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STUDY OF STRESS-STRAIN STATES OF A REGULAR HINGE-ROD CONSTRUCTIONS WITH KINEMATICALLY ORIENTED SHAPE CHANGE

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Abstract: For regular hinge-rod structures, an engineering method for analyzing the stress-strain state is developed, taking into account the transformation of the form by folding repeated fragments of the structure. For the software implementation of the proposed calculation algorithm, a macro is compiled in the APDL language, which is built into the ANSYS software package. A step-by-step procedure that simulates the transformation of the farm geometry was tested.

Keywords: hinge-rod structures, finite element method, truss element stiffness matrix, the stiffness matrix of a mechanical drive, the stress-strain state

ИССЛЕДОВАНИЕ НАПРЯЖЕННО-ДЕФОРМИРОВАННОГО СОСТОЯНИЯ РЕГУЛЯРНОЙ ШАРНИРНО-СТЕРЖНЕВОЙ КОНСТРУКЦИИ ПРИ КИНЕМАТИЧЕСКИ ОРИЕНТИРОВАННОМ ИЗМЕНЕНИИ ФОРМЫ

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Аннотация: для регулярных шарнирно-стержневых конструкций разработана инженерная методика анализа напряженно-деформированного состояния с учетом трансформации формы путем сворачивания повторяющихся фрагментов конструкции. Для программной реализации предлагаемого алгоритма расчета составлен макрос на языке APDL, встроенного в программный комплекс ANSYS. Выполнено тестирование шаговой процедуры, моделирующей процесс трансформации геометрии пространственной фермы.

Ключевые слова: шарнирно-стержневые конструкции, метод конечных элементов, матрица жесткости ферменного элемента, матрица жесткости механического привода, напряженно-деформированное состояние

INTRODUCTION

One of the creative directions in modern architecture is the so-called kinematic design, based on a controlled change in the geometry of the structure in order to obtain the required space planning decisions [1,2]. According to the principle of transformation of geometry, building structures can be divided into the following groups [3]:

- moving in space along the guides;
- performing a rotation about the axis of rotation;
- folding or rolling on the principle of a fan.

The technology of constructions made of origami, which are capable, compressing and stretching, to qualitatively change shape should be added to this. The idea of creating such building structures was taken from the field of aerospace systems such as solar panels and mir-

rors with a large surface [4].

As examples of existing original transformable building structures, Thomas Heterwick's folding bridge [5], built in London in 2004 and the folding bridge in Germany (1997), which is called «Horn», which became a landmark of Kiel [6], can be mentioned.

At the same time, there is practically no information in the literature of structural mechanics about the mathematical modeling of transformable building systems taking into account the form change. In this regard, the direction associated with the development of an engineering methodology, the calculation of geometrically variable structures using the finite element method (FEM) is relevant.

In FEM, the relationship between deformations at an arbitrary point of a finite element and the corresponding nodal displacements is generally represented in matrix form [7]

$$\{\varepsilon\}=[\Phi]\{u\} \qquad (1)$$

where $\{\varepsilon\}$ and $\{u\}$ – are the column vectors of the components of the strain tensor and nodal displacements; $[\Phi]$ – is a matrix of form functions, depending on the type of the finite element and the approximating functions.

By the hypothesis of infinitesimal deformations, it is generally accepted that the matrix does not change during loading on element. This assumption is the basis of the so-called infinitesimal theory of deformations, i.e., a theory when deformations are considered infinitesimal quantities. The use of this theory is quite justified if a change in the nodal coordinates of the finite element mesh can be neglected, during the deformation of the structure. However, in some cases, due to large elasto-plastic strains or large translational and angular displacements of the model, in order to obtain an exact solution, it is necessary to take into account the change of the matrices of finite elements. Such a theory is called the theory of finite strains. In the framework of the theory of finite strains, various step-by-step procedures are applied for the numerical implementation of calculations, the essence of which is to represent the loading process in the form of a stepwise or continuous increase in the load parameter. Moreover, at each step, as a rule, a scheme for iterative refinement of the solution is provided.

The method to describe the current deformed state of the finite element model and the initial coordinates are used is called the method of Lagrange. In case of large displacements, for example during structural modificationat the beginning of each loading step, the initial coordinates are used. This method of representing deformations is called the modified method of Lagrange [8]. The present work is devoted to the extension of the modified Lagrange method to the problem of analyzing the stress-strain state of a spatial truss structure with kinematically directed shape transformation in the plane of minimal stiffness.

2. CALCULATION METHOD

To analyze the stress-strain state of a hinged structure with a regular structure, we use the ANSYS Mechanical software [9], which implements the FEM in the form of a displacement method. As an object of research we consider a space truss structured in the form of repeating semi-octahedron (Figure 1).

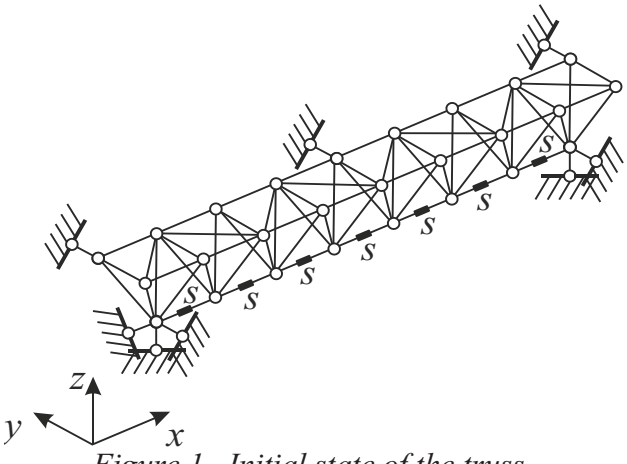


Figure 1. Initial state of the truss.

The geometrical dimension of the truss are set in the global Cartesian coordinate system

x, y, z. The truss shape transformation is provided using, located in the back bar of the structure, rods-drives with variable length. In this case, the process of forming takes place in such a way that the lengths of the rods of the top-chord and the lattice of the truss practically do not change. In Figure 1, the drive rods are indicated by s. Note that the operating mechanism of these rods provides a direct and reverse stroke, i.e., mounting and dismounting of the structure. To simulate the process of kinematically oriented structural change, we use the finite element (FE) LINK 11 (Figure 2), which allows you to change the distance s between nodes i and j.

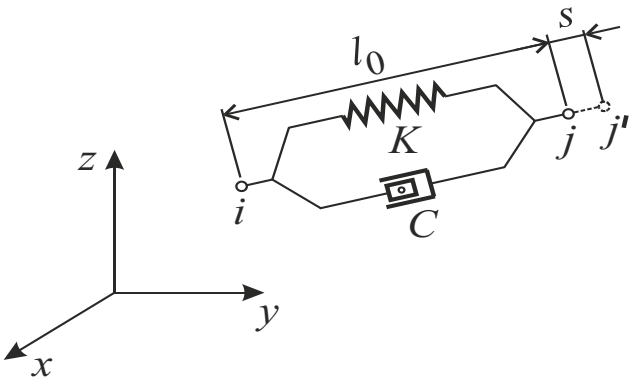


Figure 2. FE with a variable length LINK 11.

We model the rods of the top-chord and lattice with 3D truss elements of the LINK 180 type (Figure 3). In this figure the next symbols are marked: l – the length of the rod; EF – longitudinal stiffness.

The axis $\bar{x}$ of the truss CE forms with the axes x, y, z the angles directing cosines, which are determined by the formulas:

$$\cos(x\bar{x}) = \frac{x_i - x_j}{l}; \quad \cos(y\bar{x}) = \frac{y_i - y_j}{l};$$

$$\cos(z\bar{x}) = \frac{z_i - z_j}{l}. \quad (2)$$

Let s introduce the notation:

$$t_{11} = \cos(x\bar{x}),\ t_{12} = \cos(y\bar{x}),\ t_{13} = \cos(z\bar{x}).$$

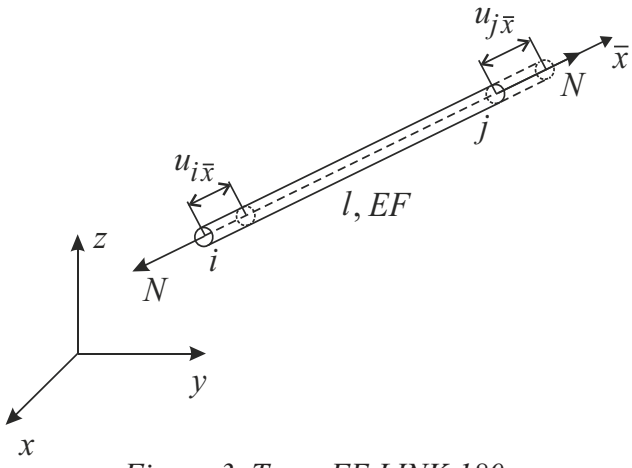


Figure 3. Truss FE LINK 180.

The relationship between the FE nodal displacements $u_{i\bar{x}}$ and $u_{j\bar{y}}$ and their projections u_{ix}, u_{iy}, u_{iz} and u_{jx}, u_{jy}, u_{jz} on the global coordinate axes is described by the relations:

$$u_{i\bar{x}} = u_{ix}t_{11} + u_{iy}t_{12} + u_{iz}t_{13};$$
$$u_{j\bar{x}} = u_{jx}t_{11} + u_{jy}t_{12} + u_{jz}t_{13}.$$

Hereinafter, the first index corresponds to the FE node number.

Axial force N in the axes x, y, z is decomposed into the following nodal components:

$$F_{ix} = -Nt_{11}; \quad F_{iy} = -Nt_{12}; \quad F_{iz} = -Nt_{13};$$
$$F_{jx} = Nt_{11}; \quad F_{jx} = Nt_{12}; \quad F_{jx} = Nt_{13}.$$

The equilibrium equation of truss FE in the axes x, y, z in matrix form has the form

$$[\mathbf{h}]\{\mathbf{u}\} = \{\mathbf{F}\}, \quad (3)$$

where column vectors of nodal displacements and forces

$$\{\mathbf{u}\} = \{u_{ix}\, u_{iy}\, u_{iz}\, u_{jx}\, u_{jy}\, u_{jz}\}^T;$$
$$\{\mathbf{F}\} = \{F_{ix}\, F_{iy}\, F_{iz}\, F_{jx}\, F_{jy}\, F_{jz}\}^T$$

(T – the symbol of matrix transposition operation); finite element stiffness matrix

$$[\mathbf{h}]=\frac{EF}{l}[\mathbf{t}], \qquad (4)$$

direction cosines matrix

$$[\mathbf{t}]=\begin{bmatrix} t_{11}^2 & t_{11}t_{12} & t_{11}t_{13} & -t_{11}^2 & -t_{11}t_{12} & -t_{11}t_{13} \\ & t_{12}^2 & t_{12}t_{13} & -t_{12}t_{11} & -t_{12}^2 & -t_{12}t_{13} \\ & & t_{13}^2 & -t_{13}t_{11} & -t_{13}t_{12} & -t_{13}^2 \\ & & & t_{11}^2 & t_{11}t_{12} & t_{11}t_{13} \\ & & & & t_{12}^2 & t_{12}t_{13} \\ & & & & & t_{13}^2 \end{bmatrix}$$

As is obvious, the matrix elements $[\mathbf{h}]$ depend on the nodal coordinates x_i, y_i, z_i and x_j, y_j, z_j, which, with large displacements of the rod, change significantly compared to the initial values.

The LINK 11 element (Figure 2), having a length in the initial state l_0, is endowed with longitudinal stiffness properties K and viscoelastic damping C. The latter is not used in this analysis. The stiffness matrix of the LINK 11 element in the global coordinate system can be represented in the form

$$[\mathbf{h}_{\text{пр}}]=K[\mathbf{t}], \qquad (5)$$

The components of the corresponding column vector of nodal forces are written as $\{\mathbf{F}_{\text{ax}}\}=N_{\text{ax}}\{-t_{11}\ -t_{12}\ -t_{13}\ t_{11}\ t_{12}\ t_{13}\}^T$ where $N_{\text{ax}}=Ks$ – axial force, due to the stroke of the drive s.

Thereafter, we accept the following assumptions:

- to describe the deformation of the structure in the process of shaping, we apply the modified method of Lagrange;
- the process of transformation of the structure represents a quasistatic sequence of steps $k=1,2,...,n$ of discrete change in the lengths of elements LINK 11 by a small amount s;
- rectilinear rods before deformation remain rectilinear after deformation;
- the cross sections of each rod remain normal to its longitudinal axis during deformation;
- we neglect the change in the longitudinal stiffness of the rods during the structural modification, i.e. we believe that the behavior of the material throughout the course of form-change obeys Hooke's law;
- in the process of transformation of the structure, the achieved level of the stress state of the rods is remain intact.

The geometry and stress state transformation of the rod modeled by the LINK 180 element is schematically shown in Figure 4. We emphasize that the transition from the current position of the rod to the subsequent position is accompanied by small increments in the values of the nodal coordinates.

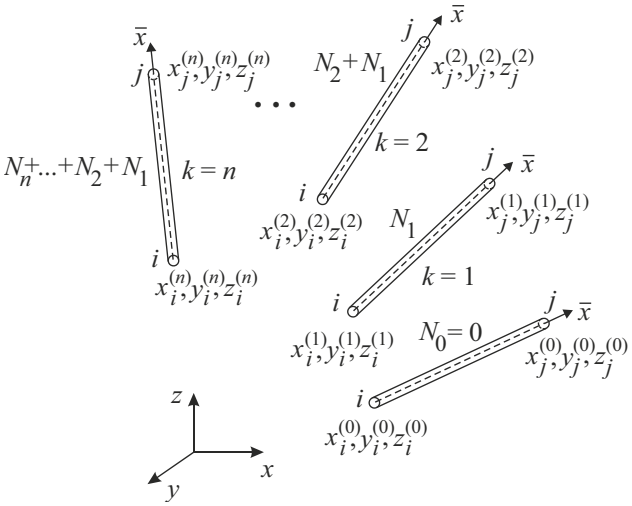


Figure 4. Visualization of the process of transformation of the rod.

The flow diagram of the algorithm developed on the basis of the accepted assumptions is shown in Figure 5. Abbreviations are introduced here: BC – boundary conditions; SLAE – simultaneous linear algebraic equations. APDL programming language is used For the software implementation of this algorithm [10], which is built into the ANSYS Mechanical. Created on the basis of this language the application macro is entered into the command window, after which

each line of the macro is processed by the APDL interpreter and, if the result is positive, is immediately launched. Thus, the macro allows you to automatically create the geometry of the structure, build a finite element mesh, set the boundary conditions and load, start the solver to perform the calculation, as well as carry out intermediate operations associated with extracting information from the ANSYS database at the current loading step and generating working arrays by performing the necessary algebraic procedures. In addition to the above actions, the macro contains commands to delete the finite element model at the current calculation step.

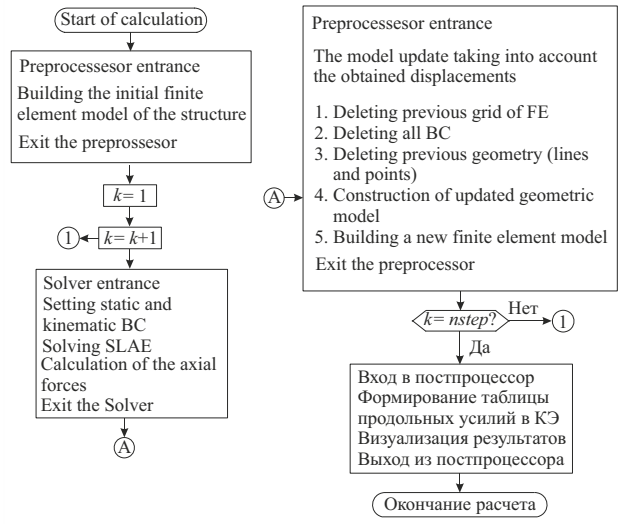


Figure 5. The block diagram of the transformable hinged structure calculating algorithm

Note that in the proposed macro the procedure of direct calculation of the axial forces in the truss rods using the formula

$$N_k = \frac{E\,F}{l_{k-1}}(l_k - l_{k-1})\,. \qquad (6)$$

This approach is explained by the fact that when using LINK 180 FE values N_k are calculated in relation to the initial (undeformed) element length. With the proposed step-by-step method of representing the process of structural modification, this method leads to incorrect results.

2. NUMERICAL CONVERGENCE OF CONVERGENCE

Testing the developed algorithm and the corresponding macro is feasible on the example of the spatial truss shown in Fig. 1. Initial data: rods of the top-chord and lattice have a tubular cross-sectional area of $F = 0{,}113 \cdot 10^{-3}$ m^2; modulus of elasticity of the material of the rods (steel) $E = 2{,}1 \cdot 10^5$ МПа ; specific density γ = 7800 kg/m^3. Overall dimensions in meters for a repeating fragment of the truss (semi-octahedron) are shown in Figure 6.

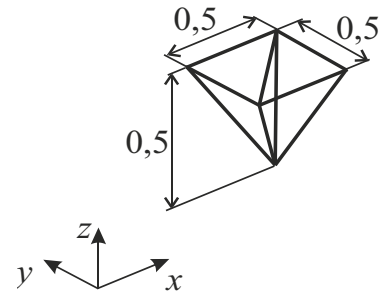


Figure 6. The repeating fragment of the truss

Stiffness coefficient and mass of the mechanical drive modeled by the LINK 11 element is $K = 1 \cdot 10^{10}$ N/m; m_{ax} = 20 kg.

In Figure 7 shows graphs of changes in the height of the lift h and the bay l_{ax} f the truss depending on the stroke of the drive s and taking into account its own weight.

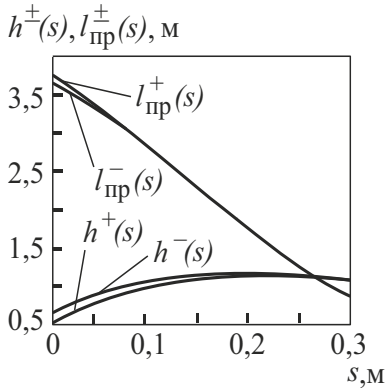


Figure 7. Graphs $h^{\pm}, l^{\pm}_{\text{пр}} \sim s$ for $s = 0{,}01$ m and nstep = 30.

Here, the «+» sign corresponds to the forward stroke (camber of truss), the «-» sign to the reverse stroke (returning the truss to the initial state). The graphs in Figure 7 are obtained with the stroke value at the transformation step $s = 0{,}01$m and the number of transformation $nstep$ = 30. Visualization of the patterns of the forward and reverse transformation of the truss for these parameters s and $nstep$ are presented in Figure 8.

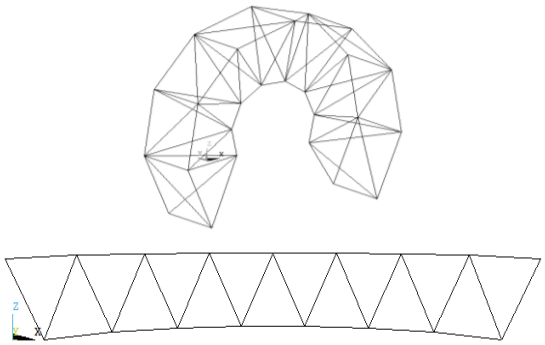


Figure 8. Visualization of the patterns of direct and reverse transformation of the farm s=0,01 m and nstep= 30.

Analyzing the graphs in Figure 7 and the type of structure after the reverse transformation, we establish that the geometry of the model as a result of the assembly-disassembly cycle is not restored to its original state and in this case there is a residual deflection of the truss top chord and truss back bar.

Figures 9 and 10 show the results of a similar calculation for the values of the parameters $s = 0{,}001$m and $nstep = 300$. For comparison, the values of the parameters $h^{+}(0,3)$ and $l^{+}_{ax}(0,3)$ amounted to: for s=0,01 m and $nstep$=30 – $h^{+}(0,3) = 1{,}071\,m$, $l^{+}_{пр}(0б3) = 0{,}8553m$; for $s = 0{,}001$m and $nstep = 300$ – $h^{+}(0,3)$= 1,036 m, $l^{+}_{ax}(0,3)$= 0,9263 m.

From the data presented it follows that with a tenfold decrease in the parameter s and the same increase in the parameter $nstep$ a satisfactory coincidence of the simulation results with a picture of the real behavior of the structure under consideration during direct and reverse transformation is observed.

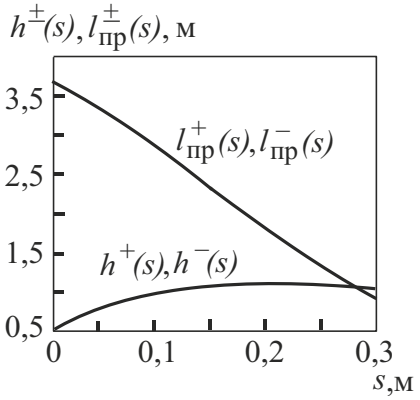


Figure 9. Graphs $h^{\pm}, l^{\pm}_{пр} \sim s$ for $s = 0{,}001m$ and $nstep = 300$.

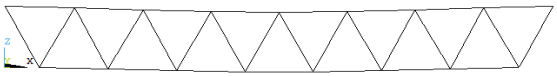


Figure 10. Visualization of the picture of the reverse transformation of the truss for s=0,001 m and nstep=300.

Axial force diagram N in the rods of the top chord and the lattice of the test truss for various values of the parameters s and $nstep$ are shown in Figures 10 and 11. From the above data it can be seen that the values N in the truss rods substantially depend on the calculated parameters s and $nstep$.

For comparison, Figure 12 shows a picture of the transformation of a test truss with a direct forward stroke of actuators by a value. $s = 0{,}3$ м. This solution was obtained as part of a linear calculation.

As you can see the picture of the farm in a deformed state, shown in Figure 12, differs qualitatively from the picture obtained in the step-by-step transformation scheme of Figure 8.

When using the option of accounting for large displacements («Large Displacement Static») in the case of simultaneous calculation (s=0,3m) we obtain a picture of the structure in the transformed state similar to that shown in Figure 8.

The values of the transformation parameters are $h^{+}(0,3) = 1{,}037$ m, $l^{+}_{пр}(0,3)$ = 0,9034 m.

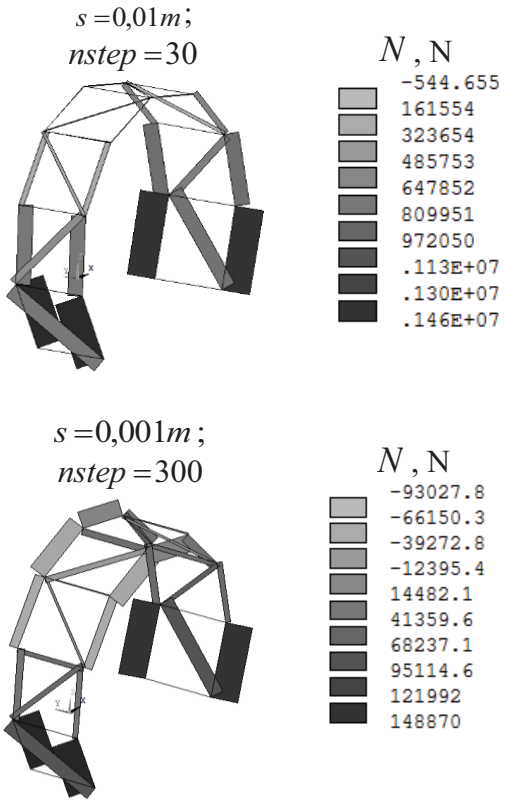


Figure 10. Axial force diagram N in the rods of the truss top chord

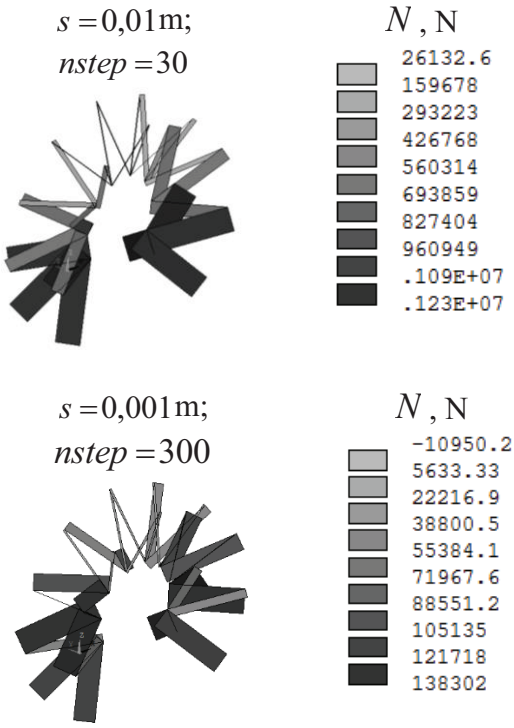


Figure 11. Axial force diagram N in the rods of the truss lattice

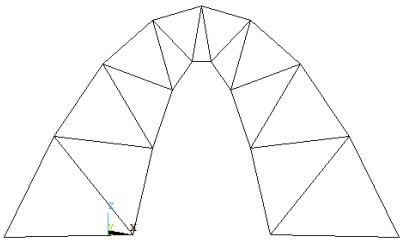


Figure 12. Simultaneous transformation of the truss for $s = 0{,}3$ м.

However, it was found that the calculation in a geometrically nonlinear setting does not allow to take into account the installation history. The obtained values of the longitudinal forces in the truss rods are very underestimated.

3. EXAMPLE.

As a demonstration example, consider an industrially significant truss formed by 24 semi- octahedron (Figure 13).

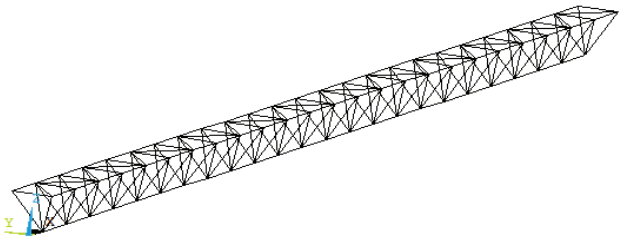


Figure 13. Initial truss position.

Initial data: rods of the top chord and lattice have a tubular cross section with an area of $F = 0{,}2901 \cdot 10^{-2}$ m^2; modulus of elasticity of the material of the rods (aluminum alloy D16T) $E = 7{,}2 \cdot 10^4$ MPa; specific density $\gamma =$ 2885 kg/m^3. he value of the temporary resistance of the material $\sigma_{tol} =$ 420MPa. The overall dimensions of the semi-octahedron are tripled in comparison with the test example (Figure 6). Mounting process is simulated by analogy with a test example, accepting $s =$ 0,001, *nstep* = 200. The initial bay $l_{\text{пр}} = 35$ m.

The results of finite element modeling are presented in Figures 13-16.

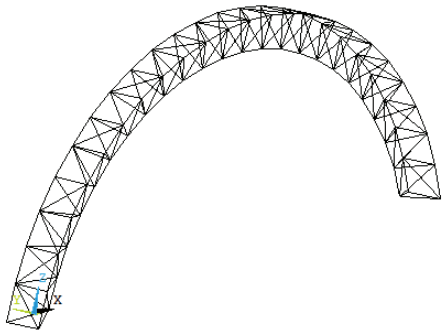


Figure 14. The truss position after the form transformation.

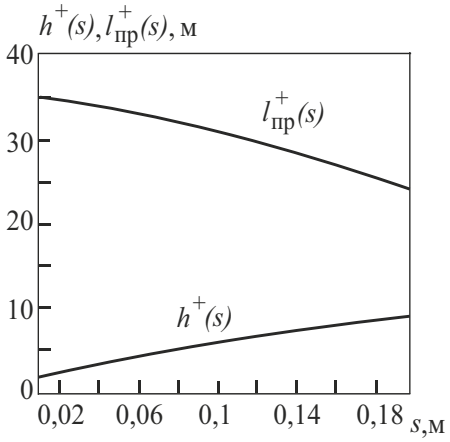


Figure 15. Graphs $h^{\pm}, l^{\pm}_{пр} \sim s$.

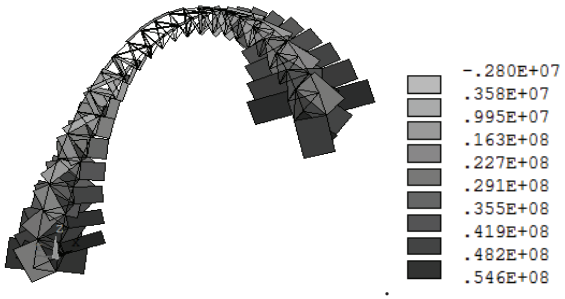


Figure 16. Axial force diagram N.

The resulting height of the farm was $h^+(0,2) =$ 8,43 m, and the total bay $l^+_{пр}(0,2) =$ 25 m (Figures 14, 15).

Figure 16 shows that the maximum value of compressive axial force $N_{max} = -2800$ kN arises in the elements of the «boss» of the arch. This value N_{max} corresponds to the maximum compressive stress $\sigma_{max} = -0{,}965$ MPa, which is significantly less than the value $\sigma_в$.

The critical value of the axial force for a axial compressed rod with a given size and mechanical characteristics is equal. $N_{cr} = 9044$ kN.

Safety factor for a compression rod

$$n_y = N_{cr} / \left| N_{max} \right| = 3{,}23.$$

Thus, the initial stress state of the arched type structure under consideration fully satisfies the requirements of operation.

CONCLUSIONS

1. An engineering method for calculating the stress-strain state of regular hingerod structures with a kinematical oriented shape change has been developed.
2. A numerical study of the convergence of the step procedure modeling the process of controlled shaping of a regular hinged rod structure was carried out.

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