible to find the features of the structures of some complex rod structures that manifest themselves in their kinematic degeneration. It is these problems that are solved in the present work as applied to a planar beam truss with a diagonal double lattice.

A review of some analytical solutions for planar trusses is contained in [12].

2. TRUSS MODEL, CALCULATION OF FORCES

In a girder truss with parallel belts, the descending and ascending braces have a different slope. In the middle of the truss there is a shortened stand with a V-shaped fastening to the upper belt (Fig. 1). In this design, it is difficult to unambiguously single out a separate panel, so we will conditionally assume that the panel is defined only by the rods of the lower and upper belts and has a length of *a*. The number of panels in the truss is 2*n*, which corresponds to $m = 8n + 10$ rods together with three supporting ones. Since the number of hinges, the equilibrium of which is considered when determining the forces in the rods is equal here is $m_s = 4n + 8$, the truss is statically definable.

As with most trusses with double grids, the usual calculation methods for this truss, such as the cross-section method or the method of successive cutting of knots, are not applicable. In [1-3], a program for computing symbolic forces from the solution of the system of equilibrium equations of all nodes is described. A program written in the language of symbolic mathematics Maple implies the assignment of coordinates of nodes and the order of connecting rods, in the same way as a graph is defined in discrete mathematics. The truss rods and nodes are numbered (Fig. 2), the origin of coordinates is chosen and the coordinates of the hinges are entered:

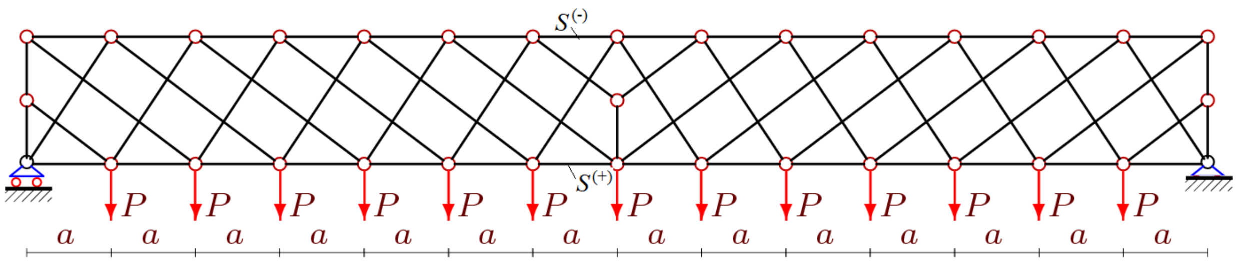

Figure 1. Truss, n=7.

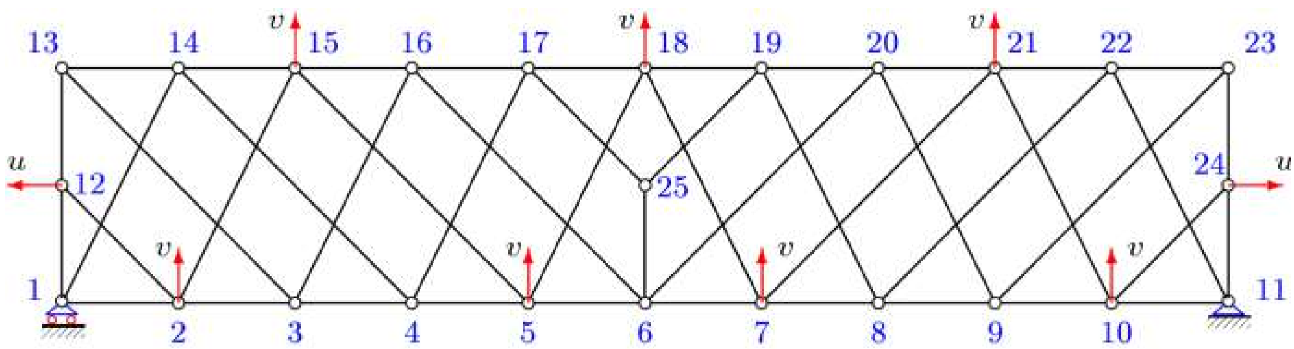

Figure 2. Numbers of rods and nodes, n=5.

$$x_i = x_{i+2n+2} = a(i-1),\ y_i = 0,$$
$$y_{i+2n+2} = 2h,\ i = 1,...,2n+1,$$
$$x_{2n+2} = 0,\ y_{2n+2} = y_{4n+4} = h,\ x_{4n+4} = 2na.$$

The order of connecting rods and nodes is given by vectors $\bar{V}$, the coordinates of which correspond to the numbers of finite hinges of the rod. Rods of the lower belt, for example, have the following coordinates:

$$\bar{V}_i = [i, i+1)],\ i = 1,...,2n.$$

Similarly, in the cycles, the remaining rods are also set, including the supporting ones:

$$\bar{V}_{m-2} = [1, m_s - 2],$$
$$\bar{V}_{m-1} = [2n+1, m_s - 1],$$
$$\bar{V}_m = [2n+1, m_s].$$

The matrix of the system of equations consists of the direction cosines of the forces, which are calculated by the formulas

$$l_i = \sqrt{l_{1,i}{}^2 + l_{2,i}{}^2},\ l_{1,i} = x_{V_{2,i}} - x_{V_{1,i}},$$
$$l_{2,i} = y_{V_{2,i}} - y_{V_{1,i}},\ \ i = 1,...,m.$$

In number $V_{j,i}$, the first index j takes values 1 or 2 and corresponds to the number of the vector $\bar{V}_i$ components, the second index corresponds to the number of the rod. Thus, taking into account, each force enters the equilibrium equations of two nodes at the ends of the rod and the projections have different signs, the matrix of direction cosines has the following elements:

$$G_{t,i} = -l_{j,i} / l_i,\ \ t = 2V_{i,2} - 2 + j,\ t \le m,\ j = 1,2,$$
$$G_{t,i} = l_{j,i} / l_i,\ t = 2V_{i,1} - 2 + j,\ t \le m,\ \ j = 1,2,$$
$$i = 1,...,m.$$

The system of equilibrium equations of knots has the form

$$\mathbf{G}\bar{S} = \bar{B},$$

where $\bar{S} = \{S_1,...,S_m\}$ is a vector of efforts, $\bar{B}$ — is a vector of external loads.

3. CASE OF KINEMATIC DEGENERATION

The very first calculations of the forces in symbolic form showed that for *n* = 2, 3, 5, 6, 8, 9 ... the determinant of the system of equilibrium equations vanishes. It is known that this corresponds to the case of instantaneous variability of the structure. Unfortunately, the known variability criteria do not apply here. There are no two disks connected by parallel rods or rods on the intersecting straight lines. There is no closed polygon of the rods of the lattice, considered by I.M. Rabinovich [13] on the basis of the work of Muller – Breslau [14]. However, for a truss you can find a kinematically consistent picture of the distribution of possible node speeds (Fig. 2), confirming that, indeed, with such a number of panels, the truss turns into a mechanism. Obviously, part of the rods, for example 2-12, 1-2, 1-12, make rotational movements, and part of them is translational: 2-15, 5-15, 5-18 and so do symmetrical rods. Most of the nodes 1, 3, 4, 6, 8, 9, 11 .., including the supporting ones, remain motionless. The velocities satisfy the obvious relation

$$u / h = v / a\ .$$

In order to exclude unacceptable values of *n* in a sequential calculation of trusses with different numbers of panels, we introduce the function $n = 3k - 2$. Changing in the process of counting the number *k* = 1,2, ..., we find by the method of induction the regularity of the formation of coefficients in the desired formula for deflection.

4. DEFLECTION

To determine the deflection of the truss, we use the Maxwell – Mohr formula,

$$\Delta = P\sum_{i=1}^{m-3} S_i^{(P)} S_i^{(1)} l_i / (EF_i),$$

where the summation is carried out only on the deformable rods of the truss and indicated: $S_i^{(1)}$ — forces from a single force applied to the middle of the lower belt (node n+1, Fig. 2), $S_i^{(P)}$ — forces in the rods from a given load, l_i — lengths of the rods. The stiffness of the rods EF_i in the General case are different. Let the area of the sections of the upper and lower zones are expressed in terms of a conditional area:

$$F_i = F / \gamma_1,\ i = 1,...,2n,\ i = 2n+3,...,4n+3.$$

The cross-sectional area of all bars of the grid, including the side racks, have the form:

$$F_i = F / \gamma_2,\ i = 2n+1, 2n+2, i = 4n+4,...,m-3.$$

A consistent analytical calculation of trusses with an increasing number of panels revealed that the formula for the deflection is the same:

$$\Delta = P\frac{C_{1,k}a^3\gamma_1 + (C_{2,k}c^3 + C_{3,k}h^3 + C_{4,k}d^3)\gamma_2}{8h^2EF}, \quad (1)$$

differing only by coefficients in degrees (cubes) of sizes

$$a, h,\ c = \sqrt{a^2+h^2} \text{ and } d = \sqrt{a^2+4h^2}.$$

In order to find the common term of the sequence of coefficients 4, 257, 2130, 8593, 24236, 55269, 109522, 196445, 327108, 514201 at a^3, you need to use the special operator **rgf_findrecur** to find the recurrence equation that this sequence satisfies:

$$C_{1,k} = 5C_{1,k-1} - 10C_{1,k-2} + 10C_{1,k-3} - 5C_{1,k-4} + C_{1,k-5},$$

and then solve it using the **rsolve** operator. The solution is

$$C_{1,k} = (135k^4 - 360k^3 + 405k^2 - 214k + 42)/2.$$

Similarly, we obtain expressions for other coefficients:

$$C_{2,k} = 4(6k^2 - 6k + 1),$$
$$C_{3,k} = 16(3k - 2),$$
$$C_{4,k} = 3(k-1)^2.$$

5. ANALYSIS OF DEFLECTION

Consider the case of a truss with a given span L and an arbitrary number of panels, so that

$$a = L/(2n).$$

The analytical form of the solution allows to clearly identify some of its features. We also fix the total load on the truss

$$P_{sum} = (2n-1)P.$$

We introduce notation for the dimensionless deflection of the

$$\Delta' = \Delta EF / (P_{sum}L).$$

In Figure 3, for $\gamma_1 = \gamma_2 = 1$ and L=100 m, curves of deflection (1) dependence on the number of panels are given, showing that in this formulation the relative deflection does not change with the increase in the number of panels.

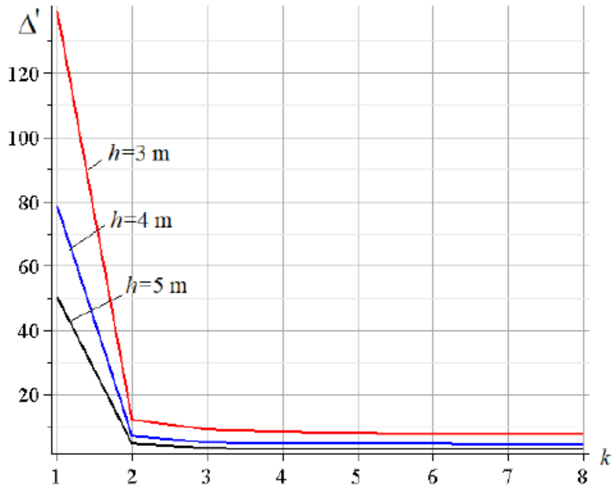

Figure 3. *Dependence of deflection on the number of panels.*

There is a mild oblique asymptote of the curve. For their analytical expression

$$\Delta' = \Delta_0 + \mu k$$

we calculate the limits:

$$\mu = \lim_{k\to\infty} \Delta'/k = \gamma_2 h / L,$$
$$\Delta_0 = \lim_{n\to\infty}(\Delta' - \mu k) = (5\gamma_1 L^3 + 256\gamma_2 h^3)/(768 L h^2).$$

Similar limits are obtained for truss in [15]. It is interesting to note that the angle of inclination of the asymptote depends only on the rigidity of the lattice γ_2, the height of h and the length of the length of the graph of the deflection decreases, and the angle of inclination of the asymptote is positive. It follows that the curves have weakly expressed extremum points, which can be used to optimize the stiffness of the structure. The dependence of the deflection on the height of the truss (Fig.4) also detects extremum. The curves are constructed under the same assumptions as the graph 3 at L=50 m.

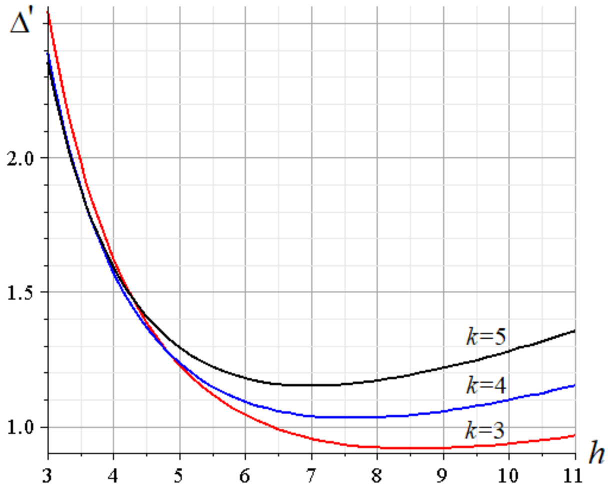

Figure 4. *The dependence of the deflection on the height of the truss.*

6. THE FORCES IN THE RODS

In the process of inductive derivation of the formula for deflection in the program the forces in the rods were also obtained. Maple graphical tools allow to give a visual representation of the distribution of forces in the rods of the belts and the grid. In figure 5, the thickness of the rod lines are proportional to the forces in them. Blue highlighted in rods subjected to compression, red — stretchable. The forces related to the magnitude of the load are also indicated.

As one would expect, with such a load, the most compressed rods are in the middle of the upper belt, the stretched ones are in the middle of the lower one.

For these forces (Fig. 1), the induction method from the analysis of six solutions for trusses with a successively increasing number k associated with the number of panels obtained formulas:

$$S^{(-)} = -Pa(9k^2 - 10k + 3)/(4h),$$
$$S^{(+)} = Pa(9k^2 - 10k + 1)/(4h).$$

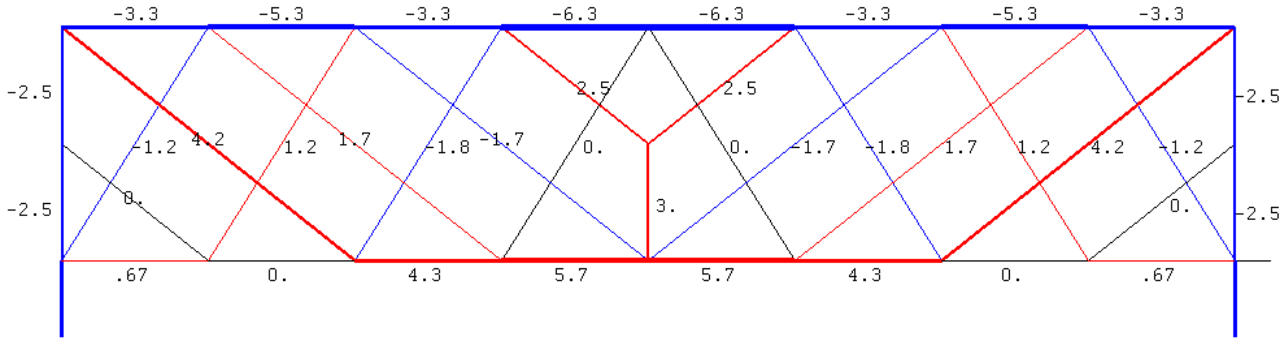

Figure 5. Distribution of relative forces, n=4.

CONCLUSION

The two main results of the study: the functional dependence of the deflection of the truss on the number of panels and the detection of a case of kinematic degeneration of the system. Both the first and the second result owe their appearance to the system of symbolic mathematics. It has been verified that, in numerical form, due to rounding errors, the fact that the determinant of the system of equilibrium equations is zero often escapes. If it is thoughtless to transfer the decision obtained for one number of panels to another, then this may lead to unpredictable consequences during the operation of the structure. Of course, replacing the joints with hard welding will soften the defect inherent in the design. However, to increase the rigidity and reliability of structures, as shown by the present study, it is better to avoid similar effects by correctly choosing the characteristics of the designed structure.

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NATURE SIMILAR TECHNOLOGIES IN CONSTRUCTION INDUSTRY

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Abstract: In modern conditions it is necessary to create high-tech, reliable and durable composites of a new generation with the required properties, and this requires qualitatively new approaches in the design, synthesis, operation, destruction and reuse of sources, that is based on the introduction of fundamentally new nature-similiar technologies. The great interest not only in Russia, but also abroad, present additive technologies.
The article proposes the technology of using water-resistant and cold-resistant quick-hardening composite on the basis of gypsum binders of a new generation with finely ground mineral additives of different genetic types, including using a new unique type of mineral additives - waste from the magnetic separation of ferruginous quartzite.

Keywords: bionics, geonics (geomimetics), 3D technologies, additive technologies, composite gypsum binders, mineral additive, construction composites

ПРИРОДОПОДОБНЫЕ ТЕХНОЛОГИИ В СТРОЙИНДУСТРИИ

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Аннотация: В современных условиях необходимо создание высокотехнологичных, надежных и долговечных композитов нового поколения с требуемыми свойствами, а для этого необходимы качественно новые подходы при проектировании, синтезе, эксплуатации, разрушении и повторном использовании сырьевых ресурсов, основанные на внедрении принципиально новых природоподобных технологий. Большой интерес не только в России, но и за рубежом представляют аддитивные технологии. В статье предлагается технология использования водостойких и морозостойких быстротвердеющих композиционных гипсовых вяжущих нового поколения с тонкомолотыми минеральными добавками разных генетических типов, в том числе с использованием нового уникального для строительного материаловедения вида минеральных добавок - отходов мокрой магнитной сепарации железистых кварцитов.

Ключевые слова: бионика, геоника (геомиметика), 3D-технологии, аддитивные технологии, композиционные гипсовые вяжущие, минеральная добавка, строительные композиты

Currently, humanity is on the threshold of a difficult stage of its development. The new century is characterized by serious problems linked with the depletion of hydrocarbons, the shortage of fresh water, the intensification of natural and man-made disasters, and environmental degradation. Human habitat are deteriorated sharply [1].

It is known that a person spends up to 90% of his life in an artificially created environment - in rooms built of various building materials, which largely determine the performance, mental activity, creative mood, psycho-emotional state and, finally, the duration of a person’s life.

Figure 1. Evolution of human habitation.

The impacts to humans are increased. That manifests itself in the form of cosmic radiation on the human body, which leads to increased fatigue, headaches, irritability. In the course of medical research, it has been established that the prolonged affecting of variable electromagnetic fields on the human body causes disturbances in the cardiovascular and nervous systems. Its manifestation affects to the decrease in performance and reduced accuracy during operation, as well as in accelerated human fatigue. The effect of a variable magnetic field on the human body is manifested in pains in the region of the heart and in headaches.

The vibrations have a negative effect on a person. A progressive increase in the values of the vibrations lead to disruption of the nervous system and rapid fatigue, disruption of the body. Often, people do not think about environmental factors, and their homes do not fit in the surrounding landscape by configuration and form [2].

The solution of such significant problems is possible through the using of a new paradigm of science – the application of transdisciplinary research, which in particular includes cybernetics, bionics, geonics [3-9]. The bionic approach to the study of living nature, and above all the morphology, ecology and physiology of living organisms, their elements and populations, is turned out to be very productive to solve the complex problems of scientific and technological progress.

The organization of the human habitat is determined, above all, the biological need to ensure the conditions of human existence. The formation of the fundamentals of building culture was determined by the natural conditions: the climate of the region, temperature fluctuations, humidity, the nature of the landscape and vegetation, the availability of building materials and others. Depending on the region features, climatic conditions, availability of materials from which it was possible to build up shelters, people created their first homes (Figure 1).

Figure 2. *Clams with shells.*

Figure 3. *Shellfish Shells.*

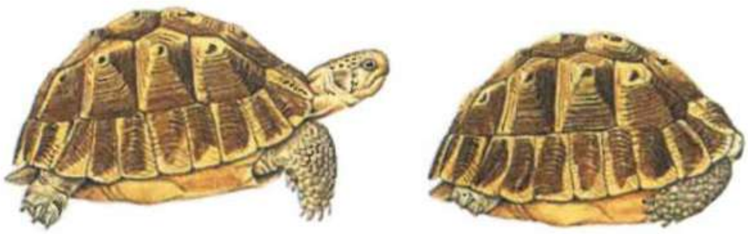

Figure 4. *Turtle transformation.*

Figure 5. *Armadillo Transformation.*

Figure 6. *Chum is the dwelling of the humans of the north.*

Figure 7. Modern residential buildings.

The configuration of the dwelling deserves special attention. Creating the safe and comfortable dwelling the humanity, since the most ancient times, imitated the animal world and nature and used the protective functions inherent in them (Fig. 2-6).

For the purpose of rational heating, heat preservation and safety, the dwellings were built of a streamlined semi-circular or conical shape, which is chosen by mollusks, foraminifera and fauna representatives for almost a billion-years period of evolution. Until now, the dwellings of northern peoples have a conical form, which provides optimal conditions for keeping warm and comfortable stay of people (Fig. 6).

All historical epochs that had left unique testimonies of human dwellings testify to the imitation of the natural forms of the organic and inorganic world by human. In the early stages, these were oval niches in caves, as the most convenient and comfortable forms for placing a person and preserving heat. Later, the human found that the oval and spherical shapes are the most durable. And human transferred this experience to the construction of dome structures.

Thus, man accepted the billionth experience of the history of the animal world as the basis to create his own shelters and dwellings.

The environment in which we live carries a huge amount of information that negatively affects us. The information coming from human's environment has the greatest influence on the his psyche and thoughts, goals and desires. During the study, certain regularities are found in the relationship between the geometric characteristics of the architectural-spatial form and the psychological response of the person to it (Figure 7).

Humanity appeared on the Earth about 70-50 millennia BC. and from the beginning of its evolutionary accumulated knowledge and improvement of the world. Over time, the sources of knowledge and research methods were differentiated.

The 21st century requires the creation of modern high-tech structures, reliable and durable composites of a new generation with the required properties, and it requires qualitatively new approaches in the design, synthesis, operation, destruction and reuse of materials based on the introduction of fundamentally new nature-similiar technologies.

Unfortunately, constructive solutions of buildings and structures at the last centuries switched to the use of vertical and horizontal elements (Figure 7), excluding the billionth experience of living nature.

3D-additive technologies are the representative card of the modern century. It has great potential in reducing the energy costs for creating a wide variety of products. Despite the many positive features of 3D printing, the introduction of these technologies in Russia has not yet reached a significant level [10-14].

The emergence of 3D printing technology or additive manufacturing is not sudden and it is generally accepted that the foundations of it were laid far in the past. The essence of additive production is summing or technique to create parts of complex shape, when the material is applied sequentially, usually, layer by layer. So it consumes as much as it is necessary to create the required forms. Using of additive production technology to construct the buildings and structures, we will significantly reduce costs by reducing material costs and increasing productivity, and will open new creative approaches in the architectural exterior of our cities and create more comfortable conditions for human habitation, that is especially important now.

New constructions should be created taking into account centuries-old building experience. The theoretical basis for the design, development and implementation of these technologies are bionic approaches, the use of the provisions of geonics, the law of the affinity of structures and human-made metasomatism in construction materials.

The theory of technogenic metasomatism in construction materials science, the law of affinity of structures (consisting in designing layered composites, mortars and repair systems, that at the nano, micro and macro levels are similar to the base matrix) are formulated within the framework of theoretical concepts of geonics (geomimetics). The development of new composites should be based on new sources.

The theoretical positions are implemented in the design and synthesis of efficient composites of a new generation based on multicomponent systems with micro, ultra and nano dispersed fillers in combination with other additives.

A whole spectrum of fast-hardening waterproof and frost-resistant composite gypsum binders (Table 1), modified by various types of mineral additives of different genetic types, has been developed [16-17].

To obtain concrete mixtures, that does not decompose to fractions, on the basis of CGB (composites on the basis of gypsum binder) for using at the manufacturing of densely reinforced or thin-walled building products and structures by the technology of layer-by-layer synthesis, the complex chemical additives are used. These additives include setting time retardants, super- and hyperplasticities that can provide the ability to control and regulate the structure formation in a plastic state and in the process of forming the structural strength of composites (Table 1).

Finely dispersed fillers with a specific surface area at least 500 ... 600 m2 / kg, obtained by fine grinding of technogenic sources (waste of magnetic separation of ferruginous quartzites, screenings of quartzitic sandstone, concrete scrap, etc.), of natural resources (silica sand, flask, perlite, tuff, chalk, etc.) contribute to the effective management of the internal structure formation of composites, providing high quality products based on this. A distinctive feature of this raw material is its activation due to air geological processes, which is manifested in the defectiveness of the crystal lattice, the presence of inclusions of the mineral-forming medium and gas-air inclusions, etc. (Figure 8).

The process of forming a single gypsum-cement matrix is also activated due to nucleating agents, which are fine ground concrete scrap and components of the proposed raw materials.

Compounds CGB with microdispersed mineral additives from technogenic raw materials, with reinforcing fibers and complex chemical additives of high water resistance and durability of compressive strength classes B5 – B30, average density D1000–2100 kg / m3, frost resistance F20 – F50, Kp = 0.65 –0.78 are developed.

The positive properties of gypsum composite materials (low cost, environmental friendliness, rapid curing, good heat and sound insulation properties, absence of shrinkage deformations, good thermal insulation and sound-absorbing ability, fire resistance, a positive effect on people's health by creating a favorable microclimate in the rooms, etc.) allow us to maintain and improve the performance of buildings and the comfort of one's internal environment.

Gypsum composite materials have a great future. The growth rates of it production and applying should be significantly higher than that is for all other building materials. It will allow not only to improve the environmental situation, reduce the energy intensity of the construction industry, but also create comfortable conditions for human existence. The gypsum-containing composite binders are most suitable for use in 3D additive technologies, due to their unique properties.

Table 1. Affecting of chemical additives on the properties of CGB (with waste MS), (W/ B = 0.46).

Additive type	Additive contains in volume, mass, %	Spread, m	Hardening time, min.,sec.		Compressive strength, MPa, after		
			begining	finish	2 h	7 days	28 days
Without additives	—	<u>0,120</u> 0,180	<u>7-40</u> 8-30	<u>9-40</u> 11-30	<u>4,6</u> 3,2	<u>14,8</u> 13,2	<u>20,0</u> 13,6
based on naphthalene							
С-3	0,1	0,160	8-30	11-30	5,5	22,0	23,3
	0,3	0,180	8-30	11-00	5,1	21,5	22,0
	0,5	0,220	8-20	11-00	4,9	19,0	20,5
Polyplast СП-1	0,1	0,160	7-45	10-45	5,0	14,5	15,7
	0,3	0,185	7-30	10-30	4,4	13,9	14,7
	0,5	0,220	7-15	10-15	4,2	13,4	13,9
melamine based							
MELMENT F10	0,1	0,165	8-15	11-50	5,8	14,4	14,7
	0,3	0,195	7-45	9-57	4,7	13,6	13,9
	0,5	0,225	7-30	10-35	3,9	13,1	13,6
based on resorcinol waste							
СБ-3	0,1	0,145	9-30	12-00	5,4	21,0	21,7
	0,3	0,200	18-00	22-20	3,8	19,2	20,0
	0,5	0,220	25-00	28-30	2,9	17,5	18,0
With lemon acid and wastes of it production (CF)							
Lemon acid	0,03	0,160	19-15	25-20	4,8	9,3	10,1
	0,05	0,162	24-20	29-00	5,1	9,9	11,2
	0,07	0,162	29-30	34-30	5,6	10,4	11,9
Citrate filtrate (CF)	0,3	0,120	10-30	15-30	5,5	17,0	17,2
	0,6	0,120	15-30	20-00	4,5	16,0	16,9
	0,9	0,120	20-00	25-00	4,2	14,2	14,5
	1,5	0,120	29-00	36-00	3,8	11,5	13,2
Complex chemical additives							
С-3-(0,5%)+ЦФ(1,5%)		0,180	45- 00	58-00	4,0	11,6	13,5
СБ-3 (0,5%+ЦФ(1,5%)		0,180	53- 00	72-00	4,3	9,0	12,3
Lemon acid – (0,05%) + Polyplast СП-1(0,3%)		0,265	30-00	35-15	4,3	13,2	13,8
Lemon acid (0,05%) + MELMENT F 10 (0,3%)		0,250	30-00	35-30	4,2	9,5	11,5

Using high-strength composite gypsum binders, the quick-hardening reaction-powder concrete reinforced with steel or polymer microfiber has been developed. A feature of this composite is the absence of coarse aggregate without loss in the binder / solid ratio, as well as high performance (concrete stregth ckass is B60 and more). Sand concrete on the reaction-powder binder have a high coefficient of structural quality, that makes it possible to create structures with a smaller volume as compared to traditional ones, respectively, with less weight and reduced material consumption [15].

The development of building composites, including powder composites, as well as the organization of it production by additive technologies will allow:

- to provide the construction industry with products of complex shape, with high performance;

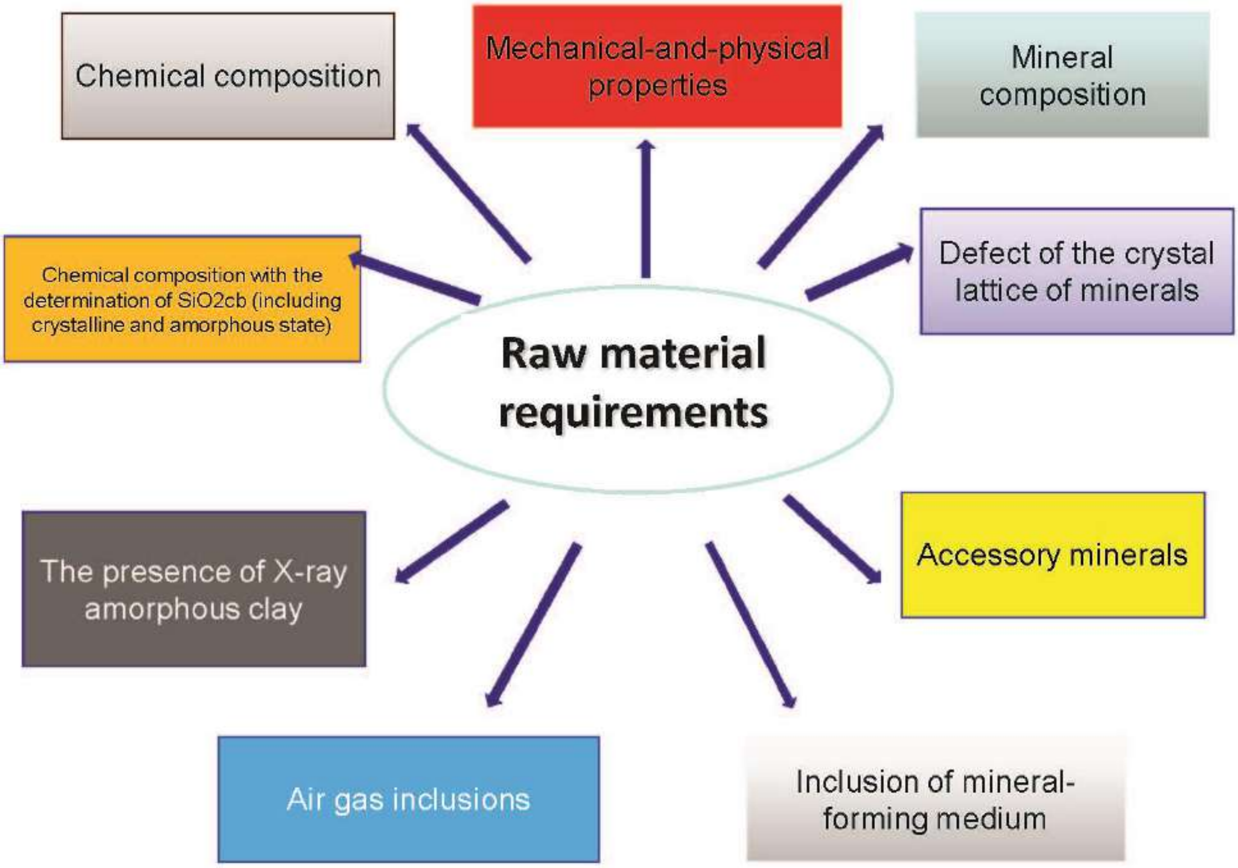

Figure 8. Raw materials requirement.

- exclude technological dependence on the foreign companies-suppliers products for domestic production;
- reduce the cost of manufacturing products of complex shape due to the rejection of expensive machining operations;
- to increase the competitiveness of high-tech products in the international and domestic markets;
- reduce building duration in many times, etc.

The introduction of 3D-technologies will allow to build residential buildings comfortable for a human living and having a rational form and configuration similar to the parameters characteristic of the animal world.

Thus, in the modern world, to create a comfortable human environment, it is necessary to develop rational forms of buildings and structures, taking into account the multimillion experience of the biological world, protective systems against negative factors, that is developed by representatives of the organic world. Highly effective quick-hardening systems based on composite waterproof and frost-resistant gypsum binders using new types of raw materials are proposed. Such composites as a result of natural technological activation of rocks due to geological or human-made processes are significantly different from the traditionally used raw materials, i.e. it is genetically activated.

The use of 3D-additive technologies for the construction of buildings and structures will allow to create nature-similiar architectural structures comfortable for human life, able to protect us from negative natural and human-made impacts.

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THE FORMATION AND DEVELOPMENT OF CONCEPTS ABOUT THE DESIGN SCHEME OF STRUCTURES

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Abstract: The history of the formation and development of design model concept of a structure accompanied structural mechanics at all stages have is analyzed in the paper. The main principles used at constructing design models are considered. New problems arisen in the process of computer-aided structural analysis also are specified in the distinctive paper.

Keywords: design scheme, structural mechanics, finite element method

СТАНОВЛЕНИЕ И РАЗВИТИЕ ПОНЯТИЯ О РАСЧЕТНОЙ СХЕМЕ СООРУЖЕНИЯ

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Аннотация: Анализируется история становления и развития понятия о расчетной схеме сооружения, которое сопровождает строительную механику на всех этапах ее существования. Рассматриваются основные принципы, используемые при построении расчетной схемы. В настоящей статье также указывается на новые проблемы, возникающие в процессе компьютеризации расчетов.

Ключевые слова*:* расчетная схема, строительная механика, метод конечных элементов

The creation of design models of structures is simultaneously a task of experts in structural mechanics and of those in structures. Various approximations of the real physical service of a structure may be created only under their joint work.

I.I. Goldenblat, V.L. Bazhenov

A design model reflects a designer's idea of the real investigation object and peculiarities of its behavior. It is a simplified object model deprived of insignificant details and closely related to a set of some physical notions of laws, which control the investigation object behavior. Nowadays great experience exists in the design model development, and, preceding from this experience, in each specific case the following "type members" are used: such as shape idealization (a bar, plate, shell), regularities of material behavior (elastic, plastic, etc.), rule of these members coupling, etc. This designer's arsenal was developed in the course of the whole history of structural mechanics as science and continues perfecting in the present.

1. BEGINNING OF THE PATH. ANALYSIS OF CERTAIN PROBLEMS

An idea of a design model probably appeared simultaneously with science of strength in 1638, when the book by Galilei *Disputes and Mathematical Proofs Concerning Two New Sciences* was published, though the term *design model* appeared much later.

Just first attempts of design analysis of structure behavior, the attempts, which were aimed at the search of failure load, proceeded from certain hypotheses on location of dangerous section and force distribution in it. A set of these hypotheses could be called now a design model or design diagram.

Galilei thought rigid bodies to be inelastic and studied the problem on the bar strength, considering it in the state of failure (limit state in terms of the present). He attributed failure to two types of deformation – tension and bending.

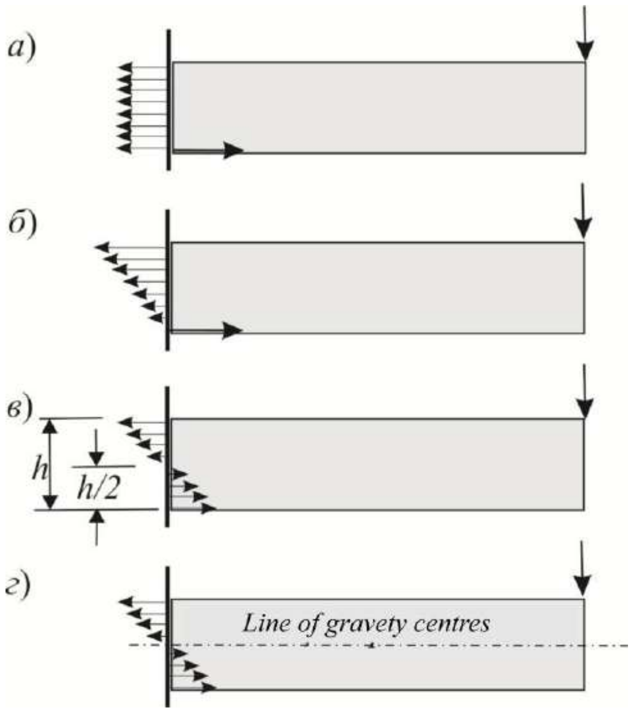

Figure 1. Bending resistance.

In the first case strength was taken as proportional to the cross-section area, Galilei bound the second case with the first one, supposing that the cantilever break occurs by crack opening displacement from above and rotation about the lower rib, the whole section being uniformly extended (Fig. 1,*a*). A question of the break place was not raised in the explicite form, Galilei probably thought it obvious.

Several laws of stress distribution throughout the section height were further offered: Mariotte [Mariotte, 1686) and Leibniz [Leibniz, 1684) considered the distribution as linear with coordinate origin at the section edge (Fig. 1,b), while Parent [Parent, 1713] used the same law, but distributed the coordinate origin in the centre of the section height (Fig. 1, c). And only Navier [Navier, 1826] placed the coordinate origin in the centre of gravity (Fig. 1, d). At last Persi, Navier's companion-in-arms at the school of bridges and roads, when developing Navier's approach, introduced an idea of the section inertia moment, which becomes and still remains a necessary attribute of the description of the schemes of bar structures.

The approach, which was based on the search for the break patterns and used a model in a form of a set of infinitely rigid blocks (the loss of link among them being connected to one or another extent with the break) prevailed in the problem of arch strain for a long period of time [Bernshtein, 1936]. An important point is that the shearing schemes appeared among possible break schemes (Fig. 2).

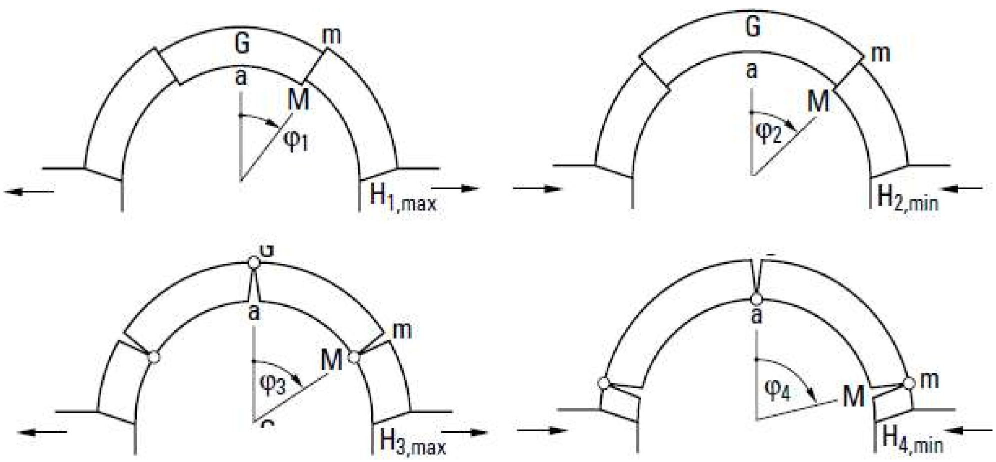

Figure 2. Coulomb's variants of arch destruction.

The problem on the shape of flexible filament also belongs to the first period of formation of structural mechanics. It developed in two directions. The first one was connected with the problem formulated by Jacob Bernoulli: "to find what shape takes a rope freely hanged at two points", and this direction has played the important part in formation of mathematical analysis.

Another direction may be connected with the name of Varignon, whose major work [Varignon, 1725] was published after his death and dedicated to the theory of funicular polygon – the design model, which is one of the principles of graphostatics.The problem on the funicular polygon attracted interest 100 years later in connection with the problem of design of suspended bridges, which chains were funicular polygons.

The discovery of Hooke's law in 1660 and the establishing of Navier's equations in 1821 are undoubtedly two important milestones in the further development of the theory initiated by Galilei. Hooke's law gave a necessary experimental substantiation of the theory.

In the period between deducing the Hooke's law and establishing general equations of elasticity theory obtained by Navier the interest of researchers was directed to solution and generalization of Galilei's problem, to allied problems which concerned vibrations of bars and plates. The first significant research in this field was made in 1705 by Jacob Bernoulli. It concerns the shape of an elastic curve of a bar and is based on the admission that the resistance of a bent bar depends on tension and compression of its longitudinal fibers.

When deriving the bar bending equation J. Bernoulli used the Hooke's law, and besides, two following hypotheses:

- the sections, plane and perpendicular to the prism ribs before its bending, remain after the bending plane and normal to these ribs and fibers or longitudinal members that become curvilinear;
- the fibers, some of them being extended others – shortened, resist independently the bending, as if they were small isolated prisms, taking no effect on one another.

The same propositions were further taken by Euler in his research, which concerned the problems of elastic line and vibrations of thin bars. The Euler-Bernoulli design model of a bar presented an elastic bar in a form of a linear set of particles resisting the bending.

The successful development of the theory of thin bars, based on special hypotheses, led to a conclusion that the theory of plates and shells may be constructed in the same way. Euler was the first, who was concerned with this problem. He offered to consider a bell as a set of thin rings, each of them behaving as a curved bar. This work was followed by the research of Jacob Bernoulli (junior). He considered a shell as a double layer of curved bars, the bars of one system intersecting with the bars of the other system at the right angle [Bernoulli, 1789). Reducing a shell to a plane plate, he obtained an equation, which, as we know it today, was incorrect (he excepted twisting for the bar).

The attempt of Jacob Bernoulli was, probably, a purpose to obtain theoretical substantiation of experimental results by Chladni [Chladni, 1802], as to the figures of nodal lines observed under plate vibrations.

These results remained unexplained, when in 1809 the French Institute offered the problem on the tones of plate vibrations as a bonus theme of scientific work. After some attempts there appeared a work by Sophie Germain awarded in 1815 and published only six years later [Germain, 1821].

But a distinct design model of a bending plate was proposed only in 1850 by Kirchhoff [Kirchhoff, 1850], who based his theory of plates on the following two hypotheses, generally recognized nowadays:

- each straight line, which was first perpendicular to the midplane of a plate, remains under bending a straight line normal to the middle surface of a bent plane;
- elements of the plate midplane are not elongated at small plate deflections under transversal load.

These admissions are close by content to hypothesis of plane sections taken today in the elementary theory of bar bending.

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2. ELASTIC BAR SYSTEMS

Before the 30's of the 19th century structural mechanics had in possession the design models of bars, arches and plates – the base elements composing real structures. All these design models were realized separately, while they interact in numerous cases, being separate fragments of a more complex structure. If in the 18th century the design and technical development of civil engineering was concentrated on stone arches, than in the 19th century the interest of engineers changed and they were oriented to analysis of skeleton constructions. In connection with a rapid development of railway engineering the transition from elementary carrying systems to composed constructive designs was much more prompt than in cast-in-place constructions (such as masonry and concrete), under these conditions geometrical and physical properties of such structures became a logic abstraction of the design model in a form of a truss.

Such a design model was used to construct bridges with various structural systems, proposed by James Warren, Stephen Harriman Long, William Howe and other inventors [Perelmuter, 2015].

The considerable number of wooden bridges in North America was described by Carl Culmann, indicating those with the signs of damage and failure, in spite of the generous use of materials [Culmann, 1851]. Culmann indicated different structural systems in those bridges and noticed that they could better perform their function on condition of correct design.

He created a theory of braced structure, based on the following admissions:

- a system of filling with bars between the top and bottom chords should be made in such way that all the bars formed triangles;
- the bars should have a possibility to turn in joints without restriction

Using the equilibrium conditions Culmann could calculate forces in the elements of any statically determined girder structure of the above type.

The work by Schwedler [Schwedler, 1851] appeared almost simultaneously, the author indicated (see Fig. 3):

"*If a structure as a whole is considered as rigid, small resistances caused by elastic bending at the points a,, d, c, etc. are insignificant compared with resistance of braces, or, in other words, it may be taken that separate components of the truss can turn at the points a, d, c, etc.*"

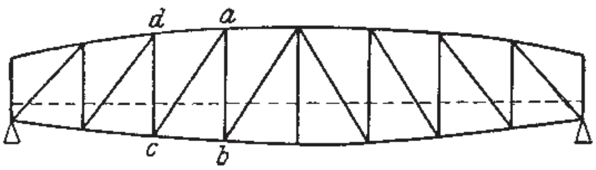

Figure 3. *Hinged model of Schwedler frame.*

Schwedler has first performed the process of abstraction, which is typical of the structural theory: from the physical carrying structure (real through system, e.g. wooden truss) through the abstract carrying system (model of a through structure or girder bar system according to Culmann) to design model (of a hinged truss), described with the help of physico-geometrical properties.

The invention of design model in a form of hinged truss has become a key concept for development of the structural theory in the second half of the 19th century. It is important that independent analysis of topologic structure might be made for this model. Such analysis was intensively developed in the works devoted to revealing kinematic properties of truss structures and then of the bar structures of any other kind.

Another important achievement, which originated from the design model of the truss, was the development of the conception of a node – a hinge joining the truss bars. Then the node hinge was considered as a material point, the equilibrium equation being formulated for it, and in this quality the node became an integral part of design model of the bar (and not only) systems. The design model of a truss was rather evident, and the truss work proved very positive. Many engineers tried to construct the truss model. Schwedler developed hinged nodes for the bridge over the Brache river (now the Brda river) near Czersk built by his project in 1861 (Fig. 4). But nine years later the other bridge over the Brache river was constructed near Bromberg (today Bydgoczsz), and now by Schwedler's project using riveted joints.

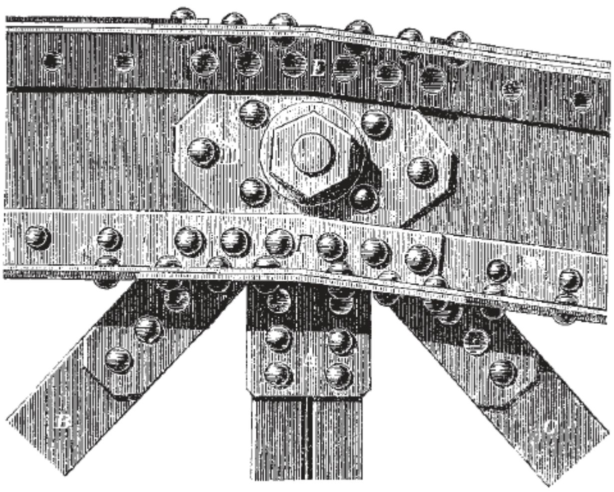

Figure 4. Bridge truss nodes.

Emil Winkler realized that the hinged model of the truss did not correspond to real work of metal trusses with riveted joint [Winkler, 1872]. Some bending moments appear due to the node rigidity in the truss bars, and as a result – additional stresses. The problem of their determining that first of all attracted attention of Manderla [Manderla, 1880], resulted in the appearance of the method of displacements.

Design models of skeleton structures, where most nodal joints are rigid, began quickly extending in connection with construction of reinforced concrete frames. And design model of a truss with ideal hinges proved to be a certain approximation to reality.

E.O. Paton estimated the degree of relative increment of stresses that appear at the expense of node rigidity [Patton, 1901], and as his studies have shown, the more precise is the "truss approximation" the more is the flexibility of the truss bar elements. In essence, there arised an important question, which soon arised in other situations: the question on usability limits of one or another design model, of the necessity of its specification or cardinal change, when its parameters are outside a certain limit.

For example, S.P. Timoshenko proposed a model of a bending beam, which difference from the Euler-Bernoulli model is that under deformation the cross-sections remain plane but not perpendicular to deformable midline of the bar, and inertial components connected with a turn of cross-sections are accounted in dynamics [Timoshenko, 1916]. E. Reissner proposed an analogous perfection of the Kirchhoff model for plates [Reissner, 1945]. In both cases the point was in the necessity of introducing some specifications, when shearing deformations begin playing a considerable part.

The transition from analysis of truss models to investigation of frame systems became a large-scale one at the end of the 19th century and especially in the first half of the 20th century. That was caused by the intensive use of cast-in-place reinforced concrete constructions in civil engineering.

Formulation of the problem on general rules of development of design models belongs to the first quarter of the 20th century, and here we should note the work of N. Gersevanov [Gersevanov, 1923], who has first formulated that:

- the design model is constructed proceeding from the expected form of failure and deformation based on the experience of building practices;
- the design model uses only hypotheses concerning the structure properties and actual loads, which allow developing the efficient methods of calculation.

The design model substantiation problem itself, besides the use of the results of experimental studies, developed in the following direction. Researchers proposed the ways to transform the design model of a more general form, e.g. a model of three-dimensional continuum of the problem of elasticity theory, to one or another model of the structural unit of a certain type. Such an approach was especially often used in development of the theory of plates and shells.

The first attempt of deriving equations of the theory of shells from the equations of elasticity theory was made by G. Aron [Aron, 1874]. Then this trend was developed in the works by A. Love [Love, 1888], A. Basset [Basset, 1892], H. Lamb [Lamb, 1890], A.I. Lurie [Lurie, 1947], et al.

After deriving the resolving equations of the shell theory and developing the corresponding design models researchers began developing various non-classical variants of the theory. Here one should recall the theory of shells of Timishenko-Reissner type that allow for longitudinal shear deformations. Besides, the theory of the ribbed and multilayered shells may be referred to non-classical ones.

First works in this field for the plates reinforced by ribs were made by I.G. Bubnov [Bubnov, 1904]. The theory of ribbed shells of a general form was presented in the works by A.I. Lurie [Lurie, 1947] and V.Z. Vlasov [Vlasov, 1949]. A.I. Lurie considered ribs as the Kirchhoff-Clebsch bars, while V.Z. Vlasov considered them as thin-walled bars.

Multilayered shells were investigated from different viewpoints in a lot of works mainly in two basic directions. The first one includes theories based on design models, where kinematic hypotheses were taken for the whole set of layers. The first-stage researches have demonstrated nonperceptibility of this approach, if properties of the layers are essentially different; that is why the works of another direction were developed in recent years; a complicated design model, where kinematic hypotheses are taken separately for each layer, was used in these works.

3. STRUCTURAL ANALYSIS

A detailed analysis of separate problems and simple objects led to development of such a notion as a material point, absolutely solid body, elastic bar, plate, etc. The properties were studied for them, which are used, when a more complex model of the problem required is constructed using such parts as of a certain “constructor”. And a problem of analysis of the composed design model of such kind appeared in natural way. Most researches were devoted to design models of trusses; they attracted by their “homogeneity” and distinct division of topologic (structure, fixing) and metric (node coordinates, section sizes) data.

As to the complete system of determining equations we should say that Alfred Clebsch has shown in his work [Clebsch, 1862] that a set of equilibrium equations and those of deformation compatibility for an arbitrary truss has the solution [Clebsch, 1883]. But the problem of solution possibility was, first of all, considered from the viewpoint of equilibrium equations issuing from the composed design model – equations of analysis of its statical determinability and invariability. Even in 1837 A.F. Möbius proved the theorem that to obtain a rigid invariable structure in a truss with *n* hinges it is necessary to have no less than 2n-3 bars in a plane system and no less than 3n-6 bars in the case of a spatial system [Möbius 1837]. In so doing he has probably first indicated a possibility of existence of exceptional configurations, when one observes infinitesimal mobility without bar deformation (a case of instant variability in the current terms)

When studying these cases Möbius has found that therewith a determinant of the set of equilibrium equations becomes zero. The connection between the variability criterion and degeneracy of the system of resolving equations became after a while the basis for computer analysis of kinematic properties of design system of any (not only truss) type. The results obtained by A.F. Möbius, which then remained unknown, were found again by P.L. Chebyshev [Chebyshev, 1870] and Otto Mohr [Mohr, 1874] and only then entered in the design practice.

Mass enthusiasm as to the method of forces, characteristic of the end of the 19^{th} and first half of the 20^{th} century, resulted in the appearance of various procedures of construction of the basic system of this method and in the problems of revealing the redundant constraints in statically undeterminable design models. Relations between the properties of static determinability, invariability and ability to realize the pre-stress were studied in detail for the bar systems. Researchers indicated methods for determining statical-kinematic properties based on reducing the system to a certain number of hard discs, bound by bars-restraints. They also introduced the notions of simple and multiple hinges and other idealized elements of the design model.

Noticeable changes in the concepts of design model are connected with the transition to displacement method analysis. In the displacement method the system elements are considered to be connected with the nodes of the design model, they are not connected directly with one another. The above peculiarity of the design model construction was often disregarded by engineers educated on the ideas of design model in the style of the force method, it is not always seen, when using the methods of design model representation that is traditional for the force method. Thus, the design system presented in Fig. 5,*a* in traditional form, inherent in the force method, can suggest the point-to-point connection of elements with one another, while a more detailed representation in Fig. 5,*b* allows avoiding such conclusion. Note also that in the detailed presentation one can also see other peculiarities of the design model implementation, in particular, a possibility to meet similar kinematic conditions with using various sets of constraints imposed on the nodes, and conditions of elements connection with the nodes.

Neglect of the above difference is not always safe. For example, from the viewpoint of kinematic properties of the problem two variants of the design model, presented in Fig. 6, have equal rights (a beam is fastened in its left end and hinged in the right one).

But from the viewpoint of giving forces these variants differ – in the scheme of Fig. 6,*b* the moment is transferred to the bar, and node 2 in this scheme turns, and in the scheme of Fig. 6,*a* the moment is not transferred, and node 2 of this scheme has zero turning angle.

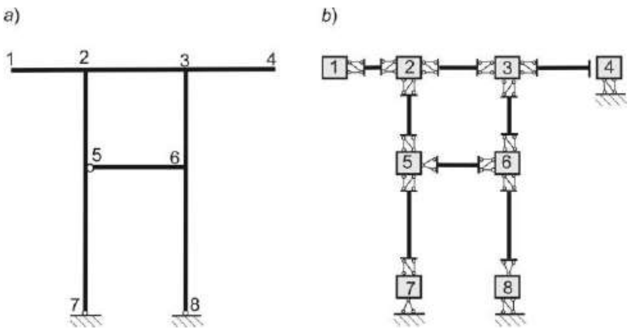

Figure 5. Presentation of design model: a – traditional; b – detailed.

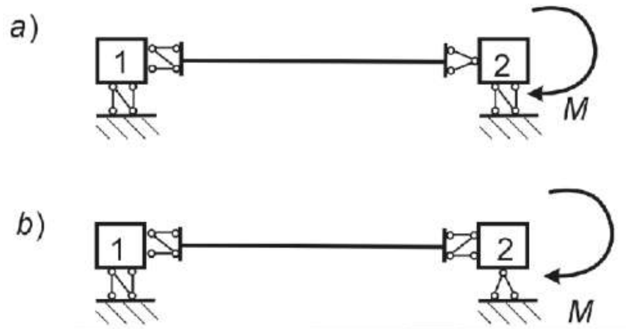

Figure 6. Two variants of presentation of one design model.

For a bending moment to appear in a bar in the scheme on Fig. 6,*a*, it should not be considered as nodal, but applied to the bar in the section near a node.

The above division of topologic and metric properties of the design model gave impetus to the works connected with the use of graph theory to analyze properties of bar systems. Such was the approach in the pioneer publications [Fenves & Branin, 1963], [Perelmuter, 1965], while in the work by Di Mattio [Di Mattio, 1963] the degree of static indeterminability is studied just as topologic property of the design system. Later on there appeared works, where topologic connectedness of design model was compared with the structure of distribution of nonzero elements of rigidity matrix of the system that is analyzed, and a possibility of optimal enumeration of unknowns [Akyjz & Utku, 1968], [Clempert, 1973]. Researchers proposed some artificial techniques aimed at the improvement of the above structure even at the expense of increasing the rigidity matrix order [Perelmuter, Slivker, 1976].

Practical interest to the analysis of suspended and rope systems, characteristic of the works of the second half of the 20th century, resulted in a detailed study of topologic and metric properties of degenerated (instantly variable and instantly rigid) systems. There appeared a number of fundamental works [Kuznetsov, 1960], a lot of researches were initiated by introduction of the systems of "tensegrity" type by Buckminster Fuller [Fuller, 1961]. He used this term to indicate the frame structures, involving continuous chains of members, which work in tension, and inserted members, which work in compression.

Study of properties of instantly rigid systems and systems of "tensegrity" type preferred to come back to general principles of analysis of static-kinematic properties of the composite design models [Shulkin, 1977], [Calladine, 1978], [Connelly, 1980]. The systems with unilateral constraints, possible combinations of static and kinematic properties being established for them [Perelmuter, 1968], were also considered.

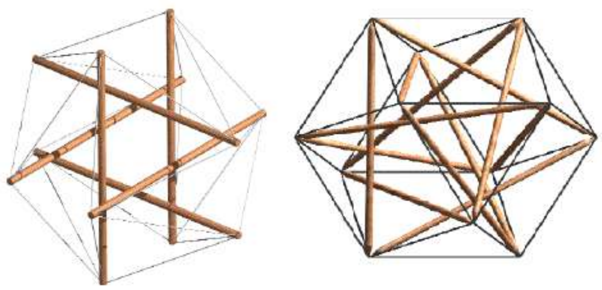

Figure 7. Examples of tensegrity structures.

4. DESIGN MODELS OF THE FINITE ELEMENT METHOD

The appearance and development of the finite element method (FEM) has told essentially on the problem of choice and substantiation of the design model. Even a description of geometrical pattern of the structure became a choice of designer, as it occurs in the problems, where a curved shell surface is modeled by a multiface set of plane finite elements. However, such a problem also appeared before, when they used an approximate description of curved bars by a certain polygon.

A possibility to present a design model as a set of finite elements, their quantity in configurations being limited by nothing but the library of finite elements at designer's disposal, raised in a new fashion the question on the number of basic unknowns, degree of kinematic and static indeterminability and other still inviolable characteristics of the design model of the building. The number of unknown displacements (the degree of kinematic indeterminability) stopped being the problem feature and became a subject of designer's self-will.

Some "standard" approaches to composing the finite element design models were developed for most types of structural systems. For example, at the first stage of using FEM a design model of a thin-walled fuselage structures and aircraft wings became popular; it was composed of shearing panels and a frame of bars, supporting them at the edges, able to take up only longitudinal forces.

In such wing model (Fig. 8) the bars simulate the work of longitudinal members of the wing structure under load; these members are subjected to compression and extension under the wing bending. Plates simulate the work of walls, which prevent shearing, as well as the external and internal wing covering.

This model was propagandized by G. Argyris [Argyris & Kelsey, 1954], and though it appeared long before FEM [Ebner & Köller, 1937], [Umansky, 1950], its implementation proved acceptable only in the framework of FME, though it existed there just for several years. Potentialities of computation complexes, which were quickly perfected, permitted even in the 70's of the 20th century specifying the design model and taking account of bending strains of the supporting frame.

There originated new approaches to design model development for the plates and shells with ribs. There also appeared competing propositions as well as the problem of their verification.

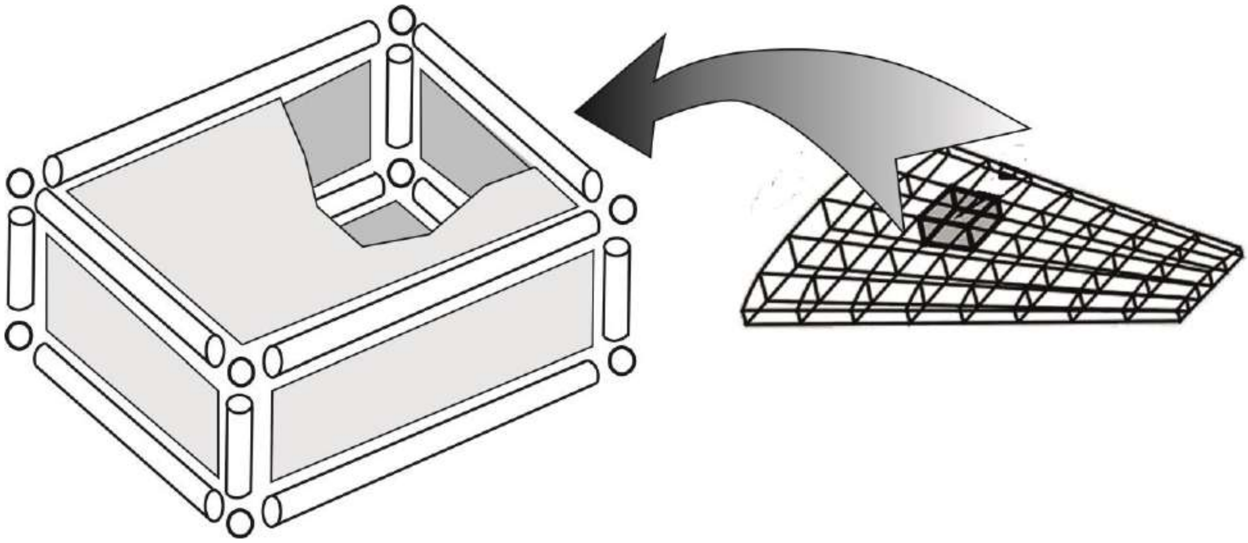

Figure 8. A thin-wall system model.

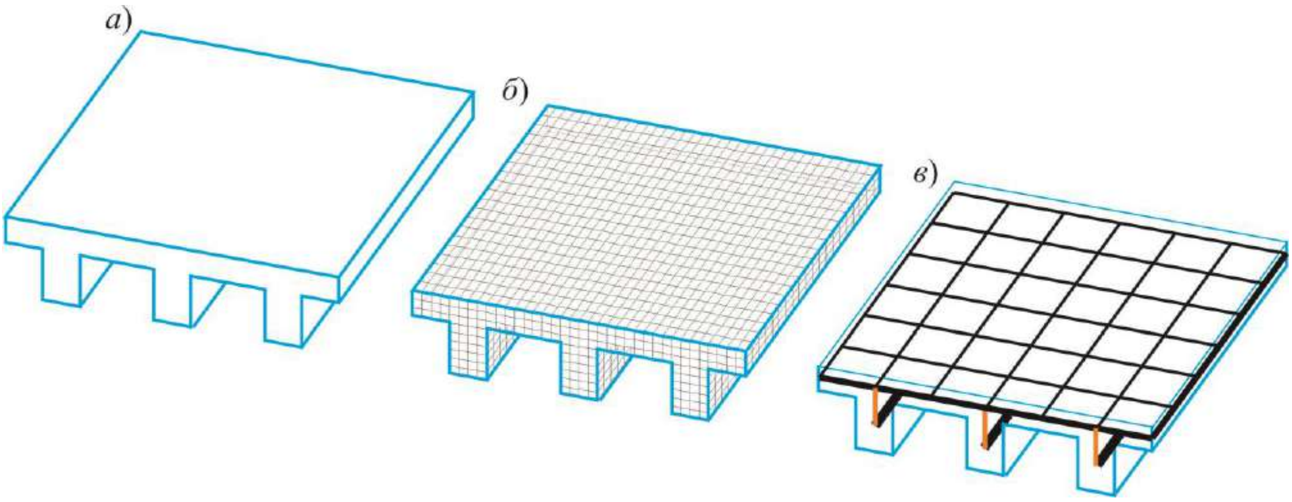

Figure 9. Possible variants of a ribbed plate modeling.

The corresponding example of the variant of a design model composed of solid elements (Fig. 9, b) and the variant of modeling using plate and bar elements connected through infinitely rigid inserts (Fig. 9, c) are presented in Fig. 9.

In industrial and civil buildings, where, e.g. the through columns are used, designers often abandon presentation of such a column as a solid rigid bar, but use a more detailed description of the structure. And above all, they practically gave up plane design models and resolve all problems with the use of three-dimensional models. The main trend of development is now the use of more outsize design models; the number of unknowns of the order of hundreds of thousands became ordinary in the design practice; in so doing such an extensive detailing is not always necessary but connected with formal construction of the design model by the data of graphical program used for making drawings.

A possibility of modeling structures of arbitrary nature, including those which separate parts are presented by bars, others – by plates or shells, the third – by three-dimensional bodies, raised questions as to connection of finite elements of different type and arising problems, caused by the difference of nodal degrees of freedom in different type elements. A necessity of using special techniques (Fig. 10), for example, such as the introduction of the bar element into the rigid disc body, is indicated [Perelmuter, Slivker, 2001].

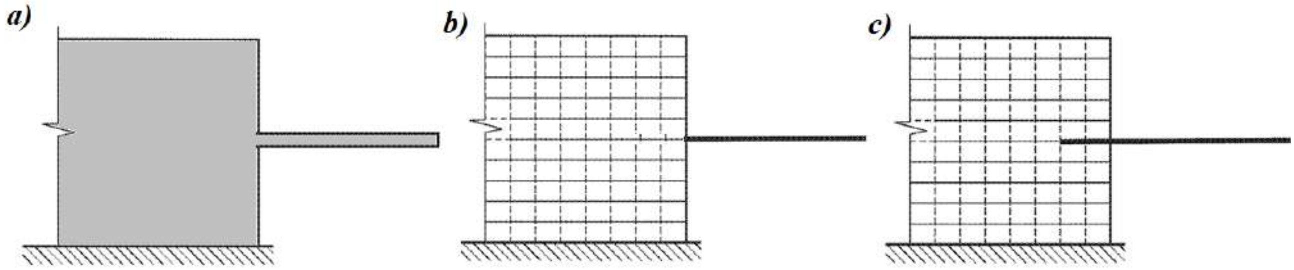

Figure 10. Modeling of interconnection of a bar and disc: a – task; b – variant, which does not provide a restraint; c – recommended decision.

But it should be allowed for that the above result may be lost because of difficulties in analysis and comprehension of excessive information. And this so-called revision often leads to shading the basic features of the structure service, and its simplified variant is considered for their analysis in parallel with a detailed design model. In so doing there sometimes arises a nontrivial problem of results comparison; it is especially difficult, when there is no exact fit between the elements of compared design models [Perelmuter, Slivker, 2001].

And finally note that the analysis of design models of the finite element method is closely connected with the problem of method convergence. Mathematical proofs of the corresponding facts (for example, a demand of compatibility of the fields of displacements) require their interpretation in terms of design models, which are successively "concentrated". In particular, in the case of incompatible elements it should be remembered that the solving of design problem is equivalent to minimization of full potential energy of the system (Lagrange functional), and approximation of the displacement field by a certain finite set of preset functions narrows a possibility of arbitrary deformation, that is it may be treated as the imposing of some constraints. If elements are incompatible, some displacements are possible at their boundaries; these displacements do not exist in the continual design model (for example, mutual rotation angles of plates), and correspond to the absence of some constraints.

When the number of finite elements increases and their sizes decrease, the total number of the structure degrees of freedom grows, and thus, the effect of the imposed nodal constraints is reduced. This process, certain conditions being fulfilled, provides the method convergence for compatible finite elements. On the other hand, the same process leads to the decrease of mutual displacements at interelement boundaries in incompatible elements that may be treated as a certain locking of the preliminarily left constraints. Thus the convergence of incompatible elements can take place only when positive tendencies of overcoming the imposed constraints prevail over this negative tendency of imposing the constraints at interelement boundaries.

Other approximations are sometimes realized simultaneously with approximation of the displacement field; those are connected with a necessity of using a finite-element model that is the substitution of the structure geometry by that similar to it. In the system geometry approximation, both the geometry and boundary conditions may change, since the latter belong now to the boundaries with other configuration. Here one can run across the reefs, since the passage to the limit of the outline form is not necessarily accompanied by passage to the limit of kinematic properties. That is evidenced by well-known Sapondzhyan paradox for a freely supported polygonal plate [Panovko, 1985].

At the outset of FEM use they discussed a so-called problem of "small length" of the bar finite element, when a stress was made on the fact that a bar was defined (in courses of material resistance or structural mechanics) as the object, which one size (length) exceeded considerably other ones that defined cross-section dimensions. But in the design model in use the bar as the design model element may be found to be very short. There seemingly appears a violence of agreements concerning a bar definition. In fact, there is no violation, since the admission on a sufficient length of the bar was only required to substantiate the type of the corresponding differential equation. As to the method of its solution, when a bar is divided into rather small parts (read interval of integration), this has no effect on the equation form.

5. SOME NEW TENDENCIES

In recent decades there appeared a branch of structural mechanics based on probabilistic analysis, which started its intensive development. The approaches to the design system itself and to development of the corresponding design model have undergone considerable changes.

The whole identity of parameters of all similar structure elements is foreseen in a determined variant. It is considered that all the headers of a three-dimensional framework have equal spans, all the columns of these headers – similar sections, etc. In so doing all such type elements are reduced to one representative, and sometimes to its one section. Such an approach is acceptable under the approach to design contained in the design norms. The approach is based on the half-probabilistic method of limiting states. Then all the probabilistic characteristics are formulated and estimated beyond the design model, and a design contains only some guaranteed the worst statistical estimates of means, standards, quantiles, etc., which are really the same for all identical elements, since one of definitions of their "identity" is the identity of the distribution law.

The transition to really probabilistic design was connected with the fact that one has not to operate on distribution parameters of random values but on distributions themselves, when random parameters operate not outside but inside the design model. And in such a model each of "identical" substructures should be defined in a form of a set of mutually independent (or correlated) random values (probably functions) with the same distribution laws [Bolotin, 1971]. In such formulation one cannot imagine a design model, e.g. of a plane problem, which provides identity (or rigid correlation with correlation coefficient equal to one) of all plane subsystems distributed in parallel.

The following should be noted: a further detailing of the models, when passing to analysis of multielement structures (a building as a whole), requires the involvement of the great number of parameters, used for the model description. If these parameters are random values, which probabilistic properties are statistically justified, the degree of design model indeterminacy as a whole increases with the number of such parameters. Thus, if a certain result of analysis is in linear relationship with N independent random parameters (e.g. external loads in the system nodes), the standard of this result is proportional to that of the input data (here loads) with a multiplier of $(N)^{1/2}$ order. It is simple to estimate what is the probability of the results of analysis at very high N values.

There are more detailed propositions as to determining the effect of output data accuracy on the results of analysis (see, for example, [Podolsky, 1984]. They evidence that the information on the input parameters being insufficient, it is expedient to use simple design models. Such a peculiarity of design modeling is connected with the fact that the loss of information because of incompleteness of output data can exceed information accumulation at the expense of refinement of the design model.

The foregoing should be never considered as a panegyric to "good old days", when everything was solved using the formula $qL^2/8$ and counted using a slide rule. The thoughtless complication of design models would be substituted by new culture of their use that includes also the estimation of possible uncertainty of solution. Now, having the modern means for analysis of complex and supercomplex systems, we study them in formulation of the problem, which rather corresponds to the 19th than to the 21st century.

At last, it should be mentioned that the above problem of output data influence on the design model form is inherent not only in the probabilistic problems.The choice and justification of the design model cannot be separated from the level of information as to the structure, which is designed as well as from the method of solving the mathematical problem, formed as a result of using the chosen design model.

And what is more, a lot of mathematical operations used, when solving a problem, often have a mechanical interpretation, which use helps understand the features of calculation process. As an example, we can refer to the interpretation of Gaussian algorithm for solving a canonical system of linear algebraic equations, as to the sequence of imposing (force method) or taking off (displacement method) constraints [Gantmacher, 1967]. Such illustration favors a better understanding of the problem.

6. DESIGN MODEL JUSTIFICATION AS A SCIENTIFIC PROBLEM

The justification of using a certain theoretical model is usually based on the following consideration. It is assumed that the result observed in a series of consistent experiments is always close to that predicted by theory. Hence, it can be concluded that the design model is applicable to the conditions similar to the experimental ones.

However, in order to decipher the concepts of "consistent experiments" and "the result is close to", and to assess the degree of similarity between the conditions of the problem and those of the experiments we have to use the theory which actually has to be justified by the experiment. For example, in order to make sure that there is no systematic error in the experimental data, it is necessary to compare the parameters of the experimental sample with the parameters of the model, and the justifying decision has to be made concerning the meaning and the number of these parameters. There are two ways to break this vicious circle:

- to ignore these logical contradictions (the conventional, so-called, "engineering" approach);
- to make an attempt towards an axiomatic theory [Truesdell, 1975].

The latter approach [Truesdell, 1975] makes the theory of structures a logically coherent science (mathematics in a sense), but it poses untypical requirements to a practicing engineer. Namely, he has to think like a mathematician, referring to axioms in obscure cases and repeating the entire chain of reasoning that led him to the considered case.

Therefore, the first approach is commonly used in practice. Its applicability is formulated as follows [Kosmodemyansky, 1969]: "*…when designing and building new structures (bridges, dams, airplanes, missiles, buildings) on the basis of tremendous experience, experts are so confident in the validity of the laws of mechanics that all the conclusions derived from the calculations are considered to be absolutely true. Any discrepancies between theory and practice are explained after a subsequent rigorous and thorough analysis either by inaccuracy of the initial data or by arithmetic errors*".

Starting with the first quarter of the 20th century, the problem of justifying a design model began to be considered as a general scientific methodological problem, the authors of the corresponding papers put forward various rules for creating design models. For example, three principles for creating design models of structures are put forward [Gersevanov, 1923]:

- calculation methods should be based on the failure and deformation modes confirmed by the construction practice experience;
- the design hypothesis should subject the model to harder conditions than those the actual structure is subjected to;
- the set of design hypotheses should provide cost-effectiveness of the structure in addition to its strength and stability.

According to the authors [Goldenblat et al., 1979] these principles, however, are not complete, and they should be supplemented as follows:

- it is reasonable to have a system of approximating models of the structural behavior with their respective limits of applicability rather than a single model (for example, one model can be used to describe the elastic behavior of a structure and the other to describe the elastic-plastic stage);
- a model approximating the structural behavior should not only correctly and fully reflect the behavior of the real structure, but it should also be simple enough, so that the calculation does not become excessively cumbersome.

General modeling issues were developed and refined in the following works: [Buslenko, 1968], [Dickson, 1969], [Kartvelishvili and Galaktionov, 1976], [Perelmuter and Slivker, 2001], [Perelmuter and Kabantsev, 2015]. J. Dickson describes the process of creating design models (Fig. 11) and indicates that "*a model is an idealized approximation to the real situation. Creating a good model involves making assumptions that take into account relative importance of various elements of the problem*".

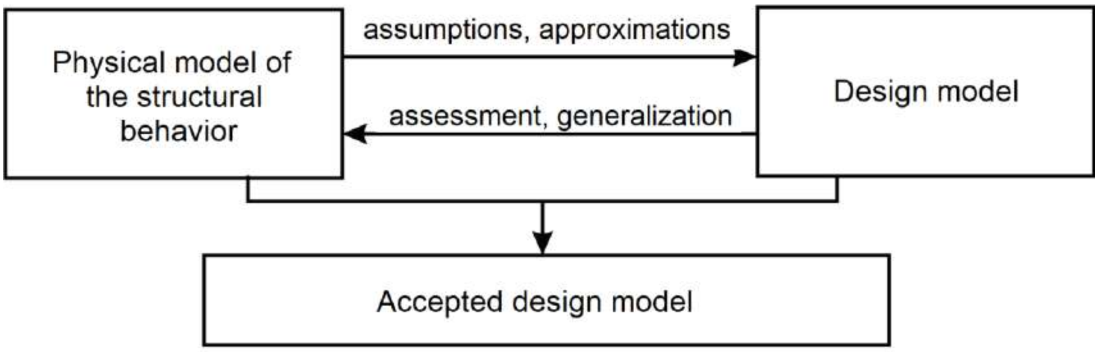

Figure 11. Logic flowchart for creating a design model.

A special role belongs to general models of typical structural elements (a bar, a plate, a shell etc.), which are used to create full design models of structures or parts of design models of other more complex structures. Such models should be studied as rigorously as possible and serve as a basis for further consideration of other design models. After such investigation, having a complete knowledge about these models makes their use quite attractive to an engineer, who can then anticipate the result of the analysis (or at least its qualitative features).

The transition from a structure to its design model made up of basic models is usually performed intuitively, and the geometric considerations ("similar" shape) are the first motive behind this transition. Though, there is room enough for a maneuver even here. The design process often involves such operations as the replacement of a lattice structure with a continuous one which has a shape only vaguely resembling that of the original structure, or "smearing" the ribs and other structural parts. There are opposite examples as well, when a continuous body is replaced with its bar analogue. Some knowledge about the peculiarities of the behavior of the selected basic models is used here in addition to their geometric shapes. For instance, if a plane section of the design model is described by purely flexural plate elements, we should keep in mind that the membrane components of the stress field cannot be determined in this case. If these components can be significant (e.g., they can cause buckling), it is more reasonable to use shell elements.

The second motive, which also plays a fundamental role in the transition to a design model, is a choice of one of the standardized idealizations of material properties (elastic, plastic, loose, etc.). These properties are also represented by previously studied basic models, and they would not be worth mentioning, if not for the following consideration: their choice requires an experimental justification even more than that of the geometric shape, but this stage is usually omitted.

The designer usually operates with the available data about the physical models of the material behavior obtained from experiments performed on other structural elements and samples. At best, such actions are justified by the fact that the results of these studies can be used in the created design model based on certain ideas about the possible nature of deformations and about the expected stress level caused by the loads of a certain level. The main role however is usually played by the tradition and the actual computational capabilities of the designer.

The use of computers has significantly expanded these capabilities, but has not made them limitless, although the finite element method implemented in the structural analysis software often creates the illusion that it is now possible to solve any problem. Quite late was it realized that the capabilities depend on the result representation methods, since the limitations of the human ability to analyze and perceive large volumes of information play an important role here.

Unfortunately, there is not system of clear integral characteristics that would alone enable to understand the structural behavior in structural mechanics. We somewhat resemble a group of imaginary experts in gas dynamics that track the movement of individual molecules without considering temperature and pressure. We can hardly expect to obtain universal design parameters like in gas dynamics, but this type of characteristics might be obtained for individual classes of problems.

The finite element method raised a new structural mechanics problem of justifying design models: creating and optimizing different types of mesh generation mechanisms.

Considerable attention was drawn to the fact that the problems of the analysis of load-bearing structures focusing on the refined prediction of the peculiarities of the behavior of the system at all stages of its operation including the stages prior to the failure cannot be usually solved by the methods of the linear structural mechanics.

Educational literature [Rudykh et al., 1998] and the majority of researchers [Novozhilov, 1958], [Lukash,1978] consider the following “set of nonlinearities”: deviation from Hooke's law (physical nonlinearity), failure to consider the equilibrium conditions in geometrical terms of the non-deformed state (geometric nonlinearity), accounting for the possible changes in the design model during the deformation process (structural nonlinearity). However, this set is not complete. It does not include the consideration of the effects caused by rheological processes in the material (for example, creep) and nonlinear effects of resistance to movements of the dry friction type or of other nature, and it does not take into account nonlinearity related to the accumulation of stress and strain during the changes of the structure as it is created (genetic nonlinearity [Perelmuter and Kabantsev, 2015]). Numerous studies focusing on justification of the design models used here were actively carried out throughout the second half of the 20th century.

The fact that it is impossible to perform a detailed justification of all parameters of a complex design model for an arbitrary structure (especially in the case of a nonlinear analysis) does not mean that we should not perform this type of analysis at all. Apparently, the most powerful strategy is to perform a thorough computer analysis of some typical models, and to compare the results of this calculation with a simplified one. This computational experimentation will let us determine (for a particular class of problems) whether a considerable discrepancy between the calculation and the experiment is a result of unsatisfactory idealization. In fact, we are dealing here with a kind of experimental justification of the design models, with the only difference being that a numerical experiment is used instead of a physical one.

Finally, it should be noted that applied research including the analysis of design models is not always mathematically rigorous, and is often based on credible assumptions [Morris, 1971]. In any case, you should keep in mind that a non-rigorous solution and an incorrect solution are fundamentally different things. And most importantly, as is customary in the natural sciences and in engineering, the results are verified by experiment or observation.

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NONLINEAR ANALYSIS OF STATICALLY INDETERMINATE WOODEN STRUCTURES AND OPTIMIZATION OF CROSS SECTION DIMENSIONS OF DOME RIBS

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Abstract: The analysis of the behaviour of natural structures of laminated wood domes and the numerous preliminary calculations have shown the possibility of saving materials by reducing the height of cross sections of meridional ribs. This is especially effective when you include in design of skins, performing a role of building shell, the collaboration with frame elements (annular and longitudinal ribs). Multiple static indeterminacy of such structure allows its non-linear work and the redistribution of forces under nonuniform loads. At the same time, the skin carries a significant part of the forces appearing in the shell and the ribs are underloaded. The stress-strain states of all elements are investigated. For the frame analysis the calculation is performed by the method of integral module that allows controlling strength resistance of a structure at any moment of its operation. The design recommendations for section dimensions of a shell are developed.

Keywords: laminated wood domes, plastic deformations, complex stress state, numerical calculations, integral module method, rational dimensions of element cross-sections

НЕЛИНЕЙНЫЙ РАСЧЕТ СТАТИЧЕСКИ НЕОПРЕДЕЛИМЫХ ДЕРЕВЯННЫХ КОНСТРУКЦИЙ И ОПТИМИЗАЦИЯ РАЗМЕРОВ СЕЧЕНИЙ РЕБЕР КУПОЛОВ

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Аннотация: Анализ работы натурных конструкций куполов из клееной древесины и многочисленные предварительные расчеты показали возможность экономии материалов за счет уменьшения высоты сечения меридиональных ребер. Особенно это эффективно при включении при проектировании обшивок, выполняющих роль ограждающих конструкций, в совместную работу с элементами каркаса (кольцевыми и меридиональными ребрами). Многократная статическая неопределимость такой конструкции допускает нелинейную её работу и перераспределение усилий при неравномерных нагрузках. При этом обшивки воспринимают значительную часть усилий, возникающих в оболочке, а ребра оказываются недогруженными. Выполнены численные расчеты оболочки в режиме реального времени при ступенчато изменяющейся односторонней (на половине покрытия) нагрузке с учетом сезонных ее колебаний в течение 5,4 лет. Исследовано НДС всех элементов. Для анализа работы каркаса расчет выполняется методом интегрального модуля, позволяющего контролировать силовое сопротивление конструкции в любой момент её эксплуатации. Разработаны рекомендации по назначению размеров сечений оболочки при проектировании.

Ключевые слова: купола из клееной древесины, пластические деформации, сложное напряженное состояние, численные расчеты, метод интегрального модуля, рациональные размеры сечений элементов

During the last two decades the production of large glued wooden structures has been formed. So it has become possible to design and build large-span buildings and structures, in particular, domes. There are already hundreds of such buildings [1]. Nowadays all stages of construction are being developed, the possibilities of new types of connections and protection of structure elements are being studied, etc.

Dome structures with a diameter of up to 100 m are usually made from frames in the form of arches, bars and fencing elements – slabs or floorings. Here slabs and floorings are fixed to a frame and further are engaged in its joint work. However, for a number of reasons this joint work is not taken into account while designing, and a part of system strength resistance is lost. The main ribs of a dome frame are meridian and traditionally with a section height of 1/40 span. They appear to be underloaded in the design taking into account the operation of slabs filling the cells between the ribs. The paper shows [2] that the meridional ribs of a conical dome can have a cross-section height of 1/70 span, what saves rib materials up to 25%. While the design and construction of the indoor skating rink in Moscow, the ribs between the diaphragms of short cylindrical shells were made with the cross-section height of 1/52,5 span equal to 42 m.

The facility has been successfully operated for 35 years (it was the first prototype [3]). The model experiments and the calculation by the integral deformation modulus [3] have showed that plastic creep deformations can be allowed under long-term loads.

The present work proposes the numerical studies of a dome with the diameter of 60 m and the cross-section height of ribs of 1/60 the span under long-term loads for up to 50 years. The reduction of loads under the absence of snow in summer time was taken into account. According to the previous publications the non-linear calculation determines the strength resistance of the structure taking into account the linear creep (the first stage by A.R.Rzhanitsyn) and the steady state (the second stage) [3]. It turns out that in the range of design loads the structures usually operate at the first stage.

The method of nonlinear calculation of a planar frame of laminated wood together with some thin sheathing is published in detail in numerous articles relating to various computational situations as well as in the monograph [3].

The calculation method is based on the method of integral module of deformations developed by V.M. Bondarenko in relation to reinforced concrete structures and it was adapted by the authors for the calculation of wooden structures. Taking into account the specific properties and structure work of wood the developed method is original and can be considered as a new theory.

The application of this method helps by iterative process to trace the changes in stress-stain states of structures under nonlinear and non-equilibrium long-term deformation, to take into account the process of forces' redistribution in individual cross sections and along the length of elements. This method makes it possible to linearize the calculation process and apply at each stage of successive approximations Betty's theorem on reciprocity of works, Maxwell's theorem on reciprocity of displacements, Moore's formula for displacements. In this case, the linearization keeps the relationship between the stiffness characteristics and the loading level. Temporary fixation takes into account the mode influence and duration of loading.

The diagram of wood operation, which was got experimentally, is shown in Fig. 1.

As the approximating function for the nonlinear relation

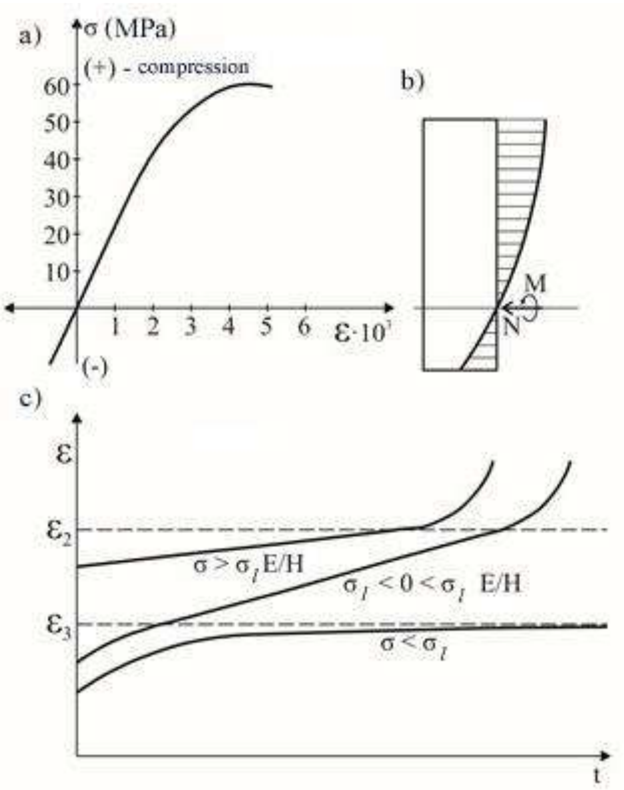
$$\sigma = f(\varepsilon)$$

the following equation is accepted:

$$\sigma = E_0\,\varepsilon - \frac{E_0^2}{4\sigma_{pp}}\varepsilon^2 \qquad (1)$$

The mechanical state equations are drawn up in relation to the three stages of creep according to Rzhanitsyn [4].

Numerous calculations have shown that in the range of design stresses the structures usually work at the first stage - linear creep, although the assumption does not reduce the generality of the solution, and it is easily possible to move to the second stage – steady creep. There is a transition time and the equation of mechanical state for this case.



<u>Figure 1.</u> Diagram of wood deformation: a – compression-tension; b – short-term bending; c –under the long-term action of a constant load.

In the method of integral estimations the process of successive approximations is a way of integral refinement of internal forces and stresses that transform in time due to their redistribution from more loaded areas to less loaded ones.

In statically indeterminate structures, where the distribution of forces is due to the nature of the change in stiffness, in addition to the process of internal iterations required to clarify the stiffness it is necessary to combine with it the process of external iterations clarifying the rule of forces distribution according to the stiffness data. The combination of internal and external iteration processes in solving the problem of stress and strain states is the following:

1) in an usual elastic-linear formulation a given statically indeterminate system is calculated and diagrams of internal forces are designed (zero approximation);
2) cross-sections are assigned, where the estimated stiffness is refined according the data of the forces of zero approximation (reduced modulus of deformation);
3) according to the new law of stiffness distribution the static calculation of the system is repeated taking into account the variability along the spans of the calculated stiffness. This clarifies the diagrams of force distribution along the axes of the system (first approximation);
4) for the forces of the first approximation, the calculated stiffness is again specified, and then force diagrams in the second approximation are found, etc. to a stable convergence with a given degree of precision.

To take into account the variability of rod stiffness, each element of the statically indeterminate system is divided into several sections, where the stiffness is considered constant and equal to the average stiffness of the boundary zones of the site. The given deformation modulus is defined as the arithmetic mean of several intermediate sites.

Changes in the external load over time in the calculation are reduced to a step scheme in such a way that within each of the intervals the load and all the characteristics of stress stain states (SSS) are considered constant. The same applies to the variability of physical and mechanical properties of materials.

In the nonlinear phase of creep instead of solving the differential equation of A.R. Rzhanitsyn the empirical dependence of Yu.M. Ivanov is assumed to be

$$\varepsilon(t) = \varepsilon(t_0)(1 + bt^{0,21}), \qquad (2)$$

where

$$b = \frac{10^{-2}}{0{,}735 - 0{,}02086\ W}, \qquad (3)$$

W – wood moisture, %.

The equation of prolonged deformation of wood when $\sigma < \sigma_l$ for a given step change of stresses can be written as following:

$$\varepsilon(t) = \varepsilon(t_0)\left[1 + b(t - t_0)^{0,21}\right] + \\ + \sum_{i=1}^{k} \frac{\Delta\sigma_i}{E_0 - \frac{E_0^2}{4\sigma_{pp}}\varepsilon_{i-1}^a}\left[1 + b(t - t_i)^{0,21}\right], \qquad (4)$$

where

$$\varepsilon_{i-1}^a = \varepsilon(t_0) + \sum_{i=1}^{k} \frac{\Delta\sigma_i}{E_0 - \frac{E_0^2}{4\sigma_{pp}}(\varepsilon_{i-2}^a + \Delta\varepsilon_{i-1}^a)}, \qquad (5)$$

ε_{i-1}^a – the total value of instantaneous (short-term) increments of relative deformations.

The equation (3) takes into account the influence of humidity W, but in the form of (4) the constancy of humidity at all stages of deformation is assumed.

Long-term strength depends on the wood moisture. Taking into account humidity E.N. Kvasnikov [7] obtained dependences of long-term strength on the duration of load action, on the humidity and on the type of stress state in the form of:

$$\sigma_l = a - b\lg t. \qquad (6)$$

Here this value is taken to be 22 MPa, t – days, the coefficients a and b are determined experimentally.

The direct use of a nonlinear equation of material mechanical states in solving problems of structural mechanics is very bulky. So the authors use S.E. Frayfeld's proposal to introduce a time deformation modulus

$$E_l(t_0, t) = \frac{\sigma(t)}{\varepsilon(t_0, t)}, \qquad (7)$$

where $\sigma(t)$ – stresses at the time of observation t; $\varepsilon(t_0,t)$ – relative deformations at the time of observation t, which are established taking into account the influence of the age of the material, its aging properties, the mode and duration of loading.

For the dependence corresponding to the linear creep the authors use the expression for the long modulus of deformation in the following form:

$$E_l(t_0, t) = \left[\frac{\varepsilon(t_0, t)}{\sigma(t)}\right]^{-1} \qquad (8)$$

and then

$$E_l(t_0, t) = \left[\frac{\varepsilon(t_0)\left(1 + b(t - t_0)^{0.21}\right)}{\sigma(t)} + \right. \\ \left. + \sum_{i=1}^{k} \frac{\Delta\varepsilon_i}{\sigma(t)}\left(1 + b(t - t_i)^{0.21}\right)\right]^{-1}. \qquad (9)$$

For a given step change of stresses $E_l(t_0,t)$ is to be:

$$E_l(t_0, t) = \frac{E_0 - \frac{E_0^2}{4\sigma_{pp}}\varepsilon(t_0)}{1 + b(t - t_0)^{0.21}} \cdot \left[1 + \right. \\ \left. + \sum_{i=1}^{k} \frac{1 + b(t - t_i)^{0.21}}{1 + b(t - t_0)^{0.21}} \cdot \frac{\Delta\sigma_i}{4\sigma_{pp}\left(1 - \frac{E_0}{4\sigma_{pp}}\varepsilon_{i-1}^a\right)^2}\right]^{-1} \qquad (10)$$

where ε_i^a – active deformation.

There would be less mathematical difficulties associated with the use of equations of mechanical state to describe SSS of elements whose materials are deformed nonlinearly and with delay, if you apply the method of integral estimations, which is based on the use of the integral deformation module.

The following approach can be considered for the most common compressed-bent wooden element in constructions analogically with the derivation of equations for determining the long-term modulus of deformation. Estimating the real deformability of elements and at the same time not operating with different deformation modules in each discrete layer, it is possible to write the deviation of the values of real deformations ε and deformations ε_{in}, identified by $\varepsilon_{in}(x, t)$. The essence of the integral estimate is to minimize the deviation, which is carried out for the section as a whole, and after performing a series of transformations of the expression for the desired deformation module, the following expression for the first stage of deformation is to be (linear creep):

$$E_{in}(x,t) = \Phi(\varepsilon_{\Phi}^{A}, b, a)\frac{E_0 - \dfrac{E_0^2 \varepsilon_{\Phi}^{A}}{4\sigma_{pp}}}{1+bt^{0,21}} \cdot \left[1 + \sum_{i=1}^{k} \frac{\Delta\sigma_i \left(1 + b(t-t_i)^{0,21}\right)}{4\sigma_{\text{пл}}(1 - \dfrac{E_0}{4\sigma_{pp}}\varepsilon_{i-1}^{a})^2 (1+bt^{0,21})}\right]^{-1}. \quad (11)$$

To validate the adopted design provisions the experimental studies have been carried out: the studies of the basic types of the coating shells of engineering constructions and residential buildings under asymmetric loadings when the most apparent redistribution of forces and nonlinear deformation take place.

THE STRUCTURES UNDER STUDY

The scheme of the ribbed ring dome is shown in Fig.2. The dome diameter is 60 m, the height - 20 m. The meridional ribs of laminated wood are located in increments of 3,926 m along the reinforced concrete support ring. The ribs are attached at the top of the dome to a metal lantern ring. The ribs via one are shortened due to the less stress because of ribs condensation. The height of the ribs is equal to 1/60 of the diameter, i.e. 1000 mm, the width – 140 mm.
The annular ribs of the cross section $b \times h$ 140x200 mm are located in increments of 2,464 m orthogonally to the meridional ribs.
The cells between the ribs are filled with plank-nail slabs resting on cranial bars. In the corner areas the gaps between the slabs and ribs are filled with the polymer solution (Fig. 3). The filling slabs are made from two layers of planks with the thickness of 15 mm. The thickness of plates was brought up to 40 mm. (Calculations were carried out also for the reduced thickness of plates of 20 mm).
The real-time calculations of the structures were carried out using the finite element method by MicroFe software complex with the control of the SSS of the shells using G.A.Geniev's strength criteria.

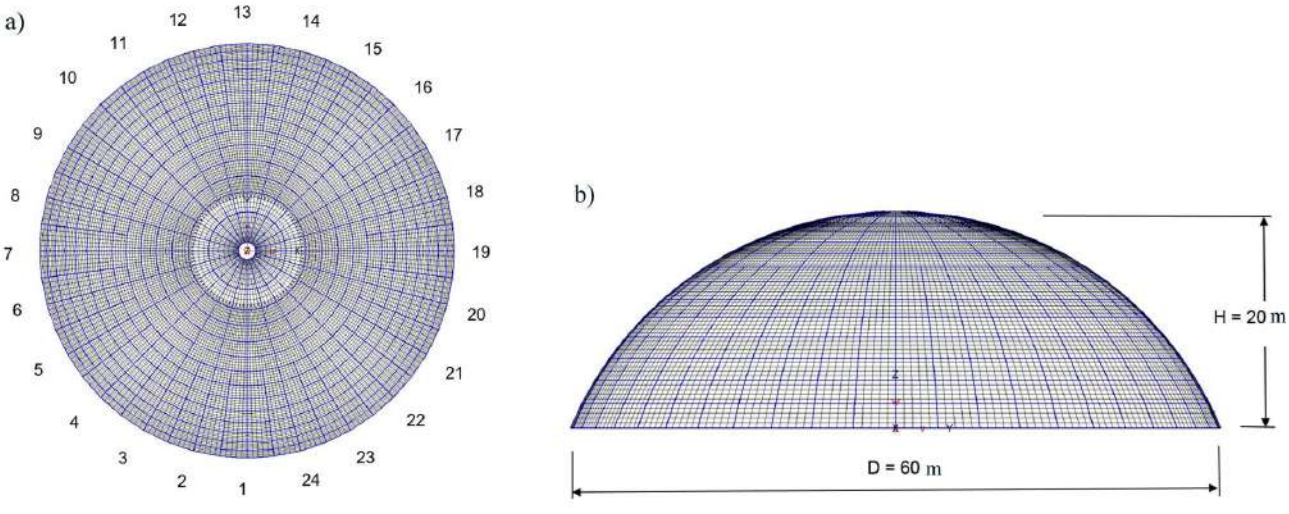


Figure 2. Scheme of ribbed ring dome: a) plan; b) facade.

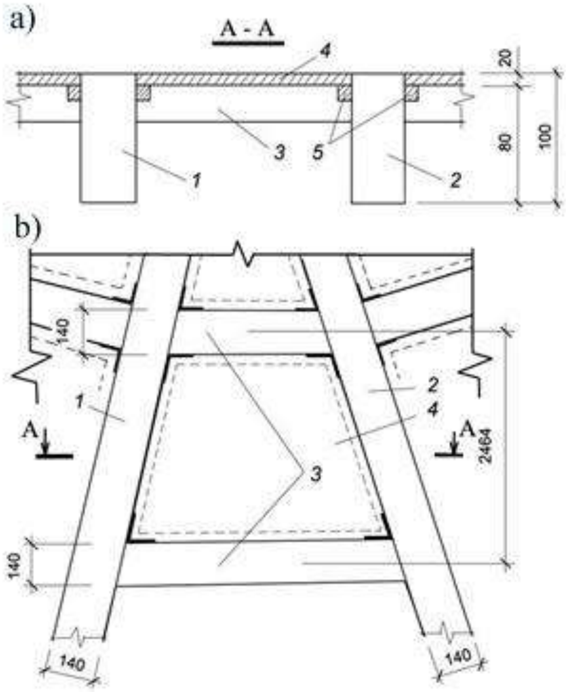


Figure 3. Scheme of plates on the dome frame: a) section; b) plan. 1, 2 – meridional ribs; 3 – annular ribs; 4 – panel; 5 – timber.

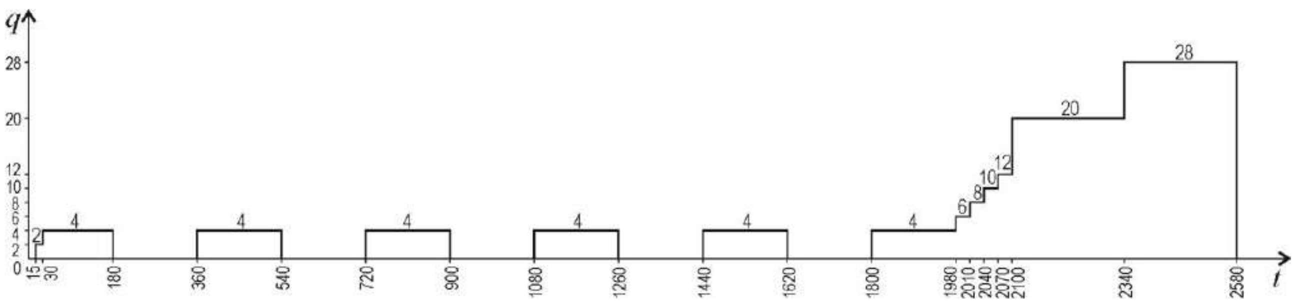


Figure 4. The mode of load application.

The program of loading at numerical calculations is shown in Fig. 4. The load is taken evenly distributed on the left half of the dome. The mode of load application mainly reflects seasonal changes in the snow load and the possibility of its uneven distribution over the surface.

Processing of the results of numerical calculations was carried out in accordance with the program determining the effect of nonlinearity of the deformations caused by prolonged action of load, seasonal change in the intensity of loads, the features of the joint work of the frame, the characteristics of redistribution of forces under uneven loads.

The main characteristics at the results processing are meridional and annular forces in the frame elements, shear forces and stresses in the skins, deflections of the entire structure and changes in the deformation modules depending on the magnitude and duration of loads.

The SSS characteristics were determined sequentially at 39 stages of loading. The maximum time of application of seasonal loads was taken to be 1980 days or 5.42 years. Further, the load was taken as incrementally increasing to assess the possibility of beyond design conditions for the structure.

The results of calculations are given in Table. 1.

Table 1. Stress-strain state of the dome with the covering of 40 mm thickness.

Stages of loading	*q, kPa*	Time exposure *t*	Stress in meridional ribs σ, *MPa*	Shear stress τ, *MPa*	Max deflections u_z, *mm*	Integral module E_{in}, *MPa*
1	Dead weight	1 min	0.0255	0.011	0	14120
3	2	15 days	0.873	0.503	-3.5	14109
5	4	180	1.45	0.97	-6.35	13785
9	4	360	1.474	0.972	-6.54	13715
13	4	720	1.46	0.972	-6.57	13685
17	4	1080	1.453	0.972	-6.58	13526
22	4	1620	1.451	0.971	-6.62	13465
27	6	1980	2.02	1.446	-9.57	13227
32	10	2070	3.20	2.378	-15.7	13212
34	12	2100	3.776	2.847	-18.7	13204
36	20	2340	6.126	4.66	-30.75	13157
38	28	2580	8.46	6.519	-43.15	13116
39	36	2580	10.67	8.37	-55.26	12457

THE CALCULATION RESULTS ACCORDING TO THE TABLE 1.

The behaviour of the dome under one-way load with intensity changing in time is investigated. It was simulated that there was no snow load in summer, and in winter the snow load was taken of higher intensity than its usual level relative to the middle European part of Russia. Calculation by the method of integral module of deformations allows defining of strength resistance of structures at any time of its operation at any changing loading.

The particular attention is paid to the effectiveness of joint work of the frame of laminated wood and wood panels.

The main indicators according to which the analysis of SSS of the investigated structure is carried out are the following: the value of the applied loads by stages and the duration of the structure exposure under this load, the stresses in meridional ribs, the shear stresses in the joints of skin elements to ribs, the maximum deflections and the value of integral module of the ribs deformations varied depending on the stress magnitude.

It was found that at the reduced thickness of skin equal to 40 mm the strength resistance of the shell is very large, and even at the maximum load exceeding the design of 3.2 kPa in 10 times, the stresses in the sections of meridional ribs did not reach the calculated value. The maximum stress under load of 36 kPa was 10.67 MPa.

Similarly, deflections throughout the loading process of 7.07 years vary by several mm and at the considered maximum load were only 55.26 mm, i.e. 1/1086 of the dome diameter. With a load of 4 kPa, the deflection was 6.35 mm, and when the shell was held under this load for 1440 days (almost 4 years), it increased up to 6.62 mm (or 1/9050 of diameter *D*), i.e. by 0.28 mm.

Similarly, the maximum value of normal and shear stresses for this period has not changed (Table 1).

Meridional ribs of the frame were taken with a reduced cross-section height to 1/60 of the span. The maximum normal stress in the supporting part of meridional ribs is 172.5 kN/m^2 $\approx$0.2 MPa.

So the shell thickness of 40 mm almost does not require ribs (sustainability is not considered here). At the same time, it should be noted that the value of integral modulus of deformation at a load of 4 kPa decreased from $1.4 \cdot 10^4$ to $1.35 \cdot 10^4$ MPa - by 3.3%.

The data of the numerical experiment of this shell with a thickness of 20 mm at a load of 3.2 kPa was taken for comparison (Table 2). As a result of the shell exposure for 50 years the deflection of the shell increased from 13.3 mm to 14.9 mm (to 1/4027 diameter). Stresses in the meridional ribs increased from 1.53 MPa to 1.6 MPa (4.1%). The absolute stress value was less than the design resistance of 13 MPa. The shear stresses at exposure under load have not increased (details of changes in shear stresses will be discussed in another article).

Table 2. Stress-strain state of the dome at a constant one-sided load of 3.2 kPa (The thickness of shell is 20 mm).

Number of stage	Time exposure, days	Max bending moment in meridional ribs, *kNm*	Max normal stress in meridional ribs, *MPa*	Max shear stresses, *MPa*	Deflection, *mm*	Integral module of deformations, *MPa*
1	0	1380.0	1.527	2.03	13.3	14399
2	1	1417.0	1.581	2.03	13.66	14200
3	180	1428.0	1.582	2.04	13.72	14027
4	730	1443.0	1.585	2.02	13.92	13884
5	18250	1462.0	1.590	2.03	14.27	13186
				1.980	14.9	

The decrease of the deformation modulus in 2 years is: 14399/13186 = 1.09 times, i.e. less than 10%.

The calculation of the frame from dome ribs without sheathing showed that at the same load of 3.2 kPa the deflection in the middle of the loaded part was 293 mm, which is 1/205 diameter, and the maximum stresses were equal to 10.95 MPa, i.e. it is quite close to the calculated resistance of 13.0 MPa. These results were obtained at a relative height of ribs' cross-section *h/D*=1/60 – reduced if you compare with the recommended norms for planar structures of 1/40 diameter.

Rib bend on opposite side is 1/206 D, i.e. it is the same with the deflection and for a planar construction (beams of attic floors and girders, trusses) and it is within the permissible limits (<1/200 D). However, for beams and trusses, deflection of covering is limited to 1/300 D.

The maximum moment is M_s = 309.386 kN m.

The corresponding longitudinal force but around support area is N = 333,3 kN when $b \times h$ =0.2·1m.

$$\sigma = \frac{N}{A} + \frac{M}{W} = \frac{333.3}{0.2} + \frac{309.4}{0.033333} = 10.95 \text{ MPa} < R_u = 13 \text{ MPa}$$

It is quite close to the calculated resistance, but it should be taken into account that this is for h/D = 1/60 < 1/40, which already indicates the spatial work of the dome frame.

The maximum stress in the annular ribs is when N = 109,88 kN; M = 4,69 kNm; $b \times h$ = 10·20 cm:

$$\sigma = \frac{109.9}{0.02} + \frac{4.7}{0.000667} = 12.5 \text{ MPa} < 13 \text{ MPa}.$$

SUMMARY

The joint work of ribs of a dome frame with elements filling the cells between them has a great influence on increasing the strength resistance of a structure as a whole.

Nonlinear calculations using the method of integral estimates allow us to analyse the strength resistance of complex modern wooden structures and possibly others, taking into account the long-term loading of any uneven loads in time and magnitude.

The calculations of domes with a reduced height of ribs' cross-section by 20% in comparison with the accepted design guidelines have been carried out. The state of a structure and its strength resistance under different loading conditions and with different stiffness of the cell filling panels is analysed.

The possibility of reducing the section height of meridional ribs and other possibilities of saving materials of these common structures are considered.

In general, the new effective calculation method and design recommendations can provide material savings up to 25%.

Thus, despite the available effective mathematical apparatus for calculating structures taking into account the nonlinear operation of wood, there are no recommendations for its application in the norms and standards and there is no indication of the need to design taking into account the joint work of elements to ensure the structural safety while reducing the consumption of materials.

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FUNDAMENTAL ERROR OF THE THEORY OF DURABLE RESISTENCE AND STANDARDS FOR REINFORCED CONCRETE

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Abstract: This article identifies and investigates the errors in the foundations of the modern theory of creep of reinforced concrete caused by the use of the principle of superposition, which is an extensive interpretation of the principle (scheme) of the linear superposition of Boltzmann. The results of the analysis published by the authors in the journal of Structural Mechanics of Engineering Constructions and Buildings No. 6 of 2017 and No. 3 of 2016 are supplemented. The article was written in accordance with the recommendations of the round table held in the RUDN University on June 9, 2016, under the guidance of D.Sc., Prof. S.N. Krivoshapko.

Keywords: elastoplastic deformations of concrete, creep theory of concrete, long-term resistance of reinforced concrete, modern building codes

ОШИБКИ В ТЕОРИИ ПОЗЛУЧЕСТИ ЖЕЛЕЗОБЕТОНА И СОВРЕМЕННЫЕ НОРМЫ

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Аннотация: В данной статье отмечаются и исследуются ошибки, имеющиеся в основах современной теории ползучести железобетона, вызванные использованием принципа суперпозиции, который представляет собой обширную интерпретацию принципа (схемы) линейной суперпозиции Больцмана. Результаты анализа, опубликованные авторами ранее в журнале «Строительная механика инженерных сооружений и зданий» (№6 за 2017 год и №3 за 2016 год), здесь существенно дополнены. Настоящая статья написана в соответствии с рекомендациями круглого стола, проведенного в Российском университете дружбы народов (РУДН) 9 июня 2016 года под руководством д.т.н., проф. С.Н. Кривошапко.

Ключевые слова: упругопластические деформации бетона, теория ползучести бетона, долговременное сопротивление железобетона, современные строительные нормы

The principle of superposition is the basis of both the modern scientific creep theory of concrete, which is called the "world harmonized format" by foreign scientists, and the developments "in recent decades of international standardization institutions ... for recommendations, norms and technical guidance documents" [1, 2, 3]. These works also indicate that McHenry in USA (1943) "substantiated this trend by experimental studies of the creep of hermetic specimens using the principle of superposition which is characteristic for the theory of Volterra".

We give the basic law of creep of concrete in the original notation [1]:

$$\varepsilon_\sigma(t)=\sigma(t_0)J(t,t_0)+ \\ +\int_{t_0}^{t}J(t,t')d\sigma(t') \qquad (1)$$

where $\varepsilon_\sigma(t)$ is the complete strain from stress $\sigma(t)$;

$$J(t,t')=\frac{1}{E_C(t')}+\frac{\varphi(t,t')}{E_C(t')}$$

– compliance function; $E_C(t')$ is nonstationary modulus of elasticity; $\varphi(t,t')$ is nonstationary creep characteristic considering ageing.

In scientific publications (1) is usually integrated by parts, thus obtaining

$$\varepsilon_\sigma(t)=\frac{\sigma(t)}{E_c(t)}- \\ -\int_{t_0}^{t}\sigma(t')\frac{\partial}{\partial t'}\left[\frac{1}{E_c(t')}+\frac{\varphi(t,t')}{E_c(t')}\right]dt'. \qquad (1')$$

The term

$$\frac{\varphi(t,t')}{E_C(t')}$$

is a measure of the creep of concrete *C(t,t')* used in publications in our country, which is preferable to application of the creep characteristics in the processing of experiments.

We emphasize that ageing of concrete is taken into account in φ (t, t ') and C (t, t'), and the modulus of elastic-instantaneous deformation Ec (t ') essentially depends on the age of the concrete.

Equations (1), (1′) are substantiated by two fundamental assumptions: the principle of linear connection between stresses and strains

$$\varepsilon_\sigma(t,t')=\sigma(t')J(t,t'); \qquad (1'')$$

the principle of superposition, verbally formulated in various versions in numerous well-known publications on the theory of creep of concrete, reference books, for example in [9]. Serious mistakes in (1) make the normative theory inconsistent with Eurocode, unreliable and uneconomical. Losses from such norms and calculations are significant as annual global volume of usage of concrete and reinforced concrete is 4 billion m^3. Let us also recall the tragedy of the collapse of the Transvaal Park (Moscow, 2004), caused by creep problems in concrete.

We note that the article has no relation to the “ongoing disputes, ... discrepancies and uncertainties” existing in this section of creep of reinforced concrete. Also, in this paper we do not discuss a different point of view. We, using the Eurocode system, identify and analyze the errors in that area of creep, where, as the leaders and developers of norms indicate, there is a “steady consensus” [1, 2, 3].

The main mathematical error in (1) lies in its basis - the principle of superposition, which appeared in the theory of reinforced concrete after the work of McHenry. This principle incorrectly builds the core of creep, incorrectly describes the processes of changing instantaneous deformations and creep strains. The errors in the principle of superposition can be determined in various ways: for example, by constructing and solving a differential equation corresponding to a linear connection (1″); sololving the inverse problem of classical mechanics; analysing the value of the total strain rate corresponding to (1″).

Applying the last method the following is obtained:

$$v_\sigma(t,t')=\dot{\sigma}(t')\cdot J(t,t')+ \\ +\sigma(t')\frac{\partial J(t,t')}{\partial t}+\sigma(t')\frac{\partial J(t,t')}{\partial t'}.$$

From this formula it is clearly seen that four terms, caused by the rate of change in the compliance factor are lost in the main law (1):

$$-\sigma(t')\frac{\dot{E}_c(t')}{E_c^2(t')}+\sigma(t')\frac{1}{E_c(t')}\frac{\partial\varphi(t,t')}{\partial t}+$$
$$+\sigma(t')\frac{1}{E_c(t')}\frac{\partial\varphi(t,t')}{\partial t'}- \qquad (2)$$
$$-\sigma(t')\varphi(t,t')\frac{\dot{E}_c(t')}{E_c^2(t')},$$

and the value of these terms is comparable with that of the remaining term. These losses cause considerable discrepancies between the theory and the experiments described in the scientific literature, e.g. [7].

Opposite mathematical actions, first differintiation and then integration, are performed (and without any need) over the known result (1″) of the classical theory in the priciple of superposition.

One term for instantaneous deformations and several terms for creep deformations are lost in the process of differentiation. After integration, the losses are included into the values of deformations, and then into the theory of design calculations.

The principle of superposition distorts the classical linear connection (1″), causing three types of errors [4, 5, 8], distorting the theory of creep of concrete:

1. incorrectly determines the values of short-term linear strains;
2. incorrectly finds the expression of a nucleus describing the process of changing linear creep strains;
3. erroneously classifies as instantaneous elastic deformations to creep strains.

Let us consider them in more detail.

1. The rate of elastic deformation equals

$$\dot{\varepsilon}_y(t')=\dot{\sigma}(t')\frac{1}{E_c(t')}+\sigma(t')\frac{\partial}{\partial t'}\frac{1}{E_c(t')}.$$

Integrating, we obtain

$$\varepsilon_y(t)-\varepsilon_y(t_0)=\int_{t_0}^{t}\frac{1}{E_c(t')}d\sigma(t')+$$
$$+\int_{t_0}^{t}\sigma(t')\frac{\partial}{\partial t'}\frac{1}{E_c(t')}dt'$$

Integrating the first term by parts, we find

$$\varepsilon_y(t)-\varepsilon_y(t_0)=\frac{\sigma(t)}{E_c(t)}-\frac{\sigma(t_0)}{E_c(t_0)}-$$
$$-\int_{t_0}^{t}\sigma(t')\frac{\partial}{\partial t'}\frac{1}{E_c(t')}dt'+$$
$$+\int_{t_0}^{t}\sigma(t')\frac{\partial}{\partial t'}\frac{1}{E_c(t')}dt'$$

Hence the short-term deformation equals

$$\varepsilon_y(t)=\frac{\sigma(t)}{E_c(t)}.$$

It is also clear that the first term under the integral sign (1′) is superfluous, and the use of the overlapping principle in (1) and (1′)

$$\varepsilon_y(t)=\frac{\sigma(t_0)}{E_c(t_0)}-\int_{t_0}^{t}\frac{1}{E_c(t')}d\sigma(t')=$$
$$=\frac{\sigma(t)}{E_c(t)}-\int_{t_0}^{t}\sigma(t')\frac{\partial}{\partial t'}\frac{1}{E_c(t')}dt' \qquad (4)$$

is strongly erroneous.

The principle of overlapping erroneously reconstructs the actual, real elastic linear model of concrete with the $E_c(t)$ module; the prinicple attaches to it a non-existent and unreal model of a linear viscous fluid with a viscosity coefficient

$$K_1(t')=\frac{E_c^2(t')}{\dot{E}_c(t')},$$

thus forming Maxwell's scheme.

Let us consider an example, putting $\sigma(t)=\sigma_0=const$ in (3), (4), we will receive

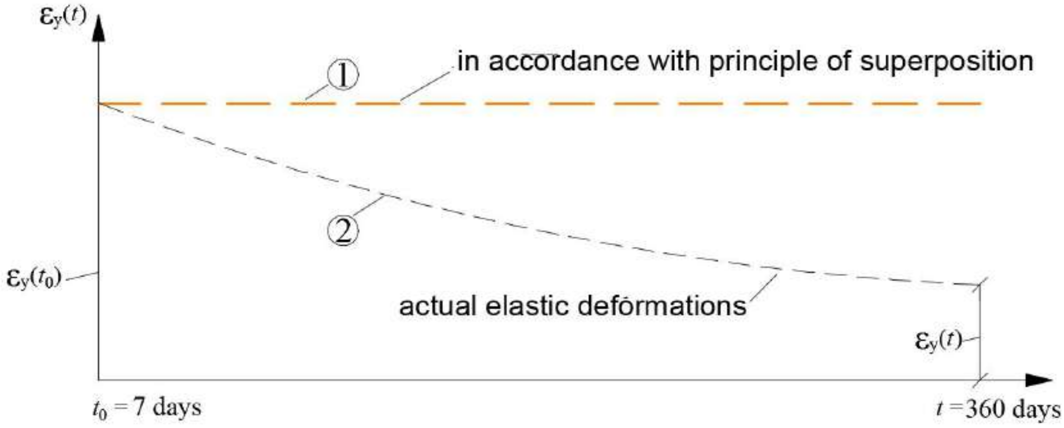

Figure 1. Comparison of $\varepsilon_y(t_0)$ *and* $\varepsilon_y(t)$.

$$\varepsilon_y(t) = \frac{\sigma_0}{E_c(t)}$$

and

$$\varepsilon_y(t_0) = \frac{\sigma_0}{E_c(t_0)} = const\ .$$

Comparison of these deformations is shown in Fig. 1.

Curve 2 in Fig. 1 corresponds to the VNIIG data on the changing of modulus of elasticity with time. Errors in the value of elastic deformation are about ≈ 300% at t = 360 days.

2. In the region of creep deformations, the number of additional (fictitious) bodies arising due to an incorrect scheme for constructing the creep kernel (hereditary function of type I) increases substantially. It depends on the form of the function *φ(t,t')* describing the nonstationary creep characteristic in the main law (1). We write this function in a well-known, widely used in the scientific literature form

$$\frac{\varphi(t,t')}{E_c(t')} = \frac{\varphi_\infty(t')\left[1 - e^{-\gamma(t-t')}\right]}{E_c(t')}, \tag{5}$$

where $\varphi_\infty(t')$ is a function considering the ageing of concrete.

In the famous monograph of Prokopovich I.E. the creep behavior *φ(t,t')* used by foreign scientists has the designation $\overline{C}(t,\tau)$, these are identical quantities.

In case (5), the fundamental law (1) forms four extra (fictitious) bodies: two Foigt type bodies and two viscous elements connected in series with each other. Deformations of these bodies are equal

$$\varepsilon_{1\phi}(t) = \int_{t_0}^{t} \sigma(t') \frac{1}{\eta_{1\phi}(t')} e^{-\gamma(t-t')} dt',$$

$$\eta_{1\phi}(t') = \frac{E_c(t')}{\dot{\varphi}_\infty(t')}; \tag{6}$$

$$\varepsilon_{2\phi}(t) = \int_{t_0}^{t} \sigma(t') \frac{1}{\eta_{2\phi}(t')} dt',$$

$$\eta_{2\phi}(t') = \frac{E_c^2(t')}{\dot{E}_c(t')} \frac{1}{\varphi_\infty(t')}; \tag{7}$$

$$\varepsilon_{3\phi}(t) = \int_{t_0}^{t} \sigma(t') \frac{1}{\eta_{3\phi}(t')} e^{-\gamma(t-t')} dt',$$

$$\eta_{3\phi}(t') = -\frac{E_c^2(t')}{\dot{E}_c(t')} \frac{1}{\varphi_\infty(t')}; \tag{8}$$

$$\varepsilon_{4\phi}(t) = \int_{t_0}^{t} \sigma(t') \frac{1}{\eta_{4\phi}(t')} dt',$$

$$\eta_{4\phi}(t') = -\frac{E_c(t')}{\dot{\varphi}_\infty(t')}, \tag{9}$$

where $\eta_{1\phi}$, ... , $\eta_{4\phi}$ are the viscosity coefficients or the coefficients of internal resistance of the fictitious bodies; moreover, the bodies (8) of Voigt and (9) of the viscous element expand under compression.

The creep deformations (6) - (9), caused by the effect of the superposition principle on the classical bond (1″), are a fiction; they are also summed up with a short-term fictitious deformation

$$\varepsilon_{5\text{ф}}(t) = -\int_{t_0}^{t} \sigma(t') \frac{\partial}{\partial t'} \frac{1}{E_{\text{c}}(t')} dt' : \qquad (10)$$

$$\varepsilon_{\sigma\text{ф}}(t) = \sum_{i=1}^{5} \varepsilon_{i\text{ф}}(t),$$

and introduce large errors in the value of the total deformation $\varepsilon_\sigma(t)$ determined by the creep law (1′). For example (Recommendations, 1988), at constant stresses, the error from applying the superposition principle for creep strains reaches 100%:

$$\frac{\varepsilon_{\text{c}\sigma}(t)_{\text{ошибки}}}{\varepsilon_{\text{c}\sigma}(t)_{\text{принцип}}} = 1 - \frac{\int_{t_0}^{t} \Omega(\tau) f(t-\tau) d\tau}{\Omega(t_0) f(t-t_0) d\tau},$$

where $\Omega(\tau)$ is "the function of the effect of ageing on the measure of creep";
$f(t\text{-}\tau)$ is - "a function that takes into account the increase in time creep measure".

3. The fact of appearance of a single short-term strain

$$\frac{1}{E_{\text{c}}(t')}$$

in the nucleus of creep of the integral equation (1′):

$$\frac{\partial}{\partial t'}\left[\varepsilon_{\text{y},1}(t') + C(t,t')\right] =$$

$$= \frac{\partial}{\partial t'}\left[\frac{\varepsilon_{\text{y}}(t')}{\sigma(t')} + C(t,t')\right]$$

led to the temptation of erroneous substitution of the properties of short-term deformation $\varepsilon_{\text{y},1}(t')$ by the properties of deformations of the hereditary type $\varepsilon_{\text{y},1}(t,t')$.

The error is corrected by making new mistakes. Concrete has essentially non-linear properties at short-term and long-term loading. The short-term load diagram has a falling section and a limited extent, see figure 2. In the main law (1), (1′) only linear deformation

$$\varepsilon_{\text{л}}(t) = \varepsilon_{\text{y}}(t)$$

is taken into account, and the nonlinear deformation $\varepsilon_{\text{н}}(t)$ is ignore, see figure 2. Aleksandrovsky S.V. indicates the reason for this circumstance: ”It is very difficult to take into account the dependence of the modulus of elasticity on stresses and age of concrete simultaneously. Therefore, the modern theory of creep of concrete takes into account only a change in the modulus in time ... “.

Let us consider two types of such substitution.

<u>The first substitution.</u> A representative forum poses the erroneous task of "taking into account the influence of the pre-history of deformation on the modulus of elastic-instantaneous deformations". The basic equation of the creep theory takes the form (in the original notation):

$$\varepsilon(t) = \frac{\sigma(t)}{E_c(t,t')} - \int_{t_0}^{t} \sigma(\tau) \frac{\partial}{\partial \tau}\left[\frac{1}{E_c(t,\tau)} + C(t,\tau)\right] d\tau \qquad (11)$$

An “experimentally valid” expression appears for the modulus of elastic deformation of concrete

An "experimentally valid" expression appears for the modulus of elastic deformation of concrete

$$E_{t,\tau} = E_t + a_{n,\tau} \varphi_t E_\tau,$$

where φ_t is characteristic of creep of concrete.

And other erroneous forms of the main creep law appear

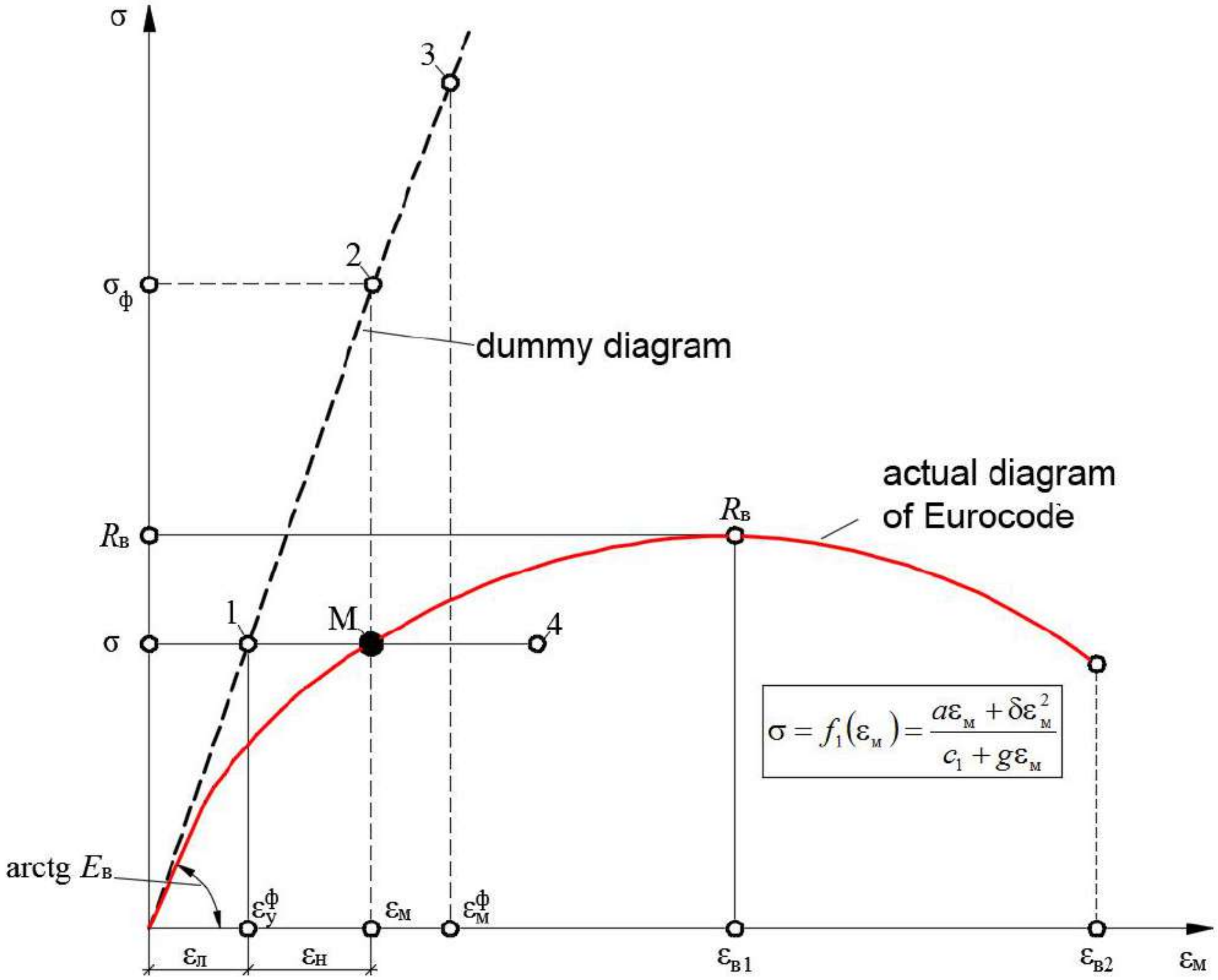

Figure 2. Distortion of the σ-ε diagram of concrete.

$$\varepsilon(t)=\frac{\sigma(t)}{E(t)}+\int_{\tau_1}^{t}\sigma(\tau)\frac{\partial}{\partial\tau}\chi(t,\tau)\,d\tau-$$
$$-\int_{\tau_1}^{t}\sigma(\tau)\frac{\partial}{\partial\tau}C^{*}(t,\tau)\,d\tau, \qquad (12)$$

where

$$\frac{\partial}{\partial\tau}C^{*}(t,\tau)=\frac{\partial}{\partial\tau}\left[\frac{1}{E(\tau)}+C(t,\tau)\right];$$

$\chi(t,\tau)$ has the name "reducing correction ... to the current specific elastic-instantaneous deformations".

The second substitution. The nonlinear short-term strain $\varepsilon_H(t)$ is erroneously attributed to the deformation properties of the hereditary type $\varepsilon_H(t,t')$, the erroneous overlapping principle is used, and, instead of the simple algebraic formula

$$\varepsilon_H(t)=B_2(t)\sigma^2(t)$$

(B_2 is a known coefficient), the integral following is contrived:

$$\varepsilon_{н}(t)=\int_{t_0}^{t}\sigma(t')\frac{\partial}{\partial t'}\frac{\varepsilon_{н}(t,t')}{\sigma(t')}dt' = \int_{t_0}^{t}\sigma(t')\frac{\partial}{\partial t'}C_{н}(t,t')dt' \quad (13)$$

where $C_{н}(t,t')$ is called the measrue of fast-flowing creep.

$$C(t,t')+C_{н}(t,t')= =\frac{1}{E_{с}(t')}\left[\varphi(t,t')+\varphi_{н}(t,t')\right], \quad (14)$$

taken into account in (1′). The gross errors in the theory from such a substitution of the short-term nonlinearity of concrete we considered in [4] and [8].

Famous foreign scientists renamed “fast-flowing creep” into "minute creep", and the erroneous idea of the Second substitution is presented as their important achievement.

The principle of superposition in the theory of creep of concrete is a mathematical error committed in the exptensive interpretation of the principle of the linear superposition of Boltzmann. In international norms of reinforced concrete, it is estimated incorrectly: it is supposedly “a tendency to study creep ... according to the principle of superposition peculiar to Volterra's theory”. Let us consider this in more detail.

We investigate the essence and the secondary nature of the Boltzmann scheme for the theory of creep of concrete on the example of concrete considered in the well-known paper of Maslov G.N. No. 4. Here the concrete has stationary properties corresponding to the classical theory. In the notation of Maslov G.N. the compliance function has the form

$$J(t-t')=\Phi(t-\tau)=a-be^{-\beta(t-\tau)},$$

where

$$a=\frac{C_0+E_0}{C_0E_0};$$

E_0 is an elastic modulus;

$$b=\frac{1}{E_0}; \quad \eta=\frac{C_0}{\beta},$$

η is a stationary coefficient of linear viscosity.

In the theory of creep, the fundamental solution of the corresponding differential equation is known to have the form

$$\varepsilon_{\sigma}(t)=\frac{\sigma(t)}{E_0}-\int_{t_0}^{t}\sigma(t')\frac{1}{E_0}\frac{\partial\varphi(t-t')}{\partial t'}dt', \quad (15)$$

where

$$\varphi(t-t')=E_0\frac{1}{C_0}\left[1-e^{-\beta(t-t')}\right]$$

is characteristic of creep.

The Boltzmann case is obtained from the solution of (15) by means of a number of its transformations mathematically valid only under the conditions of stationary properties

$$\varepsilon_{\sigma}(t)=\sigma_0\left[\frac{1}{E_0}+\frac{1}{E_0}\varphi(t-t_0)\right]+ +\int_{t_0}^{t}\left[\frac{1}{E_0}+\frac{1}{E_0}\varphi(t-t')\right]d\sigma(t') \quad (15')$$

Unlike (15), the compliance function is used in the transformation (15'), which attracted the attention of scientists. However, the transformation (15′) is possible only with substantial and very strong restrictions. In the exptensive interpretation of compliance, these restrictions were not taken into account, and the theory of creep of concrete proved to be deeply erroneous.

Here, firstly, the property of the process that creates the temptation to expand the theory and transforms into the above-mentioned gross error for nonstationary $E(t')$ accompanying the normative linear creep theory of concrete is imposed on instantaneous deformation with an extremely simple physical meaning for an arbitrary t. In scientific literature there is even an authoritative statement that "elastic-instantaneous deformations strictly obey ... the principle of superposition".

Secondly, it is necessary to integrate (15) by parts, that in the exptensive interpretation of the compliance function under the conditions of ageing of concrete (1) creates another temptation, traditionally leading to another gross error in finding the core of the integral equation. As it is known, for non-stationary properties of concrete, the creep strain is obtained from another solution of differential equation, a solution written in a more complex form

$$\varepsilon_{cc}(t)=e^{-F(t)}\left|\varepsilon_{c0}+\int_{t_0}^{t}\sigma(t)\frac{1}{\eta(t)}e^{F(t)}dt\right|,$$

$$F(t)=\int_{t_0}^{t}\beta(t)dt,$$

where the parameters $\eta(t)$ and $\beta(t)$ in (15) are functions of time.

In the concrete of Maslov G.N. the rate of deformation degenerates due to the difference kernel. In the case of an extensive interpretation of the compliance factor, the application of the Boltzmann principle usually becomes incorrect. The nonstationary model of Maslov concrete with a coefficient of viscosity

$$\eta(t)=C_0(t)/\beta$$

and a time-dependent module $E_0(t)$ demonstrates this:

– it satisfies experiments with simple loading at low levels $\sigma\approx 0{,}1R_{\text{пр}}$;
– it satisfies the requirements of classical mechanics;
– it does not satisfy the conditions of the Boltzmann principle.

The Boltzmann principle distorts the essence of the nonstationary Maslov model. It replaces one classical body of creep of concrete with a chain model of successively connected bodies with a set of erroneous properties.

In the theory of creep of concrete, there is a case when extensive interpretation of the compliance function is unacceptable even with a difference kernel. For example, the nucleus of creep in a number of known works is represented in the form (the second case)

$$K(t-t')=\frac{Ae^{-\beta(t-t')}}{(t-t')^{\alpha-1}}.$$

Certain forces correspond to this kinematic equation of motion in connection with the solution of the inverse problem of mechanics. The analysis of the differential creep equation reveals that in this nucleus there is a resistance force with a coefficient of viscosity of the linear model equal to

$$\eta(t,t')=\frac{1}{A}(t-t')^{\alpha-1},$$

which is impossible by the same reasons as in the above-mentioned case of applying the hereditary properties of the elastic modulus $E(t,t')$.

The third case corresponds to the extensive interpretation of the compliance function in the “chain model”. This case is present in theoretical rheology, and as a repetition – in the norms of reinforced concrete.

We preliminarily write the Boltzmann scheme for the Maxwell body in the form

$$\varepsilon_{\sigma}(t)=\sigma_0\left[\frac{1}{E_0}+\frac{1}{\eta}(t-t_0)\right]+\int_{t_0}^{t}\left[\frac{1}{E_0}+\frac{1}{\eta}(t-t')\right]d\sigma(t') \qquad (16)$$

where η is a stationary coefficient of viscosity.

With a variable viscosity coefficient

$$\eta(t) = \frac{E_0}{\dot{\varphi}(t)},$$

we obtain the theory of ageing of concrete (Dischinger, Whitney);

$$\varphi(t) = \varphi_\infty \left(1 - e^{-bt}\right),$$

which by series expansion gives the function of Freudenthal

$$\varphi(t) = \frac{\varphi_\infty t}{\frac{1}{b} + t},$$

substantiated by the experiments of Davis and Glanville.

In the "chain model", by successively connecting bodies (15) and (16), we have an extension record of the compliance function

$$J(t-t') = \frac{1}{E_0} + \frac{1}{E_0}\varphi(t-t') + \frac{1}{\eta}(t-t') \tag{17}$$

A pair of integral equations corresponding to the expansion hypothesis (17), and solved either with respect to deformations $\varepsilon_\sigma(t)$, or relative to the stresses σ (t), in theoretical rheology are called "Boltzmann-Volterra equations"; It is also indicated that this pair "represents a complete mathematical formulation of the principle of linear superposition."

However, such a chain model, with its extensive interpretation of the compliance coefficient, is essentially erroneous; This is evidenced by its reduction to a differential form:

$$\ddot{\varepsilon}_\sigma(t)\frac{\eta}{\beta} + \dot{\varepsilon}_\sigma(t)\eta = \ddot{\sigma}(t)\frac{\eta}{E_0\beta} + \dot{\sigma}(t)\left(\frac{\eta}{E_0} + \frac{1}{\beta} + \frac{\eta}{C_0}\right) + \sigma(t) \tag{17'}$$

It can be seen from (17 ') that there is a resistance force

$$\ddot{\varepsilon}_\sigma(t)\frac{\eta}{\beta}$$

proportional to the acceleration, which is incompatible with classical mechanics, and, in connection with Art. 5.1.1 (3) P Eurocode 0, the chain model is an inappropriate design model.

The components of the force of the computational model can be a function of position $\varepsilon_\sigma(t)$, speed $\dot{\varepsilon}_\sigma(t)$, time and other quantities. If there is (among others) a force proportional to acceleration $\ddot{\varepsilon}_\sigma(t)$, then the fundamental principle of mechanics about the independence of the action of forces is violated. The well-known scientist Pare L. has established the unacceptability of such forces in both problems of mechanics and in applications [6].

Unfortunately, in the scientific literature on concrete, in international norms, there are a number of errors analogous to those described, and consisting in an extensive interpretation of the compliance function in the form of a chain model [1], including for taking into account the rapidly flowing creep.

Thus, in the case of consistent merging of Maslov's theory and the theory of ageing of concrete (McHenry, Yashin A.V., Hansen T., Prokopovich I.E. and Ulitsky I.I.), the creep equation has the form

$$\ddot{\varepsilon}(t) + \beta\dot{\varepsilon}(t) = \ddot{\sigma}(t)\frac{1}{E_0} + \dot{\sigma}(t)\left(\frac{\dot{\varphi}_t}{E_0} + \frac{\beta}{E_0} + \frac{\beta}{C_0}\right) + \sigma(t)\left(\frac{\ddot{\varphi}_t}{E_0} + \frac{\dot{\varphi}_t}{E_0}\right).$$

If another viscous element with viscosity

$$\eta(t) = \Delta e^{-\alpha_1 t}$$

is added to this chain in order to take into account the rapidly flowing creep, that was previously assumed by the Eurocode developers before its approval, then we get another erroneous version of the theory (written without averaging)

$$\begin{aligned}&\ddot{\varepsilon}(t)+\beta\dot{\varepsilon}(t)=\ddot{\sigma}(t)\frac{1}{E_0}+\\&+\dot{\sigma}(t)\left(\frac{\dot{\varphi}_t}{E_0}+\frac{\beta}{E_0}+\frac{1}{\eta(t)}\right)+\\&+\sigma(t)\left(\frac{\ddot{\varphi}_t}{E_0}+\frac{\beta\dot{\varphi}_t}{E_0}+\frac{\beta}{\eta(t)}-\frac{\dot{\eta}(t)}{\eta^2(t)}\right).\end{aligned} \tag{*}$$

When Eurocode 2 was adopted, the theory of ageing and the viscous element were removed from this model, the error was annulled. In the Eurocode rules, only classic concrete Maslov G.N. is left; from its creep characteristics, a normative coefficient of creep development is obtained

$$\beta_c(t,t_0)=\left[\frac{t-t_0}{\beta_{\text{H}}+t-t_0}\right]^{0,3},$$

where

$$\beta_{\text{H}}=1/\beta .$$

It is obtained by decomposing

$$e^{-\beta(t-t_0)}$$

in a series using two terms. The exponent 0.3 of the power function takes into account on average the ageing of the concrete.

In the case of nonlinear creep and short-term non-linearity in Eurocodes, the use of the Boltzmann scheme is also erroneous. For nonlinear creep of concrete of Maslov G.N. (the fourth case) within the framework of generally accepted hypotheses, the rate of deformation is

$$\begin{aligned}&\mathrm{v}_\sigma\left\{t,t',F\left[\mu(t'),t'\right]\right\}=\\&\dot{\sigma}(t')\cdot F\left[\mu(t'),t'\right]\frac{1}{E_0}\varphi(t-t')+\\&+\sigma(t')\cdot\dot{\mu}(t')\frac{\partial F\left[\mu(t'),t'\right]}{\partial\mu}\frac{1}{E_0}\varphi(t-t')+\\&+\sigma(t')\cdot\frac{\partial F\left[\mu(t'),t'\right]}{\partial t'}\frac{1}{E_0}\varphi(t-t')+\\&+\sigma(t')\cdot F\left[\mu(t'),t'\right]\odot\\&\odot\frac{1}{E_0}\left[\frac{\partial\varphi(t-t')}{\partial t}+\frac{\partial\varphi(t-t')}{\partial t'}\right],\end{aligned}$$

which is not taken into account in the traditional theory. Here $F[\mu(t'),t']$ is a non-linearity function, in which the voltage

$$\mu(t')=\sigma(t')$$

is usually taken (after the work of Leaderman) as a nonlinearity parameter, which is incorrect: the methods of classical mechanics show that such an assumption is a very superficial assumption. We will devote a separate article to this problem.

For example, under this assumption, a series of multiple Volterra-Frechet integrals

$$\begin{aligned}&\varepsilon_\sigma(t)=\int_{-\infty}^{t}J_1(t-t')d\sigma(t')+\\&+\int_{-\infty}^{t}\int_{-\infty}^{t}J_2(t-t',t-t'')d\sigma(t')d\sigma(t'')+\ldots\end{aligned}$$

is a nonintegral form [10]

$$\begin{aligned}&\varepsilon_\sigma(t)=J_1(t)\sigma+J_2(t,t)\sigma^2+\\&+J_3(t,t,t)\sigma^3+\ldots\end{aligned}$$

Recently, some papers have appeared that develop "a modification of the principle of superposition of deformations for nonlinear creep" in the form

$$\varepsilon(t,t_0)=\varepsilon(t_0)+ \\ +\int_{t_0}^{t}\left[\frac{1}{E(\tau)}+C(t,\tau)\right]d\sigma_c(\tau) \quad (18)$$

where

$$\sigma_c(\tau)=S[\sigma(\tau)]$$

is the known stress function σ[τ].

The error of this formulation is similar to that used in (1). The total strain rate here is

$$v_\sigma(t,\tau)=\dot{S}\left[\sigma(\tau)\right]\left[\frac{1}{E(\tau)}+C(t,\tau)\right]+ \\ +S\left[\sigma(\tau)\right]\frac{d}{d\tau}\frac{1}{E(\tau)}+ \\ +S\left[\sigma(\tau)\right]\frac{\partial}{\partial\tau}C(t,\tau)+ \\ +S\left[\sigma(\tau)\right]\frac{\partial}{\partial t}C(t,\tau). \quad (18)$$

From this it is clear that the last three terms in (18') are lost in (18). The significance of these terms is identical to the significance that we described in paragraphs 1-3 above. We must additionally pay attention to the fact that the identity of the nonlinear function $S[\sigma(\tau)]$ for short-term and long-term deformations is also incorrect. But even if another function $S_g[\sigma(\tau)]$ is used for creep strains, then, as it is noted above, this assumption is a very superficial assumption that does not correspond to the real nonlinear creep theory of concrete, which will be published later. This theory has nothing to do with the principle of superposition.

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NON-RIGID KINEMATIC EXCITATION FOR MULTIPLY-SUPPORTED SYSTEM WITH HOMOGENEOUS DAMPING

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Abstract: This paper continues the discussion of linear equations of motion. The author considers non-rigid kinematic excitation for multiply-supported system leading to the deformations in quasi-static response. It turns out that in the equation of motion written down for relative displacements (relative displacements are defined as absolute displacements minus quasi-static response) the contribution of the internal damping to the load in some cases may be zero (like it was for rigid kinematical excitation). For this effect the system under consideration must have homogeneous damping. It is the often case, though not always. Zero contribution of the internal damping to the load is different in origin for rigid and non-rigid kinematic excitation: in the former case nodal loads in the quasi-static response are zero for each element; in the latter case nodal loads in elements are non-zero, but in each node they are balanced giving zero resulting nodal loads. Thus, damping in the quasi-static response does not impact relative motion, but impacts the resulting internal forces. The implementation of the Rayleigh damping model for the right-hand part of the equation leads to the error (like for rigid kinematic excitation), as damping in the Rayleigh model is not really "internal": due to the participation of mass matrix it works on rigid displacements, which is impossible for internal damping.

Keywords: seismic response, Rayleigh damping model, multiply-supported systems

НЕЖЕСТКОЕ КИНЕМАТИЧЕСКОЕ ВОЗДЕЙСТВИЕ ДЛЯ МНОГООПОРНОЙ СИСТЕМЫ С ОДНОРОДНЫМ ДЕМПФИРОВАНИЕМ

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Аннотация: В настоящей статье продолжается ранее начатое обсуждение вопросов вывода уравнений движения для линейных расчетов сооружений на динамические воздействия. Автор рассматривает «нежесткое» движение опор многоопорной системы, порождающее деформации уже в квазистатической реакции. Оказывается, что в уравнениях движения, записанных в относительных перемещениях (относительные перемещения определяются как абсолютные перемещения за вычетом квазистатической реакции), вклад матрицы внутреннего демпфирования в нагрузку может оказаться равным нулю даже для «нежесткого» смещения опор, - подобно тому, как это было показано ранее для «жесткого» смещения опор. Однако для этого рассматриваемая система должна быть однородной по демпфированию. Такая ситуация на практике встречается часто, хотя и не всегда. Между нулевым вкладом матрицы демпфирования в нагрузки в случаях «жесткого» и «нежесткого» движения опор для однородной по демпфированию системы есть принципиальная разница: в первом случае в квазистатической реакции соответствующие узловые силы равны нулю в каждом элементе, а во втором случае в деформированных элементах появляются усилия, но в узлах их суммы равны нулю. Демпфирование, связанное с квазистатической реакцией, не повлияет на относительные перемещения, но проявится при вычислении полных внутренних усилий. Использование модели демпфирования Рэлея для правой части уравнения движения, как и в случае «жесткого» кинематического возбуждения опор, приводит к ошибочным результатам, поскольку демпфирование в модели Рэлея благодаря участию матрицы масс работает на жестких смещениях системы, в отличие от внутреннего демпфирования.

Ключевые слова: сейсмическая реакция, модель демпфирования Рэлея, многоопорные системы

Discussion about damping at forming of seismic loadings piques interest of civil engineers [1, 2]. There are some disagreements about internal damping at right part of movement equation, written in the relative displacement form for multi-supported system. At the same time, author wrote about case of “rigid” support movement of multi-supports systems in the previous papers. It should be specified, that “rigid” movement of some points row is not equivalent of equal forward movement of such points, that can be considered as only partial case. “Rigid” movement of some points row means that one’s move as it “frozen” into absolutely rigid solid body. If this solid body rotates, then forward movement of some points of such body in local coordinate systems, linked with these points, differs in accordance with it coordinate. At the same time, such movement remains “rigid”.

Principle feature of system behavior at “rigid” movement of supports is that quasi static response to such movement is “rigid” too. In other words, at quasi static response not only support but all points of system move "rigidly". At the same time, there are not displacement in system, therefore internal forces, caused by system rigid and damping, don’t appear. This aspect does not depend on a type and homogeneous of internal damping as well as rigidity distribution in a system. Consequently, dividing movement of linear system to translational motion of whole system with it supports and additional movement of each point relative to supports, it appears that right part of movement equation contains only inertia forces. There are not rigidities or damping components in the right part of equation.

After the paper [2] publication, some authors proposed to observe alternative case, when supports movement is not “rigid”. In the paper [2], respective designations and equations are introduced for such movement.

Turning to terminology, as it seems to the author, the “translational motion” badly corresponds to description of quasi static responses in similar system, since system responses includes displacements at “non-rigid” motion of supports. It already is not “translational motion”, as it is at “rigid” motion of supports. Therefore, let us to present absolute displacements U as sum of quasi static response R and relative displacements X. Top index “+” means that it is observed whole columns, including support displacements; missing of this index means that it is observed reduced columns, which includes displacements of all points, excepting supports. Respectively, matrixes with such index and without it take different orders. Let us rewrite equation (10), taken from the paper [2], in the relative displacements form:

$$\begin{aligned}&[M]\left[\ddot{X}(t)\right]+[C]\left[\dot{X}(t)\right]+[K]\left[X(t)\right]=\\&=-\left([M][T]+[M_{sb}]\right)\left[\ddot{R}_b\right]-\left([C][T]+[C_{sb}]\right)\left[\dot{R}_b\right] \quad (1)\end{aligned}$$

Here $[M]$, $[K]$ and $[C]$ are block matrices of inertia, stiffness and viscous damping, corresponding to all system nodes, excepting supports; $[M_{sb}]$ and $[C_{sb}]$ are block matrices of inertia and viscous damping, which link supports and non-support nodes; $[R_b]$ is column of support displacements; $[T]$ is matrix, linking quasi static response of non-support nodes with support displacement:

$$[R]=[T][R_b]; \quad [T]=-[K]^{-1}[K_{sb}] \qquad (2)$$

Where $[K_{sb}]$ is block matrix of stiffness, that links non-support nodes with support nodes. Thus, conclusion of previous paper [2] can be formulated in the following form: if matrices $[C]$ and $[C_{sb}]$ describes internal damping correctly, hen at either rigid displacement of supports $[R_b]$ the last member in the right part of (1) equals to zero. It is should be noted ones more time, that this conclusion doesn’t require homogeneous damping or another special links between damping and stiffness of a system.

Current paper describes non-rigid displacement of supports $[R_b]$. Let us pay attention that second formula from (2) leads to relationship

$$[K][T]+[K_{sb}]=0 \ . \qquad (3)$$

Comparing the left part of (3) with first multiplier at the last addend in the right part of (1), we can conclude, if viscous damping matrix $[C^{+}]$ is proportional to stiffness matrix $[K^{+}]$ in partial case, then first multiplier at the last addend in the right part of (1) equals to zero. Respectively, last addend in the right side of formula (1) equals to zero at support motion or without it. It was paid attention by V.A. Semenov, speaking about proportionality between damping and stiffness.

First of all, such proportionality between damping and stiffness may appears when material of system is homogeneous by damping properties, for example, physically homogeneous, when structure made of steel or reinforced concrete only and loadings don't exceed ultimate values. As it shows by experiments, damping of construction material is not viscous, but plastic or hysteresis or material by Sorokin model. In frequencies range, the real values of elastic modulus are substituted by complex values, where imaginary parts of complex modulus don't depend on the frequency and proportional to real parts. Homogeneous damping means in this case, that proportion coefficients between real and imaginary parts of complex modulus are the same for all materials of system. It is very frequent situation, and conclusion about zero damping at right part of motion equation, written in the form of relative displacements, stay actual. Remark about material work at non-ultimate states is made, because effective modulus and effective damping are changed at large deformations. Here large deformations are not connected with geometric non-linearity, since it is described by equivalent linear models [3, 4]. If all elements are loaded by the same loads, then at deformation closing to ultimate values the homogeneous damping remains actual. However, it may occur situations, when in the system ones' structures are closer to ultimate state than other. In this case the effective damping may differs in different structures at the same material.

Thus, in a system with homogeneous damping even at non-rigid motion of supports the last member of right part of motion equation (1) equals to zero. At first sight, such result is analogous to results, obtained before for rigid motion of supports [2], however these results have principle physical difference. Physical meaning of each addend in equation (1) is nodal forces. In accordance with assembling rules of stiffness and damping matrices in FEM for chosen node these nodal forces can be spread out to force sum, coming in the node from finite elements, linked with this node. The resultant nodal force, determined by stiffness matrix or damping matrix may be null matrix by two reasons: 1) all members, determining by separate multipliers are equal to zero; 2) members are not null, but it sum equal to zero at special situation.

At “rigid” motion of supports the first reason executes. Quasi static response is rigid for entire system as well as for each finite element, therefore nodal forces, linked with stiffness and internal damping at each element are equal to zero. Respectively, resultant nodal forces at arbitrary node are equal to zero.

At “non-rigid” motion of supports the second reason exacts. Finite elements at quasi-static response are deformed, and internal forces appear, that leads to nonzero values of nodal forces. This fact relates not only with stiffness, but damping. As result, integral nodal forces at right part of equation (1) for each node, as it is shown before, are equal to zero, but such result is reached by addition of nonzero members.

Is it mean that damping, linked with quasi static response, disappears from system? No, it is. Let us remember, that equation (1) allows to determinate just relative displacement, velocities and accelerations. It should be noted, that determination of "relative" internal forces linked not only with stiffness, but also damping member in the left part of equation (1). However, these “relative” internal forces must be added to forces, caused by quasi static response, to determinate resultant dynamic internal forces. At "rigid" motion of supports forces, caused by quasi static response, are equal to zero. And here stiffness and damping, working at quasi static displacements and velocities, are applied at quasi static force calculation. Thus, part of damping, linked with quasi static response, influence to resultant forces in a system, though it occurs without “relative” part.

Now let us to discuss some questions of computational practice. Physical hysteresis damping in material forcedly substitute by non-physical damping that is proportional to velocity, using Rayleigh model, to save traditional form of differential motion equations:

$$[C^{+}] = \alpha\,[M^{+}] + \beta\,[K^{+}] \qquad (4)$$

It was already written about approximate and non-physical aspect of such substitution. First member of right part of equation (4) describes not internal, but external damping and works even at rigid displacements, that is principally impossible at internal damping. This member is introduced into Rayleigh model just to approximate restore constancy of modal factors of damping by frequency. Such constancy by frequency is observed in experiments and appears as natural consequence of plastic damping nature. This constancy disappears at introducing of viscous damping instead of plastic damping. It is necessary to apply non-physical substitution to approximate restoration.

In this case, appliance of Rayleigh model can be recognized as traditionally justified for damping matrix. But at right part of (1) in comparison with left part the modal factors of damping are unimportant. If we exclude non-physical first member of Rayleigh model from right part of the equation (1), then remaining second member that proportional to stiffness matrix in accordance with relationship (3) leads to written before right result, that is to disappearance of damping from right part of motion equation, written in the form of relative displacements for system with homogeneous damping.

In this case ultimate transformation, that was destructured by applying of full Rayleigh model (4), which is used at the right part of motion equation. Really, in accordance with physics, “rigid” motion of supports can be described as partial case of "non-rigid" motion. Respectively, equations of “non-rigid” motion of supports should be transformed to equations of "rigid" motion in this case. However non-physical first member of the relationship (4) at right part of (1) interrupts such transformation, because it does not disappear at “rigid” motion of supports. If we exclude this member from right part of the equation, ultimate transformation restores.

Let us discuss consequences of damping member disappearing at right part of equation (1) in the case of system with homogeneous damping conditions. The right part of equation (1) is simplified and takes the form:

$$\left[M\right]\left[\ddot{X}(t)\right]+\left[C\right]\left[\dot{X}(t)\right]+\left[K\right]\left[X(t)\right]= \\ =-(\left[M\right]\left[T\right]+\left[M_{sb}\right])\left[\ddot{R}_{b}\right] \quad (5)$$

The form of this equation is like a traditional equation of “rigid” displacement of supports, that allows expect that traditional linear spectral approach is applicable in this case. But there are two reasons that can break up such expectations. At first, as it is shown before, quasi static response at “non-rigid” displacements of supports makes a contribution to resultant internal forces. Thus, “relative” forces, calculated by equation (5) using analog of linear spectral method, should be added to "quasi static" forces, caused by stiffness as well as damping. There are suggestions of such addition. For example, it is supposed to use approach connected with square root of the sum of squares (SRSS) as it is contained in the codes of nuclear industry [3].

At second, at transformation of equation (5) to equations of the modal method, that is the basis of the linear spectral approach, the difficulties appear with right part of equation (1), and it left part stays traditional. At “rigid” motion of supports quasi static response R, determined by relation (2), is “rigid”. Contribution of each mode to resultant force at impact acting during chosen direction is determined by so called "involvement factor" of this mode through reviewed direction. Since there are six directions of “rigid” impact, then only six "involvement factors" are applied for each mode. Such coefficients are

generated by majority of computational programs as part of standard report at modal analysis.
At “non-rigid” motion of supports mode involvement factors, taken by quasi static responses, substitute involvement factors, taken by impact's directions, at right parts of modal motion equations. For example, if supports displacements [R_{bk}] are correspond to initial seismic impact through the k direction, then modal force q_{jk} for j-th mode [φ_j] is described by analog of involvement factor:

$$q_{jk} = -[\varphi]^T([M][T] + [M_{sb}])\,[\ddot{R}_{bk}] \qquad (6)$$

First two multipliers at right part of (6) are similar to multipliers at traditional involvement factor of j-th mode for arbitrary direction from six directions of “rigid” impact, however third multiplier differs from first ones. Additional problem is that “non-rigid” displacements of supports [R_{bk}] may depend on the frequency or wavelength, that is equivalent to first one. This aspect makes practical calculations of modal force more difficult.
Codes of nuclear industry [1] contains a few techniques to calculate inertia forces of multi supported system by spectral method. All of these methods are approximate and it accuracy significantly depends on the static correlation of impact at different supports.
Now let us refuse to suggestion about homogeneous damping and observe widely distributed partial situation when damping is heterogeneous. This situation occurs, when system with homogeneous damping supplemented by viscous dampers. Typical sample is the reinforced concrete building, built on “foundation suspension” including “foundation dampers”. This approach is frequently applied into platform models of “structure – foundation” systems [5].
The first that should be noted is that quasi static response of such system is similar to system before additional dampers appearance. The second note is that the damping matrix of such system consists of matrix of homogeneous damping and matrices of additional dampers. Consequently "homogeneous" damping disappear from right part of the motion equation, as it was before, but damping members corresponding to additional dampers are saved at the right part.
When “ground springs” are used together with ground dampers the viscous of ground dampers should be divided into two parts. The first part corresponds to “homogeneous damping” in structure and it is determined using β coefficient from equation (4), obtained for structure, fastened to rigid ground spring. And remaining viscous of ground damper is additional and remains at right part of the motion equation.
Conclusions. Initial conclusion about dumping members’ disappearance from the right part of the motion equation is spread to “non-rigid” motion of supports at system with homogeneous damping from “rigid” motion of supports in multi supported systems. It is noted that damping, linked with quasi static response of system to “non-rigid” motion of supports, does not disappear entirely in this case. It influence to results at calculation of internal forces. “Homogeneous” member disappears from the right part of the motion equation at using additional dampers in homogeneous system (for example at foundation suspension of platform models), but “additional” member remains there.

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THE EXPERIENCE DIFFERENT OF PILING TESTING ON PROBLEMATICAL SOIL GROUND OF ASTANA, KAZAKHSTAN

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Abstract: At the present time, in Astana city is going on works by construction public transport system LRT (Light Railway Transport). LRT is an overhead road with two railway lines. The first stage of construction is including construction of overhead road (bridge) with 22,4 km length and 18 stations. The foundation of bridge is the bored piles with cross-section 1.0÷1.5 m and length 8÷35 m. In these conditions, very important to control integrity of concrete body of each bored piles. For checking integrity applying two methods - Low Strain Method and Cross-Hole Sonic Logging. The aim of this paper is to discuss the advantages and disadvantages of each method using the examples of a real application. The article presents loading tests of large diameter and deep boring piles on the construction site in new capital city of the Republic of Kazakhstan. Finally, some recommendations for testing methods suitable for problematical ground conditions of Kazakhstan are introduced. Traditionally, pile load tests in Kazakhstan are carried out using static loading test methods. Static pile loading test is the most reliable method to obtain the load-settlement relation of piles. Results of static pile tests using the static compression loading test (by ASTM), static loading test (by GOST) and bi-direction static loading test (by ASTM) methods are presented in this paper. Experienced bored piles with length of 31.5 m, diameter 1000 mm. Hereafter the results of underground testing by the piles with the methods of vertical static tests of SLT, BDSLT and SCLT are presented, which had been made on Expo 2017 projects, buildings of Pavilion in Astana, Kazakhstan.

Keywords: cross hole section, testing by static vertical load, PIT, Osterberg or O-cell testing

ОПЫТ ПРОВЕДЕНИЯ ИСПЫТАНИЯ СВАЙ В СЛОЖНЫХ ГРУНТОВЫХ УСЛОВИЯХ ГОРОДА АСТАНЫ, КАЗАХСТАН

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Аннотация: В настоящее время в Астане ведутся работы по строительству системы общественного транспорта LRT (Light Railway Transport). LRT – это дорога на эстакаде с двумя железнодорожными линиями. Первый этап строительства включает в себя строительство верхнего пути (моста) длиной 22.4 километра и строительство 18 станций. Под опорной частью моста несущими подземными конструкциями являются буронабивные сваи сечением 1.0 ÷ 1.5 метра и длиной 8 ÷ 35 метров. В этих условиях очень важно контролировать целостность бетонного ствола каждой буронабивной сваи. Для проверки целостности свай применяются два метода – проверка сплошности бетона слабым импульсом и радиальный акустический каротаж. Целью данной работы является обсуждение преимуществ и недостатков каждого из этих методов на примерах их реального применения. Рассмотрены статические испытания свай большого диаметра и глубокого заложения на строительной площадке в новой столице Республики Казахстан. Представлены некоторые рекомендации по методам испытаний, подходящим для проблемных грунтов Казахстана. Традиционно, испытания с загружением свай в Казахстане проводятся методами статических испытаний. Статическое испытание свай является наиболее надежным методом получения достоверных результатов. В данной статье представлены результаты испытаний свай статическим испытанием на сжатие (по ASTM), статическим испытанием (по ГОСТу) и двунаправленным статическим испытанием (по ASTM). Были испытаны опытные буронабивные свай длиной 31.5 метров, диаметром 1000 миллиметров. Представлены результаты испытаний свай методами вертикальных статических испытаний SLT, BDSLT и SCLT, выполненных на проектах Международной специализированной выставки под эгидой Международного бюро выставок (МБВ) «ЭКСПО-2017» (Expo 2017) для здания Павильона в Астане, Казахстан.

Ключевые слова: поперечное сечение, испытание статической вертикальной нагрузкой, PIT, Osterberg или O-cell testing

1. INTRODUCTION

In the spring of 2017, in Astana city was started works by construction public transport system LRT (shown figure 1 on Light Railway Transport). The cost of the project is about 1.9 billion dollars. Construction work produce a Chinese state-owned company «China Railway Asia-Europe Construction Investment Co». LRT is an overhead road with two railway lines. The first stage of construction is including construction of overhead road (bridge) with 22,4 km length and 18 stations [1]. Height of the bridge is 7÷14 m above the ground. Overhead road based on columns every 30 meters. The foundation of each column is include 4 or 6 bored piles with cross-section 1.0÷1.5 m and length 8÷35 m. Design bearing capacity of each bored piles is from 4500 to 8000 kN.

In order to reduce the time for construction and cost of piling works Chinese companies are use Chinese drilling rigs Zoomlion without casing. To maintain the walls of boreholes in sand and gravel soils using polymer slurry. Application of polymer slurry allow reducing time for drilling, allow to use less powerful drilling rigs and equipment, but at the same time increase the risk of collapsing soil during drilling or concreting piles. In these conditions, very important to control integrity of concrete body of each bored piles. For checking integrity of bored piles applying two methods - Low Strain Method and Cross-Hole Sonic Logging.

2. SECOND MEGA PROJECT IN ASTANA EXPO-2017

The complex of Expo-2017 will comprise 4,000 apartments, a new hotel, a Congress Hall, and an indoor city stretching from the Nazarbayev University to the center of Astana (the Capital of Kazakhstan). The exhibition area will involve the national pavilion of Kazakhstan, as well as international, thematic and corporate pavilions [2-4]. There will be located shopping malls, entertainment and service facilities as well. The total area of the exhibition stands at 174 hectares (Figure 2).

Static testing with Osterberg method (O-Cell testing) was carried out for the test of deep foundations at the site of the construction of this object. Four bored piles were subjected to static tests (O-cell testing- 2 piles and SCLT- 1 pile and SLT by GOST-1 pile) (see Figure 3).

The target of this tests was obtaining of bearing capacity of piles on problematical soils ground of Expo 2017 (Astana, Kazakhstan).

Based on the field description of the soils confirmed by the results of static sounding and laboratory tests, a division of the soils composing the site of prospecting for engineering-geological elements in the stratigraphic sequence of their occurrence was carried out (Figure 4).

3. METHOD OF STATIC COMPRESSION LOAD TEST BY ASTM (SCLT)

Static compression loading testing was carried out in accordance to ASTM D 1143 [5]. Vertical static loading of piles using the SCLT method is one of the most widely used field test methods for soil used to analyze pile-bearing capacity. In the first cycle, the experimental pile was loaded to 100% of the design load, in the second cycle to 200% (12,000 kN). The holding time of intermediate loading stages was 30 minutes, unloading - 20 minutes [1-8].

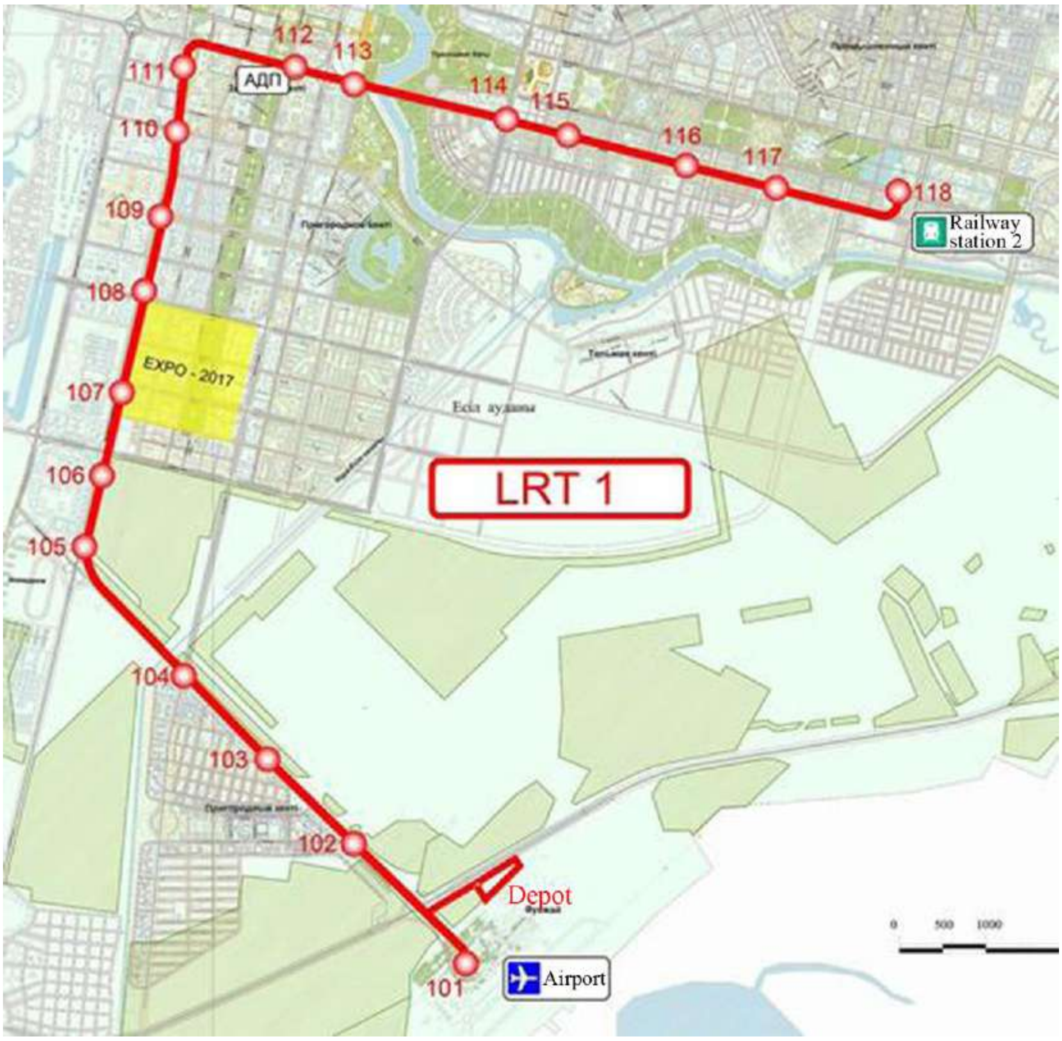

Figure 1. Map of First Line LRT in Astana, Kazakhstan.

Figure 2. Project EXPO-2017 in Astana, Kazakhstan.

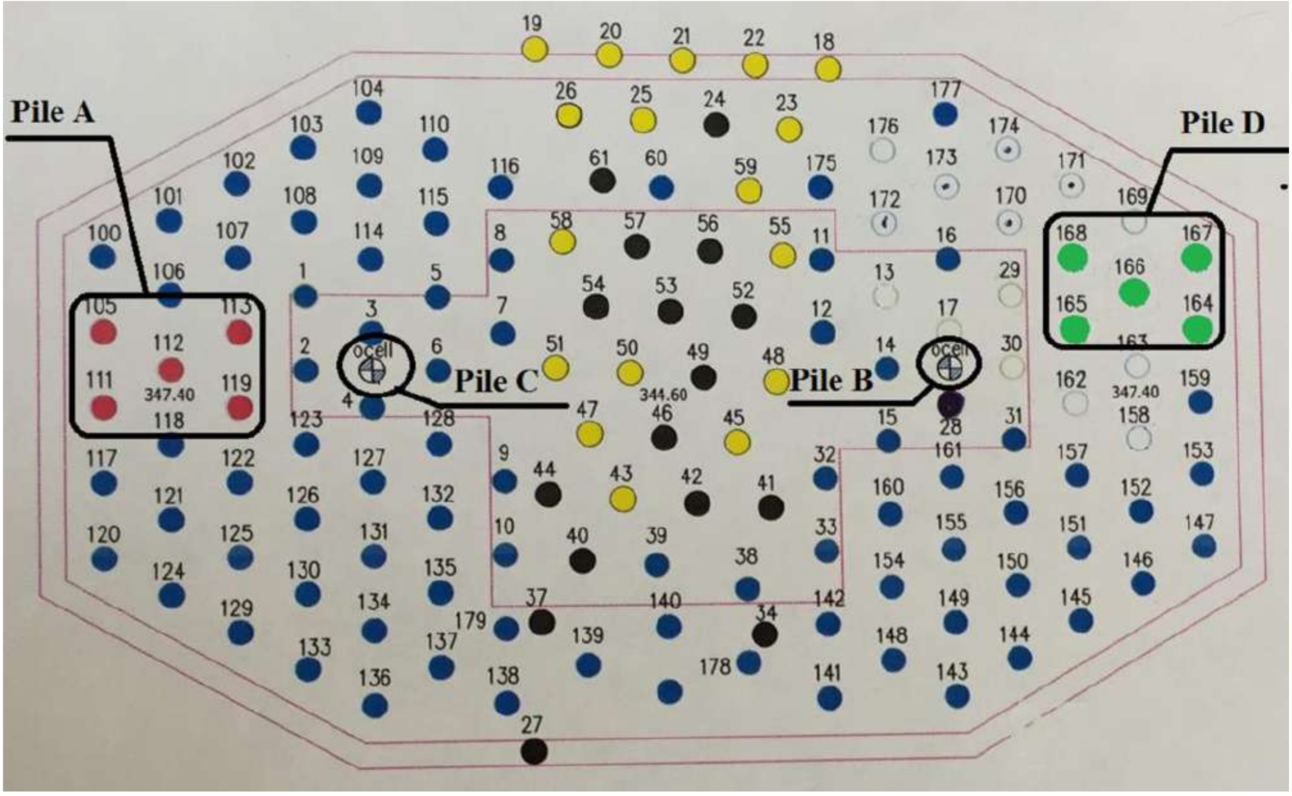

Figure 3. Plan of the pile foundations on construction site of EXPO-2017:
***Pile A** (SCLT by ASTM); **Pile B** (O-Cell-1); **Pile C** (O-Cell -2); **Pile D** (SLT by GOST).*

Number of engineering -geological element	Soil nomenclature	Design data of soil at soaking in natural state				
		E, MPa	ρ	c	φ	R0, kPa
2	Loams	12.5	1.91	38	19	-
3	Sands	17.0	1.92	2.0	35	-
5	Coarse sands	21.0	1.92	1	38	-
6	Gravel soils	23.0	-	-	-	300
7	Loams inclusions of gruse	14.0	2.04	27	27	-
8	Gruse soils	36.4	-	-	-	400
9	Rock debris soils	-	-	-	-	450

Figure 4. The physical and mechanical characteristics of the soils in Expo-2017 [1-4].

The time for maintaining peak loads was 120 and 240 minutes, respectively. The bearing capacity of the tested piles with static vertical-pressing forces, at the above construction site, was 12000 kN. It should be noted that even with a maximum test load of 12000 kN, only the elastic operation of the pile in the ground is manifested, as evidenced by a slight residual soil settlement after unloading, which is 1.4 mm (Figure 10).

4. METHOD OF BI-DIRECTIONAL STATIC LOAD TESTING (BDSLT)

Pile tests by the Osterberg method are carried out at the pre-project stage, before the design and mass penetration of the piles begins. The method makes it possible to separately determine the bearing capacity of the ground along the tip and along the lateral surface of the pile. It is usually used for testing large or large drill or ramming piles.

When testing piles using the immersed jack, the O-cell power cell is installed directly into the body of the test pile. The power cell is a system of calibrated hydraulic jacks in a protective casing. It divides the test pile into two elements: the upper one, located above the power cell, and the lower one, located under the power cell.

The monitored load in the power cell (O-cell jack) is created by the hydraulic pressure from the oil station pump located on the surface and connected to the power cell by the oil pipe. The pressure is controlled by a precision electronic pressure gauge calibrated in the general scheme of the hydraulic system. In the process of increasing the load on the walls of the jack piston, the power cell opens. The result of this disclosure is the Settlement of the upper element of the pile upward and the lower element downward. The Settlement of the upper element is measured by rod strain gages mounted on the upper plate of the jack and by displacement sensors installed in the upper part of the steel pipe. The settlement of the lower element is measured by means of rod strain gages mounted on the lower plate of the power cell (O-cell jack).

The tests are continued until one of three conditions occurs: it will be that the limit of surface friction or lateral shear is reached; the ultimate load-bearing capacity will be reached; the maximum power of the power cell (O-cell jack) will be reached. Osterberg's method allows testing piles of large dimensions without the use of anchor piles, which reduces costs at the stage of geotechnical surveys [1-8].

According to the results of engineering and geological surveys, bored piles 31.5 m long and 1000 mm in diameter were used as foundations. In order to control and evaluate the compliance of the bearing capacity of piles on the ground, the design loads were field static tests by the Osterberg method (Figure 5).

The peculiarity of the O-cell test method is that the load is applied not on the head of the pile, but in the body of the pile, where the jack (power cell) is installed, working in two directions. The power cell (O-cell jack) divides the test pile into two parts: the upper (upper test element - UTE) and the lower (lower test element-LTE). The power cell (O-cell jack) is a system of calibrated hydraulic jacks combined into one module. The hydraulic jack is installed at a depth of ½ the length of the pile - 16.8 m. The power cell is connected by hydraulic hoses to the hydraulic pump located on the ground surface.

The figure 6 shows the results of strain-measuring transducers. This figure 5 presents the load distribution along the length of the piles. The graph shows that even at maximum load, lateral resistance of the subsoil keeps the pile. Only a small part of the load accrues to the pile edge. The indicators of lateral resistance of the pile on the depth are presented in figure 6 [1-8].

When testing piles using the O-cell test, a maximum test load of 29000 kN corresponds to a draft of 18.35 mm (for the O-cell-1, pile C) and - 14.40 mm (for the O-cell-2, pile D). During the testing of the piles, both elastic and plastic deformation of the soil was observed, due to a greater test load on the pile than in the SLT method.

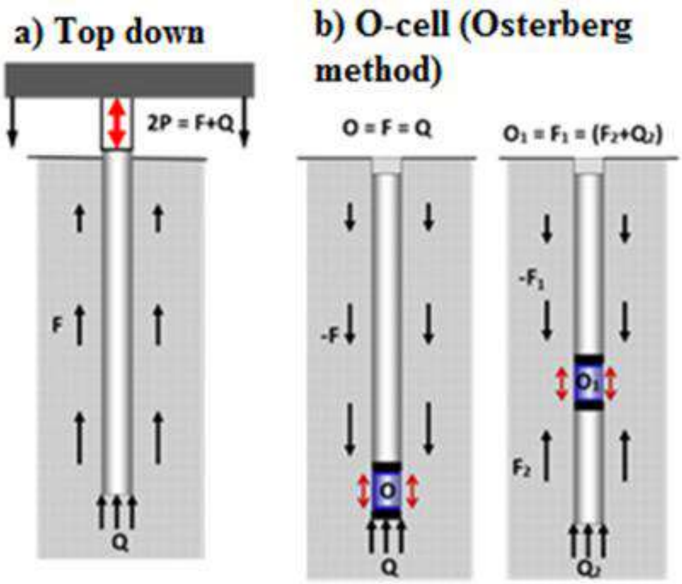

Figure 5. Scheme test load top downward and BDSLT.

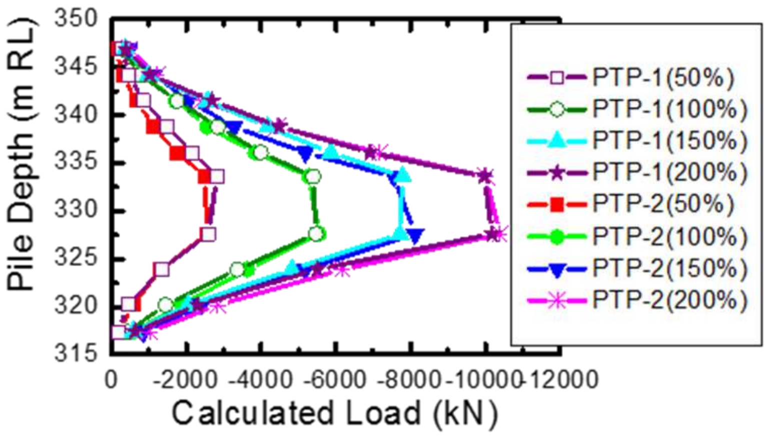

Figure 6. Calculated load distribution of piles (PTP-1 (O-cell-1) and PTP-2 (O-cell-2)).

5. STATIC TESTS IN ACCORDANCE WITH THE REQUIREMENTS OF GOST

Static tests of soils for bored piles are carried out in accordance to GOST 5686 [9]. Test was carried out after the pile concrete strength had attained more than 80% of the design value.

As part of the installation for soil testing, static pressing forces should include equipment:

- device for pile loading (jack);
- supporting structure or platform for perceiving reactive forces (for example, a system of beams with anchor piles or a platform);
- device for measuring the settlement of piles during the test (reference system with measuring instruments).

The bearing capacity of the tested piles with static vertical-pressing forces, at the above construction site, was 12000 kN (Figure 10).

6. PILE INTEGRITY TESTING

Everybody with experience in reinforced concrete construction has encountered columns that, upon dismantling of the forms, exhibit air voids and honeycombing. Although these columns may have been cast with good-quality concrete, in properly assembled forms and with careful vibration, they still exhibit defects. Cast-in-situ piles are also columns, but instead of forms made of wood or metal we have a hole in the ground. This hole may pass through layers of dumped fill, loose sand, organic matter, and ground water, which may be fast flowing or corrosive. Obviously, such conditions are not conducive to a high-quality end product. The fact that on most sites we still manage to get excellent piles is only a tribute to a dedicated team that makes this feat possible: geotechnical engineer, structural engineer, quantity surveyor, contractor, site supervisor and quality control laboratory. This is obviously a chain, the strength of which is determined by the weakest link [10].

A flaw is any deviation from the planned shape and/or material of the pile. A comprehensive list of events, each of which can lead to the formation a flaw in a pile: use of concrete that is too dry, water penetration into the borehole, collapse in soft strata, falling of boring spoils from the surface, tightly-spaced rebars etc.

Therefore, we have to face the fact that on any given site some piles may exhibit flaws. Of course, not all flaws are detrimental to the performance of the pile. Only a flaw that, because of either size or location, may detract from the pile’s load carrying capacity or durability is defined as a defect. The geotechnical engineer and the structural engineer are jointly responsible to decide which flaw comprises a defect.

The two techniques currently dominating pile integrity testing, namely the Low Strain Method and Cross-Hole Sonic Logging, both utilize sound waves.

6.1. Low Strain Method (PIT).

The low strain (sonic) method for the integrity testing of piles is aimed at routinely testing complete piling sites. To perform this test, a sensor (usually accelerometer) is pressed against the top of the pile while the pile is hit with a small hand-held hammer. Output from the sensor is analyzed and displayed by a suitable computerized instrument, the results providing meaningful information regarding both length and integrity of the pile [11].

6.2. Interpretation results obtained by PIT Method.

An assessment by this method can give a rapid and accurate appraisal of pile integrity. An integrity test will indicate when a pile should be investigated further but it cannot give information about any load carrying capacity of the pile.

Interpretation of the results obtained must take into account the specific pile circumstances, i.e. construction technique and localized soil conditions. An anomaly does not necessarily indicate a deficiency in the pile, but would certainly merit further investigation to establish the cause of the anomaly. Full interpretation of the signal responses must only be undertaken by fully trained personnel [11-15].

For interpretation ten classes are distinguished (Figure 7).

6.3. Cross-Hole Sonic Logging.

The Low Strain method belongs to the external test-methods, as it accesses only the top of the pile. Ultrasonic logging, on the other hand, is intrusive and necessitates the prior installation of access tubes (usually two or more) in the pile (Figure 8).

Before the test they have to be filled with water (to obtain good coupling) and two probes are lowered inside two of the tubes. One of these probes is an emitter and the other a receiver of ultrasonic pulses. Having been lowered to the bottom, the probes are then pulled simultaneously upwards to produce an ultrasonic logging profile. The transmitter produces a series of acoustic waves in all directions. Some of these waves do eventually reach the receiver [12].

The testing instrument then plots the travel time between the tubes versus the depth. As long as this time is fairly constant, it shows that there is no change in concrete quality. A sudden increase of the travel time at any depth may indicate a flaw at this depth [12].

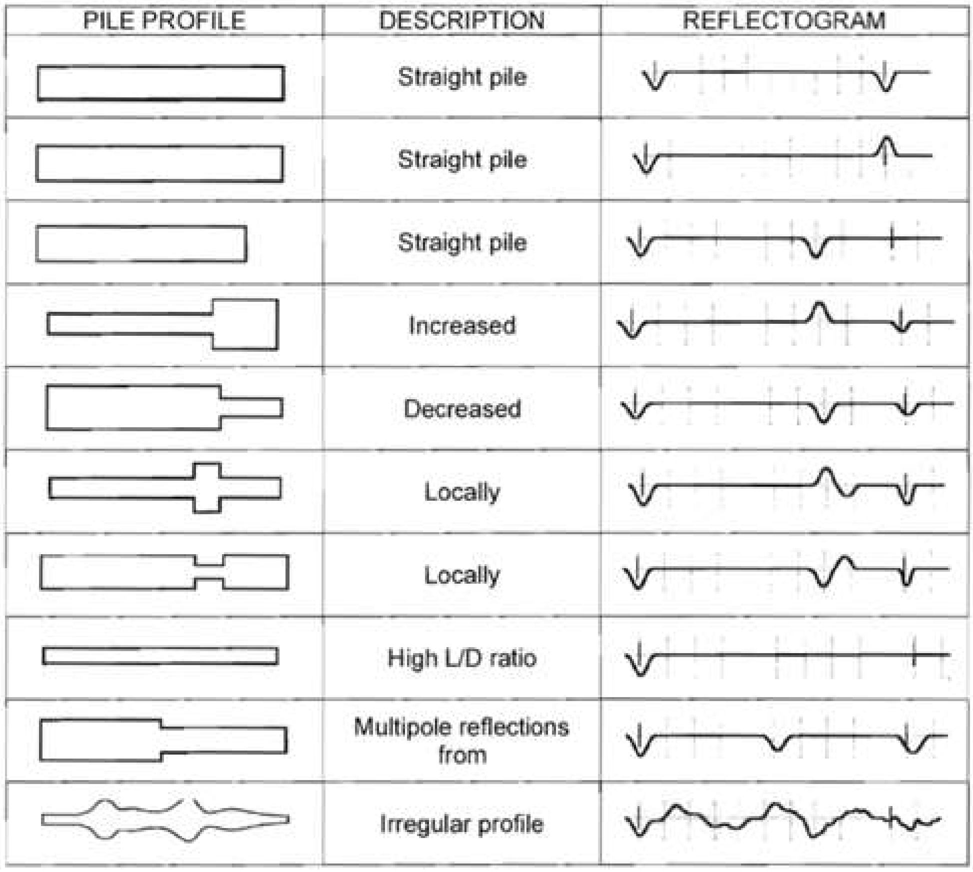

Figure 7. *Typical piles with respective reflectograms [11].*

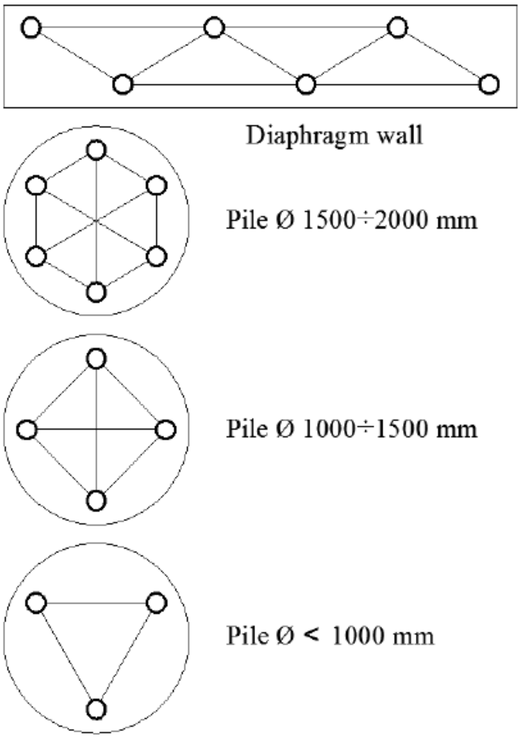

Figure 8. *Typical Access Duct Configurations.*

6.4. Cross-Hole Sonic Logging Results.

Usually the report includes presentation of Cross-Hole Sonic logs for all tested tube pairs including:

- Presentation of the traditional signal peak diagram as a function of time plotted versus depth.
- Computed initial pulse arrival time or pulse wave speed versus depth.
- Computed relative pulse energy or amplitude versus depth.

A Cross-Hole Sonic log will be presented for each tube pair. Defect zones, if any, will be indicated on the logs and their extent and location discussed in the report text. Defect zones are defined by an increase in arrival time of more than 20 percent relative to the arrival time in a nearby zone of good concrete, indicating a slower pulse velocity [12].

6.5. Tomography by the data of Cross-Hole Sonic Logging.

The same procedure, which is carried out in two dimensions on a single profile, can be used in three dimensions for the whole pile. In this case, the pile is divided into elementary voxels, or volume pixels, this process is usually called a tomography.

Tomography is a mathematical procedure that is applied to Crosshole Sonic Logging (CSL) data, providing the user with a visual image of a shaft's internal defects. The procedure involves solving a system of equations based on First Arrival Times (FAT) in order to calculate wave speeds at various points within the shaft. Tomography wave speeds distributed throughout the shaft are directly proportional to density, indicating concrete quality. PDI-TOMO is an extension of the CHA-W software designed for superior tomographic analysis results from CHAMP data with increased efficiency for the user (Figure 9).

7. RESULTS OF FIELD TRIALS USING THE STATIC LOAD TEST AND OSTERBERG METHODS

Figure 10 shows a comparison of the test results: the "load-sludge" curve obtained by the SCLT method and the equivalent "load-settlement" curve determined by the O-cell method. For the comparative criteria of Pile A (SCLT by ASTM), Pile B (O-Cell-1), Pile C (O-Cell -2) and Pile D (SLT by GOST) results fixes settlements of 10 and 14 mm had been taken [16-19].

Table 1 presents a comparative analysis of the bearing capacity of piles, obtained by different methods in this research [19].

8. CONCLUSIONS

The cost of a quality control program for each construction site is very reasonable, and in any case much lower than the potential loss caused by an undetected defect of foundation. The Low Strain test is a powerful quality-control tool, not so expensive and need about one minute for application but we must never forget that it is not omnipotent. Since the sonic method is based on the use of stress-waves, it can identify only those pile attributes that influence wave propagation and have a fairly large size.

Cross-Hole Sonic Logging method more accurate, allow to estimate the size and position of cracks. Although the access tubes introduce an extra expense item, the cross-hole test compensates for this by allowing the testing equipment to approach potential flaws. An additional advantage of this test is the enhanced resolution: while the sonic test uses a wavelength of at least two meters, the cross-hole method utilizes ultrasonic frequencies, with typical wave lengths of 50 to 100 mm. Since resolution is strongly dependent on the wave length, the cross-hole method enables us to detect much smaller flaws with high accuracy.

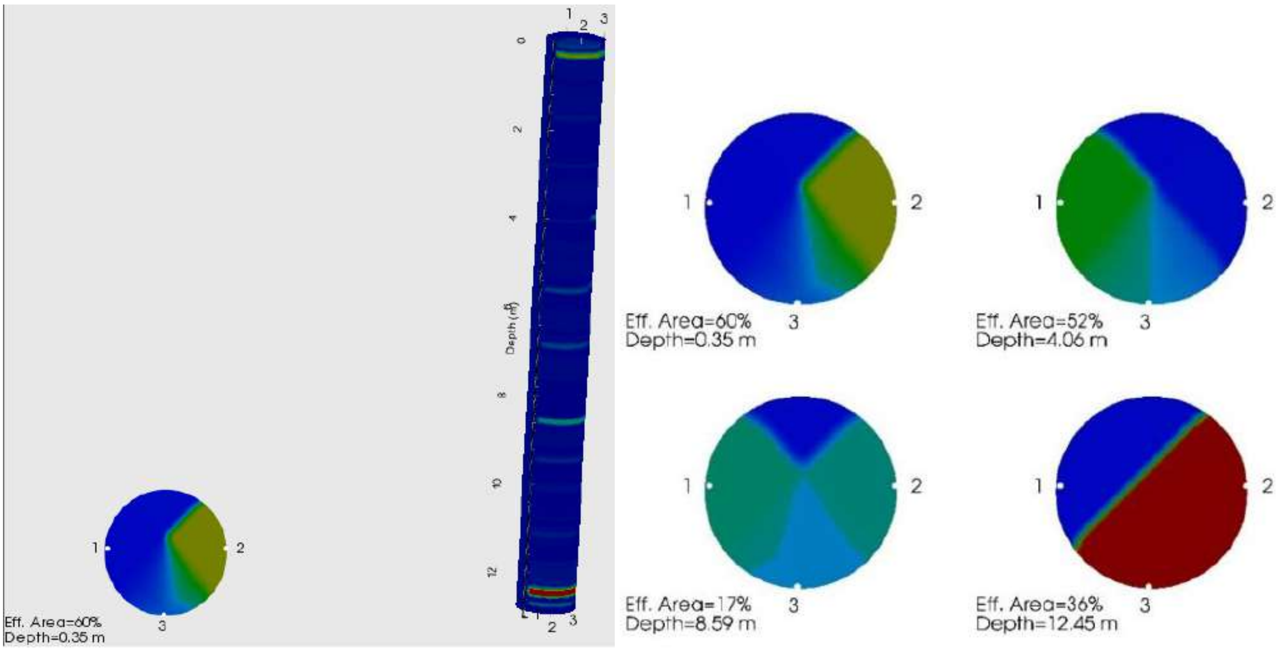

Figure 9. *Three-dimensional visualization in PDI-TOMO software and Horizontal cross-sections of pile in PDI-TOMO software.*

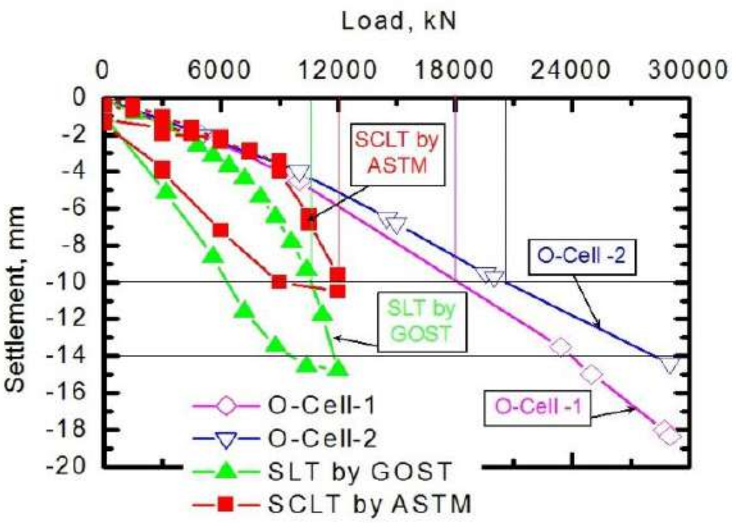

Figure 10. *Comparison of test results carried out by SCLT, SLT and O-cell methods.*

Table 1. *Different Tests.*

ID	Pile A (SCLT by ASTM)	Pile B (O-Cell-1)	Pile C (O-Cell -2)	Pile D (SLT by GOST)
The value of bearing capacity of piles, *Qd*	12000 *kN*	12000 *kN*	29000 *kN*	29000 *kN*
Results fixes settlement – 10 *mm*	11 788 *kN*	18220 *kN*	20535 *kN*	10 630 *kN*
Results fixes settlement – 14 *mm*	–	23985 *kN*	28385 *kN*	11814 *kN*

The overlay of the curves showed that the convergence of the graphs is observed only at the initial stage of loading, then a change in the trajectory of the SLT curve, characteristic of the creeping stage of soil resistance, is observed, whereas the O-cell curve (at this stage of loading) is more characteristic of the elastic resistance of the soil.

According to the results of the SCLT unloading curve, elastic work of the soil is still evident. The reason for the abrupt change in the trajectory of the SCLT curve, which is not characteristic of the elastic work of the ground, is the holding time of the loading stages (lower compared to the O-cell test method), which can also explain the almost completely elastic work of the soil during O-cell tests.

When testing piles using the SLT method "from top to bottom", a design load of 6000 kN corresponds to a draft of 2.09 mm, a maximum test load of 12000 kN is a draft of 10.51 mm. It should be noted that even with the maximum test load, only the elastic operation of the pile in the ground is manifested, as evidenced by a slight residual soil sediment after unloading, which is 1.4 mm.

When testing piles using the O-cell test, a maximum test load of 29000 kN corresponds to a draft of 18.35 mm (for the PTP-1 pile) and - 14.40 mm (for the PTP-2 pile). During the testing of the piles, both elastic and plastic deformation of the soil was observed, due to a greater test load on the pile than in the SLT method.

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К ЮБИЛЕЮ ВЛАДИМИРА АЛЕКСАНДРОВИЧА ИГНАТЬЕВА

06 декабря 2018 года исполнилось 80 лет Владимиру Александровичу Игнатьеву, доктору технических наук, профессору, заслуженному деятелю науки и техники РСФСР, почетному работнику высшего профессионального образования РФ, советнику Российской академии архитектуры и строительных наук, заведующему кафедрой Волгоградского государственного технического университета.

Владимир Александрович Игнатьев – крупный, широко известный учёный в области механики деформируемого твёрдого тела – механики дискретных и дискретно-континуальных систем, автор современных методов расчета плоских и пространственных систем на основе дискретных и континуальных моделей. Результаты его научных исследований изложены более чем в 200 печатных работах, в том числе в 12 монографиях, изданных в России и за рубежом; он имеет несколько авторских свидетельств на изобретения в области строительных конструкций. Владимир Александрович подготовил 40 кандидатов и 9 докторов наук.

В течение 25 лет – с 1983 года по 2008 год В.А. Игнатьев работал на посту ректора Волгоградского инженерно-строительного института, ставшего позже архитектурно-строительной академией, а затем университетом. За это время вуз стал лидером среди строительных вузов России по подготовке научно-педагогических кадров, международным связям, по качеству подготовки выпускаемых специалистов. По инициативе Владимира Александровича в рамках международного сотрудничества университета были организованы образовательные программы совместно с Высшей профессиональной школой (Университетом прикладных наук) города Кёльна (Германия) и Мичиганским государственным университетом (США). В настоящее время В.А. Игнатьев – заведующий кафедрой строительной механики этого университета.

Владимир Александрович является почётным профессором Ассоциации строительных вузов СНГ, членом Американской ассоциации гражданских инженеров, членом редколлегий журналов «Известия вузов. Строительство», «Вестник ВолгГАСУ» и «Строительная механика и расчет сооружений». Награждён орденом Почёта и знаком «Почётный работник высшей школы». За организацию и успешное осуществление в течение 15 лет проекта немецкоязычного учебного процесса, позволившего студентам-участникам получить одновременно дипломы российских и немецких инженеров, награжден золотой медалью университета прикладных наук г. Кёльна (Германия). За большой вклад в подготовку высококвалифицированных специалистов и научную деятельность Владимиру Александровичу объявлена благодарность Президента Российской Федерации.

Желаем Вам, Владимир Александрович, крепкого здоровья, благополучия и дальнейших творческих успехов!

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