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# ASSESSMENT OF THERMAL AND FORCE EFFECTS ON AN ORTHOTROPIC SHELL WITH POSITIVE GAUSSIAN CURVATURE

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**Abstract:** A mathematical model of thermomechanical deformation is presented for a shell with positive Gaussian curvature, made of an orthotropic composite that develops induced anisotropy during loading. The general formulation of the boundary value problems, as substantiated in a number of studies, is carried out in an uncoupled setting. The occurrence of a temperature gradient is assumed to be one-dimensional, normal to the shell surfaces. Small temperature gradients are assumed, allowing the problem to be solved in a quasi-static manner. To account for the effect of induced heterogeneity—manifested as the dependence of the deformation-strength properties of composites on the nature of the stress state—state equations formulated by one of the authors in the principal material axes of normalized tensor space are used. The developed model is implemented for the thermomechanical analysis of a single-layer shell with positive Gaussian curvature. The main solution parameters are compared with results obtained from similar problems using tested models for the theory of deformation of orthotropic materials with differing resistance proposed by other authors, as well as from the equations of orthotropic linear elasticity theory neglecting differing resistance.

**Keywords:** shell, orthotropy, thermomechanical impact, differing resistance, induced anisotropy, Gaussian curvature

# ОЦЕНКА ТЕМПЕРАТУРНОГО И СИЛОВОГО ВОЗДЕЙСТВИЯ НА ОРТОТРОПНУЮ ОБОЛОЧКУ ПОЛОЖИТЕЛЬНОЙ ГАУССОВОЙ КРИВИЗНЫ

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**Аннотация:** Представлена математическая модель термомеханического деформирования оболочки положительной Гауссовой кривизны, изготовленной на основе ортотропного композита, в котором развивается наведенная анизотропия в процессе ее нагружения. Общая постановка краевых задач, как обосновано в ряде работ, осуществлена в несвязанной постановке. Возникновение температурного перепада принято одномерным по нормали к поверхностям оболочки. При этом приняты малые градиенты распространения температурного воздействия, благодаря чему решение задач осуществлено в квазистатическом варианте. Для учета влияния наводимой неоднородности, проявляющейся как зависимость деформационно-прочностных свойств композитов от вида напряженного состояния, использованы уравнения состояния, сформулированные одним из авторов в главных материальных осях нормированного тензорного пространства. Разработанная модель реализована при термомеханическом расчете однослойной оболочки положительной Гауссовой кривизны. Основные параметры решения сравниваются с результатами аналогичных решений, полученных с использованием наиболее апробированных моделей теории деформирования ортотропных разносопротивляющихся материалов, предложенных другими авторами, а также базирующихся на уравнениях ортотропной линейной теории упругости без учета разносопротивляемости.

**Ключевые слова:** оболочка, ортотропия, термомеханическое воздействие, разносопротивляемость, наведенная анизотропия, гауссова кривизна

## INTRODUCTION

At the current stage of technological development, there is widespread use of advanced reinforced polymer composites strengthened by dispersed or continuous fibers. The key factor that has secured the leading position of these materials is their ability to combine high strength and stiffness characteristics with resistance to aggressive influences while maintaining minimal structural weight.

Analysis carried out during experiments has revealed a number of nonlinear and anisotropic effects. These include: different resistance to tension and compression, nonlinearity of deformation curves, and a pronounced dependence of deformation properties on the dominant type of stress state.

The aforementioned features have led to the development of accurate theoretical models and comprehensive verified testing. Despite significant advancements in solution methods and testing means, the issue remains open and is the subject of active scientific discussions [1-16].

A critical analysis of existing phenomenological theories intended for modeling bimodular orthotropic media has been repeatedly undertaken in scientific literature [17–19].

The identified shortcomings of the considered theories include: discontinuities in constitutive relations or on energy surfaces, insufficient justification of the physical meaning of phenomenological parameters, lack of accounting for the stress state type in mathematical formulations, the need for a priori constraints on material constants, and high dimensionality of the space of experimentally determined coefficients in polynomial approximation [14–16]. A critical factor reducing the practical value of these models is their consistent discrepancy with experimental data under multicomponent loading [8–16].

Thanks to their identified competitive advantages, fiber composite systems have become widespread in the design of thin-walled spatial structures. Of particular interest are coverings with synclastic geometry of the middle surface (positive Gaussian curvature), which operate under cyclic thermal loading regimes. The constructional solutions of such systems often provide for technological openings for organizing the installation of utilities.

In the practice of numerical modeling of thermomechanical processes in structures, both decoupled and coupled formulations are used. Nevertheless, analytical assessments of the degree of mutual influence of thermal and mechanical fields for elements made of materials with differing resistance indicate the short-term nature of transitional coupling processes, which decay after achieving steady-state temperature changes. Therefore, this work proposes a mathematical formulation for the deformation process of a thin-walled shell with positive Gaussian curvature, made of orthotropic composite with induced anisotropy, within a decoupled (separate) framework. The theoretical basis is an energy approach using a potential function adapted for materials with nonlinear property dependence on the stress state. This study demonstrates that the correct identification of all parameters of the energy expansion requires the implementation of a comprehensive test program with simultaneous shearing in three mutually perpendicular planes of anisotropy, which is beyond the capabilities of modern testing equipment. For this reason, as recommended in [22–24], a simplified version of the energy function in the normalized tensor space with quasi-linear approximation is used. Applying the Castigliano variational principle to this function, the constitutive equations are formulated as in [22–24,30]:

$$\{e\} = [C]\{\sigma\} \qquad (1)$$

## PROBLEM STATEMENT

An illustration of the structural solution for the spatial shell system of double curvature is provided in the graphical material (Figure 1b). This schematic presents a set of defining quantities: the dimensional characteristics of the structure and the parameters of the steady-state thermal regime necessary for formulating the boundary value problem.

Initial data:

1) external uniform normal pressure up to $q$ = 0,05 MPa is applied to the shallow shell (Figure 1b);

2) the initial temperature of the shell is assumed to be uniform at $T_0 = 22^\circ C$ throughout its thickness and across all surfaces, the outer surface is then cooled to a temperature $T_1 = -20^\circ C$ and maintained constant; the inner surface of the shell is heated to a temperature $T_2 = 30^\circ C$ ($\theta^\circ$ – the temperature difference at a point on the shell, occurring between the initial and final equilibrium states);

3) the principal curvatures of the shell's middle surface characterize its positive curvature: $k_1 = 1/r_1$, $k_2 = 1/r_2$, $r_1 = 7,25$ m, $r_2 = 26$ m;

4) the plan dimensions of the shell are $2a$ = 10 m, $2b$ = 20 m (Figure 1b), with the rise $f$ = 2 m (Figure 1b);

5) the shell thickness is taken as $h$ = 0,1 m (Figure 1b).

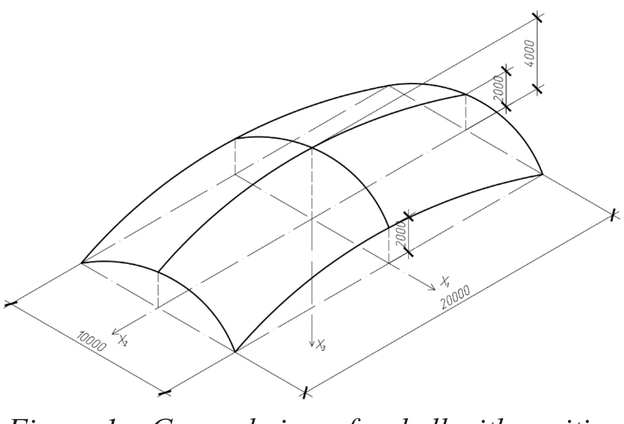


*Figure 1a. General view of a shell with positive Gaussian curvature*

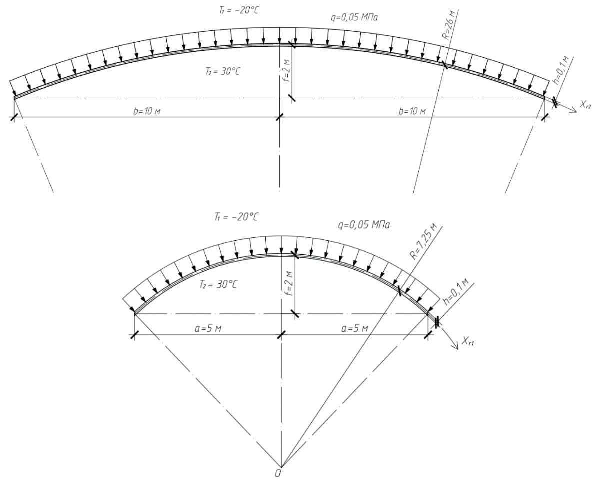


*Figure 1b. Calculation scheme of a shell with positive Gaussian curvature*

The choice of the kinematic model for the investigated problem is determined by the analysis of dimensionless geometric parameters. Despite the existence of alternative approaches—refined theories by S. P. Timoshenko, V. Z. Vlasov, and S. A. Ambartsumyan that account for transverse shear deformations—in this particular case, the thinness criterion (the ratio of thickness to a characteristic radius of curvature) satisfies the applicability conditions of the classical hypotheses of no transverse shear and normal stresses to the surface (Kirchhoff–Lyav model).

For a shell with positive Gaussian curvature, supported on a rectangular plan contour, the kinematic relations between the components of the displacement vector and the strains of the middle surface are written in the following form:

$$
\begin{gathered}
\chi_{11} = -w,_{11}; \ \chi_{22} = -w,_{22}; \\
\chi_{12} = -2w,_{12}; \\
\varepsilon_{11} = u,_1 - k_1 w + 0,5\left(w,_1\right)^2; \\
\varepsilon_{22} = v,_2 - k_2 w + 0,5\left(w,_2\right)^2; \\
\gamma_{12} = u,_2 + v,_1 + w,_1 w,_2;
\end{gathered} \qquad (2)
$$

where $\varepsilon_{11}, \varepsilon_{22}, \gamma_{12}$ – normal strains and shear strains; $u$, $v$, $w$ – displacement components; $k_1$, $k_2$ – curvature parameters of a surface ($k_1 = 1/r_1$, $k_2 = 1/r_2$)

Under the condition of strict alignment of the material's principal orthotropy axes with the axes of the attached Cartesian coordinate system, the constitutive equations (1) [30] can be transformed into a simpler form that accounts for the temperature terms:

$$e_{11} = K_{11}\sigma_{11} + K_{12}\sigma_{22} + \omega_{1T}\theta^{\circ};$$
$$e_{22} = K_{21}\sigma_{11} + K_{22}\sigma_{22} + \omega_{2T}\theta^{\circ}; \quad (3)$$
$$e_{12} = K_{12}\tau_{12},$$

where

$$K_{11} = A_{1111} + B_{1111}\alpha_{11} +$$
$$+0,5[B_{1111}\alpha_{11}(1-\alpha_{11}^{2}) - B_{2222}\alpha_{22}^{3}] +$$
$$+B_{1122}\alpha_{22}(1-\alpha_{11}^{2} - \alpha_{11}\alpha_{22});$$
$$K_{12} = K_{21} = A_{1122} + B_{1122}(\alpha_{11} + \alpha_{22});$$

In the above relations: $A_{iijj}$, $B_{iijj}$, $A_{kkkk}$, $B_{kkkk}$ – celastic constants of the material; $\alpha_{11} = \sigma_{11}/S$, $\alpha_{22} = \sigma_{22}/S$ – relative stresses; $\omega_{1T}$, $\omega_{2T}$ – oefficients of thermal expansion along the anisotropy axes.

The algorithm for determining elastic properties through mechanical testing of specimens manufactured along the principal material axes of the composite system is presented in bibliographic references [22–24].

In the considered studies, in addition to conducting experiments, approximating functions for processing experimental data were obtained. Based on these, dependencies relating the engineering constants $A_{kkkk}$, $B_{kkkk}$, $A_{iijj}$, $B_{iijj}$ to the technical parameters of the material were derived. For use within this two-dimensional boundary value problem, the specified dependencies were adapted and presented in the following form $i, j, k = 1,2$:

$$A_{kkkk} = (1/E_k^{+} + 1/E_k^{-})/2;$$
$$A_{iijj} = -(\nu_{ij}^{+}/E_j^{+} + \nu_{ij}^{-}/E_j^{-})/2;$$
$$\nu_{ij}^{+}/E_j^{+} = \nu_{ji}^{+}/E_i^{+};$$

$$B_{kkkk} = (1/E_k^{+} - 1/E_k^{-})/2;$$
$$B_{iijj} = -(\nu_{ij}^{+}/E_k^{+} - \nu_{ij}^{-}/E_j^{-})/2; \ \nu_{ij}^{-}/E_j^{-} = \nu_{ji}^{-}/E_i^{-}.$$

Furthermore, the bibliographic sources [22–24] provide reference data on the mechanical constants of a wide range of orthotropic materials that exhibit different resistance to tension and compression.

For the case of thin-walled curved structures, the derivation of the resolving system of differential equations, formulated in terms of displacements, is based on the complete set of fundamental equations of the mechanics of deformable solids. This procedure requires inverting the physical equations (3) to express the components of the stress tensor through the corresponding deformation characteristics.

$$\sigma_{11} = E_{11}e_{11} + E_{12}e_{22} - \varphi_{1T};$$
$$\sigma_{22} = E_{12}e_{11} + E_{22}e_{22} - \varphi_{2T}; \quad (4)$$
$$\tau_{12} = E_{66}e_{12};$$

where $E_{11} = K_{22}/\Psi$; $E_{22} = K_{11}/\Psi$;

$$E_{12} = -K_{12}/\Psi; E_{66} = 1/K_{12}; \Psi = K_{11}K_{22} - K_{12}^{2};$$
$$\varphi_{1T} = E_{11}\omega_{1T}\theta^{\circ} + E_{12}\omega_{2T}\theta^{\circ};$$
$$\varphi_{2T} = E_{12}\omega_{1T}\theta^{\circ} + E_{22}\omega_{2T}\theta^{\circ}.$$

The formulation of equilibrium conditions for shell systems is implemented through integral force variables of membrane and bending types. In the axisymmetric case, the model includes $N_{11}$, $N_{22}$, $M_{11}, M_{22}$ and $Q_{11}$. The numerical values of the first two forces and moments are determined by integral averaging of stresses along the normal to the middle surface (coordinate $x_3$):

$$N_{11} = \int_{-h/2}^{h/2} \sigma_{11}dx_3; \quad N_{22} = \int_{-h/2}^{h/2} \sigma_{22}dx_3;$$
$$N_{12} = \int_{-h/2}^{h/2} \sigma_{12}dx_3; \quad M_{11} = \int_{-h/2}^{h/2} \sigma_{11}x_3dx_3; \quad (5)$$

$$M_{22} = \int_{-h/2}^{h/2} \sigma_{22} x_3 d x_3 \,.$$

The transverse force $Q_{11}$ is defined by differentiating the moment $M_{11}$ with respect to the coordinate $X_1$:

$$Q_{11} = M_{11,1} + N_{11} w,_1 \,. \tag{6}$$

By jointly using expressions (1)–(4), a functional relationship is established between the given forces/moments and the deformational-geometric parameters (components of the strain tensor and changes in the principal curvatures) of the middle surface of the shell:

$$\begin{gathered} N_{11} = L_{11}\varepsilon_{11} + J_{11}\chi_{11} + L_{12}\varepsilon_{22} + J_{12}\chi_{22} - \eta_{1T} \,; \\ N_{22} = L_{12}\varepsilon_{11} + J_{21}\chi_{11} + L_{22}\varepsilon_{22} + J_{22}\chi_{22} - \eta_{2T} \,; \\ N_{12} = L_{66}\varepsilon_{12} + J_{66}\chi_{12} \,; \\ M_{11} = J_{11}\varepsilon_{11} + R_{11}\chi_{11} + J_{12}\varepsilon_{22} + R_{12}\chi_{22} - \gamma_{1T} \,; \\ M_{22} = J_{12}\varepsilon_{11} + R_{21}\chi_{11} + J_{22}\varepsilon_{22} + R_{22}\chi_{22} - \gamma_{2T} \,; \\ M_{12} = J_{66}\varepsilon_{12} + R_{66}\chi_{12} \,; \end{gathered} \tag{7}$$

where $L_{ij} = \int_{-h/2}^{h/2} E_{ij} d x_3 \,;\; J_{ij} = \int_{-h/2}^{h/2} E_{ij} z \, d x_3 \,;$

$$R_{ij} = \int_{-h/2}^{h/2} E_{ij} z^2 d x_3 \,;\; \eta_{iT} = \int_{-h/2}^{h/2} \varphi_{iT} d x_3 \,;$$

$$\gamma_{iT} = \int_{-h/2}^{h/2} \varphi_{iT} z \, d x_3 \,;\; \left(i, j = 1, 2\right).$$

Static equilibrium equations are invariant regarding the type of constitutive relations. In the context of the considered shell with positive curvature, for small parameter $zk \ll 1$ [25], they are expressed as follows:

$$\begin{cases} N_{11},_1 + N_{12},_2 = 0; \\ N_{12},_1 + N_{22},_2 = 0; \\ M_{11},_{11} + M_{22},_{22} + 2M_{12},_{12} - N_{11}(w,_{11} + k_1) - \\ -N_{22}(w,_{22} + k_2) - 2N_{12} w,_{12} = q_3; \end{cases} \tag{8}$$

The nonlinear nature of the problem, identified from the system of equations (1)–(8), manifests itself in three aspects: material properties, kinematics, and the nonlinearity of the computational model. Therefore, the governing system of differential equations is constructed and solved using the two-step method of successive parameter perturbations, developed by V. V. Petrov [26]. The first step of this method involves approximating the nonlinear dependencies with linear relations based on the incremental loading technique.

## LINEARIZATION OF THE SYSTEM OF DIFFERENTIAL EQUATIONS

As a result of the linearization of dependencies (2), equations are obtained that relate the increments of the arguments and functions:

$$\begin{gathered} \Delta\chi_{11} = -\Delta w,_{11} \,; \\ \Delta\chi_{22} = -\Delta w,_{22} \,; \\ \Delta\chi_{12} = -2\Delta w,_{12} \,; \\ \Delta\varepsilon_{11} = \Delta u,_1 - k_1 \Delta w + w,_1 \Delta w,_1 \,; \\ \Delta\varepsilon_{22} = \Delta v,_1 - k_2 \Delta w + w,_2 \Delta w,_2 \,; \\ \Delta\varepsilon_{12} = \Delta v,_1 + \Delta u,_2 + w,_1 \Delta w,_2 + w,_2 \Delta w,_1 \,; \\ \Delta e_{11} = \Delta\varepsilon_{11} + x_3 \Delta\chi_{11} \,; \\ \Delta e_{22} = \Delta\varepsilon_{22} + x_3 \Delta\chi_{22} \,; \\ \Delta e_{12} = \Delta\gamma_{12} + x_3 \Delta\chi_{12} \,; \end{gathered} \tag{9}$$

where $\Delta$ – is the increment of the arguments. Following the approach of [27], we will linearize the constitutive equations (1) by expanding them into a Taylor series and retaining first-order smallness terms:

$$\begin{gathered} \Delta e_{11} = \frac{\partial e_{11}}{\partial \sigma_{11}} \Delta\sigma_{11} + \frac{\partial e_{11}}{\partial \sigma_{22}} \Delta\sigma_{22} + \omega_{1T} \frac{\partial e_{11}}{\partial \theta^\circ} \Delta\theta^\circ \,; \\ \Delta e_{22} = \frac{\partial e_{22}}{\partial \sigma_{11}} \Delta\sigma_{11} + \frac{\partial e_{22}}{\partial \sigma_{22}} \Delta\sigma_{22} + \omega_{2T} \frac{\partial e_{22}}{\partial \theta^\circ} \Delta\theta^\circ \,; \\ \Delta e_{12} = \frac{\partial e_{12}}{\partial \tau_{12}} \Delta\tau_{12} \end{gathered} \tag{10}$$

By inverting equations (10) with respect to the increments of stresses, we obtain:

$$\Delta\sigma_{11} = D_{11}\Delta e_{11} + D_{12}\Delta e_{22} - \Delta\varphi_{1T};$$
$$\Delta\sigma_{22} = D_{21}\Delta e_{11} + D_{22}\Delta e_{22} - \Delta\varphi_{2T}; \quad (11)$$
$$\Delta\tau_{12} = D_{66}\Delta e_{12};$$

где $D_{11} = \Omega_{22}/\Omega;$

$$D_{12} = D_{21} = -\Omega_{21}/\Omega = -\Omega_{12}/\Omega;$$
$$D_{22} = \Omega_{11}/\Omega; \quad D_{66} = \Omega_{66}/\Omega;$$
$$\Omega_{11} = \frac{\partial e_{11}}{\partial\sigma_{11}}; \; \Omega_{12} = \Omega_{21} = \frac{\partial e_{11}}{\partial\sigma_{22}} = \frac{\partial e_{22}}{\partial\sigma_{11}};$$
$$\Omega_{22} = \frac{\partial e_{22}}{\partial\sigma_{22}}; \; \Omega_{66} = \frac{\partial e_{66}}{\partial\sigma_{66}}; \; \Omega_{1T} = \frac{\partial e_{11}}{\partial\theta^{\circ}};$$
$$\Omega_{2T} = \frac{\partial e_{22}}{\partial\theta^{\circ}};$$
$$\Omega = \Omega_{11}\Omega_{22} - \Omega_{12}\Omega_{21};$$
$$\Delta\varphi_{1T} = (\Omega_{12}\Omega_{2T} - \Omega_{1T}\Omega_{22})/\Omega;$$
$$\Delta\varphi_{2T} = (\Omega_{21}\Omega_{1T} - \Omega_{2T}\Omega_{11})/\Omega.$$

The transition from stress increments (11) to increments of forces and moments is carried out according to the same rules as (5):

$$\Delta N_{11} = \int_{-h/2}^{h/2} \Delta\sigma_{11} dx_3;$$
$$\Delta N_{22} = \int_{-h/2}^{h/2} \Delta\sigma_{22} dx_3; \quad (12)$$
$$\Delta M_{11} = \int_{-h/2}^{h/2} \Delta\sigma_{11} z dx_3;$$
$$\Delta M_{22} = \int_{-h/2}^{h/2} \Delta\sigma_{22} z dx_3;$$

where $\Delta N_{11}$, $\Delta N_{22}$, $\Delta M_{11}$, $\Delta M_{22}$ – the increments of the corresponding forces and moments. By applying expressions (12) to the increment equations (11), we obtain:

$$\Delta N_{11} = L_{11}{}^{\Delta}\Delta\varepsilon_{11} + J_{11}{}^{\Delta}\Delta\chi_{11} + L_{12}{}^{\Delta}\Delta\varepsilon_{22} +$$
$$+J_{12}{}^{\Delta}\Delta\chi_{22} - \Delta\eta_{1T};$$
$$\Delta N_{22} = L_{12}{}^{\Delta}\Delta\varepsilon_{11} + J_{21}{}^{\Delta}\Delta\chi_{11} + L_{22}{}^{\Delta}\Delta\varepsilon_{22} +$$
$$+J_{22}{}^{\Delta}\Delta\chi_{22} - \Delta\eta_{2T};$$
$$\Delta N_{12} = L_{66}{}^{\Delta}\Delta\varepsilon_{12} + J_{66}{}^{\Delta}\Delta\chi_{12}; \quad (13)$$
$$\Delta M_{11} = J_{11}{}^{\Delta}\Delta\varepsilon_{11} + R_{11}{}^{\Delta}\Delta\chi_{11} + J_{12}{}^{\Delta}\Delta\varepsilon_{22} +$$
$$+R_{12}{}^{\Delta}\Delta\chi_{22} - \Delta\gamma_{1T};$$
$$\Delta M_{22} = J_{12}{}^{\Delta}\Delta\varepsilon_{11} + R_{21}{}^{\Delta}\Delta\chi_{11} + J_{22}{}^{\Delta}\Delta\varepsilon_{22} +$$
$$+R_{22}{}^{\Delta}\Delta\chi_{22} - \Delta\gamma_{2T};$$
$$\Delta M_{12} = J_{66}{}^{\Delta}\Delta\varepsilon_{12} + R_{66}{}^{\Delta}\Delta\chi_{12};$$

where $L_{ij}^{\Delta} = \int_{-h/2}^{h/2} D_{ij} dx_3; \; J_{ij}^{\Delta} = \int_{-h/2}^{h/2} D_{ij} z\, dx_3;$

$$R_{ij}^{\Delta} = \int_{-h/2}^{h/2} D_{ij} z^2 dx_3; \; \Delta\eta_{iT} = \int_{-h/2}^{h/2} \Delta\varphi_{iT} dx_3;$$
$$\Delta\gamma_{iT} = \int_{-h/2}^{h/2} \Delta\varphi_{iT} z\, dx_3; \; (i, j = 1, 2).$$

The statics equations (8) after transformation through increments are written in the form:

$$\begin{cases} \Delta N_{11},_1 + \Delta N_{12},_2 = 0; \\ \Delta N_{12},_1 + \Delta N_{22},_2 = 0; \\ -N_{11}\Delta w,_{11} - \Delta N_{11} w,_{11} - \Delta N_{11} k_1 - \\ -N_{22}\Delta w,_{22} - \Delta N_{22} w,_{22} - \Delta N_{22} k_2 - \\ -2N_{12}\Delta w,_{12} - 2\Delta N_{12} w,_{12} + \\ +\Delta M_{11},_{11} + \Delta M_{22},_{22} + 2\Delta M_{12},_{12} = \Delta q_3; \end{cases} \quad (14)$$

where $\Delta q_3$ – increment of transverse load intensity.

The linearized equations in increments of displacements of the middle surface of the Gaussian curvature shell were obtained by reducing expressions (9), (13), and (14) to a common form:

$$L_{11}^{\Delta},_1 [\Delta u,_1 - k_1\Delta w + w,_1 \Delta w] + L_{11}^{\Delta}[\Delta u,_{11} - k_1\Delta w,_1 +$$
$$+w,_{11} \Delta w,_1 + w\Delta w,_{11}] - J_{11}^{\Delta},_1 \Delta w,_{11} - J_{11}^{\Delta}\Delta w,_{111} +$$
$$+L_{12}^{\Delta}[-\Delta w k_2 + w,_2 \Delta w + \Delta v,_2] + L_{12}^{\Delta}[-\Delta w,_1 k_2 +$$
$$+w,_{12} \Delta w,_2 + w,_2 \Delta w,_{12}] + \Delta v,_{12} - J_{12}^{\Delta},_1 \Delta w,_{22} -$$

$$
\begin{aligned}
&-J^{\Delta}_{12}\Delta w_{,122}-\Delta\eta_{1T},_{1}+L^{\Delta}_{66},_{2}[w_{,1}\Delta w_{,12}+\Delta w_{,1}w_{,2}+\\
&+\Delta u_{,2}+\Delta v_{,1}]+L^{\Delta}_{66}[w_{,12}\Delta w_{,2}+w_{,1}\Delta w_{,12}+\\
&+w_{,12}\Delta w_{,12}+\Delta w_{,1}w_{,22}+\Delta u_{,22}+\Delta u_{,12}]-\\
&-2L^{\Delta}_{66}[\Delta w_{,122}]=0
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
&L^{\Delta}_{66},_{1}[w_{,1}\Delta w_{,2}+\Delta w_{,1}w_{,2}+\Delta u_{,2}+\Delta v_{,1}]+\\
&+L^{\Delta}_{66}[w_{,11}\Delta w_{,2}+w_{,1}\Delta w_{,2}+w_{,11}\Delta w_{,2}+\Delta w_{,1}w_{,12}+\\
&+\Delta u_{,122}+\Delta v_{,11}]-2J^{\Delta}_{66}\Delta w_{,112}+L^{\Delta}_{12},_{2}[\Delta u_{,1}-k_1\Delta w+\\
&+w_{,1}\Delta w]+L^{\Delta}_{12}[\Delta u_{,12}-k_1\Delta w_{,2}+\Delta w_{,1}w_{,12}+\\
&+w_{,1}\Delta w_{,12}]-J^{\Delta}_{12}\Delta w_{,112}+L^{\Delta}_{22},_{2}[-wk_2+w_{,2}\Delta w_{,2}+\\
&+\Delta v_{,2}]+L^{\Delta}_{22},_{2}[-\Delta w_{,2}k_2+\Delta w_{,22}+w_{,2}\Delta w_{,22}+\\
&+\Delta v_{,22}]-J^{\Delta}_{22}\Delta w_{,22}-J^{\Delta}_{22}\Delta w_{,222}-\Delta\eta_{2T},_{2}=0
\end{aligned}
$$

$$
\begin{aligned}
&-\Delta\gamma_{1T},_{11}-\Delta\gamma_{2T},_{22}-R^{\Delta}_{11}\Delta w_{,1111}+J^{\Delta}_{12},_{11}[-wk_2+\\
&+w_{,2}\Delta w_{,2}+\Delta v_{,2}]++2J^{\Delta}_{12},_{1}[-\Delta wk_2+w_{,12}\Delta w_{,12}+\\
&+w_{,2}\Delta w_{,12}+\Delta v_{,12}]+J^{\Delta}_{12}[-\Delta w_{,11}k_2+w_{,112}\Delta w_{,2}+\\
&+2w_{,12}\Delta w_{,12}+w_{,2}\Delta w_{,112}+\Delta v_{,112}]-\\
&-2\{L^{\Delta}_{66},_{1}[w_{,1}\Delta w_{,2}+\Delta w_{,1}w_{,2}+\Delta u_{,2}+\Delta v_{,1}]-\\
&-2J^{\Delta}_{66}\Delta w_{,12}\}w_{,12}-\{L_{11}[u_{,1}-k_1w+0,5(w_{,1})^2]-\\
&-J_{11}w_{,11}+L_{12}[v_{,2}-k_2w+0,5(w_{,2})^2]-\\
&-J_{12}w_{,22}-\eta_{1T}\}\Delta w_{,11}-\{L^{\Delta}_{11}[\Delta u_{,1}-k_1\Delta w+\\
&+w_{,1}\Delta w_{,1}]-J^{\Delta}_{11}\Delta w_{,11}+L^{\Delta}_{12}[-\Delta wk_2+w_{,2}\Delta w_{,12}+\\
&+\Delta v_{,2}]-J^{\Delta}_{12}\Delta w_{,22}-\eta_{1T}\}w_{,11}-\{L_{11}[\Delta u_{,1}-k_1\Delta w+\\
&+w_{,1}\Delta w_{,1}]-J^{\Delta}_{11}\Delta w_{,11}+L^{\Delta}_{12}[-\Delta wk_2+w_{,2}\Delta w_{,12}+\\
&+\Delta v_{,2}]-J^{\Delta}_{12}\Delta w_{,22}-\eta_{1T}\}k_1-\{L_{12}[u_{,2}-k_2w+\\
&+0,5(w_{,1})^2]-J_{12}w_{,11}+L_{22}[v_{,2}-k_2w+0,5(w_{,2})^2]-\\
&-J_{22}w_{,22}-\eta_{2T}\}\Delta w_{,22}-\{L^{\Delta}_{12}[\Delta u_{,1}-k_1\Delta w+\\
&+w_{,1}\Delta w_{,1}]-J^{\Delta}_{12}\Delta w_{,11}+L^{\Delta}_{22}[-\Delta wk_2+w_{,2}\Delta w_{,2}+\\
&+\Delta v_{,2}]-J^{\Delta}_{22}\Delta w_{,22}-\Delta\eta_{2T}\}w_{,22}-\{L^{\Delta}_{12}[\Delta u_{,1}-\\
&-k_1\Delta w+w_{,1}\Delta w_{,1}]-J^{\Delta}_{12}\Delta w_{,11}+L^{\Delta}_{22}[-\Delta w,k_2+\\
&+w_{,2}\Delta w_{,2}+\Delta v_{,2}]-J^{\Delta}_{22}\Delta w_{,22}-\eta_{2T}\}k_2-\\
&-2\{L_{66}[u_{,2}-v_{,1}+w_{,1}\Delta w_{,2}]-2J_{66}w_{,12}\}\Delta w_{,12}-\\
&-R^{\Delta}_{11},_{11}\Delta w_{,11}-2R^{\Delta}_{11}\Delta w_{,111}+J^{\Delta}_{11},_{11}[\Delta u_{,1}-\\
&-k_1\Delta w+w_{,1}\Delta w_{,1}]+2J^{\Delta}_{11}[\Delta u_{,11}-k_1\Delta w_{,1}+\\
&+w_{,11}\Delta w_{,1}+w_{,1}\Delta w_{,11}]+J^{\Delta}_{11}[\Delta u_{,111}-k_1\Delta w_{,11}+\\
&+w_{,111}\Delta w_{,1}+2w_{,11}\Delta w_{,11}+w_{,1}\Delta w_{,111}]+\\
&+J^{\Delta}_{12},_{22}[\Delta u_{,1}-k_1\Delta w+w_{,1}\Delta w_{,1}]+\\
&+2J^{\Delta}_{12},_{2}[\Delta u_{,12}-k_1\Delta w_{,2}+w_{,1}\Delta w_{,12}+w_{,1}\Delta w_{,12}]+\\
&+J^{\Delta}_{12}[\Delta u_{,122}-k_1\Delta w_{,22}+2w_{,12}\Delta w_{,12}+\\
&+\Delta w_{,1}w_{,122}+w_{,1}\Delta w_{,122}]-R^{\Delta}_{12}\Delta w_{,1122}+\\
&+J^{\Delta}_{22},_{22}[-\Delta wk_2+w_{,2}\Delta w_{,2}+\Delta v_{,2}]+\\
&+2J^{\Delta}_{22}[-\Delta w_{,2}k_2+w_{,22}\Delta w_{,2}+w_{,2}\Delta w_{,22}+\\
&+\Delta v_{,22}]+J^{\Delta}_{22}[-\Delta w_{,22}k_2+w_{,222}\Delta w_{,2}+\\
&+2w_{,22}\Delta w_{,22}+w_{,2}\Delta w_{,222}+\Delta v_{,222}]+\\
&+2J^{\Delta}_{66}[w_{,112}\Delta w_{,2}+w_{,11}\Delta w_{,22}+2w_{,12}\Delta w_{,12}+\\
&+w_{,1}\Delta w_{,122}+w_{,12}\Delta w_{,122}+\Delta w_{,11}w_{,22}+\\
&+\Delta w_{,1}w_{,112}+\Delta u_{,122}\Delta v_{,112}]-4R^{\Delta}_{66}\Delta w_{,1122}-\\
&-R^{\Delta}_{12},_{11}\Delta w_{,22}-2R^{\Delta}_{12}\Delta w_{,122}-R^{\Delta}_{12}\Delta w_{,1122}+\\
&+2J^{\Delta}_{66},_{2}[w_{,11}\Delta w_{,2}+w_{,1}\Delta w_{,12}+\Delta w_{,11}w_{,2}+\\
&+\Delta w_{,1}w_{,12}+\Delta u_{,12}+\Delta v_{,11}]+2J^{\Delta}_{66},_{1}[w_{,12}\Delta w_{,2}+\\
&+w_{,1}\Delta w_{,22}+\Delta w_{,2}w_{,12}+\Delta w_{,1}w_{,22}+\\
&+\Delta u_{,22}+\Delta v_{,12}]-R^{\Delta}_{22},_{22}\Delta w_{,22}-2R^{\Delta}_{22},_{2}\Delta w_{,222}-\\
&-R^{\Delta}_{22}\Delta w_{,2222}+2J^{\Delta}_{66},_{12}[w_{,1}\Delta w_{,2}+\\
&+w_{,1}\Delta w_{,2}+\Delta u_{,2}+\Delta v_{,1}]=\Delta q
\end{aligned}
$$

To ensure a closed formulation of the problem, boundary conditions must be added to the differential equations (15), also expressed in terms of increments of displacements.

The rigid clamping of the support contour is specified by the following relations:

$$\Delta u=0;\quad \Delta w=0;\quad \Delta w,_{r}=0. \tag{16}$$

## THE TEMPERATURE COMPONENT OF THE PROBLEM

The differential heat conduction equation [28] adequately describes the heat propagation process in structurally orthotropic solids:

$$T,_{t}=\Lambda_1\cdot T,_{11}+\Lambda_2\cdot T,_{22}+\Lambda_3\cdot T,_{33}, \tag{17}$$

where $\Lambda_1$, $\Lambda_2$, $\Lambda_3$ – thermophysical material constants along the principal axes, defining the thermal inertia of the material;; $t$ – time; $T$ – temperature.

Within the framework of the given problem, the temperature distribution is formed exclusively in the cross-section of the shell, and the heat transfer process is one-dimensional along the $x_3$-axis. In this regard, the differential heat conduction equation (17) admits significant simplification and can be presented in the following form:

$$T_{,t} = \Lambda_3 \cdot T_{,33}\,, \qquad (18)$$

where $\Lambda_3 = \lambda / c$ – thermal diffusivity coefficient in the direction normal to the shell surface $\lambda$ – eat transfer coefficient; $c$ – specific heat capacity per unit volume.

The subject of numerical investigation were shell systems made of fiberglass materials and composites with similar properties, possessing significant heat transfer coefficients. Due to high thermal conductivity, in thin-walled structures, the formation of a steady temperature gradient across the thickness occurs rapidly, and the spatial temperature distribution is characterized by linearity. This fact allows the related thermoelastic problem to be considered after reaching a steady-state regime with a constant temperature gradient along the coordinate $x_3$. As a result, the temperature across the shell section is determined by the expression (Figure 1b):

$$T(x_3) = (T_2 - T_1)\,x_3/h + (T_1 + T_2)/2 - T_0\,. \qquad (19)$$

## SOLUTION OF THE PROBLEM AND ANALYSIS OF THE RESULTS

As a representative orthotropic material, fiberglass composite was investigated. The mechanical property values used in this study, including the tensile elastic moduli along the principal anisotropy axes, are consistent with data from references [13, 14]: $E_1^+$ =140 GPa, $E_2^+$ =280 GPa, at axial compressions: $E_1^-$ =70 GPa, and $E_2^-$ =140 GPa; corresponding coefficients of transverse deformation: $\nu_{12}^+ = 0,2$; $\nu_{12}^- = 0,3$; coefficients of linear thermal expansion in the directions of the principal material axes: $\omega_{1T} = 3,3 \cdot 10^{-6} (°C)^{-1}$; $\omega_{2T} = 4,0 \cdot 10^{-6} (°C)^{-1}$ [13, 14].

Integration of the linearized system of differential operators (15) satisfying boundary conditions (16) and considering the temperature profile through the thickness of the structure (19) was implemented based on a high-order accuracy finite difference scheme, including central approximations at internal nodes and one-sided formulas in boundary regions. Spatial discretization was performed by placing 200 nodal points along the radial coordinate with simultaneous division of the thickness into 21 layers to implement Simpson's quadrature formula. The resulting algebraic system of linear equations was solved by the Gauss-Seidel iterative method with the application of Lüsternik relaxation acceleration. Numerical integration along the loading path was performed using Adams extrapolation scheme. The developed computational algorithm with branched logic was implemented as part of specialized software built on the open environment "SCILAB".

Verification of the developed approach was carried out by comparing the calculated data with results based on alternative constitutive models from publications [8–13]. A generalized analysis was conducted in the following directions:

a) with results obtained using advanced constitutive relations for orthotropic materials (Model 1);

b) calculation employing averaged mechanical characteristics while neglecting induced deformation anisotropy properties (Model 2);

c) results from the developed model with thermal effects excluded (Model 3);

d) data obtained following the theory of C. W. Bert – J. N. Reddy [8, 9] (Model 4);

e) solutions based on the model by R. M. Jones – D. A. R. Nelson [10, 13] (Model 5).

f) with solutions of S. A. Ambartsumyan (Model 6).

Figures 2-5 and 8-11 show the calculated results of the normal stresses $\sigma_{11}$ and $\sigma_{22}$; Figures 14-18 show the displacements and deflections of the middle surface of the considered shell, obtained using various state equations for structurally orthotropic materials exhibiting deformational inhomogeneity (variable resistance) during loading. Figures 6, 7, 12, and 13 present characteristic stress distributions through the thickness of the shell in a specified cross-section.

The results of calculating the stress-strain state of a shell with positive Gaussian curvature under combined transverse loading and temperature gradient revealed their significant dependence on the choice of constitutive relations for orthotropic media that exhibit different behavior under various stress state types.

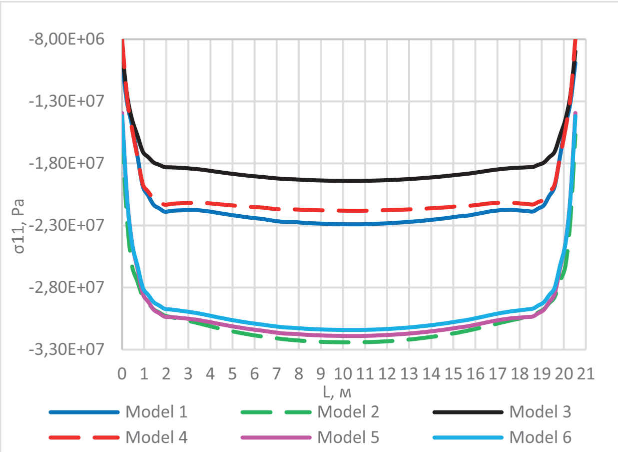


*Figure 2.* *Normal stresses* $\sigma_{11}$ *at the top along the long side along the Y axis, Pa*

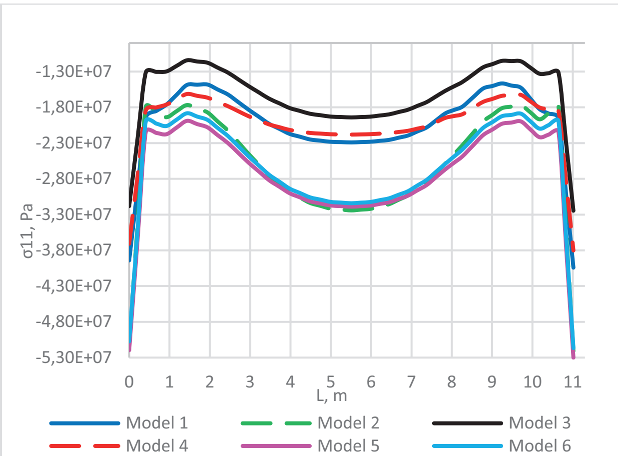


*Figure 3.* *Normal stresses* $\sigma_{11}$ *at the top along the short side along the X axis, Pa*

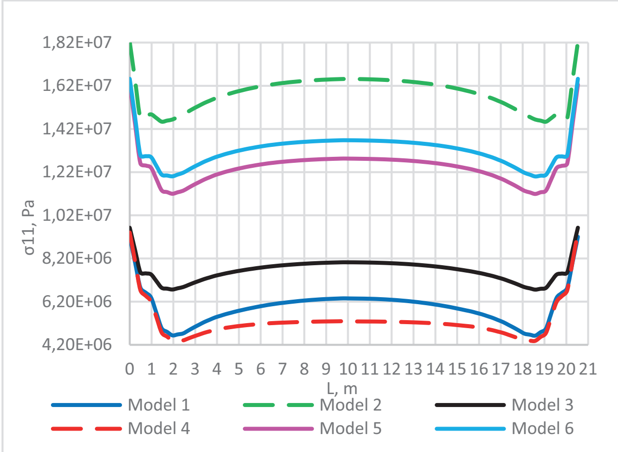


*Figure 4.* *Normal stresses* $\sigma_{11}$ *at the bottom along the long side along the Y axis, Pa*

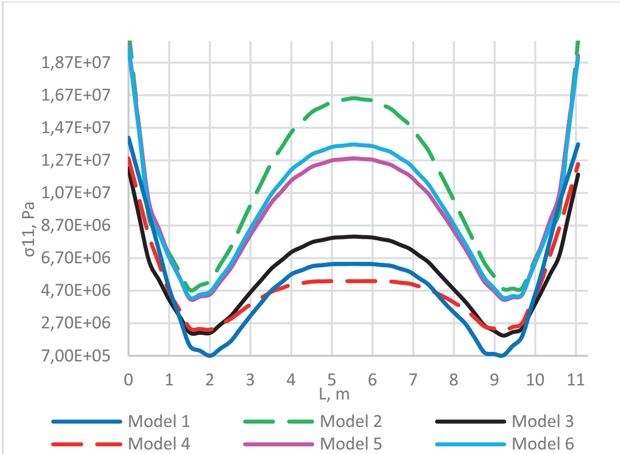


*Figure 5.* *Normal stresses* $\sigma_{11}$ *at the bottom along the short side along the X axis, Pa*

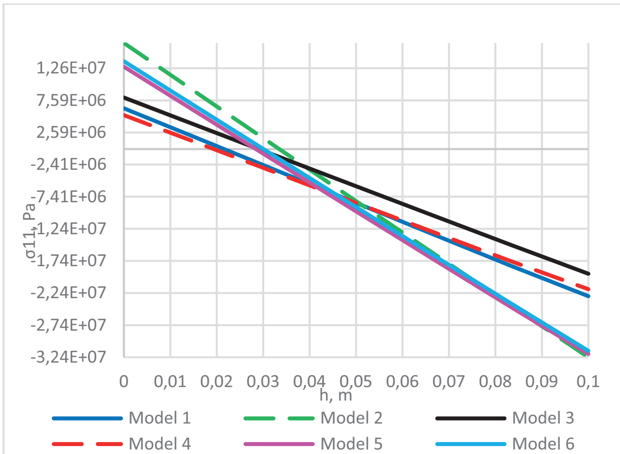


*Figure 6.* *Normal stresses* $\sigma_{11}$ *at the center through the thickness, Pa*

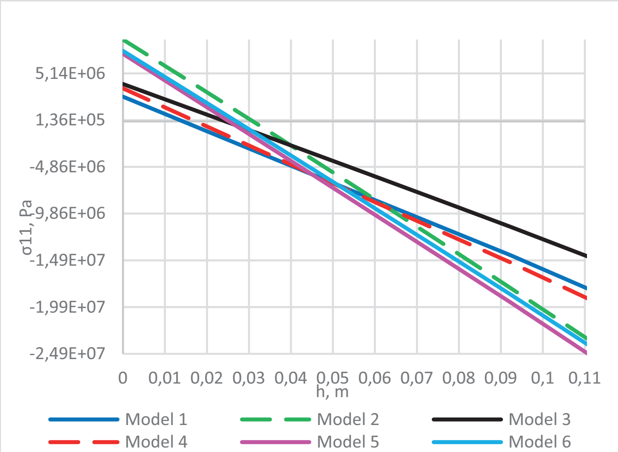


*Figure 7. Normal stresses $\sigma_{11}$ at ¼ length to the left along the X axis through the thickness, Pa*

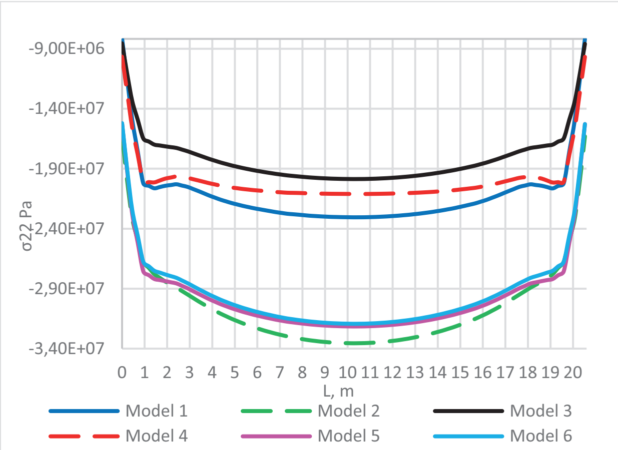


*Figure 8. Normal stresses $\sigma_{22}$ at the top along the long side along the Y axis, Pa*

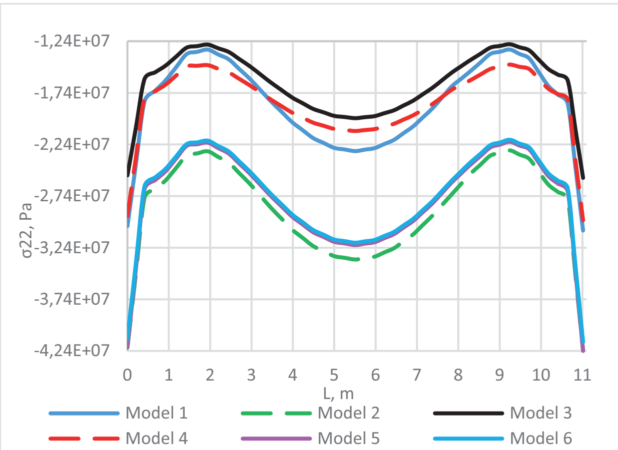


*Figure 9. Normal stresses $\sigma_{22}$ at the top along the short side along the X axis, Pa*

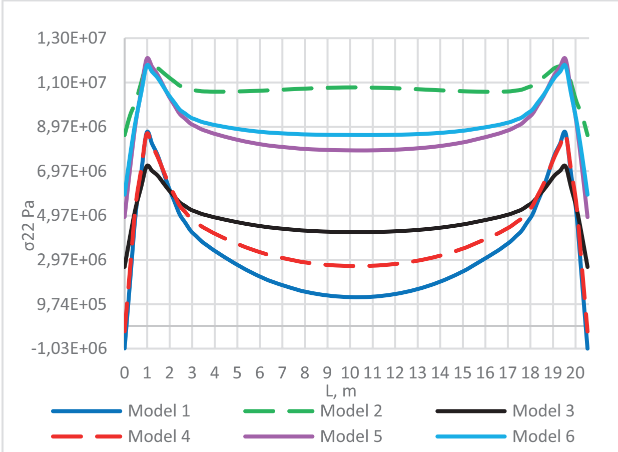


*Figure 10. Normal stresses $\sigma_{22}$ at the bottom along the long side along the Y axis, Pa*

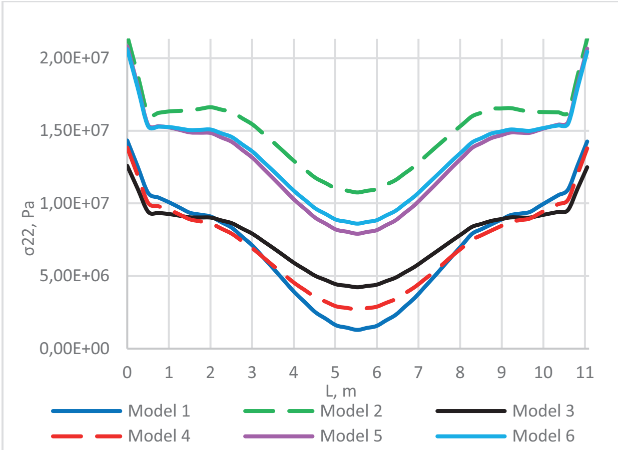


*Figure 11. Normal stresses $\sigma_{22}$ at the bottom along the short side along the X axis, Pa*

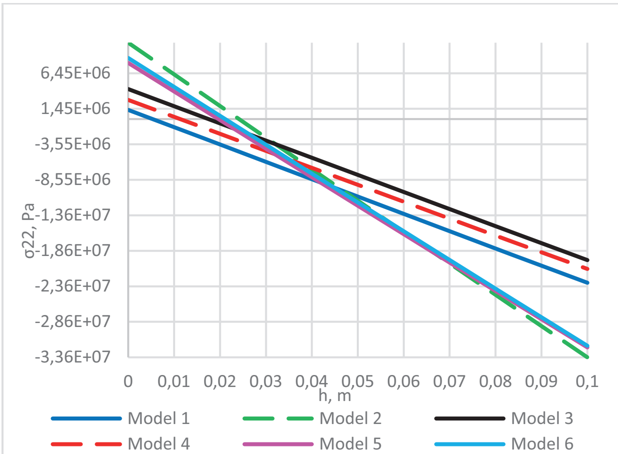


*Figure 12. Normal stresses $\sigma_{22}$ at the center through the thickness, Pa*

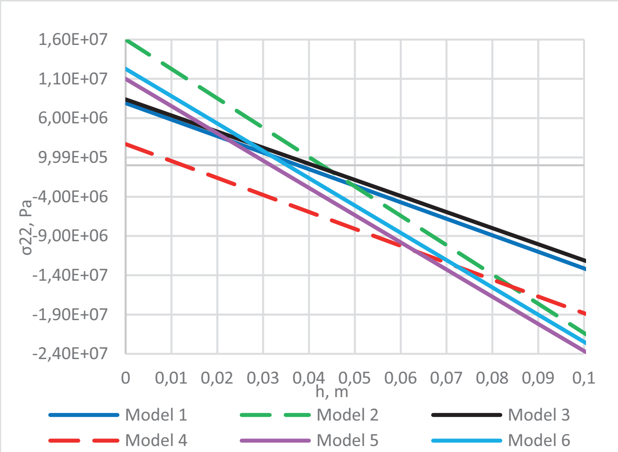


*Figure 13. Normal stresses $\sigma_{22}$ at ¼ to the left along the X axis through the thickness, Pa*

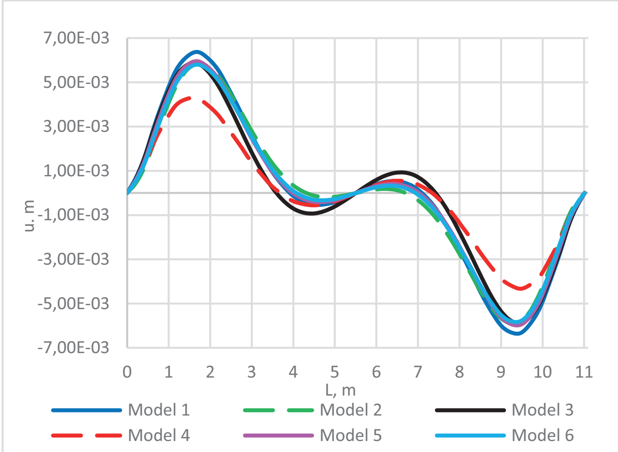


*Figure 16. Displacements U along the long side, m*

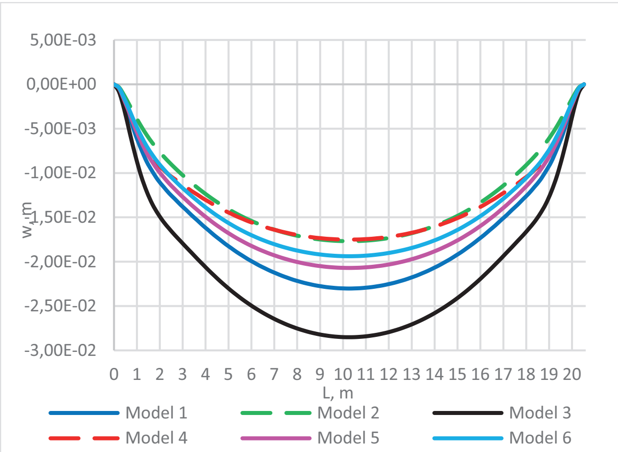


*Figure 14. Displacements W along the long side, m*

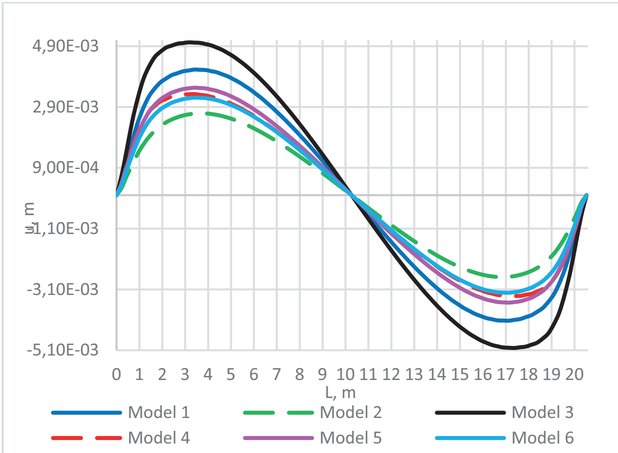


*Figure 17. Displacements U along the long side, m*

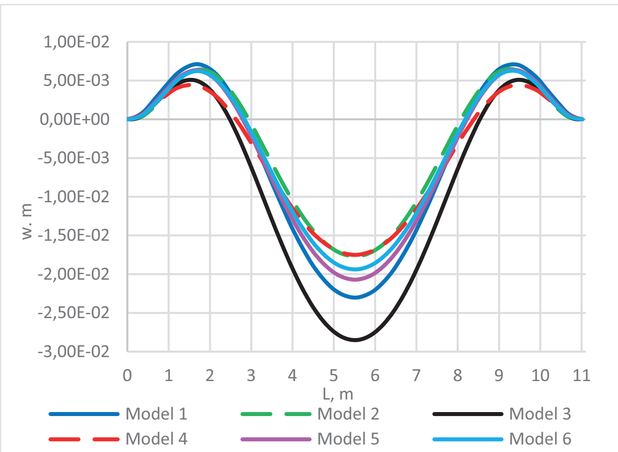


*Figure 15. Displacements W along the short side, m*

Through numerical analysis of the thermomechanical deformation of a positively curved orthotropic shell made of fiberglass, a set of characteristic patterns was established:

1.The discrepancy in the values of normal stresses $\sigma_{11}$ compared to the classical theory, which ignores the anisotropy of deformation properties and uses averaged constants, reaches up to 41,5%, ignoring thermal effects yields deviations of 15,3%; compared to the theory of C. W. Bert – J. N. Reddy, the deviation is 4,8%; with the application of the relations by R. M. Jones – D. A. R. Nelson, it is 39,3%; and under all equal thermomechanical loading conditions, it is 37,2% compared to calculations based on equations by S. A. Ambartsumyan;

2. For the normal stresses $\sigma_{22}$, the differences from the classical approach reach 45,9%; without considering thermal effects, 13,7%; compared to C. W. Bert – J. N. Reddy's theory, 8,3%; using R. M. Jones – D. A. R. Nelson's relations, 39,8%; and under equal thermomechanical loading conditions, 38,8% relative to calculations by S. A. Ambartsumyan;
3. For displacements W, the discrepancy reaches 23,1% relative to calculations by traditional orthotropic shell theory with averaged material characteristics, 24,1% without thermal gradient consideration, 23,9% compared to C. W. Bert – J. N. Reddy's model, 10% relative to R. M. Jones – D. A. R. Nelson's variant, and 15,7% compared to S. A. Ambartsumyan's theory;
4. For displacements U, the discrepancy is 12,5% relative to traditional orthotropic shell theory with averaged characteristics, 22% without temperature gradient, 23% compared to S. A. Ambartsumyan's theory, 6,5% compared to C. W. Bert – J. N. Reddy's model, and 17,5% relative to R. M. Jones – D. A. R. Nelson's variant.

## CONCLUSIONS SUPPORTED BY RESEARCH

The implemented computational experiment program for evaluating the stress-strain state parameters of an orthotropic shell structure with positive Gaussian curvature, formulated in the normalized tensor space of stresses, the correctness of which was demonstrated by verification with experimental data in publications [13], provides the basis for the following conclusions:
1. The application of the proposed mathematical model guarantees enhanced reliability and engineering accuracy in the analysis of thin-walled spatial systems compared to traditional approaches presented in other researchers' works;
2. Computational experiments clearly demonstrate that the correct accounting of induced deformational anisotropy in numerical algorithms reveals significant limitations of traditional methods for predicting the stress state of curved shell elements. Ignoring this phenomenon can generate substantial deviations in forecasting the mechanical response of spatial structures.

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