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STUDY OF THE INFLUENCE OF THE KINETICS OF HYDROGEN SATURATION ON THE STRESS-DEFORMED STATE OF A SPHERICAL SHELL MADE FROM TITANIUM ALLOY

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Abstract. In this paper, a mathematical model is considered that allows one to determine the stress-strain state of a spherical shell made of titanium alloy VT1-0, the external load is assumed to be transverse uniformly distributed, acting on the outer surface, the medium is assumed to act on the inner surface of the shell. For this, a nonlinear model was used, presented in normalized stress spaces. Fastening along the contour of the shell is rigid. Nonlinear resolving equations for calculating a spherical shell are obtained. An algorithm for solving the problem of hydrogenation of titanium alloy shells has been developed. A practical solution was made by a two-step method of sequential perturbations of parameters using the MatLab and Maple software packages. To solve the system of the obtained differential equations, the method of finite differences is applied. The calculation of the stress-strain state of the shell is obtained taking into account the diffusion process of an aggressive hydrogen-containing medium, and the obtained solution is compared with the results of the classical nonlinear theory without taking into account the aggressive medium. The results of comparing these solutions demonstrate quantitative differences in the parameters of the shell deformation process, which are explained by a more accurate account of the influence of the type of stress state. This approach has a rather flexible mechanism for considering the initial and induced differential resistance, demonstrates a high accuracy of matching the obtained theoretical results with empirical data on loading a wide range of materials under complex types of stress state.

Keywords: flat shell, titanium alloy, finite differences, nonlinear equations, initially isotropic material, large deflections.

ИССЛЕДОВАНИЕ ВОЗДЕЙСТВИЯ КИНЕТИКИ НАВОДОРОЖИВАНИЯ НА НАПРЯЖЕННО-ДЕФОРМИРОВАННОЕ СОСТОЯНИЕ СФЕРИЧЕСКОЙ ОБОЛОЧКИ ИЗ ТИТАНОВОГО СПЛАВА

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Аннотация. В данной работе рассмотрена математическая модель, позволяющая определять напряжённо-деформированное состояние сферической оболочки из титанового сплава ВТ1-0, внешняя нагрузка принята поперечной равномерно распределённой, действующей на внешнюю поверхность, среда принята действующей на внутренную поверхность оболочки. Для этого использована нелинейная модель, представленная в нормированных пространствах напряжений. Закрепление по контуру оболочки жёсткое. Получены нелинейные разрешающие уравнения расчёта сферической оболочки. Разработан алгоритм решения задачи наводороживания оболочек из титанового сплава. Практическое решение производилось двухшаговым методом последовательных возмущений параметров с использованием пакетов прикладных программ MatLab и Maple. Для решения системы полученных дифференциальных уравнений применён метод конечных разностей. Получен расчет НДС оболочки с учетом процесса диффузии агрессивной водородосодержащей среды, произведено сравнение полученного решения с результатами классической нелинейной теории без учета агрессивной среды. Результаты сравнения указанных решений демонстрируют количественные различия в параметрах процесса деформирования оболочки, которые объясняются более точным учетом влияния вида напряженного состояния. Данный подход имеет

достаточно гибкий механизм учета изначальной и наведенной разносопротивляемости, демонстрирует высокую точность согласования получаемых теоретических результатов с эмпирическими данными по нагружению широкого круга материалов при сложных видах напряженного состояния.

Ключевые слова: пологая оболочка, титановый сплав, конечные разности, нелинейные уравнения, начально изотропный материал, большие прогибы.

INTRODUCTION

One of the first theories for calculating structural elements operating in aggressive hydrogen-containing media, taking into account the change in material properties over time, apparently, should be noted the model proposed in the works [1 – 4].

In previous studies, to construct a mathematical model of the behavior of materials in a hydrogen-containing medium, it was proposed to use the theory of Yu.N. Rabotnov [5, 6] taking into account physical and chemical effects on the surface and in the volume of the deformable But, as practice has shown, this theory does not take into account a number of effects inherent in the problem under consideration, such as the presence of triple nonlinearity, as well as induced differential resistance, which undoubtedly leads to a decrease in the accuracy of the results. This study also considers the change in the properties of materials under the influence of a changing concentration of an aggressive medium, but initially nonlinear relationships were used built in normalized stress spaces, which consider a continuous change in the state of a structural material depending on the type of stress state and quantitative characteristics at a point. A more effective mathematical model for solving the problem of the effect of hydrogenation on the stress-strain state of a flat spherical shell made of titanium alloy is proposed, a numerical solution of the problem is constructed based on the finite difference method. To solve the problem with triple nonlinearity, a two-step method of sequential perturbations of parameters was adopted [7], which allows linearizing the resolving equations and also has a high accuracy. The numerical implementation of this approach was carried out by the finite difference method of increased accuracy. The integration of the functions of the stress-strain state and rigidity parameters over the thickness was carried out by the Simpson method [8].

FORMULATION OF THE PROBLEM

The object of research is a flat spherical shell made of titanium alloy VT1-0, loaded with an external uniformly distributed transverse load with an intensity of up to 5 MPa, rigidly fixed along the perimeter, the radius of curvature of the shell is taken equal to $R = 3$ m, the radius of the base of the shell is taken equal to $a = 1{,}5$ m, and a hoist arrow – $f = 0{,}4$ m.

The location of any point on the median surface of a spherical shell is determined by a Gaussian coordinate system α_1, α_2, and the position of an arbitrary point in thickness is determined by the coordinate α_3, considering that u – horizontal displacements along a radial coordinate r (the projection α_1), ϑ – radial displacements, w – vertical displacements (deflections) under the action of a lateral load q. The design scheme of the shell is shown in the figure 1.

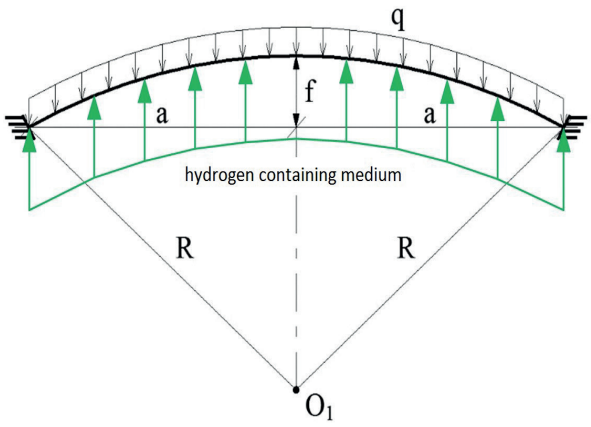


Figure 1. The design scheme of the shell

For a flat spherical shell, the constancy of the main radius of curvature of its middle surface within the plan is valid:

$$R_1 = R_2 = R, \quad (1)$$

$k_1 = k_2 = k = 1/R$ – main curvature.

Consider the equilibrium of a spherical shell with thickness $h = 0,05$ m, under the influence of a transverse axisymmetric uniformly distributed load q and a hydrogen-containing medium with a concentration λ. We accept the kinetic potential of deformations in the form [9, 10]:

$$W_1 = (A_e(\lambda) + B_e(\lambda)\xi)\sigma^2 + (C_e(\lambda) + D_e(\lambda)\xi + E_e(\lambda)\eta Cos3\varphi)\tau^2 + [(A_p(\lambda) + B_p(\lambda)\xi)\sigma^2 + (C_p(\lambda) + D_p(\lambda)\xi + E_p(\lambda)\eta Cos3\varphi)\tau^2]^n, \quad (2)$$

where $A_e(\lambda)$, $B_e(\lambda)$, $C_e(\lambda)$, $D_e(\lambda)$, $E_e(\lambda)$, $A_p(\lambda)$, $B_p(\lambda)$, $C_p(\lambda)$, $D_p(\lambda)$, $E_p(\lambda)$ – functions that determine the physical and mechanical characteristics of the material that appear in the recording of the potential of the quasilinear and nonlinear parts and depend on the degree of hydrogen saturation. The quantitative characteristics of the stress state are determined by the modulus of the total stress vector on the deviator site:

$$S_0 = \sqrt{\sigma^2 + \tau^2},$$

but a quality picture – is determined by the normalized stresses on this site, which depend on the angle ψ between normal and vector S_0, and also the harmonic phase invariant φ:

$$\xi = \cos\psi = \sigma / S_0; \quad \eta = \sin\psi = \tau / S_0;$$
$$\cos 3\varphi = \sqrt{2}\det(S_{ij}) / \tau^3,$$

where $\sigma = \sigma_{ij}\delta_{ij}/3$ – medium stress or normal octahedral; $\tau = (S_{ij}S_{ij}/3)^{1/2}$ – tangential octahedral stress; δ_{ij} – the Kronecker symbol; $S_{ij} = \sigma_{ij} - \delta_{ij}\sigma$ – stress deviator $i, j = (1, 2, 3)$.

The dependences of the mechanical properties on the degree of hydrogenation of the material are presented in the form of a polynomial expansion of the coefficients of the kinetic potential in powers of the concentration of the medium λ, and the expansion coefficients of the polynomials are determined by least-squares processing of empirical data on the deformation of titanium alloy specimens to axial tension and compression at different levels λ (0; 0,02; 0,04 и 0,08%), which for the VT1-0 alloy take the form:

$$V_{ek}(\lambda) = e_{0k} + e_{1k}\cdot\lambda + e_{2k}\cdot\lambda^2;$$
$$V_{pk}(\lambda) = p_{0k} + p_{1k}\cdot(p_{2k})^{\lambda}; \quad (3)$$
$$A_e(\lambda) = V_{e1}(\lambda); \quad B_e(\lambda) = V_{e3}(\lambda);$$
$$C_e(\lambda) = V_{e2}(\lambda); \quad D_e(\lambda) = V_{e4}(\lambda);$$
$$E_e(\lambda) = V_{e5}(\lambda);$$
$$A_p(\lambda) = V_{p1}(\lambda); \quad B_p(\lambda) = V_{p3}(\lambda);$$
$$C_p(\lambda) = V_{p2}(\lambda); \quad D_p(\lambda) = V_{p4}(\lambda);$$
$$E_p(\lambda) = V_{p5}(\lambda), \quad (4)$$

where e_{ik}, p_{ik} – polynomial coefficients, $i = 0...2$; $k = 1...5$ [9, 10].

Connections of strain and stress tensors are established from potential (2) using Castigliano's formulas:

$$e_{kk} = \frac{\partial W_1}{\partial \sigma_{kk}}; \quad \gamma_{ij} = \frac{\partial W_1}{\partial \tau_{ij}}; \quad (5)$$
$$(i, j, k = 1,2,3;\ i \neq j);$$

Taking into account the axisymmetry of the problem and the fact that the shell is subjected to pressure q on the outer surface of the shell, the geometric dependences take the form:

$$\varepsilon_1 = u_{,1} - kw + 0{,}5(w_{,1})^2; \quad \varepsilon_2 = \frac{u}{r} - kw;$$
$$\chi_1 = -w_{,11}; \quad \chi_2 = -\frac{w_{,1}}{r}; \qquad (6)$$
$$e_{11} = \varepsilon_1 + z\chi_1; \quad e_{22} = \varepsilon_2 + z\chi_2,$$

where ε_1, ε_2 – relative deformations in the middle surface; χ_1, χ_2 – mid-surface curvatures.

Considering equations (2) - (6) and Kirchhoff-Lav's hypotheses, the relationship between the simplified form of deformations and stresses is represented in the form:

$$\begin{Bmatrix} e_{11} \\ e_{22} \end{Bmatrix} = [\mathrm{A}] \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \end{Bmatrix}; \; [\mathrm{A}] = \begin{bmatrix} A_{11}(\lambda) & A_{12}(\lambda) \\ A_{21}(\lambda) & A_{22}(\lambda) \end{bmatrix}. \qquad (7)$$

Inverting matrix equations (7), we obtain the dependences of stresses on deformations:

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \end{Bmatrix} = [\mathrm{B}] \begin{Bmatrix} e_{11} \\ e_{22} \end{Bmatrix}; \; [\mathrm{B}] = \begin{bmatrix} B_{11}(\lambda) & B_{12}(\lambda) \\ B_{21}(\lambda) & B_{22}(\lambda) \end{bmatrix}, \qquad (8)$$

where $[\mathrm{B}] = [\mathrm{A}]^{-1}$; A_{11}, A_{12}, … – components of the symmetric compliance matrix [A], which are functions containing the deformation potential W_1 (2), depending on the type of stress state and the degree of hydrogenation of the titanium alloy.

These components are determined according to [9, 10] as follows:

$$A_{11}(\lambda) = \{2[R_1(\lambda) + 2R_3(\lambda)]/3 + R_2(\lambda)\xi[3 - 2\xi^2]/3 + R_4(\lambda)[\xi(2-\eta^2)/3 + 4(\sigma_{11} - 2\sigma_{22})/9S_0] + R_5(\lambda)[\eta Cos3\varphi(1+\xi^2) + 2\sqrt{2}\xi - 2Cos3\varphi - \sqrt{2}\sigma_{22}/S_0]\}/3;$$
$$A_{12}(\lambda) = \{2[R_1(\lambda) - R_3(\lambda)]/3 + [R_2(\lambda) + R_4(\lambda)/3]\xi + R_5(\lambda)[Cos3\varphi(1-\eta) - \sqrt{2}\xi]\}/3;$$
$$A_{22}(\lambda) = \{2(R_1(\lambda) + 2R_3(\lambda))/3 + R_2(\lambda)\xi[3 - 2\xi^2]/3 + R_4(\lambda)[\xi(2-\eta^2) + 4(\sigma_{22} - 2\sigma_{11})/9S_0] + R_5(\lambda)[\eta Cos3\varphi(1+\xi^2) + 2\sqrt{2}\xi - 2Cos3\varphi - \sqrt{2}\sigma_{22}/S_0]\}/3;$$
$$A_{12}(\lambda) = A_{21}(\lambda);$$
$$R_k(\lambda) = L_{ek}(\lambda) + n[(A_p(\lambda) + B_p(\lambda)\xi)\sigma^2 + \left(C_p(\lambda) + D_p(\lambda)\xi + E_p(\lambda)\eta\cos 3\varphi\right)\tau^2]^{n-1} L_{pk}(\lambda);$$
$$L_{m1}(\lambda) = A_m(\lambda); \quad L_{m2}(\lambda) = B_m(\lambda);$$
$$L_{m3}(\lambda) = C_m(\lambda); \quad L_{m4}(\lambda) = D_m(\lambda);$$
$$L_{m5}(\lambda) = E_m(\lambda); \quad m = e, p; \quad k = 1,2,3.$$

The forces and moments are determined by integrating the stresses across the shell thickness in the traditional way:

$$N_1 = \int_{-h/2}^{h/2} \sigma_{11} dz; \quad N_2 = \int_{-h/2}^{h/2} \sigma_{22} dz;$$
$$M_1 = \int_{-h/2}^{h/2} \sigma_{11} z dz; \quad M_2 = \int_{-h/2}^{h/2} \sigma_{22} z dz. \qquad (9)$$

The moments and forces are expressed in terms of the components of the deformations of the middle surface of the shell in the following way:

$$\begin{aligned} N_1 &= K_{11}(\lambda)\varepsilon_1 + K_{12}(\lambda)\varepsilon_2 + P_{11}(\lambda)\chi_1 + P_{12}(\lambda)\chi_2; \\ N_2 &= K_{12}(\lambda)\varepsilon_1 + K_{22}(\lambda)\varepsilon_2 + P_{21}(\lambda)\chi_1 + P_{22}(\lambda)\chi_2; \\ M_1 &= P_{11}(\lambda)\varepsilon_1 + P_{12}(\lambda)\varepsilon_2 + D_{11}(\lambda)\chi_1 + D_{12}(\lambda)\chi_2; \\ M_2 &= P_{12}(\lambda)\varepsilon_1 + P_{22}(\lambda)\varepsilon_2 + D_{21}(\lambda)\chi_1 + D_{22}(\lambda)\chi_2, \end{aligned} \qquad (10)$$

where the integral characteristics of material functions, taking into account the influence of the degree of hydrogenation, are calculated through its concentration λ as follows:

$$K_{ij} = \int_{-h/2}^{h/2} B_{ij}(\lambda) dz, \quad P_{ij} = \int_{-h/2}^{h/2} B_{ij}(\lambda) z dz;$$
$$D_{ij} = \int_{-h/2}^{h/2} B_{ij}(\lambda) z^2 dz.$$

In connection with the triple nonlinearity of the problem, the resolving equations are formulated in a linearized form using the two-step method of sequential perturbations of the parameters of V.V. Petrov [7]. Physical dependencies are presented in the following linearized form:

$$\delta e_{11} = \frac{\partial e_{11}}{\partial \sigma_{11}}\delta\sigma_{11} + \frac{\partial e_{11}}{\partial \sigma_{22}}\delta\sigma_{22};$$
$$\delta e_{22} = \frac{\partial e_{22}}{\partial \sigma_{11}}\delta\sigma_{11} + \frac{\partial e_{22}}{\partial \sigma_{22}}\delta\sigma_{22}; \qquad (11)$$
$$\delta\varepsilon_1 = \delta u_{,1} - k\delta w + w_{,1}\,\delta w_{,1}; \quad \delta\varepsilon_2 = \frac{\delta u}{r} - k\delta w;$$
$$\delta\chi_1 = -\delta w_{,11}; \; \delta\chi_2 = \frac{-\delta w_{,1}}{r}.$$

The inversion of relations (11) leads to the following dependences of stresses on deformations in increments of the following form:

$$\delta\sigma_{11} = B_{11}(\lambda)\delta e_{11} + B_{12}(\lambda)\delta e_{22};$$
$$\delta\sigma_{22} = B_{21}(\lambda)\delta e_{11} + B_{22}(\lambda)\delta e_{22}, \qquad (12)$$

where $B_{11}(\lambda) = \frac{\Delta_{22}}{\Delta}$;

$$B_{12}(\lambda) = B_{21}(\lambda) = -\frac{\Delta_{21}}{\Delta} = -\frac{\Delta_{12}}{\Delta};$$
$$B_{22}(\lambda) = \frac{\Delta_{11}}{\Delta}; \quad \Delta_{11} = \frac{\partial e_{11}}{\partial \sigma_{11}}; \quad \Delta_{22} = \frac{\partial e_{22}}{\partial \sigma_{22}};$$
$$\Delta_{12} = \Delta_{21} = \frac{\partial e_{11}}{\partial \sigma_{22}} = \frac{\partial e_{22}}{\partial \sigma_{11}}; \quad \Delta = \Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21};$$

Dependences of deformation increments at a point on the deformation increments of the middle surface $\delta\varepsilon_1$ and $\delta\varepsilon_2$ and it's curvature $\delta\chi_1$, $\delta\chi_2$ are presented in the form:

$$\delta e_{11} = \delta\varepsilon_1 + z\delta\chi_1; \quad \delta e_{22} = \delta\varepsilon_2 + z\delta\chi_2 .$$

Then the equations for the connection of efforts with deformations of the middle surface in increments have the form:

$$\delta N_1 = K_{11}(\lambda)\delta\varepsilon_1 + K_{12}(\lambda)\delta\varepsilon_2 + P_{11}(\lambda)\delta\chi_1 + P_{12}(\lambda)\delta\chi_2;$$
$$\delta N_2 = K_{12}(\lambda)\delta\varepsilon_1 + K_{22}(\lambda)\delta\varepsilon_2 + P_{21}(\lambda)\delta\chi_1 + P_{22}(\lambda)\delta\chi_2; \qquad (13)$$
$$\delta M_1 = P_{11}(\lambda)\delta\varepsilon_1 + P_{12}(\lambda)\delta\varepsilon_2 + D_{11}(\lambda)\delta\chi_1 + D_{12}(\lambda)\delta\chi_2;$$
$$\delta M_2 = P_{12}(\lambda)\delta\varepsilon_1 + P_{22}(\lambda)\delta\varepsilon_2 + D_{21}(\lambda)\delta\chi_1 + D_{22}(\lambda)\delta\chi_2,$$

The axial symmetry of the problem under consideration makes it possible to simplify the equilibrium equations in increments as follows:

$$\delta M_{1,11} - \delta M_{2,1}/r + 2\delta M_{1,1}/r + k(\delta N_1 + \delta N_2) + \delta N_1 w_{,11} + N_1\delta w_{,11} = -\delta q;$$
$$\delta N_{1,1} + (\delta N_1 - \delta N_2)/r - k[\delta M_{1,1} + (\delta M_1 - \delta M_2)/r] = 0. \qquad (14)$$

Integrating equations (12) over the thickness of the shell according to the rules (9), and introducing the results into equilibrium equations (14), we arrive at two linearized differential equations in displacements:

$$2r^2 D_{12,11}\,\delta w_{,1} + 2r^2 D_{12,1}\,\delta w_{,11} - 2r^2 P_{12,11}\,\delta u - 2r^2 P_{12,1}\,\delta u_{,1} + 2rP_{22,1}\,\delta u + 2rP_{22}\delta u_{,1} - 2P_{22}\delta u - 2rD_{22,1}\,\delta w_{,1} - 2rD_{22}\delta w_{,11} + 2D_{22}\delta w_{,1} - 4r^2 P_{11,1}\,\delta u_{,1} - 4r^2 P_{11}\delta u_{,11} + 4r^2 D_{11,1}\,\delta w_{,11} + 4r^2 D_{11}\delta w_{,111} - 2r^3 P_{11,11}\,\delta u_{,1} - 4r^3 P_{11,1}\,\delta u_{,11} - 2r^3 P_{11}\delta u_{,111} + 2r^3 P_{11}(\delta w_{,11})^2 + 2r^3 D_{11,11}\,\delta w_{,11} + 4r^3 D_{11,1}\,\delta w_{,111} + 2r^3 D_{11}\delta w_{,1111} - 2r^3 kK_{11}w_{,1}\,\delta w_{,1} - 2r^3 kK_{12}w_{,1}\,\delta w_{,1} + 2r^3\delta w_{,11}\,K_{11}k\delta w - 2r^3\delta w_{,11}\,K_{11}w_{,1}\,\delta w_{,1} + 2r^3\delta w_{,11}\,K_{12}k\delta w + 2r^3\delta w_{,11}\,K_{11}kw + 2r^3\delta w_{,11}\,K_{12}kw - 2r^3 kK_{12}\delta u_{,1} + 2r^3 K_{22}k^2\delta w - 2r^3\delta w_{,11}\,K_{11}\delta u_{,1} - 2r^3\delta w_{,11}\,K_{11}u_{,1} - r^3\delta w_{,11}\,K_{11}w_{,1}{}^2 - 4r^2 P_{11}w_{,1}\,\delta w_{,11} - 2r^2 kK_{12}\delta u - 2r^2 kK_{22}\delta u - 2r^2\delta w_{,11}\,K_{12}\delta u + 2r^2\delta w_{,11}\,P_{12}\delta w_{,1} - 2r^2\delta w_{,11}\,K_{12}u + 2r^2 P_{12,1}\,k\delta w + 2r^2 P_{12,1}\,w_{,1}\,\delta w_{,1} + 4r^2 P_{12}k\delta w_{,1} + 2r^2 P_{12}w_{,11}\,\delta w_{,1} + 4r^2 P_{12}w_{,1}\,\delta w_{,11} -$$

$$
\begin{gathered}
-2r^2P_{22},_1\,k\delta w+4r^2P_{11},_1\,k\delta w-4r^2P_{11},_1\,w,_1\,\delta w,_1+\\
+4r^2P_{11}k\delta w,_1-4r^2P_{11}w,_{11}\,\delta w,_1+\\
+2r^3P_{11},_{11}\,k\delta w-4r^3P_{11},_1\,w,_{11}\,\delta w,_1-2r^3P_{11,11}w,_1\,\delta w,_1+\\
+4r^3P_{11},_1\,k\delta w,_1-4r^3P_{11},_1\,w,_1\,\delta w,_{11}+\\
+4r^3P_{11}k\delta w,_{11}-2r^3P_{11}w,_{111}\,\delta w,_1-2r^3P_{11}w,_{11}\,\delta w,_{11}-\\
-2r^3P_{11}w,_1\,\delta w,_{111}+2r^3P_{12},_{11}\,k\delta w+\\
+4r^3P_{12},_1\,k\delta w,_1+4r^3P_{12}k\delta w,_{11}-2r^3kK_{11}\delta u,_1+\\
+2r^3K_{11}k^2\delta w+4r^3K_{12}k^2\delta w=2r^3\delta q\,;\quad (15)\\
r^2\left(kD_{11}-P11\right)\delta w,_{111}-\\
-\left(rP_{11},_1-krD_{11},_1+r\left(kP_{11}-K11\right)w,_1-kD_{11}+P_{11}\right)r\delta w,_{11}-\\
-r^2\left(kP_{11}-K_{11}\right)\delta u,_{11}-\\
-\delta w,_1\,r^2\left(kP_{11}-K_{11}\right)w,_{11}+\\
+kr^2\left(k\delta w-w,_1\,\delta w,_1-\delta u,_1\right)P_{11},_1-\\
-r^2\left(k\delta w-w,_1\,\delta w,_1-\delta u,_1\right)K_{11},_1+\\
+r\left(-\delta w,_1+k\left(kr\delta w-\delta u\right)\right)P_{12},_1+\\
+\left(-kr^2\delta w+r\delta u\right)K_{12},_1+\delta w,_1\,D_{12},_1\,kr+\\
+\left(-r\left(kP_{11}-kP_{12}-K_{11}+K_{12}\right)w,_1\right)\delta w,_1+\\
(k^2P_{11}r^2+k^2P_{12}r^2-kr^2K_{11}-kr^2K_{12}-kD_{22}+P_{22})\delta w,_1-\\
-r\left(kP_{11}-K_{11}\right)\delta u,_1+\delta wP_{11}k^2r-\\
-\delta wK_{11}kr-\left(kr\delta w-\delta u\right)\left(kP_{22}-K_{22}\right)=0\,.
\end{gathered}
$$

The resulting gradient system of equations (15) needs to be supplemented with boundary conditions in increments. Due to the axial symmetry of the problem, at the center of the shell the rotation of the normal to the middle surface, radial displacements and their increments will be equal to zero ($w,_1=0$, $u=0$, $\delta w,_1=0$, $\delta u=0$).

In the process of chemical adsorption, hydrogen molecules disintegrate into atoms that diffuse deep into the material [9, 10]. The flux density J is proportional to the spatial concentration gradient λ, the diffusion equation takes the form:

$$J=-Dgrad\lambda=-D\frac{\partial\lambda}{\partial z},\qquad (16)$$

where $D=const$ – diffusion coefficient, z – coordinate in the direction of diffusion that corresponds to the axis α_3.

In accordance with the experimental data presented in [11], as well as in connection with unidirectional diffusion, the kinetic equation of hydrogenation corresponds to Fick's second law, and its solution is known due to the double Fourier transform (direct and inverse), the result of which has the form:

$$\frac{\partial\lambda(z,t)}{\partial t}=D\frac{\partial^2\lambda(z,t)}{\partial z^2},\qquad (17)$$

where t – is a current time.

The solution of equation (17) for the process of one-sided diffusion has a well-known approximate analytical solution presented in the works [9, 10]:

$$
\begin{gathered}
\lambda(z,t)=\lambda_1+(\lambda_2-\lambda_1)z/h+\\
+(2/\pi)\sum_{i=1}^{\infty}\sin(i\cdot\pi\cdot z/h)\exp(-F_o\pi^2i^2)\times\\
\times[\lambda_2\cos(i\cdot\pi)-\lambda_1]/i,
\end{gathered}
$$

where $F_O=Dt/h^2$ – is a Fourier number; i – is a row member number; λ_1 and λ_2 – boundary values of the concentration of the medium on the lower and upper surfaces of the shell.

For the shell, the following boundary conditions are accepted: in the event of an aggressive medium acting on the side of the application of a power load:

$$\lambda(-h/2,t)=\lambda_\infty=\lambda_1,\quad \lambda(+h/2,t)=0=\lambda_2,$$

where λ_∞ – equilibrium concentration of a hydrogen-containing medium.

The initial conditions are:

$$\lambda(z,0)=0\,.$$

The calculations were performed using the MATLAB and Maple software packages. The results of calculating a spherical shell operating in an aggressive environment of hydrogen with various concentrations from 0 to 0.08% using the proposed model are presented below.

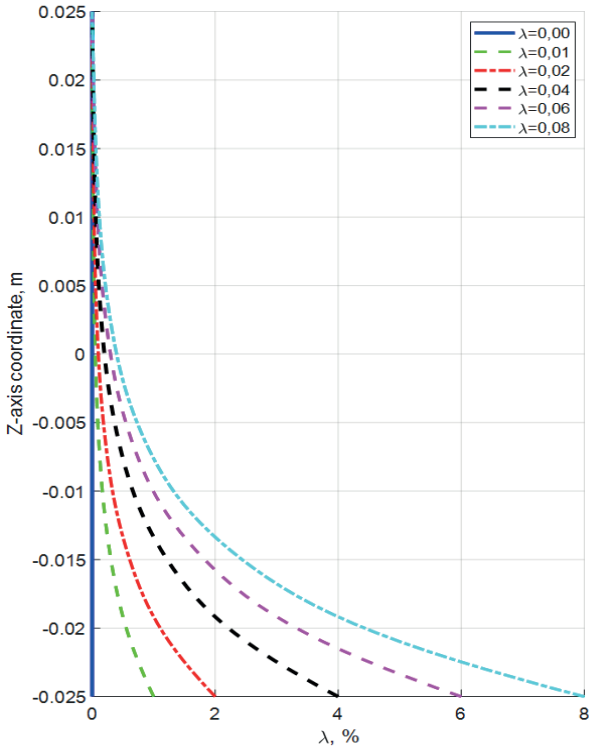


Figure 2. Distribution of hydrogen concentration by shell thickness

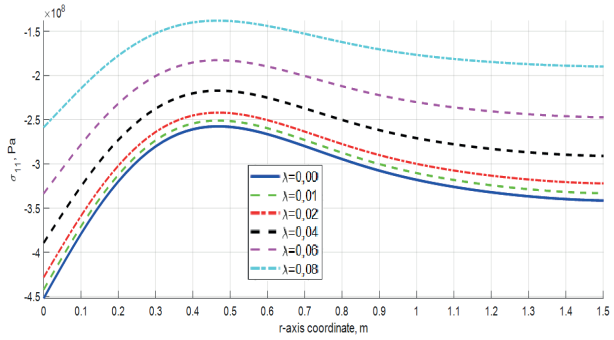


Figure 3. Stresses σ11 along the radius from below

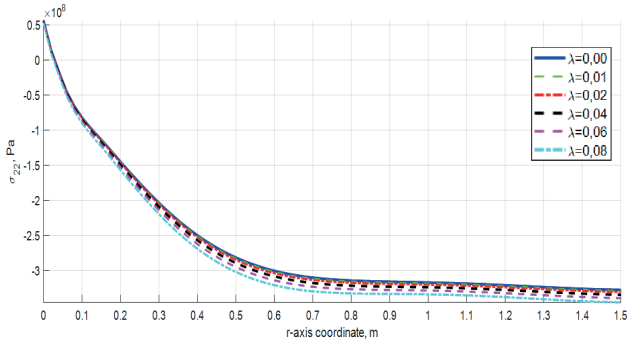


Figure 4. Stresses σ22 along the radius from above

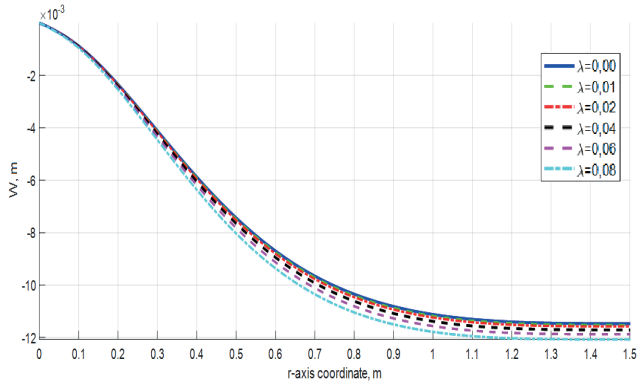


Figure 5. Deflections in the shell

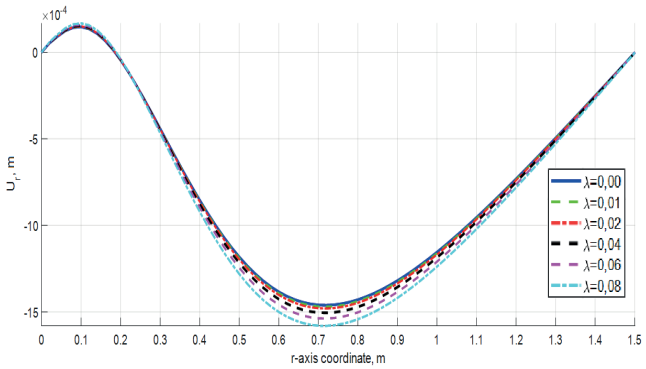


Figure 6. Horizontal displacements

CONCLUSION

After analyzing the presented graphs shown in figures 2 - 6, it is easy to note the similarity of the results calculated using the model considered in the presented article with classical nonlinear solutions without taking into account hydrogen saturation. The conducted research is fully consistent with the experimentally established facts, which shows that during a certain time interval corresponding to large gradients of hydrogen concentrations, there is an intense change in the nature of the stress-strain state of structures. In this case of a spherical shell, quantitative changes reach 20% for stresses in compressed and 24% in stretched zones. The control of the impact of an aggressive hydrogen environment in the work was organized on the basis of nonlinear relationships that consider the induced sensitivity to hydrogen saturation in a wide range of changes in the types of stress state [8, 9, 12, 13].

In this work, a mathematical model of the effect of hydrogen saturation on the stress-strain state of a shallow spherical shell is constructed and a numerical solution to the problem is presented with an illustration of deflections, displacements and stresses.

The model of the influence of gas saturation, constructed in this work, is based on the approaches to constructing constitutive relations for materials with different resistance, proposed in works [8, 9, 12, 13]. This approach uses a rather flexible mechanism for taking into account a variety of stress states and demonstrates a high accuracy of agreement between the results obtained and experimental data on the deformation of a wide range of materials under complex types of stress states. In turn, the model for accounting for materials with different resistance, proposed in the works of I.G. Ovchinnikov [1 – 4], is based on the simplest nonlinear theory of elasticity and is built considering the uniaxial stress state and, therefore, has an approximate mechanism of the influence of the type of stress state on the strength of materials.

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