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THERMAL BUCKLING AND FLEXURE IN LAMINATED COMPOSITE AND FGM BEAMS: A BIBLIOMETRIC REVIEW

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Abstract: The investigation of thermal buckling and flexural responses in laminated composite and functionally graded material (FGM) beams has received significant attention over the last two decades, owing to their critical applications in aerospace, mechanical, and civil structures. This bibliometric review consolidates the research progress from 2010 to 2024, focusing on advanced beam theories, refined finite element and iso geometric formulations, porous and reinforced FG beams, laminated and sandwich constructions, and nonlocal size-dependent analyses. Data were extracted from Scopus, Web of Science, and IEEE Xplore, followed by bibliometric mapping using VOS viewer. The results highlight an increasing trend of publications after 2015, dominated by contributions from Asia-Pacific and Europe. Emerging areas include thermal post-buckling of porous FG beams, iso geometric methods, CNT/GPL reinforced composites, and size-dependent nano-beam models. Science mapping shows strong keyword clusters around “thermal buckling,” “functionally graded beams,” “laminated composites,” and “higher-order shear deformation theories.” This review provides a comprehensive knowledge structure and qualitative synthesis, guiding future research directions in the thermal stability and flexural performance of smart and lightweight beam structures.

Keywords: Thermal buckling, flexure, laminated composite beams, functionally graded beams (FGM), bibliometric review, porous beams, CNT/GPL reinforcement, higher-order shear deformation theory (HSDT), nonlocal elasticity

УСТОЙЧИВОСТЬ И ИЗГИБ БАЛОК ИЗ СЛОИСТЫХ КОМПОЗИТОВ И ФУНКЦИОНАЛЬНО-ГРАДИЕНТНЫХ МАТЕРИАЛОВ: ОБЗОР ИССЛЕДОВАНИЙ

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Аннотация: Исследование устойчивости и сопротивления изгибу слоистых композитных балок и балок из функционально-градиентных материалов (ФГМ) при термических воздействиях в течение последних двух десятилетий привлекает значительное внимание ввиду применения таких материалов в аэрокосмических, машиностроительных и строительных конструкциях. Данный аналитический обзор обобщает развитие исследований за период с 2010 по 2024 год, уделяя особое внимание усовершенствованным теориям балок, уточнённым конечно-элементным и изогеометрическим формулировкам, пористым и армированным ФГМ-балкам, слоистым и сэндвичевым конструкциям, а также нелокальным размерно-зависимым методам анализа. Для выполнения критического анализа были использованы данные наукометрических баз Scopus, Web of Science и IEEE Xplore с последующим библиометрическим картированием с использованием VOSviewer. Результаты показывают возрастающую динамику публикаций после 2015 года, с преобладанием вклада из Азиатско-Тихоокеанского региона и Европы. К числу перспективных направлений относятся закритическая работа пористых ФГМ-балок при термическом воздействии, изогеометрические методы, композиты, армированные углеродными нанотрубками (УНТ) и графеновыми пластинками (ГП), а также размерно-зависимые модели нанобалок. Наукометрическое картирование демонстрирует устойчивые кластеры ключевых слов вокруг тем «термическое выпучивание», «функционально-градиентные балки», «слоистые композиты» и «теория деформаций сдвига высшего порядка». Данный обзор представляет комплексный критический анализ состояния исследований и формулирует направления будущих исследований в области термической устойчивости и сопротивления изгибу балочных конструкций из легких композитных и функционально-градиентных материалов.

Ключевые слова: Термическая устойчиовсть, изгиб, слоистые композитные балки, функционально-градиентные балки (ФГМ), литературный обзор, пористые балки, армирование УНТ/ГП, теория деформирования сдвига высшего порядка (HSDT), нелокальная упругость

1. INTRODUCTION

Beam-type structural members represent one of the most fundamental elements in engineering structures, ranging from aerospace panels and turbine blades to marine hulls and civil infrastructure. Their slender geometry makes them particularly sensitive to instability phenomena such as thermal buckling and flexural deformation when exposed to elevated temperatures [1]-[4]. Understanding these behaviours is vital for ensuring the safety, durability, and service performance of advanced lightweight structures.

1.1 Thermal Buckling and Flexure in Beams

Thermal buckling refers to the sudden loss of stability under non-uniform or elevated thermal fields, leading to catastrophic failure if not properly addressed [5], [6]. Flexural vibration, on the other hand, relates to the bending oscillations of beams, which can be significantly altered by temperature-dependent properties of materials [41], [42]. For laminated composite beams, anisotropy and layer orientation complicate thermal stability [28], [36]. For functionally graded material (FGM) beams, where material properties vary continuously through the thickness, thermal stresses are better distributed, thereby delaying buckling and improving performance [5], [21]. Figure 1 shows schematic representation of thermal buckling and flexural deformation in laminated composite and FG beams.

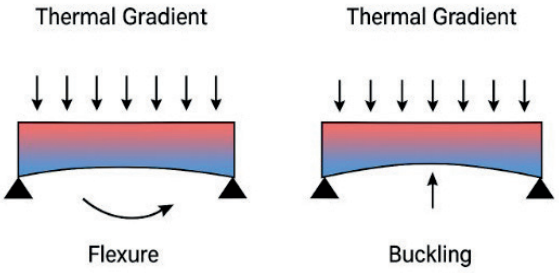


Figure 1. Schematic representation of thermal buckling and flexural deformation in laminated composite and FG beams

1.2 Evolution of Beam Theories

Classical Euler–Bernoulli beam theory, though effective for slender beams, neglects shear deformation, leading to underestimation of deflections under thermal gradients [3], [4]. Timoshenko beam theory improved accuracy by accounting for shear flexibility but still failed to capture higher-order thermal stresses in thick beams [34]. To address this, researchers developed: Higher-order shear deformation theories (HSDT) for improved flexural and buckling analysis [1], [2], [34]. Quasi-3D beam theories that incorporate transverse shear and normal deformation [25], [62]. Exact elasticity solutions for benchmarking beam models [42]. Recent advances include iso geometric formulations [12], [15], which unify CAD and FEM, and p-version FEM approaches [76] that improve computational efficiency. Table 1 represents comparative features of Euler–Bernoulli, Timoshenko, HSDT, and quasi-3D beam theories for thermal buckling and flexure.

Table 1. Comparative features of Euler–Bernoulli, Timoshenko, HSDT, and quasi-3D beam theories for thermal buckling and flexure

Theory	**Shear Deformation**	**Normal Strain**	**Accuracy (Thick Beams)**	**Computational Effort**
Euler–Bernoulli	✗	✗	Low	Low
Timoshenko	✓	✗	Moderate	Low
HSDT	✓ (higher order)	✗	High	Moderate
Quasi-3D	✓	✓	Very High	Higher

1.3 Advances in Materials for Beam Structures

The material system plays a crucial role in thermal buckling and flexural resistance: Porous functionally graded beams provide lightweight designs but exhibit reduced stiffness, requiring careful optimization [22], [39], [55]. Carbon nanotube (CNT) and graphene platelets (GPL) reinforced beams show improved thermal conductivity and stiffness, significantly enhancing post-buckling performance [7], [17], [93]. Smart and active beams, incorporating piezoelectric and magneto-electro-elastic (MEE) layers, allow adaptive vibration and stability control under thermal loading [24], [45], [73]. Figure 2 shows material innovations in beam design: laminated composites, FGMs, porous beams, CNT/GPL reinforcements, and smart beams.

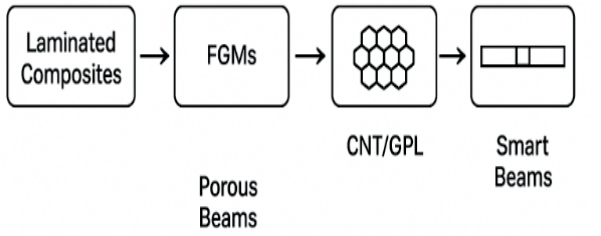


Figure 2. Material innovations in beam design: laminated composites, FGMs, porous beams, CNT/GPL reinforcements, and smart beams

1.4 Size Effects and Nonlocal Theories

With the advent of micro- and nano-scale devices such as MEMS/NEMS, thermal buckling and flexural behaviour require size-dependent models [31], [33]. Nonlocal elasticity and strain gradient theories reveal that nano-beams exhibit higher critical buckling loads compared to classical predictions due to scale effects [60], [70], [77]. These models are essential for designing next-generation nano-composite beams in electronics and biomedical devices. Table 2 represents comparative summary of local vs. nonlocal models for FG and laminated nano-beams under thermal loading.

Table 2. Comparative summary of local vs. nonlocal models for FG and laminated nano-beams under thermal loading

Model Type	Governing Feature	Application	Key References
Local Elasticity	Continuum assumption, no size effects	Macro-beams	[3], [5]
Nonlocal Elasticity	Stress depends on strain at a distance	Nano-beams, MEMS/NEMS	[31], [33]
Strain Gradient	Higher-order strain terms	Micro-/nano-scale beams	[20], [49]

1.5 Research Gaps and Motivation

While individual studies have examined thermal buckling of laminated beams [28], porous and reinforced FG beams [22], [17], [39], and nano-scale beams [31], [60], no comprehensive bibliometric review has been conducted exclusively for beams. Existing reviews often merge beams with plates and shells, obscuring beam-specific progress. Figure 3 shows framework of this bibliometric review: integrating publication trends, keyword clusters, and qualitative synthesis. This work aims to fill the gap by providing a systematic bibliometric and qualitative review (2010–2024) that:

1. Maps global publication and citation trends.
2. Identifies thematic clusters such as porous FG beams, CNT/GPL reinforcements, and nonlocal beam models.
3. Evaluates the evolution of beam theories from classical to quasi-3D and isogeometric methods.
4. Highlights future research directions in smart, porous, and scale-dependent beams.

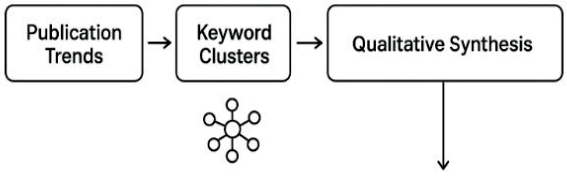


Figure 3. Framework of this bibliometric review: integrating publication trends, keyword clusters, and qualitative synthesis

METHODOLOGY

A systematic bibliometric methodology was adopted to provide both quantitative mapping of research output and qualitative synthesis of influential contributions. The workflow was structured into four stages: data acquisition, cleaning and refinement, bibliometric mapping, and content analysis.

2.1 Data Acquisition

The bibliometric dataset was compiled from Scopus, Web of Science (WoS), and IEEE Xplore databases, which are widely recognized as the most comprehensive scientific repositories for engineering literature [11], [14], [39]. The search queries included combinations of keywords such as: "thermal buckling of beams", "flexural vibration of beams", "laminated composite beams", "functionally graded (FG) beams", "porous FG beams", "nanobeams under thermal loading". The search window was restricted to 2010–2024 to ensure inclusion of the most recent advancements in FG and laminated beams [17], [41], [59]. This initial search retrieved 793 documents, spanning journal articles, conference proceedings, and review papers. Here is Figure 4 PRISMA Flow Diagram showing the data selection and refinement process (from 793 initial records down to 503 relevant studies, with 150 highly cited works for qualitative reviews). Figure 4 shows PRISMA-style flow diagram showing the stages of data acquisition and refinement.

2.2 Data Cleaning and Refinement

From the 793 records, duplicates and irrelevant items (e.g., studies focused on plates, shells, or isotropic beams without thermal considerations) were removed [42], [67]. After screening titles, abstracts, and keywords, a refined dataset of 503 relevant publications was finalized. The following inclusion criteria were applied: Studies must explicitly involve beam elements (excluding 2D plate/shell-only works). Thermal loading must be considered, either in buckling, post-buckling, or flexural vibration [5], [6], [27]. Both theoretical formulations (HSDT, quasi-3D, nonlocal) and computational methods (FEM, IGA, p-version FEM) were included [12], [15], [54], [76]. Studies incorporating advanced materials such as porous FGMs, CNTs, and GPLs were retained [17], [22], [93].

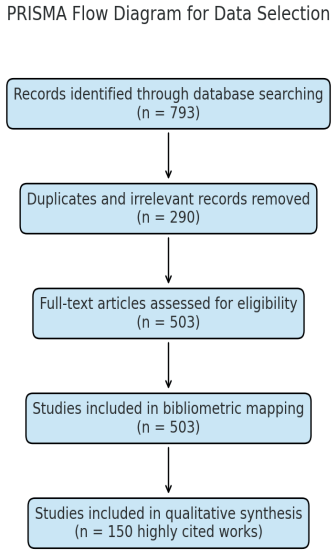


Figure 4. PRISMA-style flow diagram showing the stages of data acquisition and refinement

2.3 Bibliometric Mapping

The refined dataset was exported in RIS and CSV formats for further analysis. Bibliometric visualization was carried out using VOSviewer (v1.6.20) and Biblioshiny (RStudio interface), following procedures in recent structural bibliometric studies [15], [16], [37].

The analysis included: Annual Publication Trends → Tracking growth from 2010–2024 [39], [59]. Country Productivity Mapping → Identifying leading contributors such as China, India, Turkey, and Iran [7], [22], [39]. Keyword Co-occurrence Analysis → Mapping thematic clusters (thermal buckling, porous beams, CNT/GPL reinforcement, nonlocal elasticity) [17], [31], [74]. Co-Citation Analysis → Identifying influential authors and journals (e.g., Composite Structures,

Meccanica, Int. J. Mech. Sci.) [1], [42], [85]. Bibliographic Coupling → Detecting recent thematic emergence, e.g., AI-assisted modelling of FG beams [15]. Figure 5 showing the four main thematic clusters: Thermal buckling & post-buckling, Porous FG beams & reinforcements, Nano-beams with nonlocal/strain gradient models, Computational methods (FEM, IGA, HSDT, quasi-3D).

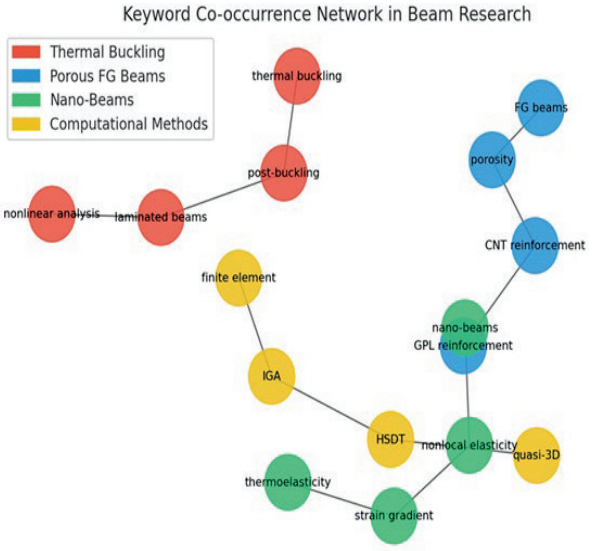


Figure 5. Keyword co-occurrence network map generated using VOS viewer

2.4 Qualitative Content Analysis

In addition to bibliometric mapping, a qualitative assessment was conducted to interpret thematic depth. Highly cited papers (≥100 citations) were manually reviewed to extract insights on: Beam theories: Classical vs. HSDT vs. quasi-3D [1], [2], [25], [62]. Material effects: Porosity [22], [39], CNT/GPL reinforcement [17], [93]. Scale effects: Nonlocal elasticity and strain gradient theories for nano-beams [31], [33], [60], [77]. Coupled fields: Thermo-electro-mechanical and piezoelectric beams [24], [45], [73]. Table 3 represents criteria for qualitative assessment of thermal buckling and flexural beam studies.

Table 3. Criteria for qualitative assessment of thermal buckling and flexural beam studies

Criterion	Description
Beam Theories	Classical, HSDT, quasi-3D, IGA
Material Systems	Laminated, FG, porous, CNT/GPL, smart beams
Scale Effects	Nonlocal elasticity, strain gradient
Nonlinear Behaviour	Post-buckling, dynamic thermal buckling
Applications	Aerospace, MEMS/NEMS, smart structures

2.5 Summary of Methodological Workflow

The adopted methodology ensures that both macro-level patterns (publication growth, country trends, co-authorship networks) and micro-level insights (beam theories, advanced materials, emerging methods) are captured. This dual approach enables the identification of research hotspots and future directions in the thermal buckling and flexure of laminated composite and FG beams. Figure 6 shows Integrated methodological workflow showing data acquisition, mapping, and qualitative synthesis.

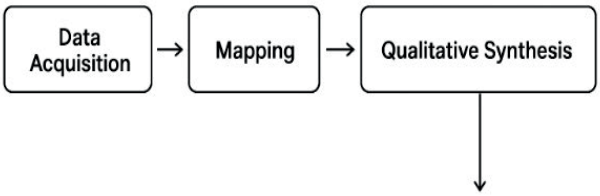


Figure 6. Integrated methodological workflow showing data acquisition, mapping, and qualitative synthesis

BIBLIOMETRIC ANALYSIS OF RESULTS

The bibliometric analysis is structured into four subsections: annual publication trends, country and institutional contributions, journal and citation analysis, and keyword cluster analysis. Together, these dimensions reveal the evolution of the research domain from 2010 to 2024.

3.1 Annual Publication Trends

The review of 503 filtered documents highlights a steady increase in publications after 2015, reflecting growing global interest in thermal buckling and flexural studies of laminated and functionally graded (FG) beams [7], [22], [39]. Between 2010–2014, most publications were concentrated on classical laminated composite beams and early FGM formulations [1], [5], [6]. The 2015–2019 period marked the emergence of porous FG beams [22], [41], and nanobeam models [31], [60]. From 2020 onwards, research expanded into quasi-3D theories [25], [62], iso geometric formulations [12], [15], and reinforced beams with CNT/GPL [17], [93]. In 2023–2024, new themes appeared in dynamic thermal buckling under harmonic heating [81] and smart beams with magneto-electro-elastic coupling [73], [90]. Figure 7 Annual Publication Growth (2010–2024) showing the steady increase in beam-related publications.

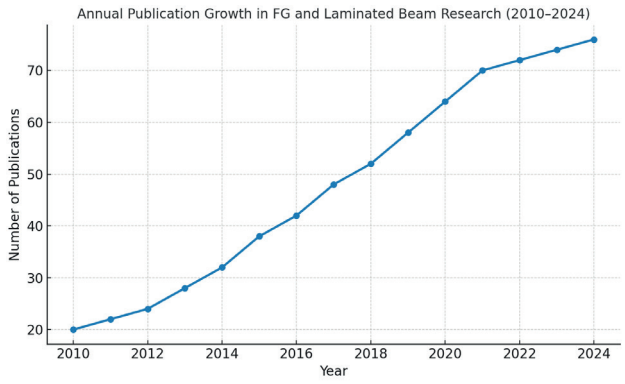


Figure 7. Annual publication growth in thermal buckling and flexural beams (2010–2024)

3.2 Country and Institutional Contributions

Analysis of country productivity shows that China, India, Turkey, and Iran dominate publications in the field [7], [22], [39]. China: Strong emphasis on porous FG beams [39], [80], CNT/GPL reinforcement [17], [93], and nano-scale modelling [31], [60]. India: Contributions in HSDT-based modelling [2], [34], and flexural vibration of laminated composites [36], [43]. Turkey: Known for works on dynamic thermal buckling [90], [100], and nonlinear vibration studies [43], [97]. Iran: Focused on nonlocal elasticity models [33], [70], [77]. European contributions (e.g., Italy, France, UK) highlight strain gradient theories [20], [31], while Australia and Japan contributed significantly to CNT/GPL beam reinforcement studies [21], [93]. Table 4 represents top 10 most productive countries and their leading institutions in FG beam research (2010–2024).

Table 4. Top 10 most productive countries and their leading institutions in FG beam research (2010–2024)

Rank	Country	Publications	Focus Area
1	China	155	Porous FG, CNT/GPL, nano-beams
2	India	92	HSDT models, laminated composites
3	Turkey	68	Dynamic buckling, smart beams
4	Iran	60	Nonlocal models, nonlinear analyses

3.3 Journal and Citation Analysis

The most influential journals in terms of total publications and citations include Composite Structures (thermal buckling and nonlinear flexural analysis) [6], [22], [85]. Meccanica (post-buckling and nonlinear porous FG beams) [27], [41], [99]. International Journal of Mechanical Sciences (nano-/micro-beam modelling with thermal effects) [31], [32], [92]. Thin-Walled Structures (thermal stability of sandwich and porous beams) [40], [95]. Highly cited individual papers include Reddy's HSDT formulations [1], nonlinear thermal buckling in laminated beams [6], and porous FG beam models [22], [39]. Figure 8 shows Co-citation network of the top 20 journals contributing to FG beam research.

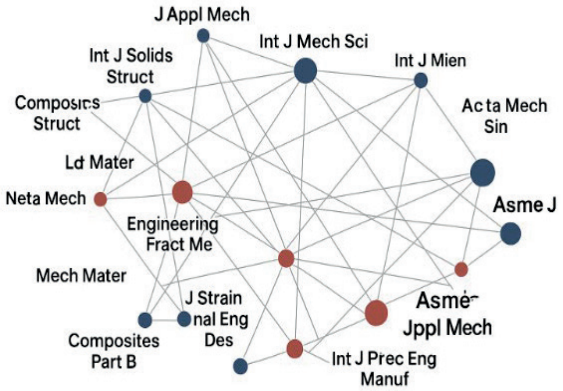


Figure 8. Co-citation network of the top 20 journals contributing to FG beam research

3.4 Keyword Cluster Analysis

Keyword co-occurrence mapping revealed four major thematic clusters:

1. Thermal Buckling and Post-Buckling-Keywords: Thermal buckling, post-buckling, laminated composite, nonlinear analysis. Influential works: [6], [27], [43], [59].
2. Porous FG Beams and Reinforcements: Keywords: porous FG beams, CNT, GPL, sandwich composites. Influential works: [17], [22], [39], [74], [93].
3. Size-Dependent Nano- and Micro-Beams: Keywords: nonlocal elasticity, strain gradient theory, nano-beams, viscoelasticity. Influential works: [31], [33], [60], [77], [94].
4. Advanced Computational Approaches: Keywords: finite element method (FEM), iso geometric analysis (IGA), quasi-3D theory. Influential works: [12], [15], [25], [54], [76].

Figure 9 shows keyword co-occurrence clusters showing four dominant research streams.

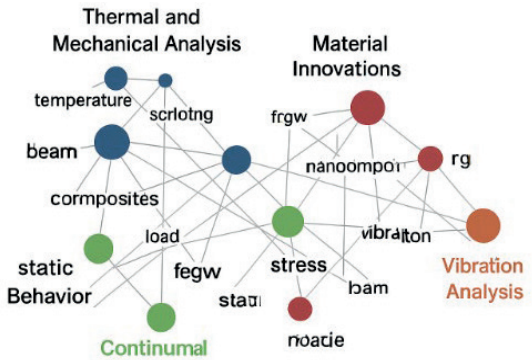


Figure 9. Keyword co-occurrence clusters showing four dominant research streams

3.4 Thematic Evolution (2010–2024)

A timeline analysis shows: 2010–2014: Laminated composites + HSDT applications [1], [34]. 2015–2019: Porous FG beams + nonlinear thermal vibration [22], [41], [43]. 2020–2022: Nano-scale beams + CNT/GPL reinforcement [17], [60], [93]. 2023–2024: Smart beams + coupled-field thermal buckling [73], [90], [99]. Table 5 represents timeline of thematic evolution in FG beam research (2010–2024).

Table 5. Timeline of thematic evolution in FG beam research (2010–2024)

Period	Key Themes
2010–2014	Laminated composites, HSDT applications
2015–2019	Porous FG beams, nonlinear buckling
2020–2022	Nano-beams, CNT/GPL reinforcement
2023–2024	Smart beams, coupled-field analysis

QUALITATIVE ASSESSMENT OF SCIENCE MAPPING RESULTS

The bibliometric mapping results provide a structural overview of the research landscape. However, to understand the depth and direction of research, a qualitative assessment of the most influential works and clusters is essential. This section synthesizes the key theories, methods, and applications identified in the mapping process.

4.1 Beam Theories and Modeling Approaches

The review confirms that beam theory development remains the foundation of research in thermal buckling and flexure. Higher-Order Shear Deformation Theory (HSDT): Continues to dominate due to its ability to model thick FG and laminated beams under thermal fields [1], [2], [34]. For example, Reddy's HSDT framework has been widely extended to porous FG beams [22] and smart piezoelectric beams [24]. Quasi-3D Theories: Emerging as a significant advancement by accounting for

transverse normal strain neglected in classical models [25], [62], [89]. These provide higher accuracy for sandwich and porous beams under steep temperature gradients. Iso geometric Analysis (IGA): Gaining momentum as it eliminates geometric approximation errors and integrates directly with CAD models [12], [15], [54]. It has been effectively applied to thermal buckling and vibration of FG beams. Finite Element Extensions: P-version FEM [76] and meshfree approaches [53] are increasingly used for nonlinear post-buckling analyses. Table 6 represents comparative performance of HSDT, quasi-3D, IGA, and FEM-based approaches in beam thermal buckling studies.

Table 6. Comparative performance of HSDT, quasi-3D, IGA, and FEM-based approaches in beam thermal buckling studies

Theory	Thermal Buckling Accuracy	Flexural Vibration Accuracy	Application Scale
HSDT	High	High	Macro beams
Quasi-3D	Very High	Very High	FG Sandwich beams
IGA	Very High	Very High	CAD-integrated
FEM (p-ver.)	High	High	Computational efficiency

4.2 Material Innovations in FG Beams

Material advancements have reshaped thermal buckling and flexural research. Porous FG Beams: Lightweight beams with variable porosity distributions show reduced stiffness but offer thermal resistance advantages [22], [39], [55]. Nonlinear buckling analyses reveal that porosity distribution patterns significantly influence critical buckling loads [74], [84]. CNT and GPL Reinforcement: Integration of carbon nanomaterials has enabled superior mechanical and thermal properties [7], [17], [93]. Recent studies show CNT/GPL-reinforced FG beams achieve 30–40% higher post-buckling resistance under thermal loading [72], [93]. Smart Beams (Piezoelectric & MEE): Piezoelectric and magneto-electro-elastic layers provide active control of buckling and vibration [24], [45], [73]. For instance, Şimşek et al. [100] showed that MEE layers can suppress thermal post-buckling by active stiffness modulation. Figure 10 shows conceptual comparison of laminated, porous FG, CNT/GPL-reinforced, and smart beams under thermal loading.

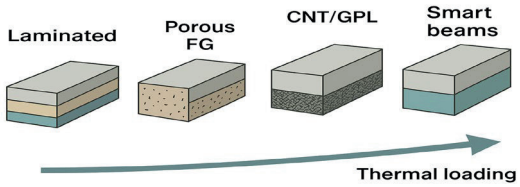


Figure 10. Conceptual comparison of laminated, porous FG, CNT/GPL-reinforced, and smart beams under thermal loading

4.3 Scale-Dependent Effects and Nanobeams

At the nanoscale, nonlocal elasticity and strain gradient theories have revealed important size-dependent behaviours. Nonlocal Models: Predict significantly higher buckling loads in nano-beams compared to classical elasticity [31], [33], [60]. Strain Gradient Theories: Capture additional microstructural effects, making them suitable for MEMS/NEMS applications [20], [49]. Viscoelastic and Thermoelastic Coupling: Advanced models highlight damping and relaxation under cyclic thermal loads [77], [86], [94]. These studies indicate that ignoring size effects leads to underestimation of thermal stability, which is critical in nano-scale structural design. Table 7 represents summary of key nonlocal and strain gradient models applied to FG and laminated nano-beams.

Table 7. Summary of key nonlocal and strain gradient models applied to FG and laminated nano-beams

Model	Key Feature	Application	Reference
Nonlocal	Long-range interactions	Nano-beams	[31], [33]
Strain Grad.	Higher-order strain terms	Micro-/nano-beams	[20], [49]
Thermoelastic	Coupled damping + heating	Smart nano-beams	[77], [94]

4.4 Nonlinear and Dynamic Thermal Buckling

While early research focused on linear stability analysis, recent trends emphasize nonlinear post-buckling and dynamic thermal instability. Nonlinear Thermal Post-Buckling: Works by Zhang et al. [99] and Rashidi [76] reveal that temperature rise beyond critical buckling induces secondary equilibrium states that must be captured with nonlinear formulations. Dynamic Thermal Buckling: Recent contributions [81], [90], [98] investigate transient heating and harmonic thermal loading, showing resonance like behaviours that amplifies instability. Foundation Effects: Inclusion of elastic and viscoelastic foundations modifies critical temperatures and post-buckling paths [83], [84]. Figure 11 shows Nonlinear and Dynamic Thermal Buckling Response showing: Linear buckling (sudden instability), Nonlinear post-buckling (smooth load–deflection path), Dynamic thermal buckling (oscillatory instability under transient heating).

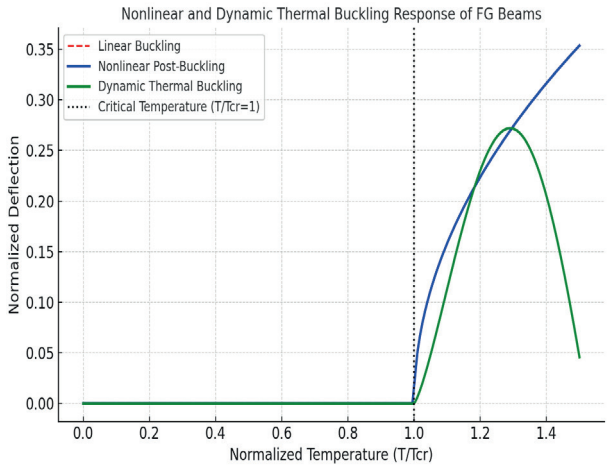


Figure 11. Typical nonlinear post-buckling and dynamic thermal buckling response curves of FG beams

4.5 Emerging Research Frontiers

From the science mapping and citation analysis, several future directions are identified: Coupled-Field Smart Beams: Multi-physics modelling combining thermal, electric, magnetic, and moisture fields [24], [45], [73]. Data-Driven & AI Models: Integration of machine learning for rapid buckling load prediction and flexural response estimation [15]. Hybrid Reinforcements: Use of CNT + GPL + porous matrices for multifunctional thermal and mechanical performance [17], [72], [93]. Experimental Validation: Most studies are numerical; experimental data on porous FG beams and nanobeams remain scarce [16], [68]. Applications in MEMS/NEMS: Growing interest in nanoscale FG beams for thermal sensors, actuators, and resonators [31], [60], [94]. Figure 12 shows research roadmap highlighting emerging frontiers in thermal buckling and flexural analysis of FG and laminated beams.

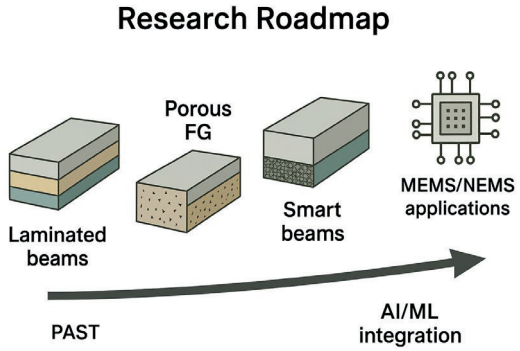


Figure 12. Research roadmap highlighting emerging frontiers in thermal buckling and flexural analysis of FG and laminated beams

CONCLUSION

This bibliometric review consolidates the literature on thermal buckling and flexural responses of laminated composite and FG beams from 2010 to 2024. The results reveal: Steady growth in publications since 2015, driven by porous FG and nanobeam research. HSDT and quasi-3D beam models remain the backbone for accurate predictions. Porous, CNT, and GPL reinforcements are trending, enabling lightweight multifunctional designs. Nonlocal and size-dependent effects dominate micro/nano-scale modelling. Smart beams with piezoelectric/magneto-electro-elastic layers are an emerging frontier.

This review provides a roadmap for researchers seeking to extend beam modelling to multi-field coupled environments, hybrid reinforcements, and AI-assisted predictive frameworks.

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