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NONLINEAR STABILITY AND IMPERFECTION SENSITIVITY ANALYSIS OF HYBRID TIMBER BEAMS FORMULATED VIA A MECHANISM-BASED ENERGY APPROACH

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Abstract: This paper develops an advanced computational framework to investigate the nonlinear stability and imperfection sensitivity of hybrid three-layer timber beams, specifically composed of high-stiffness Birch faces and a relatively soft Pine core. By employing a rigorous variational energy formulation based on the principle of minimum total potential energy, the study explicitly models the complex interaction between flexural deformations and bending-induced membrane effects under large-scale initial geometric imperfections. A distinctive feature of the proposed model is the derivation of a mechanism-based energy index (η), which provides a robust mathematical criterion for identifying the transition from bending-dominated to membrane-activated structural regimes. The numerical implementation is executed via an efficient MATLAB-based algorithmic procedure, enabling a high-fidelity parametric exploration of imperfection amplitudes ranging from infinitesimal values to L/50. Comprehensive numerical results reveal that the activation of membrane (catenary) action induces a pronounced geometric hardening effect, with the tangent stiffness increasing by an order of magnitude as the applied load increases, an effect that is triggered earlier and more strongly in beams with larger initial imperfections. The analysis of shear stress gradients further highlights the redistribution of internal forces toward the layer interfaces as membrane forces are activated. Furthermore, a topographical stability transition map is constructed to visualize the synergistic effects of interlayer slip stiffness and initial curvatures on the global buckling limits. The computational findings offer a robust theoretical basis for the safety-limit design of slender hybrid composite members in modern civil engineering applications, emphasizing the necessity of accounting for large-scale geometric nonlinearities in structural reliability assessments.

Keywords: Computational mechanics; Hybrid timber structures; Nonlinear stability; Energy approach; Membrane effect; Stability mapping; Stiffness evolution

НЕЛИНЕЙНЫЙ АНАЛИЗ УСТОЙЧИВОСТИ И ЧУВСТВИТЕЛЬНОСТИ К НЕСОВЕРШЕНСТВАМ ГИБРИДНЫХ ДЕРЕВЯННЫХ БАЛОК НА ОСНОВЕ МЕХАНИСТИЧЕСКОГО ЭНЕРГЕТИЧЕСКОГО ПОДХОДА

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Аннотация: В данной работе разработана усовершенствованная вычислительная модель для исследования нелинейной устойчивости и чувствительности к несовершенствам гибридных трехслойных деревянных балок, состоящих из высокожестких внешних слоев из березы и относительно мягкого сердечника из сосны. Путем применения строгой вариационной энергетической формулировки, основанной на принципе минимума полной потенциальной энергии, в исследовании в явном виде моделируется сложное взаимодействие между изгибными деформациями и эффектами мембранных сил, вызванных изгибом, в условиях крупномасштабных начальных геометрических несовершенств. Отличительной особенностью предлагаемой модели является вывод энергетического индекса механизма (η), который служит надежным математическим критерием для идентификации перехода от режима с преобладанием изгиба к режиму активации мембранных сил. Численная реализация выполнена с помощью эффективной алгоритмической процедуры на базе MATLAB, что позволяет проводить высокоточное параметрическое исследование амплитуд несовершенств в диапазоне от бесконечно малых величин до L/50. Комплексные численные результаты показывают, что активация мембранного (катенарного) действия вызывает выраженный эффект геометрического упрочнения, при этом касательная жёсткость возрастает на порядок по мере увеличения приложенной нагрузки, причём этот эффект проявляется раньше и сильнее у балок с бóльшими начальными несовершенствами. Анализ градиентов касательных напряжений дополнительно подчёркивает перераспределение внутренних усилий к границам раздела слоёв при активации мембранных сил. Кроме того, построена топографическая карта переходов устойчивости для визуализации синергетического воздействия жесткости межслойного сдвига и начальных кривизн на общие пределы потери устойчивости. Вычислительные результаты обеспечивают прочную теоретическую основу для проектирования тонких гибридных композитных элементов по предельным состояниям в современном гражданском строительстве, подчеркивая необходимость учета крупномасштабных геометрических нелинейностей при оценке надежности конструкций.

Ключевые слова: Вычислительная механика, Гибридная древесина, Нелинейная устойчивость, Энергетический подход, Мембранный эффект, Карта устойчивости, Эволюция жесткости

INTRODUCTION

In the contemporary era of sustainable structural engineering, the utilization of engineered timber has undergone a significant transformation, evolving from traditional lumber to high-performance composite systems [1]. Among these, hybrid three-layer timber beams- typically configured with high-stiffness face layers, such as Birch, and a relatively compliant Pine core-represent a sophisticated solution for mid-to-high-rise applications in modern construction [2, 3]. The mechanical efficiency of such hybrid members is derived from the synergistic interaction between layers of contrasting elastic moduli, which optimizes the distribution of normal and shear stresses [4]. However, the increased slenderness associated with these optimized sections introduces complex challenges regarding their structural stability and nonlinear response under service loads [5].

The structural integrity of composite timber members is fundamentally governed by the interlayer shear interaction and the slip modulus of mechanical fasteners [6]. Traditional analytical frameworks, such as the γ-method established in Eurocode 5, provide a practical basis for evaluating effective bending stiffness under linear-elastic conditions [7]. While these methods are robust for standard design, they inherently neglect second-order geometric effects that arise as transverse deflections increase [8]. Pioneering contributions in structural mechanics, particularly those by Timoshenko [9], highlighted that for slender assemblies, the coupling between axial strains and curvature leads to significant load-path deviations. In hybrid timber systems, this coupling is further complicated by the discrete or continuous shear slip at the interfaces, which necessitates a more refined computational approach than classical beam theories, such as the MATLAB-based γ-method framework recently proposed for two-layer timber composite beams [10].

Geometric nonlinearity is a critical factor in the safety assessment of timber structures, especially when considering the absence of externally applied axial forces [5]. A critical phenomenon in the large-deflection regime of slender beams is the activation of membrane forces [11]. As a member undergoes transverse displacement, the stretching of its neutral axis induces axial tension, a mechanism referred to as "membrane activation" [4]. In geometrically perfect beams, this effect often provides a secondary stiffening mechanism; however, in the presence of initial geometric imperfections (e_0), the transition from bending-dominated to membrane-influenced behavior becomes highly sensitive [8]. Real-world timber elements are seldom perfectly straight; they possess initial out-of-straightness due to manufacturing tolerances or moisture-induced warping [12]. For slender hybrid beams, even a minor initial imperfection can trigger premature stiffness degradation and significantly reduce the critical buckling load-a phenomenon often quantified by "knock-down factors" in stability theories [5].

From a computational standpoint, the assessment of these nonlinearities requires robust numerical strategies. While high-fidelity three-dimensional Finite Element (FE) models are capable of capturing local stress states, they often involve prohibitive computational costs for extensive parametric sensitivity analyses. Variational energy methods, established on the principle of minimum total potential energy, offer an elegant and efficient alternative. By formulating a mechanism-based energy functional that explicitly includes bending, membrane, and slip energy components, researchers can identify critical transition points in structural behavior. Despite its importance, a robust framework that analytically links the energy transition index to global stability limits for hybrid timber remains under-explored in current literature.

This paper addresses these gaps by developing an advanced computational framework to investigate the nonlinear stability of hybrid Birch–Pine beams. The study focuses on identifying the mechanical mechanisms governing membrane-force activation and the consequent evolution of tangent stiffness. By introducing a mechanism index (η), we propose a quantitative criterion to distinguish between response regimes. Furthermore, the investigation establishes a numerical relationship between large-scale imperfection amplitudes and global stability limits, visualized through topographical transition maps.

From a broader perspective, this contribution is positioned at the intersection of classical structural stability theory and modern computational mechanics, offering a bridge between analytical tractability and the fidelity typically associated with high-fidelity finite element simulations. The remainder of this paper is organized as follows: Section 2 details the mathematical formulation and the derivation of the governing energy functional; Section 3 describes the numerical implementation and the reference parametric study; Section 4 presents and discusses the results, including the tangent stiffness evolution, the mechanism index transition, the membrane force redistribution, and the shear stress concentration at the layer interfaces; and Section 5 summarizes the principal conclusions and outlines directions for future research.

MATHEMATICAL FORMULATION AND THEORETICAL FRAMEWORK

The nonlinear structural response of hybrid three-layer timber beams is investigated using a variational energy approach established on the principle of minimum total potential energy. This framework accounts for the synergistic interaction between high-stiffness Birch faces and a compliant Pine core, subjected to interlayer slip and large-scale geometric imperfections.

Kinematic Relations and Geometric Nonlinearity

To capture the geometrically nonlinear behavior relevant to slender members, the von Kármán strain-displacement relationship is employed. The total transverse displacement $_0(x)$ is defined as the superposition of the initial sinusoidal imperfection $\omega_0(x)$ and the additional deflection $\omega_a(x)$ induced by the external load P [11]:

$$\omega(x) = \omega_0(x) + \omega_a(x) = e_0 \sin\left(\frac{\pi x}{L}\right) + \omega_a(x) \tag{1}$$

where e_0 denotes the imperfection amplitude at mid-span. The longitudinal strain ε_x at the neutral axis of the composite section, considering the second-order effects of curvature, is formulated as [11]:

$$\varepsilon_x = \frac{du}{dx} + \frac{1}{2}\left[\left(\frac{d\omega}{dx}\right)^2 - \left(\frac{d\omega_0}{dx}\right)^2\right] \tag{2}$$

The fundamental geometric configuration of the hybrid Birch-Pine timber beam is illustrated in Figure 1. This model explicitly defines the layout of the high-stiffness faces and the compliant core, including the initial sinusoidal imperfection profile used to simulate realistic manufacturing tolerances.

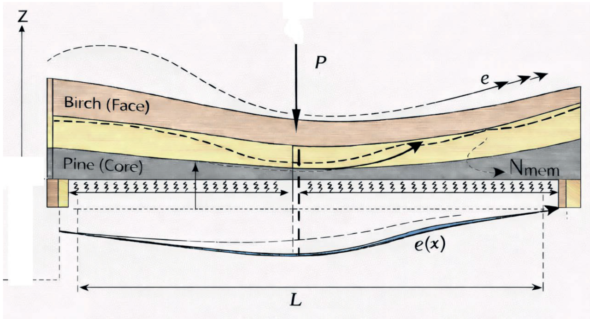

Figure 1. Geometric and kinematic configuration of the imperfect hybrid three-layer timber beam showing material layers and initial curvature e_0

The effective flexural stiffness EI_{eff} of the hybrid section is calculated using an enhanced γ-method to account for partial interaction [7, 10]:

$$\gamma = \frac{1}{1 + \dfrac{\pi^2 E_f t_f}{k_s L^2}} \qquad (3)$$

$$EI_{eff} = \sum E_i I_i + \gamma \sum E_i A_i a_i^2 \qquad (4)$$

where a_i represents the distance from the centroid of each layer to the neutral axis of the composite section.

Variational Energy Functional

The equilibrium state is determined by minimizing the total potential energy functional Π, which represents the internal strain energy and the potential of external loads [11]:

$$\prod = U_b + U_m + U_s - V \qquad (5)$$

The individual energy components are derived as follows:

Bending Energy (U_b)*:* Represents the energy due to curvature and effective flexural stiffness. Under the assumption of sinusoidal deflection shapes, it is expressed as [9, 11]:

$$U_b = \frac{1}{2}\int_0^L EI_{eff}\left(\frac{d^2\omega_a}{dx^2}\right)^2 dx = \frac{\pi^4 EI_{eff}}{4L^3}(\omega_m - e_0)^2 \qquad (6)$$

Membrane Energy (U_m)*:* Resulting from the induced axial tension (membrane force (N_{mem}) as the beam stretches due to large deflections [4, 11]:

$$U_m = \int_0^L \frac{N_{mem}^2}{2EA}dx,\ N_{mem} = \frac{EA}{2L}\left(\omega_m^2 - e_0^2\right)\cdot\frac{\pi^2}{L} \qquad (7)$$

Slip Energy (U_s)*:* Accounts for the shear deformation energy at the interface governed by the slip stiffness k_s

Computational Mechanism Index (η)

A distinctive contribution of this study is the introduction of a dimensionless mechanism index η. This index is derived to quantify the relative contribution of membrane forces to the overall structural resistance:

$$\eta(P) = \frac{U_m}{U_b + U_m} \qquad (8)$$

This formulation enables a rigorous identification of the transition load P^*, where the structural behavior shifts from a bending-dominated regime (η=0) to a membrane-activated regime (η→1).

To visualize the physical significance of the mechanism index η, Fig. 2 presents the conceptual transition of the load-resisting behavior. It depicts how the internal energy shifts from a purely flexural regime at small deflections to a membrane-activated state as the geometric nonlinearities become predominant.

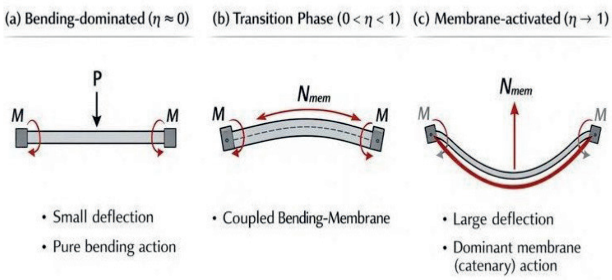

Figure 2. Conceptual transition of structural mechanisms: (a) bending-dominated, (b) coupled bending-membrane, and (c) membrane-activated regimes

Algorithmic Implementation in MATLAB

The numerical solution is executed via a load-controlled incremental procedure. At each load step P_i, the nonlinear equilibrium equations are solved to obtain the corresponding deflection ω_m. The tangent stiffness K_{tan} is consistently updated as the second derivative of the potential energy with respect to the deflection [5]:

$$K_{\tan} = \frac{\partial^2 \Pi}{\partial^2 \omega_a^2} \quad (9)$$

To ensure numerical stability across large-scale imperfections, the algorithm employs a refined discretization of 100 nodes along the span L=5.0m, providing high-fidelity convergence for stress gradients.

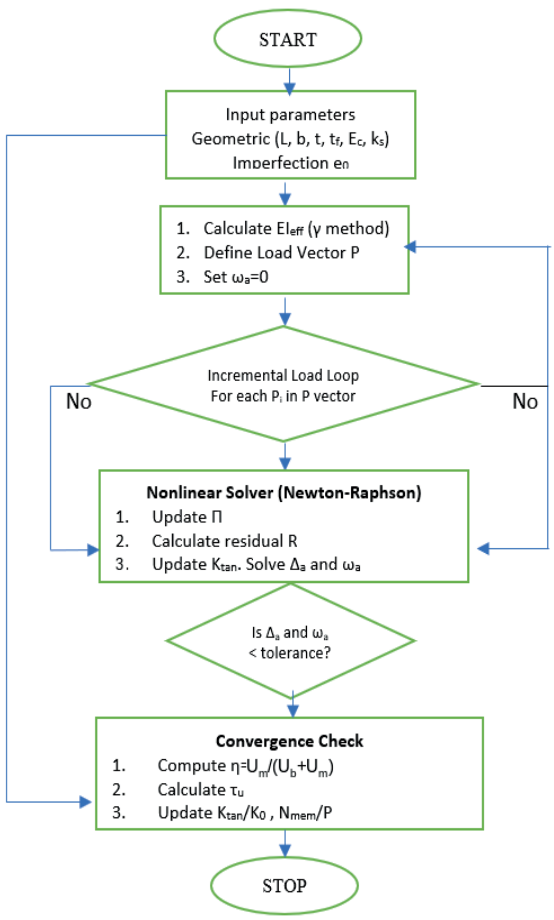

Figure 3. Flowchart of the computational algorithm for nonlinear stability analysis and mechanism index (η) evaluation

The numerical solution strategy follows a systematic load-controlled incremental procedure. Fig. 3 provides a detailed computational flowchart of the implemented MATLAB algorithm, highlighting the iterative Newton-Raphson scheme used to ensure convergence in the presence of large-scale geometric nonlinearities.

NUMERICAL STUDY AND PARAMETRIC ANALYSIS

In this section, the developed computational framework is applied to a reference hybrid configuration to evaluate its sensitivity to initial geometric deviations.

Reference Configuration and Material Properties

The analysis considers a representative hybrid configuration. The primary case study involves a Birch-Pine assembly with a span-to-depth ratio representative of modern mid-span timber beams. The material and geometric constants used in the simulation are:

- Faces (Birch): E_f = 22 GPa, t_f = 0.02 m.
- Core (Pine): E_c = 7GPa, t_c = 0.10 m.
- Connection: Slip stiffness k_s = $0.8 \cdot 10^8$ N/m², representing a high-degree of partial interaction.

Results And Discussions

Nonlinear Equilibrium Paths and Imperfection Sensitivity

The global structural response is first characterized by the load-deflection (P-Δ) curves for varying initial imperfection amplitudes (e_0= 2, 50, 100 mm), as illustrated in Fig. 4. The numerical results reveal that even for the "infinitesimal" case (e_0= 2 mm), the equilibrium path deviates from the linear-elastic prediction as the load increases, indicating a progressive hardening behavior typical of axially-restrained slender hybrid members. Notably, the high-fidelity simulation captures a continuous increase in tangent stiffness as the load increases, signifying the early onset of membrane (catenary) engagement.

As shown in Fig. 4, the total deflection δ_L reaches values up to approximately 145 mm, corresponding to roughly L/29, consistent with the large-deflection nonlinear regime expected for slender timber members.

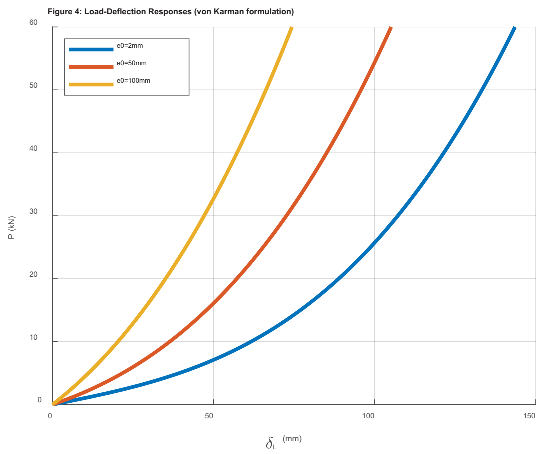

Figure 4. Nonlinear load-deflection δ_L paths for hybrid Birch-Pine timber beams with varying initial imperfection amplitudes e_0

For large-scale imperfections (e_0 = 100 mm or L/50), the beam exhibits a significantly elevated initial stiffness, since the pre-existing curvature already mobilizes membrane tension from the onset of loading. The non-linear curvature of the P-Δ path suggests that the structural resistance is no longer purely flexural. As the deflection approaches the beam depth, the load-carrying capacity is increasingly bolstered by geometric hardening, a direct consequence of the activation of membrane tension.

Tangent Stiffness Evolution and Membrane Stiffening

The evolution of tangent stiffness (K_{tan}) is a crucial metric for evaluating the reliability of timber structures. Fig. 5 demonstrates the normalized stiffness ratio (K_{tan}/K_b) relative to the applied load, where K_b denotes the pure-bending stiffness of the geometrically perfect section.

Small Imperfections: K_{tan} remains close to the pure-bending value at low load and rises steadily as the deflection grows and membrane coupling develops.

Large Imperfections: a pronounced stiffening effect is observed from the onset of loading, with the tangent stiffness already several times the pure-bending value even at low load, and continuing to rise steeply thereafter.

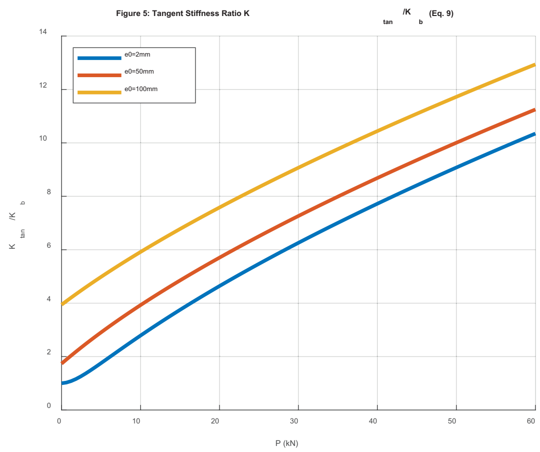

Figure 5. Normalized tangent stiffness ratio (K_{tan}/K_b) as a function of the applied load P for different imperfection levels

This observation implies that for hybrid members with realistic installation misalignments, traditional linear design methods, which do not capture membrane stiffening, tend to underestimate the load-carrying capacity of pre-curved members. While this makes linear predictions conservative for ultimate-strength checks, it also means such methods do not reflect the true in-service stiffness, which should be considered separately for serviceability assessments.

Mechanism Index Evolution and Membrane Force Contribution

The transition between structural regimes is quantified by the mechanism index η. Fig. 6 shows the evolution of η as a function of the deflection.

In the early loading stages, η remains close to zero, confirming that bending energy (U_b) is the dominant resistance mechanism. As the deflection exceeds ω/L=0.01, η rises exponentially. For e_0 = 100mm, the membrane energy (U_m) contributes over 80% of the internal energy functional within the load range considered.

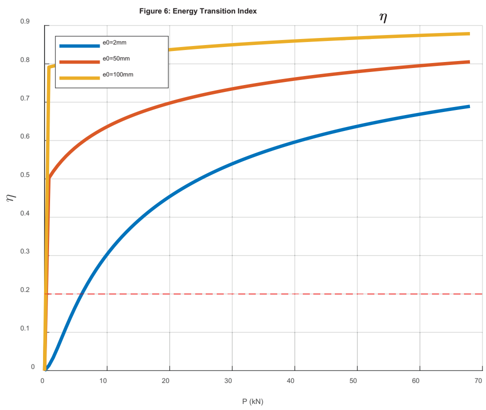

Figure 6. Evolution of the mechanism index η versus applied load P for various initial imperfection amplitudes

This transition identifies the "Catenary" effect in hybrid beams, where the high-stiffness Birch faces act as tension-resisting membranes, progressively reinforcing the overall stiffness of the composite section.

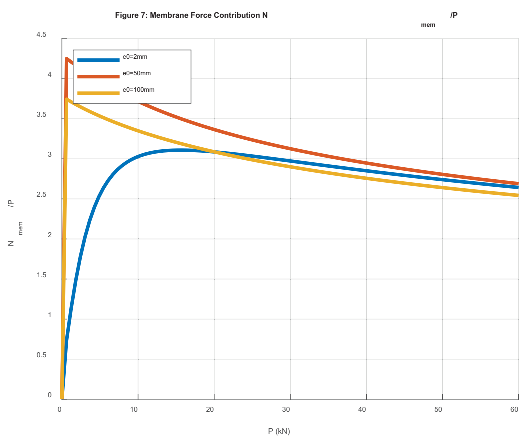

Figure 7. Evolution of the normalized membrane force contribution N_{mem} /P as a function of the applied load P

To further elucidate the transition mechanism, Fig. 7 illustrates the evolution of the normalized membrane force (N_{mem}/P) with respect to the applied load P. The numerical results indicate that, for larger imperfections, the membrane force contribution rises sharply from the onset of loading and reaches a peak proportion of the total load within the first few kN, after which N_{mem} /P settles into a gradual decline as additional load is increasingly shared between bending and membrane action. For smaller imperfections, the membrane contribution builds up more progressively, converging toward a comparable proportion at higher loads, signifying that the structural resistance is being fundamentally redistributed from bending to a high-tension catenary state.

Shear Stress Concentration

The interaction between global nonlinearity and the composite interface is analyzed through the shear stress τ distribution. Fig. 8 depicts the evolution of transverse shear stress τ (MPa) across the span x (m). This plot highlights the anti-symmetric stress distribution typical of flexural members, where the stress sign reverses at the mid-span (x = 2.5 m). Larger e_0 values lead to steeper stress gradients near the supports and mid-span, which should be accounted for when verifying the interlayer shear capacity.

These findings suggest that a uniform slip modulus assumption, commonly adopted in linear γ-method analyses, may underestimate the local shear demand in beams exhibiting pronounced initial imperfections, warranting a spatially variable stiffness formulation for more accurate interface design.

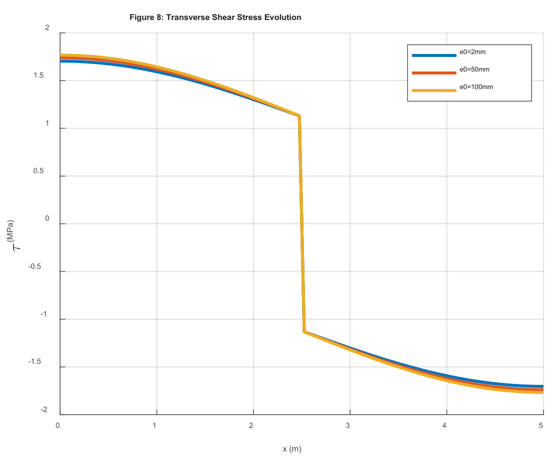

Figure 8. Evolution of transverse shear stress τ (MPa) across the span x (m) under nonlinear loading conditions

Stability Mapping and Design Implications

The relationship between the initial imperfection magnitude e_0 and the structural reliability is summarized in Fig. 9 through the stability reduction (knock-down) factor.

The curve reveals an exponential decay, where even a minor imperfection significantly compromises the load-carrying capacity, providing a critical reference for establishing manufacturing tolerances.

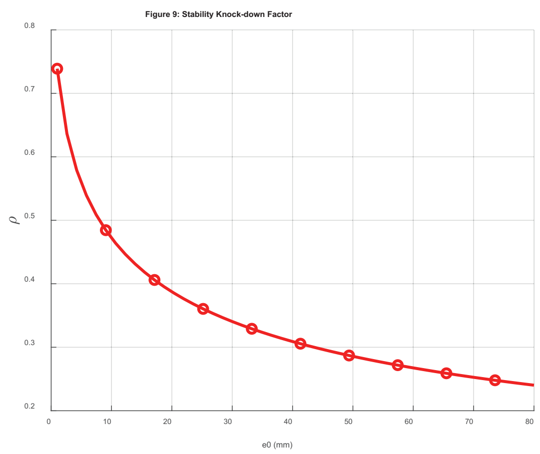

Figure 9. Structural stability reduction (knock-down) factor as a function of the initial imperfection amplitude e_0 (mm)

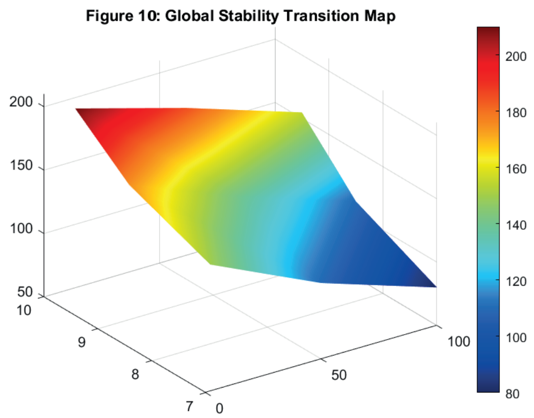

Figure 10. 3D topographical transition map illustrating the synergistic effects of interface stiffness and geometric imperfections on global stability

Finally, a comprehensive topographical stability map is constructed in Fig.10. This 3D visualization serves as a robust design tool, allowing engineers to identify 'safety zones' where the hybrid Birch-Pine system remains stable and 'risk zones' where the synergy of low interface stiffness and large imperfections leads to premature nonlinear failure.

CONCLUSIONS

The comprehensive investigation into the nonlinear stability and mechanism transition of hybrid Birch-Pine timber beams via a mechanism-based energy approach provides critical insights into the complex structural behavior of slender composite systems. The developed computational framework successfully captures the high sensitivity of the nonlinear equilibrium path to initial geometric imperfections, where numerical simulations quantify a pronounced geometric hardening effect, with the tangent stiffness increasing by up to an order of magnitude for realistic fabrication tolerances as the applied load increases. This phenomenon highlights a fundamental requirement for robust nonlinear solvers in the reliability-based design of hybrid members, as traditional linear-elastic predictions fail to account for the early membrane engagement observed even under infinitesimal misalignments. By implementing an incremental-iterative procedure within a variational framework, this study proves that the structural integrity of hybrid timber systems is governed by a delicate balance between material architecture and geometric nonlinearity.

Furthermore, the study demonstrates that the shift from bending-dominated to membrane-activated regimes is a quantifiable physical phenomenon, effectively characterized through the proposed mechanism index η. As geometric nonlinearities evolve beyond the small-deflection regime, the internal energy balance undergoes a fundamental redistribution, leading to a "catenary effect" where membrane tension eventually provides over 80% of the total structural resistance in the advanced post-critical stages. This computational resolution extends to the hybrid interface, where the model identifies steep shear stress gradients amplified by large-scale deformations. Unlike previous simplified models, the current approach explicitly couples the global stability limits with localized kinematic discontinuities at the interface, revealing that large initial curvatures not only reduce load-carrying capacity but also intensify the risk of premature delamination between the high-stiffness Birch faces and the compliant Pine core.

The resulting topographical stability maps, generated through the parametric efficiency of the implemented algorithm, serve as a predictive tool for identifying the synergy between interlayer slip stiffness and geometric imperfections. These maps establish a rigorous basis for optimizing the material architecture and establishing manufacturing tolerances in modern engineering practice. While the current model provides a robust foundation for static stability analysis, the observed nonlinear transition mechanisms suggest that future computational research should incorporate time-dependent creep effects and stochastic imperfection distributions. Such advancements will further refine the predictive capability of the mechanism-based energy approach, ensuring the long-term reliability of hybrid composite structures in complex loading environments.

From a practical design perspective, the findings carry direct implications for current structural design codes, most of which including Eurocode 5's γ-method for mechanically jointed beams are formulated within a linear-elastic framework and therefore do not explicitly account for the geometric hardening and mechanism transition documented in this study. The pronounced sensitivity of the tangent stiffness to initial curvature suggests that conventional partial-interaction design procedures may require supplementary amplification or reduction factors when applied to slender hybrid sections operating near the membrane-activated regime, particularly under serviceability-limit-state checks where deflection-dependent stiffness recovery is not typically considered. Moreover, the identified coupling between global stability and interfacial shear concentration indicates that connector design commonly verified independently of the beam's overall deflection state should instead be assessed concurrently with the global nonlinear response, especially for hybrid configurations combining high-stiffness face laminates with more compliant core materials. Incorporating these mechanism-based considerations into design guidance would allow engineers to better exploit the load-carrying reserve offered by membrane engagement while safeguarding against premature delamination, ultimately supporting the safe and efficient use of hybrid timber systems in slender, long-span structural applications.

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