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PROBABILISTIC METHOD OF STRUCTURAL RELIABILITY ANALYSIS BY SAMPLING WITH LIMITED STATISTICAL DATA

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Abstract: The article presents an approach to modernizing the data sampling algorithm for structural reliability analysis. This approach enables obtaining more conservative estimates of the failure probability in cases of incomplete or limited statistical data. Numerical examples demonstrate that an incorrect hypothesis regarding the probability distribution type of a random variable in structural reliability problems can lead to estimation errors of up to 5% in the reliability index. Furthermore, neglecting the confidence limits for the parameters of the assumed probability distribution function can result in errors of up to 60% (in a strict mathematical solution). An analysis of scientific and technical publications reveals varying preferred probability distributions for steel yield strength, depending on the steel type and stress-strain conditions. This variability complicates unambiguous selection of a specific probability distribution in real-world tasks during construction site assessments. The proposed data sampling algorithm is based on a modification of the classical Monte Carlo method. It involves selecting a random distribution function and estimating the confidence intervals for its parameters via a bootstrap approach for each generated sample value. The algorithm produces an arbitrary-sized sample containing values representing the lower and upper confidence bounds of the studied parameter. Consequently, the structural element's failure probability is expressed as an interval of values.

Keywords: reliability, probabilistic analysis, safety, sampling, yield strength, failure probability, reliability index, incomplete data

ВЕРОЯТНОСТНЫЙ МЕТОД АНАЛИЗА НАДЕЖНОСТИ СООРУЖЕНИЙ ГЕНЕРАЦИЕЙ ДАННЫХ ПРИ НЕПОЛНОЙ СТАТИСТИЧЕСКОЙ ИНФОРМАЦИИ

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Аннотация: В статье представлен подход к модернизации алгоритма генерации (сэмплинга) данных анализа надежности элементов строительных конструкций, позволяющий получать более достоверные оценки вероятности отказа в случае неполной статистической информации. На численных примерах показано, что неверная гипотеза о типе распределения вероятностей случайной величины в задачах анализа надежности может привести к ошибке в оценке до 5% индекса надежности. В то же время отсутствие учета доверительных границ изменчивости параметров принятой функции распределения вероятностей может привести к ошибкам до 60% (в строгом математическом решении). Анализ научно-технических публикаций показывает различные варианты предпочтительных распределений для предела текучести стали в зависимости от вида стали и условий эксплуатации, что затрудняет принятие однозначного решения о конкретном распределении вероятностей в реальных задачах при обследовании строительных объектов. Предложенный алгоритм генерации данных основывается на модернизации классического метода Монте-Карло и заключается в выборе случайной функции распределения и доверительной оценке ее параметров путем бутстреп-подхода для каждого генерируемого значения выборки. Результатом использования предлагаемого алгоритма является выборка произвольного размера, содержащая значения нижней и верхней доверительной границы значений исследуемого параметра. Вероятность отказа элемента сооружения будет представлена в виде интервала значений.

Ключевые слова: надежность, вероятностный анализ, безопасность, генерация, предел текучести, вероятность отказа, индекс надежности, неполная информация

INTRODUCTION

Reliability is one of the key indicators in substantiating the adopted design decision of the structure. There are many reliability measures, but the failure probability P_f and the reliability index β are most often used in practice of structural design. The calculation of these parameters is based on the principles of probability theory and mathematical statistics.

The fundamental theory of structural reliability in a probabilistic context has been widely developed since the middle of the 20th century [1]. However, the use of many well-developed theoretical provisions faces difficulties in practical application due to limited (incomplete) statistical information and data. It is noted in [2], that "The probability distribution of random variables is usually incomplete owing to a limited amount of data, making it a challenging problem for structural reliability assessments". Modeling of uncertainties in the tasks of assessing and analyzing the reliability of building structures is one of the key problems. As noted in research [3], "despite the highly detailed numerical predictions that can be obtained, these results often do not achieve a satisfactory level of agreement with 'reality', i.e., the actual physical behavior of the considered continuum in the effective operational environment. This discrepancy is caused by epistemic (reducible) and aleatory (caused by variation) uncertainty in the model".

Let's consider a practical problem – the formation of a random variable model for the yield strength of steel of an existing structural steel element. There are several options for selecting a cumulative probability distribution function. In Section 3.02 of Joint Committee on Structural Safety (JCSS) is noted that the log-normal distribution is recommended for yield strength of steel, but bi-modal distributions are preferred in some cases. For the probabilistic description of the yield strength of steel, the normal and lognormal distributions are most often used [4].

Also, an optimal function of probability distribution may change over time and with changing operating conditions. The paper [5] presents the results of studying probability distribution function of yield strength of steel, and it concluded that "the choice of probability distribution for material properties that captures scatter in data over a range of temperatures can influence the simulated structural response" and logistic and beta distribution are suggested. In paper [6], it is noted that "The Shapiro-Wilk test indicates that elastic modulus, yield strength, tensile strength, peak strain, and yield plateau slope of corroded steel with different corrosion loss ratios all fitted the normal distribution".

The authors of research [7] are noted that "according to our experience, the statistic output file is approximated, in the optimal way, by the Hermite four-parameter distribution, which takes into account even the skewness and kurtosis of random yield strength". In the article [8] it is annotated that "the yield stress as well as ultimate steel strength random parameters do not follow a Gaussian distribution".

So, the above literature review shows that there are many possible probabilistic models for the yield strength of steel. The choice of a specific probability distribution function in the real task of structural reliability analysis imposes a certain responsibility on the civil engineer. The problem becomes more complicated when you have to analyze a small statistical sample size. In conditions of limited statistical data, the parameters of the probability distribution functions are confidence intervals. The use of common complexes in which data sampling tasks are solved is based on a specific function with precise parameters. Using the parameters of a sample data set in this case leads to a decrease in the accuracy of the results and a difficult-to-estimate calculation error.

In this study, it is proposed to consider an algorithm for structural reliability analysis with incomplete statistical information, when data is obtained by the new approach of sampling.

METHODS

The key task of probabilistic reliability analysis of structures is the preliminary formation of random variable models that will describe the variability of calculated parameters. As a rule, classical probability distribution functions of random variables known from mathematical statistics and probability theory are used for this purpose. Choosing an unsuitable distribution function can lead to significant errors in estimating the reliability index or the probability of failure. This problem is particularly acute in practical tasks, when statistical samples of only a few dozen values are available based on the results of tests/studies. Let's first analyze how big relative errors can be if an analytical engineer selects a suboptimal probability distribution function or its statistical parameters.

Let us consider the problem of selecting a random variable model for the yield strength of steel based on the results of its tests. Let C255 steel be given, and its statistical characteristics are obtained from the study [9].

The following 30 samples were obtained when sampling values from the normal distribution with parameters: expected value (mean) $\mu_x = 288.18$ MPa and standard deviation $\sigma_x = 20.17$ MPa (the coefficient of variation is 7 percent): $x \in$ {303.49, 293.85, 290.28, 286.92, 272.97, 282.52, 254.03, 286.20, 268.24, 265.92, 291.81, 316.26, 306.70, 256.50, 268.22, 307.16, 268.36, 283.65, 299.27, 268.65, 290.31, 261.27, 276.05, 294.63, 256.05, 297.06, 287.78, 299.32, 292.70, 315.69} MPa.

Let's also assume that the task is to evaluate the reliability index β when a stress equal to the normative value of the yield strength $R_{y,n} = 255$ MPa.

If the true probability distribution function and its exact parameters are known, then the reliability index can be expressed as:

$$\beta = \frac{\mu_x - R_{y,n}}{\sigma_x}. \quad (1)$$

Using eq. (1), it is possible to estimate the true reliability index $\beta = 1.645$ and following no-failure probability $P = \Phi(\beta = 1.645) = 0.95$, where Φ(.) denotes Laplace function (the Z table). But in practice, the probability distribution function and its parameters are unknown in advance.

There are various approaches for choosing the most appropriate probability distribution function for a data sample. For example, the Kolmogorov–Smirnov statistic D_n quantifies a distance between the empirical distribution function of the sample $F_n(x)$ and the cumulative distribution function of the reference distribution $F(x)$ as:

$$D_n = \sup_x \left| F_n(x) - F(x) \right|. \quad (2)$$

The lower the value of the statistic D_n, the more likely it is that the sample values are derived from the function $F(x)$. The critical value of the Kolmogorov-Smirnov statistic for 30 values is $D_{cr} = 0.248$ (with the confidence probability $\alpha = 0.05$) or $D_{cr} = 0.223$ (with the confidence probability $\alpha = 0.10$).

The choice of the optimal distribution for the available statistical data sample is related to the selection of both the probability distribution function and its parameters. The Distribution Fitter App of the MATLAB mathematical software package or the SciPy library of the Python programming language can be used to assign the parameters.

It is possible to obtain following parameters of different types of distributions (Table 1). Using the value of the distribution function at $F(0.05)$, as well as the inverse normal distribution $P = \Phi(\beta)$, it is possible to estimate the non-failure probability and the reliability index β of the yield strength of steel at the point of characteristic value.

The cumulative distribution functions plots are presented in Figure 1.

Table 1. Analysis of different distribution functions for steel yield strength

Type of the probability distribution function	Fitted parameters*	K-S statistics D_n	Reliability index β	ε, %
Normal	$\mu_x = 284.73$ MPa; $\sigma_x = 17.32$ MPa	0.123454	1.716766	4.18
Lognormal	$s = 0.0611$; loc = 0 MPa; scale = $\ln(\mu_x) = 5.6497$ MPa	0.121157	1.716777	4.18
Logistic	$\mu_x = 285.13$ MPa; $S_x = 10.33$ MPa	0.131419	1.632292	0.78
GEV	$c = 0.379$; loc = μ_x =279.422 MPa; scale = σ_x =17.877 MPa	0.121072	1.650278	0.32
Gamma	$a = 268.49$; scale = 1.05 ($\mu_x = 284.3$ MPa; $\sigma_x = 17.34$ MPa)	0.121829	1.732762	5.06

*The parameters fitted in Python using SciPy library

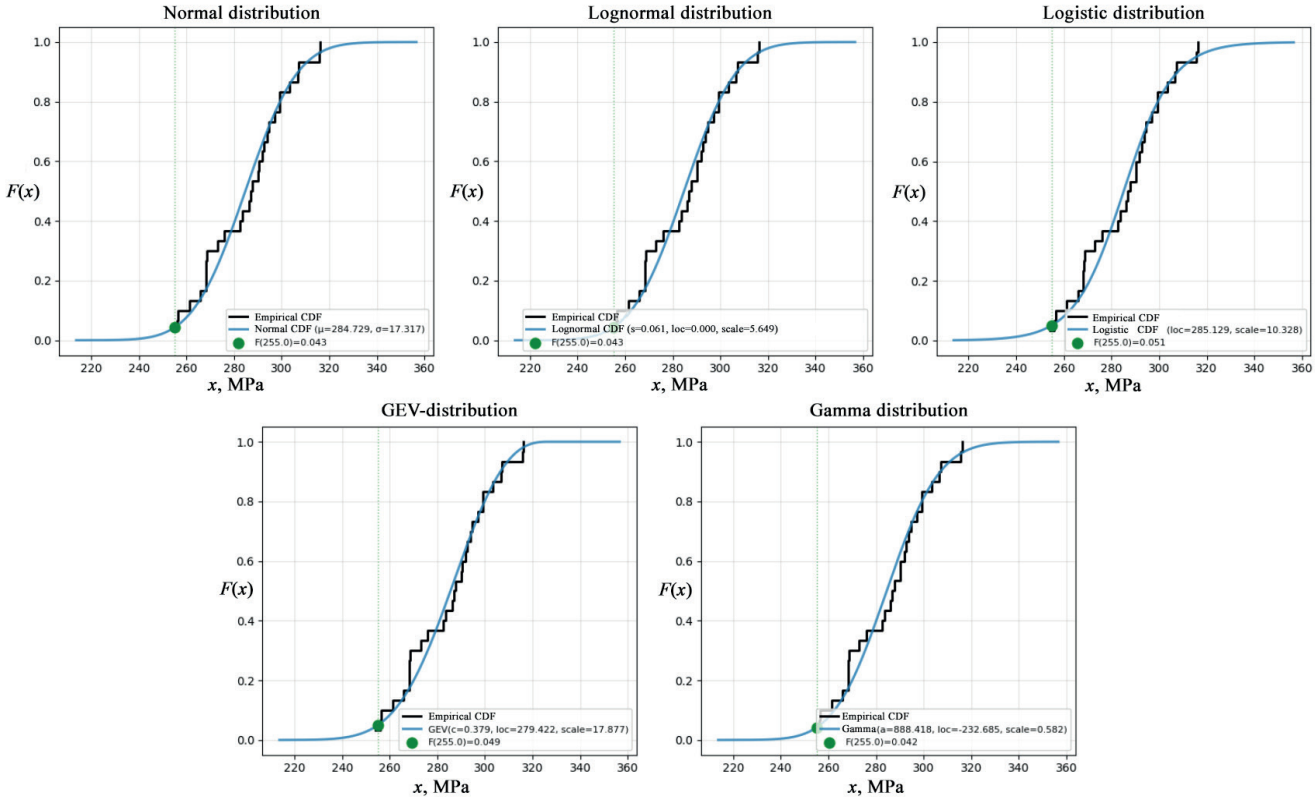


Figure 1. Cumulative distribution functions (CDF) plots of different probability distributions

As can be seen from Table 1, preferred distributions for above sample dataset are Generalized Extreme Value (GEV) with $D_n = 0.121$. The lognormal and gamma distributions also have similar values of Kolmogorov-Smirnov statistics and can be assumed, but in this case the error (which in practice cannot be estimated, since we know the real probability distribution function and parameters only in the current conditional example) would be 4-5%.

Let's assume that an analytical engineer reliably knows the probability distribution function based on a long-term analysis of a random variable at a steel manufacturing plant. According to probability theory and mathematical statistics, the values of the parameters of the general data set can be estimated from a sample population at confidence intervals. Fig. 2 shows the boundaries of the confidence intervals for the mean and standard deviation normal distribution and the above data sample with different confidence levels.

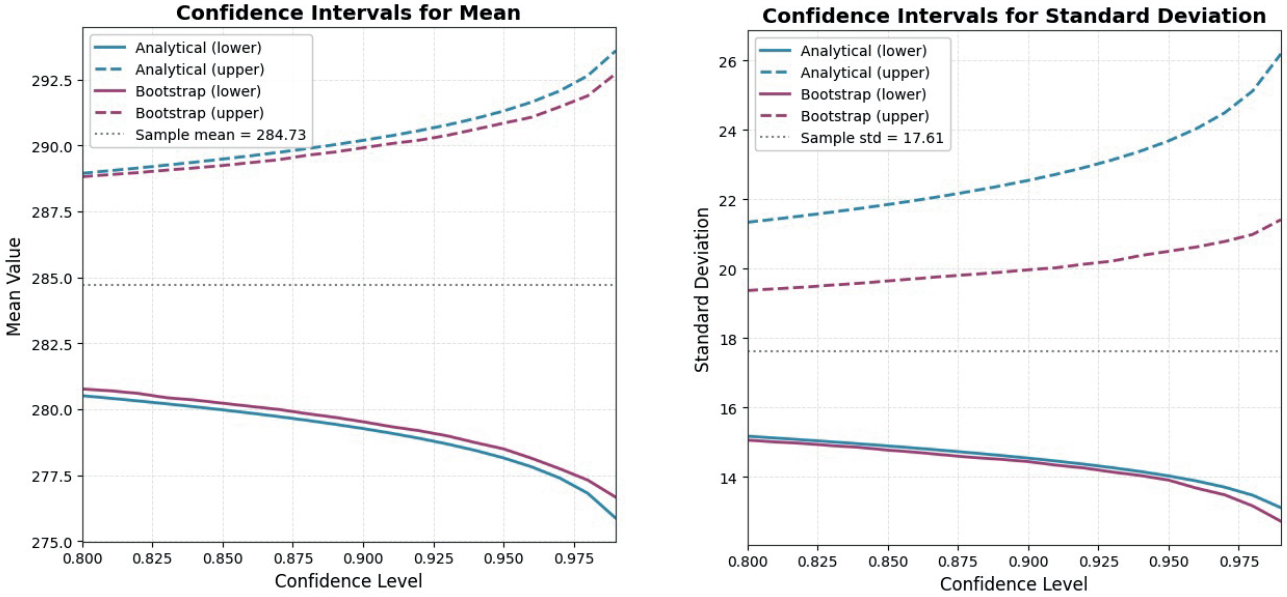


Figure 2. Confidence intervals for mean (a) and standard deviation (b) depending on confidence level

Confidence intervals for the reliability index β at various levels can be calculated based on confidence estimates of statistical parameters and equation (1). The best-case (maximum mean and minimum standard deviation) and worst-case (minimum mean and maximum standard deviation) scenarios are considered (Table 2) [10].

Table 2. Confidence intervals for reliability index β

Confidence level	0.80	0.90	0.95	0.99
Analytical β	[1.202; 2.238]	[1.077; 2.421]	[0.998; 2.590]	[0.780; 2.944]
[min ε, max ε], %	[-26.5, 36.9]	[-32.1, 59.7]	[-36.5, 68.3]	[-44.1, 105.7]
Bootstrap β	[1.330; 2.246]	[1.229; 2.418]	[1.146; 2.577]	[1.012; 2.966]
[min ε, max ε], %	[-26.7, 23.7]	[-31.9, 33.9]	[-36.2, 43.6]	[-44.5, 62.6]

As a rule, the lowest value of β is taken and confidence level 0.95. The difference in this case is about 30-60% between real and estimated value of the reliability index β (in a strict mathematical solution).

Thus, a preliminary analysis shows the importance of choosing a random variable distribution function and assigning its parameters. The following algorithm is proposed to modernize the random variable model in practical problems:

1. Five (k=1, 2, …, 5) cumulative distribution functions (CDFs) of random variables are selected based on scientific and technical analysis of previous experience in statistical data analysis of the random parameter under study;
2. Samples of random variables $x = \{x_1,\ x_2,\ ...,\ x_n\}$ are obtained by measuring/testing. At least $n = 20$ values are recommended;
3. Values of K-S statistics D_n are estimated for every CDFs by eq. (2). Parameters of CDFs are fitted using Distribution fitter in MATLAB or SciPy in Python;
4. Weights of CDFs $w_{KS,k}$ are estimated based on the K-S statistics D_n as $w_{KS,1} = \dfrac{1/D_{n,1}}{1/\sum_{k=1}^{5} D_{n,k}}$, where $\sum_{k=1}^{5} w_{KS,k} = 1$;
5. Weights of CDFs $w_{emp,k}$ are estimated based on empirical data. These weights are assigned by an expert, depending on the scientific and technical analysis, the data of the manufacturing plant, the characteristics of the material, etc. The assignment of weights for design parameters during design is considered as a general decision of the engineering community, approved in the design recommendations. As a result $\sum_{k=1}^{5} w_{emp,k} = 1$;
6. Based on the data from points 3-4, a weight $w_k = \dfrac{w_{KS,k} + w_{emp,.k}}{2}$ is assigned to each probability distribution function;
7. The bootstrap method [11] estimates confidence intervals for the parameters of the selected distribution functions;
8. Confidence intervals for probability distribution functions are constructed using probability boxes [12, 13] approach based on the confidence intervals of CDFs parameters;
9. Then the values of the random variable are sampled as follows:
9.1. A random cumulative distribution function is selected according to its weight according to p. 6 (a uniform distribution is used);
9.2. One value is sampled for the lower confidence bound of this CDF and one value for the upper bound. The reverse transformation method is used for sampling;
9.3. Paragraphs 9.1-9.2 are repeated as many times as samples need to be obtained for reliability analysis, depending on the required accuracy;
10. As a result of the implementation of the algorithm, the results are obtained in the form of the required number of samples N for the lower and upper confidence bounds of the cumulative distribution function of a random variable x.

Using the above algorithm, it is possible to obtain the confidence bounds for cumulative distribution functions for all random variables for which a sample set of data is given (snow load, steel/concrete strength measurements, etc.).

Further reliability analysis can be performed using the Interval Monte Carlo (IMC) method [14, 15]. The limit state function $g(\mathbf{X})$ is formed, where N samples are performed for each random variable in the vector of random variables $\mathbf{X}$ (according to the above algorithm or based on known cumulative distribution functions). In the case of confidence interval estimates, the vector will also be represented in interval form $\underline{\mathbf{X}}$ and $\overline{\mathbf{X}}$. Then the probability of failure is calculated as follows:

$$\underline{P}_f \approx \frac{1}{N}\sum_{j=1}^{N} I\left[g\left(\underline{\mathbf{X}}\right) \le 0\right], \tag{3}$$

$$\overline{P}_f \approx \frac{1}{N}\sum_{j=1}^{N} I\left[g\left(\overline{\mathbf{X}}\right) \le 0\right], \tag{4}$$

where is an indicator function that has a value of 1 if I[.] is equal to "true", and the value is 0 if I[.] is equal to "false".

There are various approaches to assigning the required probability of failure, for example [16, 17, etc.]

RESULTS AND DISCUSSION

For the above data, based on the bootstrap method (1000 samples and a confidence probability of 0.95), the following interval estimates of the parameters of probability distributions were obtained:

- Normal distribution: $\mu_x \in \lfloor 278.39,\ 290.66 \rfloor$ MPa, $\sigma_x \in \lfloor 13.76,\ 20.09 \rfloor$ MPa;
- Lognormal distribution: $s \in \lfloor 0.0488,\ 0.0703 \rfloor$, $\mu_x \in \lfloor 277.98,\ 289.94 \rfloor$ MPa;
- Logistic distribution: $\mu_x \in \lfloor 277.74,\ 291.89 \rfloor$ MPa, $S_x \in \lfloor 7.75,\ 12.21 \rfloor$ MPa;
- GEV distribution: $loc = \mu_x \in \lfloor 273.67,\ 285.95 \rfloor$ MPa, $scale = \sigma_x \in \lfloor 13.88,\ 20.71 \rfloor$ MPa, $c = 0.3789$;
- Gamma distribution: $scale \in \lfloor 1.0380,\ 1.0846 \rfloor$, $a = 268.5$.

Based on the interval estimations of parameters, following probability boxes [12] can be constructed (Fig. 3).

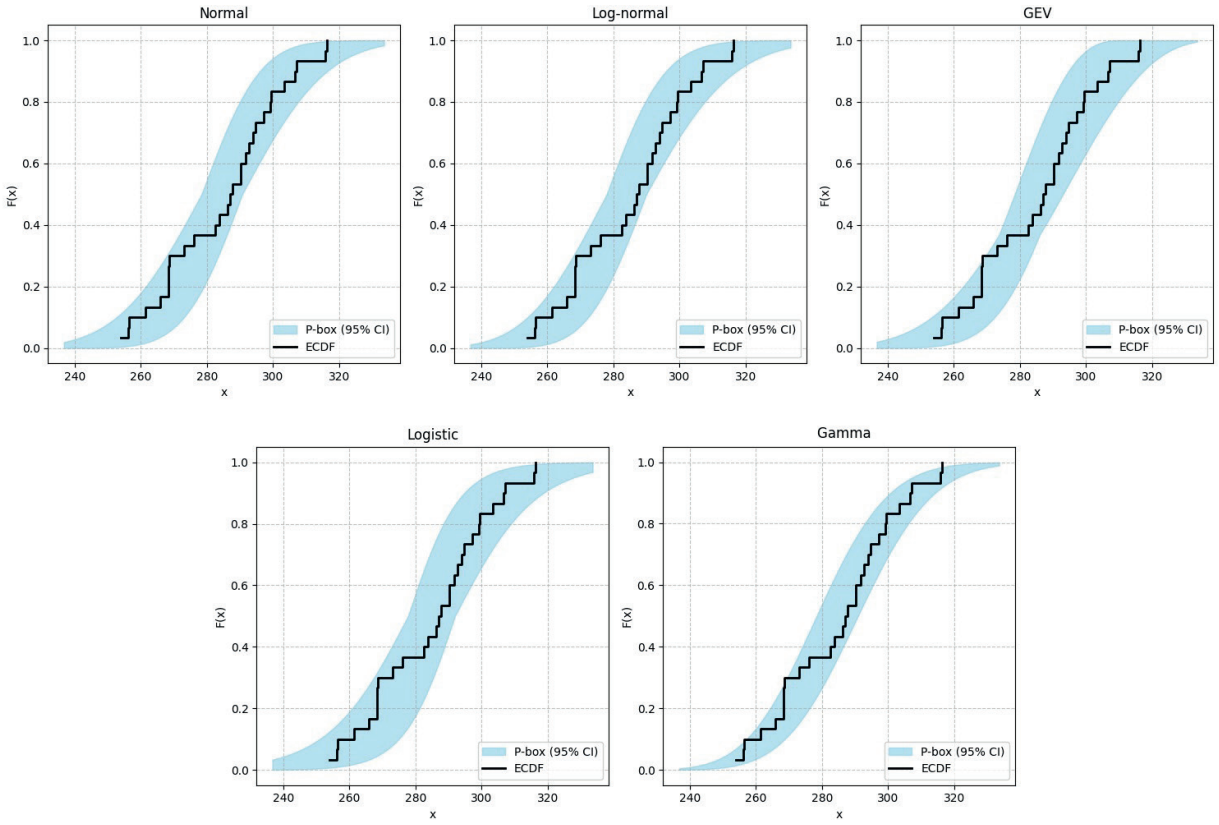


Figure 3. Probability boxes for different types of cumulative distribution functions

Probability box is a confidence interval for the real cumulative distribution function. The left and right boundaries of the functions are accepted as the possible worst and best options for various combinations of interval parameters, and are not always accepted for all the smallest and largest values of the interval.

Let's assume that based on a priori data, as well as Kolmogorov-Smirnov statistics, the following weights were obtained for probability distributions: GEV-distribution $w_{GEV} = 0.30$, lognormal distribution $w_{\ln} = 0.30$, normal distribution $w_{norm} = 0.20$, logistic $w_{\log} = 0.10$, gamma distribution $w_{gamma} = 0.10$.

Further, according to the calculated statistical parameters and weights of the probability

distribution functions, sampling of 1 million values for the lower and upper boundary functions of the probability distribution was performed using the above algorithm. Graphs of these functions, as well as empirical and theoretical ones, are shown in Fig. 4.

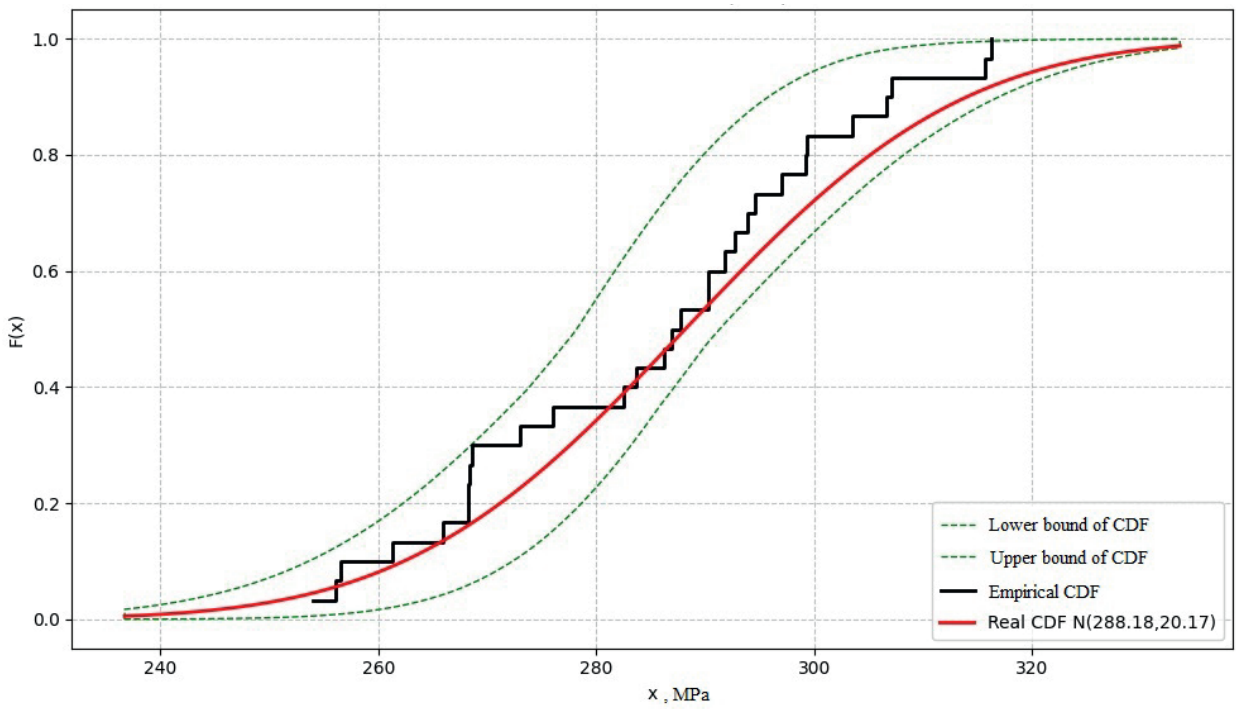


Figure 4. Graphs of confidence bounds for CDF by proposed algorithm, empirical CDF and true CDF

As can be seen from Figure 4, the confidence bounds for the cumulative distribution function according to the proposed algorithm completely cover both the real cumulative distribution function and the empirical one (the sample from which was obtained using the true distribution function).

In cases of complex mathematical models of the limit state, including numerical models, modern approaches can be used to reduce the required number of calculations based on meta-models and surrogate models [18-20].

CONCLUSION

1. Choosing a not optimal cumulative distribution function or its parameters can lead to significant errors (up to 60%) in estimating the reliability index or the probability of failure. This problem is particularly acute in practical tasks, when statistical samples of only a few values are available based on the results of tests/experimental research.

2. The proposed algorithm for modeling random variables for probabilistic reliability analysis of building structures is based on a mixture of probability distribution functions with interval estimation of its parameters by bootstrapping. A numerical example shows the efficiency of using the algorithm – the confidence domain covers both the real distribution function and the empirical one.

3. The estimate of the probability of failure or the reliability index will be presented in an interval form. For a narrower interval response, it is necessary to increase the quantity/quality of available statistical information. If it is impossible to obtain the necessary information, reliability can be improved using classical methods – increasing cross-sections, increasing strength, reducing loads, etc.

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