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# DYNAMIC BEHAVIOR OF REINFORCED CONCRETE COLUMN UNDER ACCIDENTAL IMPACT

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**Abstract**: The analysis of scientific literature shows that to date, the physical parameters of the deformation of reinforced concrete bar structures during their dynamic buckling and the influence of the dissipative properties of the structural system on this process remain insufficiently studied. In this regard, the paper proposes an analytical solution to the problem of dynamic buckling of a reinforced concrete column when it is loaded with an impact load, taking into account the presence of initial geometric and (or) physical imperfections and damping properties of the system, as well as an analysis and assessment of the column deformation parameters based on the obtained analytical solution. An expression for the dynamic deflection of a bar element under its axial loading with a high-speed shock load, taking into account damping, is obtained in an analytical form. For practical calculations in a quasi-static formulation, the paper proposes an expression for the dynamic factor kd of bar structures under axial shock load. A numerical example of calculating a reinforced concrete column using the obtained analytical expressions with and without damping is considered. It was found that the maximum deflection of the elastic axis of the column under high-speed loading was achieved at t = 0.04 s. In this case, the total dynamic deflection taking into account damping was 4.8% less than the deviation without taking into account damping and 1.18 times more than the corresponding static value.

**Keywords**: reinforced concrete, column, buckling, accidental impact, progressive collapse, deflection, velocity, acceleration

# ДЕФОРМИРОВАНИЕ ЖЕЛЕЗОБЕТОННОЙ ВНЕЦЕНТРЕННО СЖАТОЙ КОЛОННЫ ПРИ ДОГРУЖЕНИИ УДАРНОЙ НАГРУЗКОЙ

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**Аннотация**: Анализ научной литературы показывает, что к настоящему времени остаются недостаточно исследованными физические параметры деформирования железобетонных стержневых конструкций при их динамическом продольном изгибе и влияние на этот процесс диссипативных свойств конструктивной системы. В связи с этим, в рассматриваемой работе предложено аналитическое решение задачи о динамическом продольном изгибе железобетонной колонны при ее догружении ударной нагрузкой с учетом наличия начальных геометрических и (или) физических несовершенств и демпфирующих свойств системы, а также дан анализ и оценка параметров деформирования колонны на основе полученного аналитического решения. В аналитической форме получено выражение динамического отклонения упругой оси стержневого элемента при его продольном нагружении высокоскоростной ударной нагрузкой с учетом демпфирования. Для практических расчетов в квазистатической постановке предложено выражение коэффициента динамичности kd стержневых конструкций на продольную ударную нагрузку. Рассмотрен численный пример расчета железобетонной колонны с использованием полученных аналитических выражений при учете демпфирования и без него. Установлено, что наибольшее отклонение упругой оси рассматриваемой колонны при высокоскоростном нагружении было достигнуто при t = 0.04 s. При этом полное динамическое отклонение с учетом демпфирования на 4,8 % меньше отклонения без учета демпфирования и в 1,18 раз больше соответствующей статической величины.

**Ключевые слова**: железобетон, колонна, продольный изгиб, ударная нагрузка, прогрессирующее обрушение, отклонение, скорость, ускорение

## INTRODUCTION

During the entire service life, the buildings and structures are subject to power and environmental influences of various nature and intensity. In some cases, such influences can lead to a loss of the bearing capacity of the structural elements of a building, which in turn can lead to a disproportionate failure of the entire structural system that got name of progressive collapse. Major accidents such as the collapse of a section of the Ronan Point high-rise residential building (London, 1968) [1], the Sampoong Department Store (Seoul, 1995) [2], the Transvaal Park (Moscow, 2004) [3], the WTC building (New York, 2011) [4], the federal buildings of Alfred Murray (Oklahoma City, USA, 1995) [5], the residential building Champlain Towers South (Florida, USA, 2021), etc., clearly demonstrated the relevance of this problem.

In the event that the removal time of the bearing element is counted in fractions of a second, then this process is accompanied by the emergence of significant inertial forces, which leads to dynamic loading of the remaining elements of the bearing system of the building. This is confirmed by the results of field tests carried out by Sasani and Sagiroglu [6] during the demolition of buildings, as well as by testing scale models of flat and spatial frames of buildings made by V.I. Kolchunov, N.V. Fedorova, P.A. Korenkov, N.T. Vu et al. [7–10], A.I. Demyanov and Alcadi S.A. [11] and others. According to the results of the aforementioned studies, the elements of coatings (floors) over the structure to be removed are the first to be included in the dynamic process of redistribution of power flows through the alternate load paths.

However, a scenario is possible in which the removal of the structural element of the building frame can lead to the buckling of the eccentrically compressed elements of the deformed structural system. Such a scenario for the development of progressive collapse can be associated with the accumulation of environmental damage (corrosion, high temperatures) [12-14], a change in the nature of the stress-strain state of eccentrically compressed elements due to an increase in the span of the floor structure or an increase in the eccentricity of the application of a longitudinal force, an increase in the effective length of the considered eccentrically compressed element, due to removal of vertical ties or degradation of the fastening [15,16].

V.A. Gordon and V.I. Kolchunov [17] investigated the stability of the columns of the reinforced concrete building frames under the degradation of the conditions of fastening in the nodes. V.I. Kolchunov, N.O. Prasolov and Kozharinova L.V. [16] carried out numerical and experimental studies of the stability of reinforced concrete frames with a sudden change in the effective lengths of the elements. V.M. Bondarenko [18] investigated the issues of survivability exposure of reinforced concrete corrosively damaged eccentrically compressed bar elements. Investigations of the dynamic stability of reinforced concrete compressed bar elements taking into account the partial absence of adhesion of reinforcement to concrete were carried out by A.G. Tamrazyan, D.S. Popov. and Ubysz A. [19]. D.G. Utkin [20] carried out the study of the bearing capacity and methods of strengthening reinforced concrete columns subjected to transverse impact. In the work of Alekseytsev A.V. [21], it was carried out a numerical study of the mechanical safety of reinforced concrete frames of buildings under transverse impact using NX Nastran, in which, among other things, the bearing capacity of eccentrically compressed elements was assessed according to the strength and stability criteria.

A brief analysis of the mentioned above and other scientific publications on the issue of buckling and stability of columns of reinforced concrete frames of buildings during structural alterations caused by the occurrence of local damage or destruction of load-bearing structures allows us to conclude that by now the parameters of dynamic deformation of reinforced concrete bar structures remain insufficiently studied. during their buckling and the influence on this process of the dissipative properties of the bearing structural system of a building or structure.

In this regard, the purpose of the presented study was to obtain an analytical solution to the problem of dynamic buckling of a reinforced concrete column when it is loaded with a shock load, taking into account the presence of initial geometric and (or) physical imperfections and damping properties of the system, as well as to estimate the column deformation parameters based on the obtained analytical solution

## MATERIALS AND METHODS

Let us consider a reinforced concrete frame of a multi-storey civil building. Figure 1 shows a variant of its secondary design scheme for the analysis of resistance to progressive collapse. The column of the first floor in the axes "A" - "2" of such a frame with a sudden removal of the column of the first floor in the axes "A" - "1" can be considered as an eccentrically compressed element, loaded with a rapidly increasing load. As shown by the results of experimental studies [6], the redistribution of power flows along alternative paths can occur in tenths, and in some cases hundredths parts of a second.

Material parameters adopted in the study are the follows: concrete B25, reinforcement bars A400. The axial force in the column from the operating load $P$ = 5000 kN. After a sudden failure of the outermost column of the frame, the considered column is loaded with an axial force of 2000 kN during 0.01 s.

It's should be noted that the issue of assessing the time of redistribution of efforts during a sudden failure of the structural member is not considered in this study. Therefore, we have assumed that the magnitude of the force and the time of its application to the considered eccentrically compressed bar element of the column are known preliminary. The problem is solved under the assumption that there are no longitudinal inertial forces in the bar when it is dynamically loaded.

The differential equation of motion of an eccentrically compressed bar (Figure 2, a) when it is loaded with a high-speed shock load, taking into account the damping properties of the system, can be written in the following form:

$$EJ\frac{\partial^4\left(w_1-w_0\right)}{\partial x^4}+P\frac{\partial^2 w_1}{\partial x^2}+\frac{\gamma\cdot A}{g}\frac{\partial^2 w}{\partial t^2}+k\frac{\partial w}{\partial t}=0, \qquad (1)$$

where $w_0 = w_0\ (x)$ is the initial deviation of the elastic axis of the rod from the vertical axis passing through the center of gravity of one of the end sections. The appearance of the initial deviation is due to the geometric and (or) physical imperfections of the column sections, acquired during manufacture and operation, as well as buckling from a statically applied operational load.

$w_1 = w_1\ (x, t)$ is the total deflection of the elastic axis of the bar at time $t$;

$w = w_1 - w_0$ - additional deflection of the elastic axis of the rod at time $t$;

$E$ is the initial modulus of elasticity of concrete;

$J$ is the moment of inertia of the reduced section of a reinforced concrete bar element, taken constant along the length of the bar along the section with the lowest bending stiffness;

$P = P\ (t)$ - the law of variation in time of the longitudinal force (Figure 2, b), adopted in the form:

$$P=\begin{cases}P_0+\Omega\cdot t, & for \quad 0\le t\le t_1;\\ P_0+\Omega\cdot t_1, & for \quad t>t_1,\end{cases} \qquad (2)$$

where $P_0$ is a statically attached axial force;

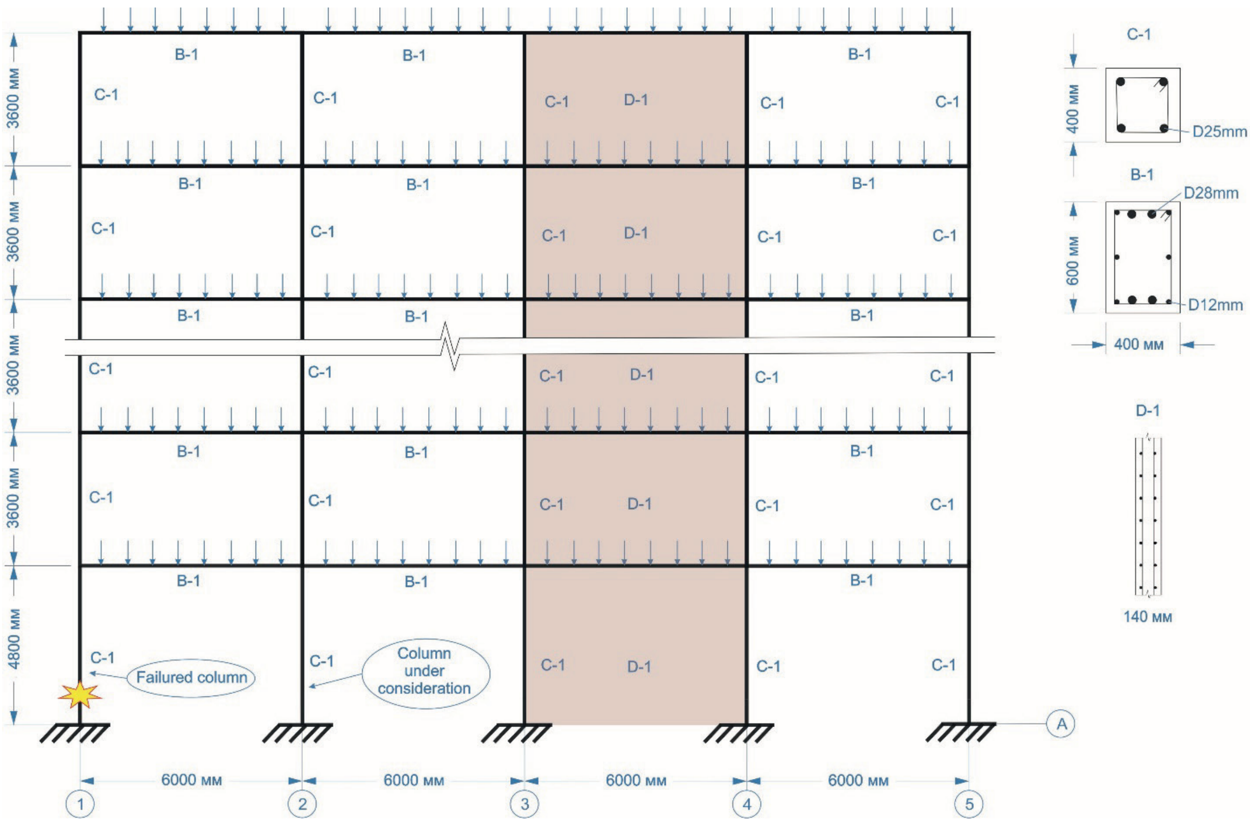


*Figure 1. Secondary design scheme of a reinforced concrete frame of a building for analysis of resistance to progressive collapse*

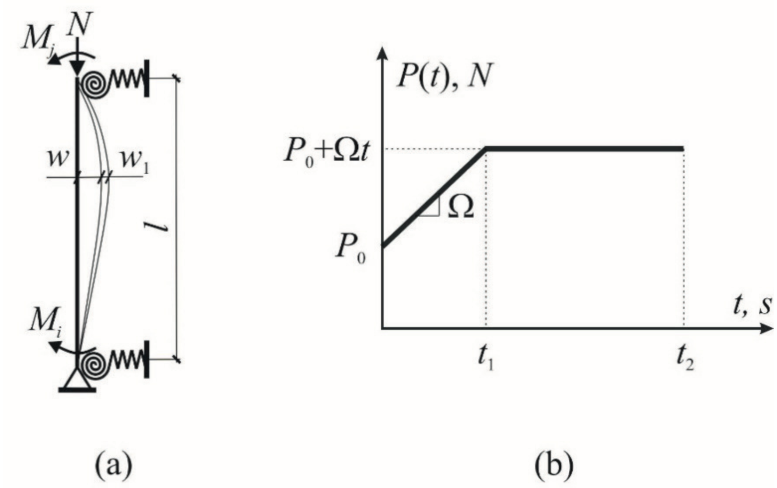


*Figure 2. Design scheme of an eccentrically compressed bar member (a) and the law of variation of the axial compressive force in time (b)*

$\Omega$ is a velocity of the bar member dynamic loading, N/s;
$t_1$ is a time of dynamic loading of the bar member by shock load;
$\gamma$ is a specific gravity of the material;
$A$ is an area of reduced cross section;
$g$ is the acceleration of gravity;
$k$ is proportionality factor accounting damping properties, taken as a first approximation

$$k = \frac{\delta \cdot \omega \cdot \gamma \cdot A}{\pi \cdot g}.$$

Here $\delta = 0.31$ is – vibration decrement for reinforced concrete structures, adopted according to [22], $\omega$ is the circular vibration frequency.

In order to solve equation (1), we used the separation of variables, setting the shape of the elastic line of the deformed column bar, as well as the shape of the initial deviation in the form of a half-wave of a sinusoid:

$$w_0 = f_0 \cdot sin\left(\frac{m \cdot \pi \cdot x}{l}\right);$$

$$w_1 = f_1(t) \cdot sin\left(\frac{m \cdot \pi \cdot x}{l}\right), \qquad (3)$$

where $f_0$ is amplitude value of the initial deviation along the length of the rod;

$f_1(t)$ is amplitude value of total deflection along the length of the rod at time $t$;

$m$ is the number of half-waves of a sinusoid along the length of the bar;

$l$ is an effective length of the bar.

Substituting (3) into equation (1) and excluding the variable x, we obtain:

$$\frac{df_1^2}{dt^2} + \frac{k \cdot l^2 \cdot \omega^2}{\pi^2 \cdot P_E}\frac{df_1}{dt} + \omega^2 m^2\left(m^2 - \frac{P}{P_E}\right)f_1 = \omega^2 m^4 f_0. \qquad (4)$$

In equation (4), the following designations are adopted:

$$\omega = \frac{\pi^2}{l^2}\sqrt{\frac{E \cdot g}{\gamma} \cdot \frac{J}{A}}; \quad P_E = \frac{\pi^2 \cdot E \cdot J}{l^2}.$$

Here $\omega$ is the frequency of the first vibration tone,

$P_E$ is Euler critical force.

We solved equation (4) during two stages. At the first stage, accepting the law of variation of the axial force in time $P(t) = P_0 + \Omega t$, we obtain a solution to equation (4) for the time interval $0 \le t \le t_1$. At the second stage, taking the longitudinal force $P = P_0 + \Omega t_1 = \text{const}$, we obtain a solution to equation (4) for the time interval $t > t_1$. In this case, the initial conditions at the second stage are the deviation of the elastic axis and the velocity of transverse displacement at time $t = t_1$, obtained from the solution of the differential equation at the first stage. When performing operations of differentiation, integration and search for numerical values of variables, we used the MathCAD software package.

## RESULTS

**Solution to the problem of dynamic buckling with a load increasing in time and damping.** In order to simplify the form of writing the differential equation (4), taking into account the law of variation of the longitudinal force in time $P(t) = P_0 + \Omega t$, we introduce the change of the variable $t_\omega = t$

$$f_1 = y \cdot e^{-\frac{\alpha \cdot t_\omega}{2}}, \text{ where } \alpha = \frac{k \cdot \omega \cdot l^4}{\pi^4 \cdot E \cdot J}.$$

Then, taking into account the adopted law of the change in the longitudinal force in time, we obtain:

$$y''+\left(m^4-\frac{P_0}{P_E}m^2-\frac{\alpha^2}{4}-\frac{\Omega\cdot m^2}{\omega\cdot P_E}t_\omega\right)y=m^4\cdot f_0\cdot e^{\frac{\alpha\cdot t_\omega}{2}}. \quad (5)$$

Let us the following substitute into the equation (5):

$$y(t_\omega)=\eta(\zeta);\quad t_\omega=b-a^{\frac{1}{2}}\cdot\zeta,$$

$$b=\frac{\omega}{\Omega}\left(P_E\cdot m^2-P_0-\frac{\alpha^2\cdot P_E}{4\cdot m^4}\right);\quad a=\left(\frac{\omega}{\Omega}\frac{P_E}{m^2}\right)^2,$$

taking into account which we get:

$$\frac{d^2\eta}{d\zeta^2}+a\cdot\zeta\cdot\eta=m^4\cdot f_0\cdot e^{\frac{\alpha}{2}\left(b-a^{\frac{1}{2}}\cdot\zeta\right)}. \quad (6)$$

We exclude the parameter $a$ from the left-hand side of Eq. (6) by performing an additional substitution $\psi(z)=\eta(\zeta)$, $z=a^{1/3}\zeta$, as a result of which we obtain the inhomogeneous equation:

$$\frac{d^2\psi}{dz^2}+z\cdot\psi=a^{\frac{1}{3}}\cdot m^4\cdot f_0\cdot e^{\frac{\alpha}{2}\left(b-a^{\frac{1}{6}}\cdot z\right)}, \quad (7)$$

the solution of which we found in the form of the sum of the solution of the homogeneous equation known in the scientific literature [23] and the particular solution, selected according to the form of the right-hand side:

$$\psi=C_1\cdot z^{\frac{1}{2}}\cdot J_{1/3}\left(\frac{2}{3}z^{\frac{3}{2}}\right)+C_2\cdot z^{\frac{1}{2}}\cdot J_{-1/3}\left(\frac{2}{3}z^{\frac{3}{2}}\right)+\frac{4\cdot a^{\frac{1}{3}}\cdot m^4\cdot f_0}{\alpha^2\cdot a^{\frac{1}{3}}+4\cdot z}\cdot e^{\frac{\alpha}{2}\left(b-a^{\frac{1}{6}}\cdot z\right)}. \quad (8)$$

In the expression (8), $J_{1/3}\left(\frac{2}{3}z^{\frac{3}{2}}\right)$, $J_{-1/3}\left(\frac{2}{3}z^{\frac{3}{2}}\right)$ are Bessel functions;
$C_1$, $C_2$ are the arbitrary constants determined from the initial conditions of the problem:

$$\psi=f_0,\quad \frac{d\psi}{dz}=0\quad for\quad t=0\quad or\quad z=b\cdot a^{-\frac{1}{6}}.$$

It should be noted that the parameter $z$ decreases with increasing time $t$. Therefore, when it goes into the negative range of values in expression (8), the Bessel functions $J_{1/3}\left(\frac{2}{3}z^{\frac{3}{2}}\right)$, $J_{-1/3}\left(\frac{2}{3}z^{\frac{3}{2}}\right)$ must be replaced by modified Bessel functions $I_{1/3}\left(\frac{2}{3}z^{\frac{3}{2}}\right)$, $I_{-1/3}\left(\frac{2}{3}z^{\frac{3}{2}}\right)$, and the arbitrary constants $C_1$, $C_2$ must be redefined under the initial conditions for $z = 0$ obtained by solving the original equation (8).
For the reinforced concrete column under consideration, the graphs of the change in the parameter $z$, lateral deviation $f_1$, velocity and acceleration for the time interval $0 \le t \le t_1$ with and without damping are shown in Figure 3.

**Solution to the problem of dynamic buckling of a bar with a constant axial force and damping.** At the second stage of the study, taking the longitudinal force constant $P = P_0 + \Omega t_1 = \text{const}$, for $P < P_E$ we obtain a solution to equation (4) for the time interval $t > t_1$:

$$f_1 = e^{-\frac{\beta \cdot t_2}{2}} \cdot \left( D_1 \cdot cos\left(\frac{\lambda \cdot t_2}{2}\right) + D_2 \cdot sin\left(\frac{\lambda \cdot t_2}{2}\right)\right) + \frac{f_0}{1 - \dfrac{P}{m^2 \cdot P_E}}, \tag{9}$$

in which a particular solution (the last term) is a static deflection of an eccentrically compressed rod with an initial deflection $f_0$;

$D_1$, $D_2$ - arbitrary constants determined from the initial conditions, which are the deviation of the elastic axis and the speed of transverse displacement at time $t = t_1$, obtained from expression (8) and its first derivative, respectively;

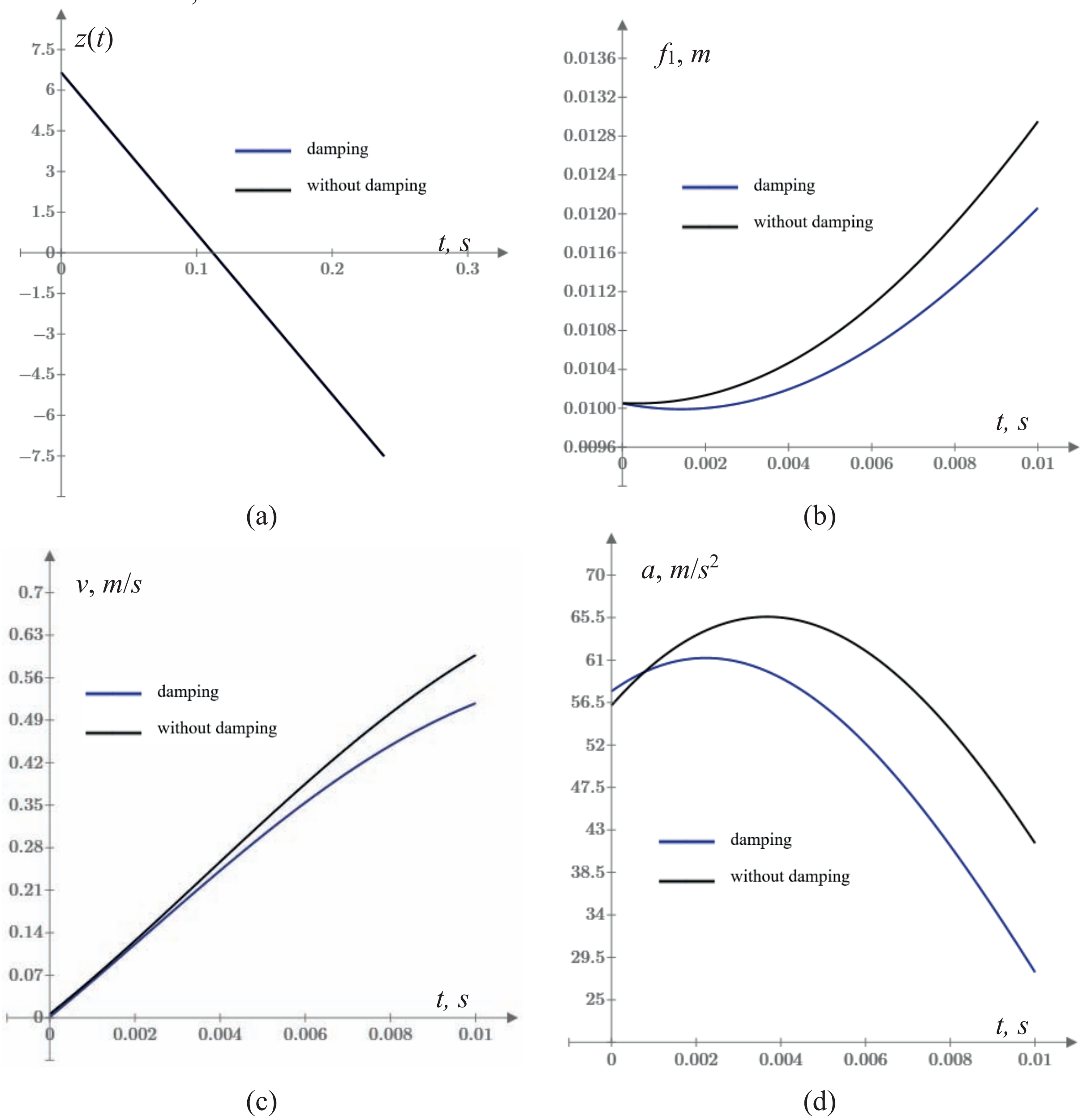


*Figure 3. Parameters of reinforced concrete column deformation for the time interval $0 \le t \le t_1$: dimensionless time parameter z (a), deflection of the elastic axis of the bar element (b), lateral displacement rate (c), acceleration of the lateral displacement (d)*

$$\beta = \frac{k \cdot l^2 \cdot \omega^2}{P_E \cdot \pi^2}, \quad t_2 = t - t_1 \quad for \quad t > t_1,$$

$$\lambda^2 = \beta^2 - 4\omega^2 \cdot m^4 \left(1 - \frac{P}{m^2 \cdot P_E}\right).$$

It should be noted that the above assumption $P < P_E$ corresponds to the case $\lambda^2 < 0$. Analysis of the expression for the parameter $\lambda^2$ shows that if the value of the longitudinal force approaches $P_E$, then $\lambda^2$ changes its sign, and the first term in expression (9) must be written in hyperbolic functions.

For the reinforced concrete column under consideration, the graphs of changes in the lateral deflection $f_1$, speed, acceleration and the ratio of dynamic to static deflection for the time interval $t > t_1$ (in the presence and absence of damping are shown in Figure 4.

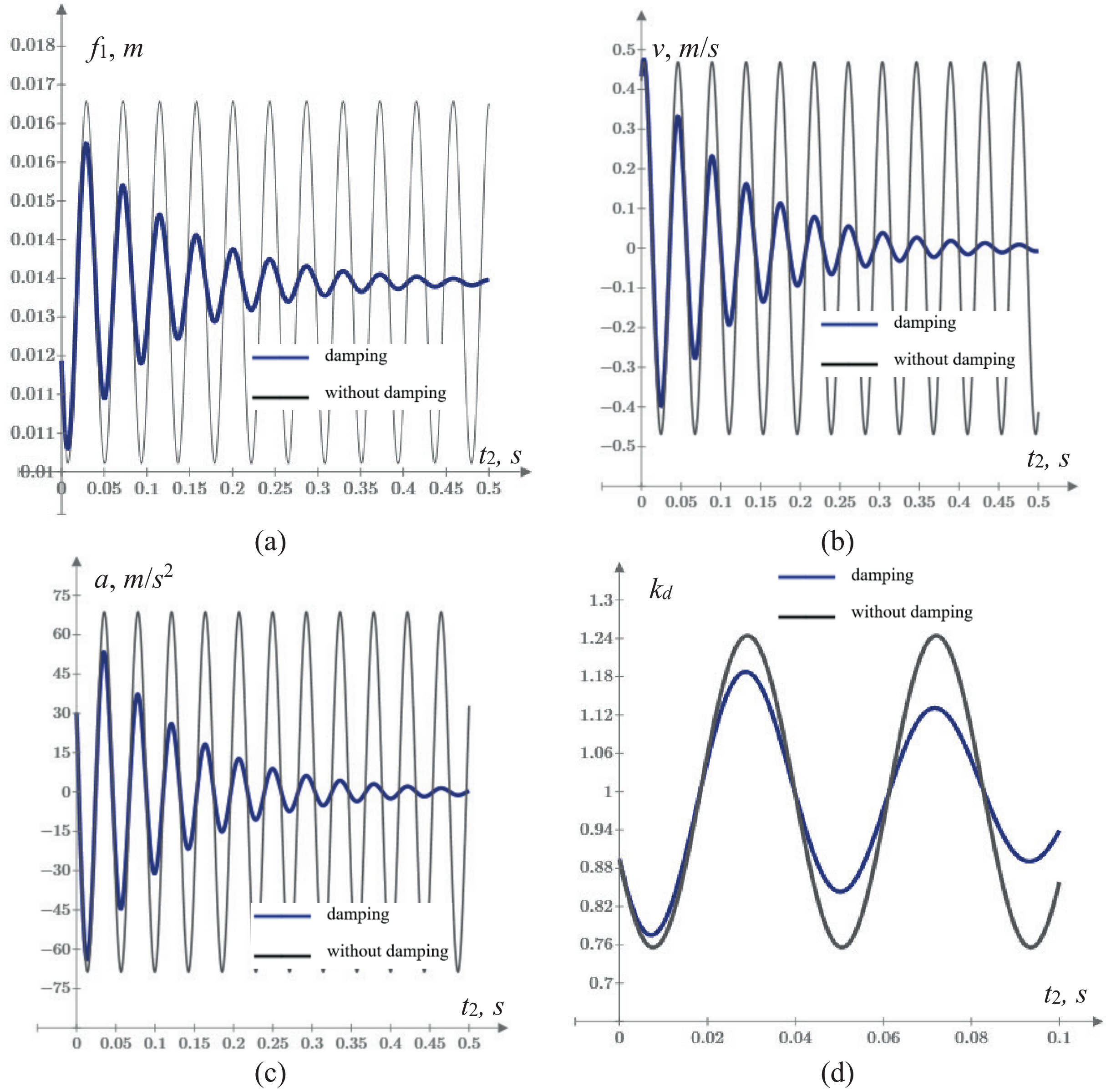


*Figure 4. Parameters of deformation of a reinforced concrete column for the time interval $t > t_1$: deviation of the elastic axis of the bar element (a), the rate of lateral displacement (b), acceleration of the lateral displacement (c); the ratio of the dynamic deflection of an eccentrically compressed bar element to the static one (d)*

Considering that on the first wave of transverse vibrations in the time interval $t > t_1$, the decrease in the value of dynamic deflection when taking into account damping compared to dynamic deflection without taking into account damping is within 5% (see Figure 4, d), then for practical calculations of bar structures in a quasi-static formulation, the dynamic factor $k_d$ can be calculated by the formula:

$$k_d = 1 + \left( D_1 \cdot cos\left( \frac{\lambda \cdot t_2}{2} \right) + D_2 \cdot sin\left( \frac{\lambda \cdot t_2}{2} \right) \right) \frac{1 - \dfrac{P}{m^2 \cdot P_E}}{f_0}. \tag{10}$$

## DISCUSSION

Analysis of the plots presented in the previous section shows that the maximum deviation of the elastic axis of the column under the considered shock impact was achieved at $t = t_1 + t_2 = 0.04$ s. In this case, the total dynamic deflection taking into account damping was 16 mm, which is 4.8% less than the deviation without taking into account damping and 1.18 times more than the corresponding static value.
Attention should be paid to the fact that the time to reach the maximum deflection of the elastic axis of the column is comparable to the time the elastic wave running the length of the bar. Therefore, in further studies, it is advisable to take into account the forces of longitudinal inertia.
Based on the results obtained in the previous section, the amplitude values of the dynamic transverse deflections of the elastic axis of the bar, it can be assumed that the transverse vibrations of the bar have practically no effect on the magnitude of the longitudinal forces due to the small deviations compared to the dimensions of the cross-section of the bar. Thus, a possible way to solve the problem can be the search for the function $P(t)$ from the solution of the wave equation with substitution this one into the differential equation of dynamic buckling.

## CONCLUSION

Summarizing the results of the study, the following conclusions can be drawn:

1. In an analytical form, an expression was obtained for the dynamic deflection of the elastic axis of a bar element under its longitudinal loading with a high-speed shock load, taking into account damping.
2. For practical calculations in a quasi-static formulation, an expression is proposed for the dynamic factor $k_d$ of bar structures under axial shock load.
3. It was found that for the reinforced concrete column under consideration, the maximum deviation of its elastic axis under impact action was achieved at t = 0.04 s. In this case, the total dynamic deflection taking into account damping was 16 mm, which is 4.8% less than the deviation without taking into account damping and 1.18 times more than the corresponding static value.
4. In the considered example, the time to reach the maximum dynamic deflection of the elastic axis of the column is comparable to the time the elastic wave running the length of the bar. Therefore, when studying the dynamic buckling from the shock load, it is advisable to take into account the forces of longitudinal inertia.

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