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NUMERICAL MODELING OF PLANE FRAME VIBRATIONS TAKING INTO ACCOUNT NONLOCAL IN TIME ELASTIC PROPERTIES OF THE MATERIAL

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Abstract: This paper presents a model for the dynamic deformation of plane composite frames accounting for the time-nonlocal elastic behavior of the material. The time-nonlocal model, previously developed for beam elements, is extended to frame structures and implemented within the finite element method. The influence of the nonlocal model scale parameter on the dynamic response is investigated. A U-shaped frame made of thermosetting vinyl ester glass-fiber-reinforced polymer is used as a case study to calibrate the scale parameter against reference three-dimensional finite element analysis results using the least-squares criterion and the golden-section search method. The calibrated time-nonlocal model significantly reduces the root-mean-square error compared with the classical local model and provides results that are in close agreement with the reference three-dimensional finite element analysis. The results provide additional evidence that the scale parameter of the time-nonlocal model can be regarded as an intrinsic material property independent of the structural geometry.

Keywords: Nonlocal mechanics, nonlocal damping, numerical simulation, finite element method

ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ КОЛЕБАНИЙ ПЛОСКИХ РАМ С УЧЕТОМ НЕЛОКАЛЬНЫХ ВО ВРЕМЕНИ УПРУГИХ СВОЙСТВ МАТЕРИАЛА

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Аннотация: В статье рассматривается модель динамического деформирования плоских рам из композитных материалов с учетом нелокальности упругих свойств материала во времени. Нелокальная модель, разработанная ранее для балочных элементов, распространена на случай рамных систем и выполнена её конечно-элементная реализация. Исследовано влияние изменения масштабного параметра нелокальной модели на характеристики колебательного процесса. На примере расчета П-образной рамы из термоактивного винилэфирного стеклопластика выполнена калибровка масштабного параметра по результатам трехмерного конечно-элементного моделирования с использованием метода наименьших квадратов и метода золотого сечения. Показано, что применение откалиброванной нелокальной модели позволяет существенно снизить среднеквадратичную ошибку по сравнению с классической локальной моделью и получить результаты, сопоставимые с трехмерным моделированием. Установлено, что масштабный параметр нелокальной модели может рассматриваться как характеристика материала, не зависящая от геометрии конструкции.

Ключевые слова: Нелокальная механика, нелокальное демпфирование, численное моделирование, метод конечных элементов

INTRODUCTION

Technological development plays a key role in many fields, including construction, mechanical engineering, and manufacturing. Particular attention is given to the development and application of composite and nanomaterials with controllable physical, mechanical, and chemical properties, as these materials are essential for the design and development of innovative engineering products. However, such materials often exhibit orthotropic or anisotropic behavior, requiring the development of reliable theoretical models supported by experimental validation. These models must accurately describe the material response under various loading conditions, including dynamic loading. Three-dimensional finite element models are widely used for this purpose because they can account for the internal structure of the material and its influence on mechanical behavior. However, their high computational cost limits their use in practical engineering applications. An alternative approach is provided by one-dimensional models based on specific hypotheses, including those derived from the principles of nonlocal mechanics.

Examples of such models include the approach presented in [1], which considers several damping mechanisms during the vibration of a quasi-isotropic pultruded composite beam, including external damping, strain rate-dependent internal damping, spatial hysteresis, and temporal hysteresis. The study also investigates different combinations of these damping mechanisms. Other models based on the principles of nonlocal mechanics include the nonlocal elasticity model proposed in [2] and the model presented in [3], which incorporates the nonlocality of both elastic and damping properties. The identification of the parameters of a nonlocal damping model using numerical simulations of the dynamic response of a composite beam is described in [4], whereas [5] presents a time-nonlocal internal friction model integrated into a finite element algorithm. These studies demonstrate that one-dimensional models based on the nonlocal approach can serve as an efficient alternative to detailed three-dimensional finite element models.

In the classical local formulation, the defining relations for viscoelastic material, as described by the Kelvin-Voigt model, take the following form:

$$\sigma^{ij} = C^{ijkl}\varepsilon_{kl} + X^{ijkl}\frac{\partial \varepsilon_{kl}}{\partial t}, \qquad (1)$$

where C^{ijkl} is elastic moduli tensor, $X^{ijkl} = C^{ijkl}t_e$ is viscosity moduli tensor, t_e is the retardation time, σ^{ij} is stress tensor, ε_{kl} is strain tensor.

The Cauchy relation:

$$\varepsilon_{ij} = \frac{1}{2}\left(\nabla_i u_j + \nabla_j u_i\right), \qquad (2)$$

where ∇_i is the differential operator and u_i is the components of displacements.

When transitioning to a nonlocal formulation, relation (1) takes the following form:

$$\sigma^{ij} = C^{ijkl}\int_0^t R(t-\tau)\varepsilon_{kl}(\tau)d\tau + X^{ijkl}\dot{\varepsilon}_{kl} \qquad (3)$$

In Eq. (3), $R(t-\tau)$ denotes the kernel of the nonlocal elasticity operator. It characterizes the decay of the contribution of strains at previous time instants τ to the elastic forces at the current time t. The kernel must satisfy the normalization requirement: $\int_0^t R(t-\tau)\,d\tau = 1$

In the present model, only the elastic properties of the material are assumed to be time-nonlocal. Under this assumption, the elastic forces at the current time depend not only on the current deformation state but also on the deformation history of the material.

A time-nonlocal model of the elastic properties of materials was previously developed for Euler–Bernoulli beam elements and integrated into a finite element algorithm [6].

The incorporation of the model into the finite element framework enables its application to the

analysis of rod systems with relatively complex geometries. The present study extends this approach to frame structures and demonstrates the applicability of the time-nonlocal model for predicting their dynamic response.

CONSTRUCTION OF A TIME-NONLOCAL DYNAMIC DEFORMATION MODEL FOR A ROD SYSTEM.

In order to make the nonlocal in time dynamic deformation model suitable for practical calculations, it has been incorporated into the finite element method algorithm. The physical interpretation of the finite element dynamic problems can be based on the principle of virtual work [7]:

$$\int_V \rho \ddot{u}_i \delta u_i dV + \int_V \sigma_{ij} \delta \varepsilon_{ij} dV = \int_A q_i \delta u_i dA + \int_V F_i \delta u_i dV. \quad (4)$$

Here ρ is a material density; q_i and F_i are surface and body forces; δu_i is the field of virtual displacements; $\delta \varepsilon_{ij}$ is the field of virtual strains. The two dots denote the second time derivative. Using the defining relation of a nonlocal in time dynamic deformation model (3) and considering the components of elastic moduli tensor constant in time, we can rearrange (4) as:

$$\int_V \rho \ddot{u}_i \delta u_i dV + \int_V C^{ijkl} \int_0^t R_t(t-\tau) \varepsilon_{kl}(\tau) d\tau \, \delta \varepsilon_{ij} dV + \int_V X^{ijkl} \dot{\varepsilon}_{kl} \delta \varepsilon_{ij} dV = \int_A q_i \delta u_i dA + \int_V F_i \delta u_i dV. \quad (5)$$

When employing the finite element approach, the actual strain and displacement fields for an element are replaced by approximate ones [8]:

$$\tilde{u} \approx \boldsymbol{N}\boldsymbol{d}, \quad (6)$$

where $\boldsymbol{N}$ is the shape functions matrix, $\boldsymbol{d}$ is the vector of displacements along the nodal degrees of freedom.

With the matrix operator of Cauchy relations $\boldsymbol{A}$ we can write $\boldsymbol{A}\boldsymbol{N} = \boldsymbol{B}$, then assumed element strain field is:

$$\tilde{\varepsilon} = \boldsymbol{B}\boldsymbol{d}. \quad (7)$$

Considering this and omitting the virtual nodal displacements according to the principle of virtual work [7] we can represent (5) in matrix form:

$$\int_V \rho \boldsymbol{N}^{\boldsymbol{T}} \boldsymbol{N} dV \ddot{\boldsymbol{d}} + \int_V \boldsymbol{B}^{\boldsymbol{T}} \boldsymbol{X} \boldsymbol{B} dV \dot{\boldsymbol{d}} + \int_V \boldsymbol{B}^{\boldsymbol{T}} \boldsymbol{C} \int_0^t R(t-\tau) \boldsymbol{B}\boldsymbol{d}(\tau) d\tau \, dV = \int_A \boldsymbol{N}\boldsymbol{q} dA + \int_V \boldsymbol{N}\boldsymbol{F} dV \quad (8)$$

For a plane frame, the constitutive matrix $\boldsymbol{C}$ contains two components corresponding to the axial and bending stiffness of the element, while the strain vector consists of two components: the axial strain and the curvature. The bending stiffness matrix was derived previously in [9]. Therefore, in the present study, only the axial component is derived:

$$\varepsilon_{11} = \frac{du}{dx}. \quad (9)$$

Here, x is the longitudinal coordinate of the beam, ε_{11} is the axial strain, and u is the axial displacement of the frame cross-sections. The assumed solution $\tilde{u} \approx \boldsymbol{N}\boldsymbol{d}$ for each frame

element can be written using interpolation functions:

$$\boldsymbol{N} = \left[1 - \frac{s}{l} \quad \frac{s}{l}\right], \tag{10}$$

where l is an element length, $s \in [0, l]$ is an element longitudinal coordinate.
Using $\frac{d}{ds}$ as $\boldsymbol{A}$ we have:

$$\boldsymbol{B} = \frac{d\boldsymbol{N}}{ds} = \left[-\frac{1}{l} \quad \frac{1}{l}\right]. \tag{11}$$

Since all elements of $\boldsymbol{B}$ matrix are time independent we can take it out of the integral over time in (8). Then taking into account that cross-sectional moment of inertia is $A = \int_A dA$ we obtain:

$$\begin{aligned} &\int_l \rho A \boldsymbol{N^T N} ds \ddot{\boldsymbol{d}} + \int_l \boldsymbol{B^T} \chi I_z \boldsymbol{B} ds \dot{\boldsymbol{d}} \\ &+ \int_l E I_z \boldsymbol{B^T B} \int_0^t R(t-\tau)\boldsymbol{d}(\tau) d\tau \, ds \\ &= \boldsymbol{F}. \end{aligned} \tag{12}$$

Here $\boldsymbol{M_i} = \int_l \rho A \boldsymbol{N^T N} ds$ is the beam element mass matrix, $\boldsymbol{D_i} = \int_l \boldsymbol{B^T} \chi A \boldsymbol{B} ds$ is the element damping matrix, $\boldsymbol{K_i} = \int_l E A \boldsymbol{B^T B} ds$ is the element stiffness matrix, $\boldsymbol{F}$ – is the load vector.
To derive the stiffness matrices of a rod element subjected to axial deformation, the shape function matrix $\boldsymbol{N}$ and the strain–displacement matrix $\boldsymbol{B}$ are substituted into the integral expressions for the stiffness matrices, respectively, previously obtained in Eq. (12).

$$\boldsymbol{K_i} = E \begin{bmatrix} \frac{\mathrm{A}}{l} & -\frac{\mathrm{A}}{l} \\ -\frac{\mathrm{A}}{l} & \frac{\mathrm{A}}{l} \end{bmatrix}, \tag{13}$$

In (12) the damping matrix is proportional to the stiffness matrix. The same result was obtained in [5], where the damping matrix was derived using the stationary condition of the system's total energy during vibration.
After assembling $\boldsymbol{M_i}$, $\boldsymbol{D_i}$, $\boldsymbol{K_i}$ to a global mass matrix $\boldsymbol{M}$, damping matrix $\boldsymbol{D}$ and stiffness matrix $\boldsymbol{K}$ of the beam, we can write the finite element formulation of equation of motion:

$$\boldsymbol{M}\ddot{\boldsymbol{d}} + \boldsymbol{D}\dot{\boldsymbol{d}} + \boldsymbol{K} \int_0^t R(t-\tau)\boldsymbol{d}(\tau) d\tau = \boldsymbol{F}. \tag{14}$$

When modeling the dynamic behavior of a plane frame, it is necessary to take into account that its elements are subjected not only to bending but also to tension and compression. Therefore, each node of the finite element model should have three degrees of freedom (Fig. 1). The bending stiffness matrix for nonlocal in time FEA model also has a standard form, which was demonstrated in [мы].
The stiffness matrix $\boldsymbol{K}_i$ of a plane frame element with three degrees of freedom at each node is a combination of axial and bending stiffness matrices and has the following form:

$$\begin{bmatrix} \frac{EA}{l} & 0 & 0 & -\frac{EA}{l} & 0 & 0 \\ 0 & \frac{12EI}{l^3} & \frac{6EI}{l^2} & 0 & -\frac{12EI}{l^3} & \frac{6EI}{l^2} \\ 0 & \frac{6EI}{l^2} & \frac{4EI}{l} & 0 & -\frac{6EI}{l^2} & \frac{2EI}{l} \\ -\frac{EA}{l} & 0 & 0 & \frac{EA}{l} & 0 & 0 \\ 0 & -\frac{12EI}{l^3} & -\frac{6EI}{l^2} & 0 & \frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ 0 & \frac{6EI}{l^2} & \frac{2EI}{l} & 0 & -\frac{6EI}{l^2} & \frac{4EI}{l} \end{bmatrix} \tag{15}$$

Here, **E** is the modulus of elasticity, **I** and **A** are the moment of inertia and the cross-sectional area of the element, respectively, and $\boldsymbol{l}$ is the length of the element.

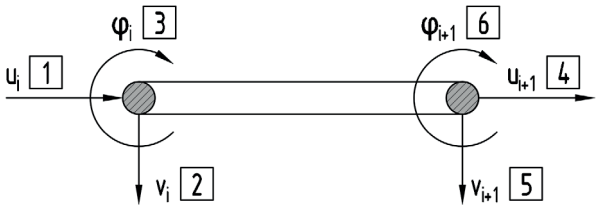


Figure 1. Numbering of the degrees of freedom of a frame finite element

To account for the time-nonlocal properties of the material, it is assumed that the elastic forces in the system at time t depend not only on the current displacement $\boldsymbol{d}(t)$, but also on the displacement history, i.e., the displacement values at previous time instants τ. The influence of past states decreases as the time interval between t and τ increases [10].

Various kernel functions have been proposed to model the memory function [2]. In the present study, the nonlocal operator is defined by an error function kernel, which is expressed in the following form accounting for the normalization requirement:

$$R(t-\tau) = \frac{2\eta}{\sqrt{\pi}} \cdot e^{-\eta^2(t-\tau)^2} \qquad (16)$$

Like for all non-local models, in this case, a defining characteristic is the scale parameter η, which determines the level of nonlocality in the elastic properties of the material over time. Figure 2 illustrates how this parameter influences the shape of the kernel.

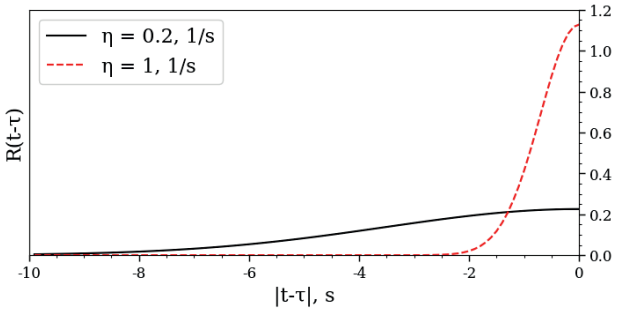


Figure 2. Graphs showing the effect of the parameter η on the nonlocality of the model

ASSESSMENT OF THE INFLUENCE OF THE SCALE PARAMETER ON THE NATURE OF THE OSCILLATORY PROCESS

To further develop the proposed model, it is necessary to investigate the effect of the scale parameter on the dynamic response of the rod system. The equation of motion was solved using an implicit Newmark integration scheme [11]. Nodal displacements were computed following the procedure previously developed for the vibration analysis of Euler–Bernoulli beam elements with time-nonlocal elastic material properties [6].

As a numerical example, a U-shaped frame with fixed supports was analyzed (Fig. 3). The frame was assumed to be made of a Class I thermoactive vinyl ester fiberglass composite, the material properties of which are listed in Table 1. The columns have a square cross-section with a side length of 0.4 m, whereas the horizontal beam has a rectangular cross-section measuring 0.3 m in height and 0.4 m in width. An instantaneous uniformly distributed load with an intensity of −11 kN/m was applied along the horizontal beam.

Table 1. Properties of thermoactive vinyl ester glass-reinforced plastic Class I (plane model):

Young's Modulus, Elw	17.2 GPa
Poisson's Ratio, μlw	0.32
Density, ρ	1.9 kg/m3
Damping Coefficient, γ	0.042

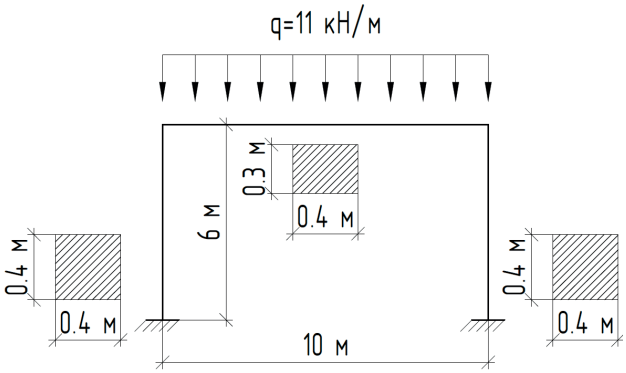


Figure 3. Computational model of the frame adopted for numerical simulation

The viscosity coefficient for the considered frame is $\chi = 0.021$ Pa·s.
To solve the equation of motion (14) in matrix form, an implicit scheme [11] was employed:

$$\boldsymbol{Q}\boldsymbol{d}_{i+1} = \boldsymbol{F}_{i+1} + \mathbf{M}\ddot{\boldsymbol{d}}_i + \boldsymbol{Q_1}\dot{\boldsymbol{d}}_i + \boldsymbol{Q_2}\boldsymbol{d}_i - \boldsymbol{K}\beta, \quad (17)$$

where:

$$\boldsymbol{Q} = \frac{2}{\Delta t^2}\boldsymbol{M} + \frac{1}{\Delta t}\boldsymbol{D} + \boldsymbol{K}\alpha, \qquad \boldsymbol{Q_1} = \frac{2}{\Delta t}\boldsymbol{M},$$
$$\boldsymbol{Q_2} = \frac{2}{\Delta t^2}\boldsymbol{M} + \frac{1}{\Delta t}\boldsymbol{D}, \quad (18)$$

The integral kernel (16) has been discretized and divided into two parts: α - the weighting coefficient associated with $\boldsymbol{d}$i+1, β - the sum of all other weights (fig.2):

$$\alpha = \frac{2\eta}{\sqrt{\pi}} \cdot e^{-\eta^2(t_{i+1}-t_i)^2} \cdot \Delta t;$$
$$\beta = \Delta t \sum_{j=1}^{i} \frac{2\eta}{\sqrt{\pi}} \cdot e^{-\eta^2(t_i-t_{j-1})^2} \quad (22)$$

Computer programs were developed in Python programming language. Based on the above initial data, a series of calculations was performed for different values of the scale parameter in order to assess its influence on the characteristics of the oscillatory process. The obtained results are presented in Fig. 4.
Analysis of the presented plots shows that a decrease in the parameter η (an increase in the degree of nonlocality) leads to an increase in the vibration amplitude.

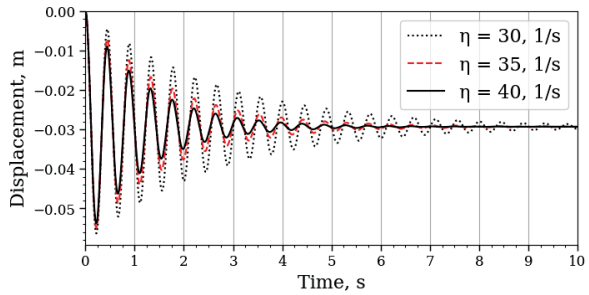


Figure 4. Displacement plots of the central node of the upper chord of the frame obtained from three computational cases with different values of the scale parameter

DETERMINATION OF THE NONLOCAL PARAMETER BASED ON THE RESULTS OF A NUMERICAL EXPERIMENT

For the proposed nonlocal material model to be used in engineering applications, the value of the nonlocal parameter must first be identified. In the present study, this parameter is determined through a calibration procedure based on the least squares method using the results of a numerical experiment. The reference data for the calibration were obtained from a three-dimensional finite element analysis of the frame structure described in the previous section. The numerical model was developed in the MIDAS Civil NX software package and accounts for the orthotropic properties of the composite material. The material properties used in the analysis are listed in Table 2.

Table 2. Properties of thermoactive vinyl ester glass-reinforced plastic Class I (3D solid model):

Young's Modulus (Longitudinal), E_{lw}	17.2 GPa
Young's Modulus (Transverse), E_{cw}	12.2 GPa
Poisson's Ratio (Longitudinal), μ_{lw}	0.32
Poisson's Ratio (Transverse), μ_{cw}	0.15
Density, ρ	1.9 kg/m^3
Damping Coefficient, γ	0.042

The finite element analysis provided the displacement history of the midpoint of the frame girder. The corresponding displacement data obtained in MIDAS Civil NX were exported to a Python program developed for identifying the scale parameter. The program implements a least squares calibration procedure, in which the scale parameter of the nonlocal model is identified by minimizing the sum of squared differences between the reference data and the numerical predictions. The optimization was performed using the golden section method.
The comparison of the results is shown in Figs. 5 and 6. Compared with the classical local model, the calibrated nonlocal rod model shows better agreement with the results of the three-dimensional finite element analysis.

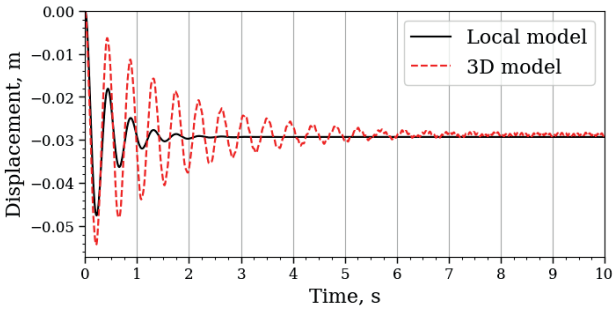


Figure 5. Comparison of the results of the three-dimensional numerical experiment with the results of one-dimensional modeling based on the classical local model. The relative error is 13.56%

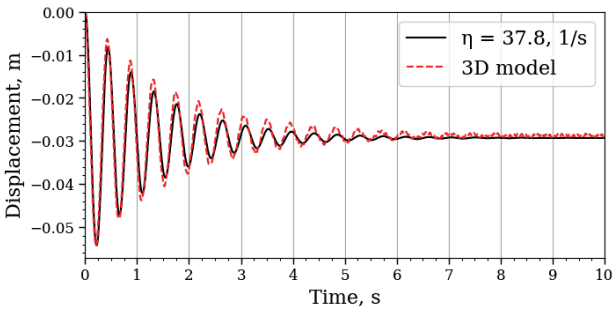


Figure 6. Comparison of the results of the three-dimensional numerical experiment with the results of one-dimensional modeling using the calibrated time-nonlocal deformation model with the error function kernel. The relative error is 5.41%

The results clearly demonstrate that the calibrated nonlocal rod model allows to reduce the root mean square error by nearly one order of magnitude compared with the corresponding local model. Thus, the proposed approach achieves an accuracy comparable to that of the three-dimensional finite element model.

Paper [6] presents the results of a series of numerical studies on Euler–Bernoulli beam elements with time-nonlocal elastic material properties.

The calibration data for the 12 meters long beam with a fixed ends and with a cross-section 20x30 cm was obtained in MIDAS Civil NX similarly as it was done for the frame. The material properties were taken as for the GFRP (Table 2)

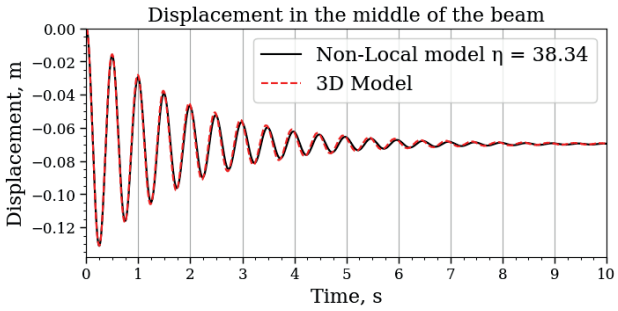


Figure 7. Calibration results for the beam presented in [6]

The beam has the similar firs Eigen frequency as the considered frame (fig. 3), and thus a similar viscosity coefficient of 0.021 Pa·s. It was identified that the value of the scale parameter for the considered beam was 38.34 s^{-1}. Although the calibration presented in this study was performed independently for a frame structure, the resulting combination of viscosity coefficient and scale parameter (0.022 Pa·s and 37,8 s^{-1}, respectively) is in close agreement with the corresponding values reported in [6]. This finding provides additional evidence that the scale parameter is an intrinsic material property of the nonlocal model and, therefore, does not require recalibration when the geometry of the structure changes.

CONCLUSION

The previously developed time-nonlocal dynamic deformation model, originally formulated for Euler–Bernoulli beam elements, has been extended to describe the dynamic response of frame structures. The influence of the scale parameter η on the structural response has been investigated, showing that an increase in the degree of temporal nonlocality leads to higher vibration amplitudes.

Compared with the classical local formulation, the calibrated nonlocal rod model shows significantly better agreement with the reference three-dimensional finite element analysis. The calibration results provide additional evidence that the scale parameter is an intrinsic material property of the proposed nonlocal model and is independent of the structural geometry.

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