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A FINITE ELEMENT MODEL FOR CALCULATING A NON-THIN ORTHOTROPIC CYLINDRICAL SHELL, TAKING INTO ACCOUNT THE INDUCED INHOMOGENEITY

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Annotation. The formulation of a model for the deformation and strength calculation of an open cylindrical shell of circular shape rigidly pinched along contour planes formed by generators is considered. The ratios of the geometric parameters of the shell differ significantly from thin-walled structures, to which traditional technical hypotheses could be applied. The formulation of a mathematical model defining the stress-strain states of a thick-walled structure was carried out within the framework of a nonlinear three-dimensional theory. The peculiarity of the developed model was that orthotropic composites were adopted as the structural materials of the shell, the deformation and strength properties of which manifest themselves in different ways in accordance with the types of stress states being realized. The manifestation of such mechanical properties of materials is interpreted as induced heterogeneity, which significantly complicates the methodology for calculating spatial structures. Due to the specified features of the research object, isoparametric finite elements of a three-dimensional tetrahedral shape, the nodes of which had three degrees of freedom, were used to develop a computational model. In order to adequately account for the stiffness properties of orthotropic composites exhibiting induced heterogeneity, the basis of the determining relationships for them was the deformation potential formulated in the normalized stress tensor space associated with the main axes of orthotropy of materials. Due to the nonlinearity of the problem under consideration, the process of solving it was based on the method of variable elasticity parameters. As a result of the implementation of the computational model, comprehensive information was obtained on the stresses, deformations and displacements of the shell, the main of which are presented in the text of the article with the addition of their analysis.

Keywords: finite element method, open cylindrical shell, structural orthotropy, induced inhomogeneity, dependence of stiffness on the type of stress state

КОНЕЧНО-ЭЛЕМЕНТНАЯ МОДЕЛЬ РАСЧЕТА НЕТОНКОЙ ОРТОТРОПНОЙ ЦИЛИНДРИЧЕСКОЙ ОБОЛОЧКИ С УЧЕТОМ НАВОДИМОЙ НЕОДНОРОДНОСТИ

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Аннотация. Рассмотрена формулировка модели деформационно-прочностного расчета незамкнутой цилиндрической оболочки кругового очертания, жестко защемленной по контурным плоскостям, сформированным образующими. Соотношения геометрических параметров оболочки существенно отличаются от тонкостенных конструкций, которым могли быть применены традиционные технические гипотезы. Формулировка математической модели, определяющей напряженно-деформированные состояния толстостенной конструкции, была осуществлена в рамках нелинейной трехмерной теории. Особенностью разработанной модели явилось то, что в качестве конструкционных материалов оболочки были приняты ортотропные композиты, деформационные и прочностные свойства которых проявляются по-разному в соответствии с реализуемыми видами напряженных состояний. Проявление подобных механических свойств материалов трактуется как наведенная неоднородность, что существенно усложняет методику расчета пространственных конструкций. Ввиду указанных особенностей объекта исследований для разработки расчетной модели были приняты изопараметрические конечные элементы объемной тетраэдрической формы, узлы которой имели три степени свободы. Для адекватного учета жесткостных свойств ортотропных композитов, проявляющих наведенную неоднородность, основой определяющих соотношений для них был принят потенциал деформаций, сформулированный в нормированном тензорном пространстве напряжений, связанном с главными осями ортотропии материалов. Ввиду нелинейности рассматриваемой задачи процесс ее решения строился по методу переменных параметров упругости. В результате реализации расчетной модели получена исчерпывающая информация о возникающих напряжениях, деформациях и перемещениях оболочки, основные из которых представлены в тексте статьи с добавлением их анализа.

Ключевые слова: метод конечных элементов, незамкнутая цилиндрическая оболочка, структурная ортотропия, наведенная неоднородность, зависимость жесткости от вида напряженного состояния

1. INTRODUCTION

Modern technology and construction use all new structural materials, among which composites have become widespread. Their peculiarity often lies in the presence not only of structural orthotropy, but also in the manifestation of deformation anisotropy, such as the dependence of deformation and strength properties on the kind of stress state. These materials are widely used in the manufacture of spatial structures, the elements of which cannot be classified as thin-walled. In particular, this applies to cylindrical shells of various configurations and other curved structures. Calculations for the strength and deformability of similar structures should be based on reliable calculation models based on the specifics of the mechanical properties of these composites and be adequate to the real picture of the loading processes and geometry of structures. It is obvious that the accuracy of the calculation methods used depends on the reliability of the determining ratios, which do not contradict established and approved mechanical deformation experiments of the studied composites. Obviously, the dependences of strain and stress tensors arising from the potential proposed in the works can serve as the most reliable equations of state [1 – 3]. In the same works and in monographs [4, 5], a detailed analysis of the serious shortcomings of widely known models published by many authors [6 – 14] is presented and, therefore, their assessment is not given here.

2. EQUATIONS OF STATE FOR ORTHOTROPIC MATERIALS EXHIBITING INHOMOGENEITY DURING DEFORMATION

As a result of the use of the author's technique of the normalized tensor stress space associated with the main axes of the structure of materials, the deformation potential of an orthotropic substance sensitive to the kind of stress state was postulated in [2, 3]:

$$W = 0,5(A_{1111} + B_{1111}\alpha_{11})\sigma_{11}^2 + 0,5(A_{2222} + B_{2222}\alpha_{22})\sigma_{22}^2 + 0,5(A_{3333} + B_{3333}\alpha_{33})\sigma_{33}^2 + [A_{1122} + B_{1122}(\alpha_{11} + \alpha_{22})]\sigma_{11}\sigma_{22} + [A_{2233} + B_{2233}(\alpha_{22} + \alpha_{33})]\sigma_{22}\sigma_{33} + [A_{3311} + B_{3311}(\alpha_{33} + \alpha_{11})]\sigma_{33}\sigma_{11} + 0,5(A_{1212}\tau_{12}^2 + A_{2323}\tau_{23}^2 + A_{3131}\tau_{31}^2), \quad (1)$$

where $\alpha_{ij} = \sigma_{ij} / S$ – stresses in the normalized space of the tensor associated with the main axes of the orthotropy of the material; $S = \sqrt{\sigma_{ij}\sigma_{ij}}$ – the norm of the tensor stress space; $\alpha_{ij}\alpha_{ij} = 1$ – tensor space normalization condition; material parameters A_{ijkm}, B_{ijkm} are determined after identification of the equations of state arising from the potential (1) using the results of mechanical tests performed; (i, j, k, m= 1, 2, 3).

Having differentiated the potential (1) according to the Castigliano rules, we establish the equations of connection between the tensors of the second rank of the deformed and stressed states for orthotropic structures acquiring deformation anisotropy:

$$e_{ij} = C_{ijkm}\sigma_{km}, \quad i, j, k, m = 1, 2, 3, \qquad (2)$$

where $C_{1111} = (A_{1111} + B_{1111}\alpha_{11}) +$
$+0,5[B_{1111}\alpha_{11}(1 - \alpha_{11}^2) - B_{2222}\alpha_{22}^3 -$
$-B_{2222}\alpha_{22}^3 - B_{3333}\alpha_{33}^3 - A_{1212}\alpha_{12}^3 - A_{2323}\alpha_{23}^3 -$
$-A_{1313}\alpha_{13}^3] + B_{1122}\alpha_{22}(1 - \alpha_{11}^2 - \alpha_{11}\alpha_{22}) +$
$+B_{1133}\alpha_{33}(1 - \alpha_{11}^2 - \alpha_{11}\alpha_{33}) -$
$-B_{2233}\alpha_{22}\alpha_{33}(\alpha_{22} + \alpha_{33});$

$$C_{1122} = A_{1122} + B_{1122}(\alpha_{11} + \alpha_{22});$$
$$C_{1133} = A_{1133} + B_{1133}(\alpha_{11} + \alpha_{33});$$

$C_{2222} = (A_{2222} + B_{2222}\alpha_{22}) +$
$+0,5[B_{2222}\alpha_{22}(1 - \alpha_{22}^2) - B_{1111}\alpha_{11}^3 - B_{3333}\alpha_{33}^3 -$
$-A_{1212}\alpha_{12}^3 - A_{2323}\alpha_{23}^3 - A_{1313}\alpha_{13}^3] +$
$+B_{1122}\alpha_{11}(1 - \alpha_{22}^2 - \alpha_{11}\alpha_{22}) +$
$+B_{2233}\alpha_{33}(1 - \alpha_{22}^2 - \alpha_{22}\alpha_{33}) -$
$-B_{1133}\alpha_{11}\alpha_{33}(\alpha_{11} + \alpha_{33});$

$$C_{2233} = A_{2233} + B_{2233}(\alpha_{22} + \alpha_{33});$$

$C_{3333} = (A_{3333} + B_{3333}\alpha_{33}) +$
$+0,5[B_{3333}\alpha_{33}(1 - \alpha_{33}^2) - B_{1111}\alpha_{11}^3 -$
$-B_{2222}\alpha_{22}^3 - A_{1212}\alpha_{12}^3 - A_{2323}\alpha_{23}^3 - A_{1313}\alpha_{13}^3] +$
$+B_{1133}\alpha_{11}(1 - \alpha_{33}^2 - \alpha_{11}\alpha_{33}) +$
$B_{2233}\alpha_{22}(1 - \alpha_{33}^2 - \alpha_{22}\alpha_{33}) -$
$-B_{1122}\alpha_{11}\alpha_{22}(\alpha_{11} + \alpha_{22});$

$C_{1212} = 0,5\{A_{1212} - (B_{1111}\alpha_{11}^3 + B_{2222}\alpha_{22}^3 +$
$+B_{3333}\alpha_{33}^3) - 2[B_{1122}\alpha_{11}\alpha_{22}(\alpha_{11} + \alpha_{22}) +$
$+B_{2233}\alpha_{22}\alpha_{33}(\alpha_{22} + \alpha_{33}) + B_{1133}\alpha_{11}\alpha_{33}(\alpha_{11} + \alpha_{33})]\};$

$C_{2323} = 0,5\{A_{2323} - (B_{1111}\alpha_{11}^3 + B_{2222}\alpha_{22}^3 +$
$+B_{3333}\alpha_{33}^3) - 2[B_{1122}\alpha_{11}\alpha_{22}(\alpha_{11} + \alpha_{22}) +$
$+B_{2233}\alpha_{22}\alpha_{33}(\alpha_{22} + \alpha_{33}) + B_{1133}\alpha_{11}\alpha_{33}(\alpha_{11} + \alpha_{33})]\};$

$C_{1313} = 0,5\{A_{1313} - (B_{1111}\alpha_{11}^3 + B_{2222}\alpha_{22}^3 +$
$+B_{3333}\alpha_{33}^3) - 2[B_{1122}\alpha_{11}\alpha_{22}(\alpha_{11} + \alpha_{22}) +$
$+B_{2233}\alpha_{22}\alpha_{33}(\alpha_{22} + \alpha_{33}) + B_{1133}\alpha_{11}\alpha_{33}(\alpha_{11} + \alpha_{33})]\};$

$C_{1212} = C_{2121}$, $C_{2323} = C_{3232}$, $C_{1313} = C_{3131}$, the other components of the tensor C_{ijij}, characterizing the malleability of the material are identically equal to zero

As a result of the analysis of experimental data on the proportional loading of standard samples made of orthotropic composites of various classes, the identification of equations of state (2) was carried out in [2, 3, 5]. In this case, the constants of the compliance tensor (2) were calculated using technically measured parameters established during tensile testing, compression of standards cut along the main axes of orthotropy and in shear experiments in the main planes of materials:

$$A_{kkkk} = (1 / E_k^+ + 1 / E_k^-) / 2;$$
$$B_{kkkk} = (1 / E_k^+ - 1 / E_k^-) / 2;$$
$$A_{iijj} = -(\nu_{ij}^+ / E_j^+ + \nu_{ij}^- / E_j^-) / 2;$$
$$B_{iijj} = -(\nu_{ij}^+ / E_j^+ - \nu_{ij}^- / E_j^-) / 2;$$
$$A_{ijij} = 1 / G_{ij}; \quad \nu_{ij}^+ / E_j^+ = \nu_{ji}^+ / E_i^+;$$
$$\nu_{ij}^- / E_j^- = \nu_{ij}^- / E_i^-; \quad i, j, k = 1, 2, 3, \qquad (3)$$

where $E_k^{\pm}$, $\nu_{ij}^{\pm}$ – elastic modulus and transverse strain coefficients calculated as a result of processing data from experiments conducted in the directions of the main axes of orthotropy using the least squares method (a positive sign corresponds to tensile experiments, and a negative sign corresponds to compression); G_{ij} – shear modules in the main orthogonal planes of orthotropy.

Along with the identification of the model (1) – (3), in the same works [2, 3, 5], the boundaries of possible potential constants were determined in accordance with Drucker's postulate and the proof of the theorems of the existence and uniqueness of solutions to boundary value problems. These procedures confirmed the inviolability of energy conservation [2, 3, 5]. Particular values of the mechanical characteristics of orthotropic materials with acquired heterogeneity are available in well–known publications [1 – 13, 15].

3. PROBLEM STATEMENT AND SHELL CALCULATION

Using the physical and mathematical correctness of potential dependencies (1) – (3), they can be used as the basis for a finite element model for calculating the stress-strain state of thick-walled cylindrical shells. Considering that it is supposed to investigate shells that do not meet the criteria of thin ones, three-dimensional finite elements of isoparametric interpretation were used in the form of a tetrahedron having three degrees of freedom at the node. The mathematical apparatus of these finite elements is fully developed in [16, 17]. The presented article is based on the transformation of only the stiffness matrix of the specified elements, into which the equations of state (2) were embedded, previously converted into the matrix form of the stress-strain relationship:

$$\{\sigma\} = [P]\{e\} \tag{4}$$

where $\{\sigma\}$ and $\{e\}$ – six-dimensional stress and strain vectors composed of components of the corresponding tensors (2); $[P]$ – a rigid matrix in size 6×6, the inverse of the material's compliance matrix, consisting of tensor components C_{ijij} (2), depending on the type of stress state, $[P] = [C]^{-1}$.

A short, unclosed cylindrical shell of circular shape with rigid clamping along the reference planes was selected for analysis. The breakdown of the shell into finite elements is shown in Fig. 1, and the calculation scheme is shown in Fig. 2. In the process of sampling the shell continuum, the convergence of the FE model was checked, which is shown in Fig. 3.

The developed FE model was used not only taking into account the equations of state (2), (4) introduced, but also calculations were carried out on its basis with embedded determining ratios of widespread theories of deformation of different resistant orthotropic materials. In order to compare the results of shell calculations, the equations proposed in the works of such authors as R.M.Jones – D.A.R.Nelson [8, 9, 11, 12] and S.A.Ambartsumyan [14] were used. In addition, to estimate the error given by the generalized Hooke's law formulated for orthotropic materials [18], if it is used as physical dependencies of deformation mechanics, it was also used in calculations. At the same time, the average characteristics obtained from experiments on uniaxial tension and compression in the main axes of orthotropy were taken as elastic modulus and transverse deformation coefficients. To calculate the shell, a composite material based on graphite of the ATJ-S brand, widely used in technology, was adopted. A complete set of stiffness parameters characterizing its properties is given in [11]. As a result of the calculations, complete information was obtained on the distribution of shell SSC parameters. Here we will focus on the analysis of individual parameters, which are presented in Fig. 4 – 8.

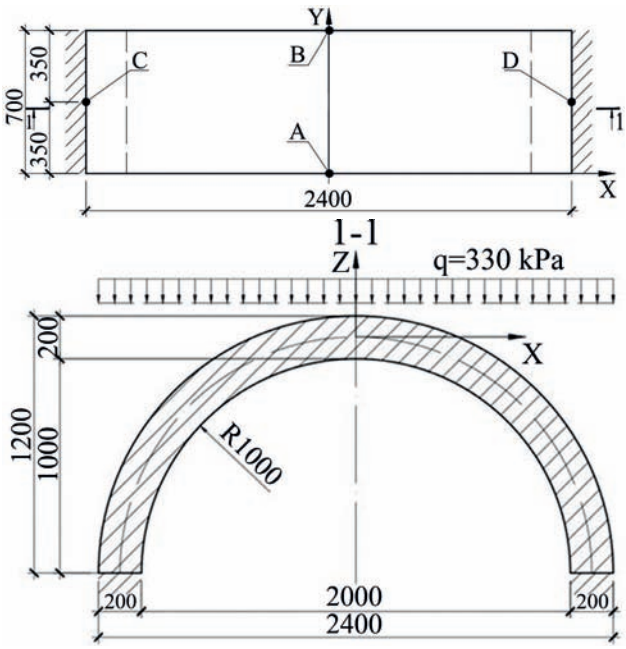


Figure 1. The design scheme of the cylindrical shell

Figure 2. Discretization of the shell on the FE

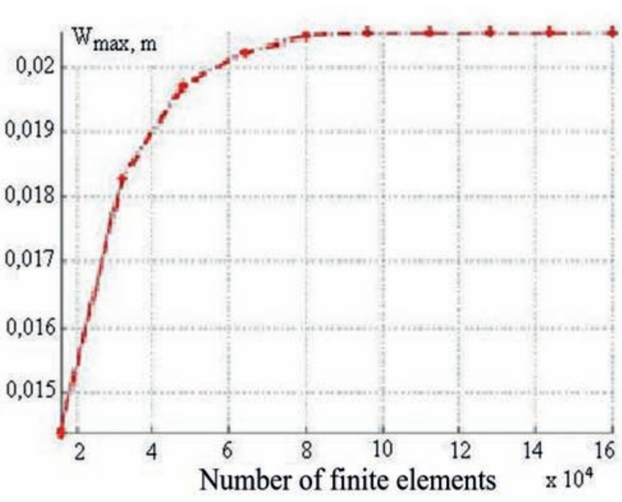


Figure 3. Convergence of the FE model with respect to maximum deflections

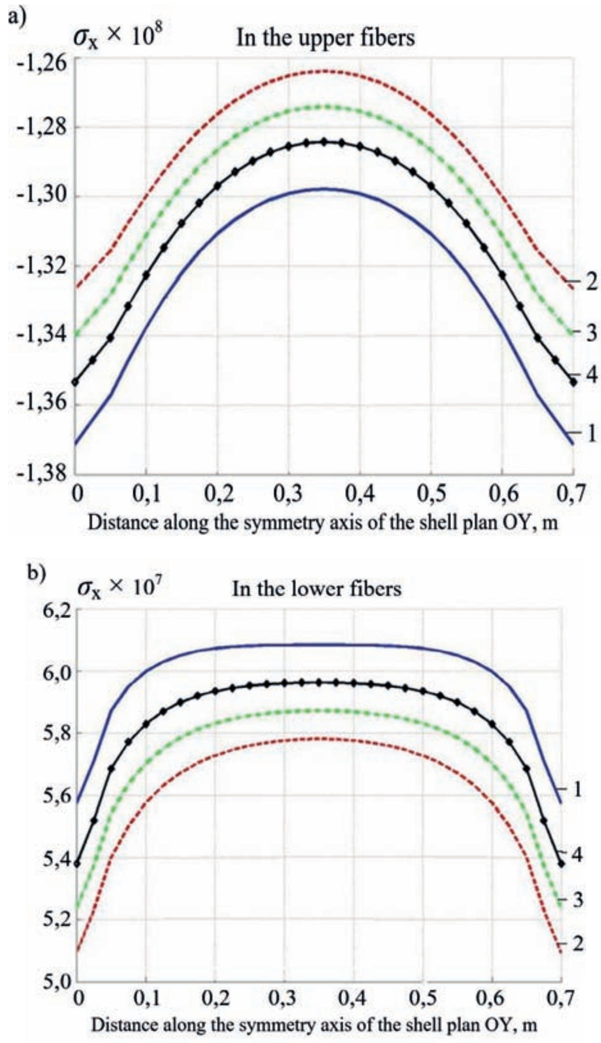
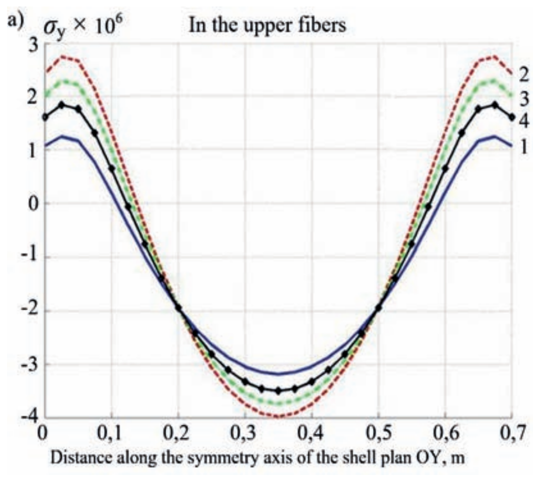


Figure 4. Normal stress distribution σ_x along the line A-B, Pa: in the upper fibers (a); in the lower fibers (b)



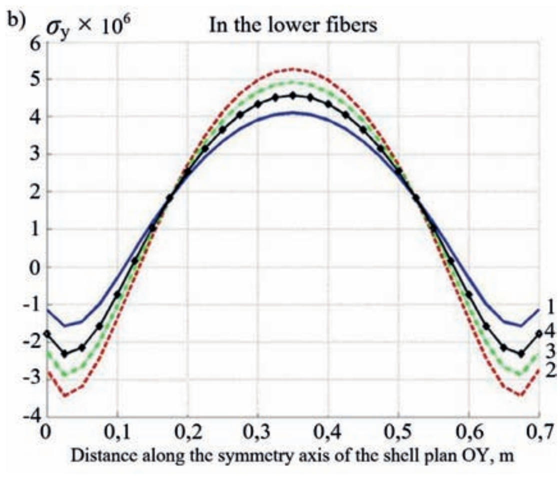


Figure 5. Normal stress distribution σ_y along the line A-B, Pa: in the upper fibers (a); in the lower fibers (b)

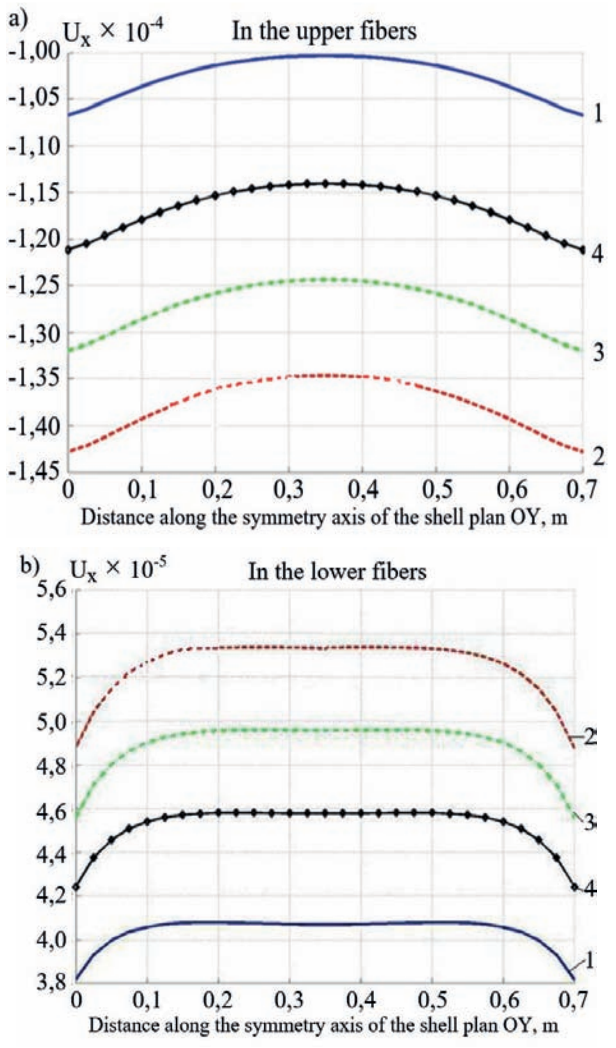


Figure 6. Distribution of displacements U_X along the line A-B, m: in the upper fibers (a); in the lower fibers (b)

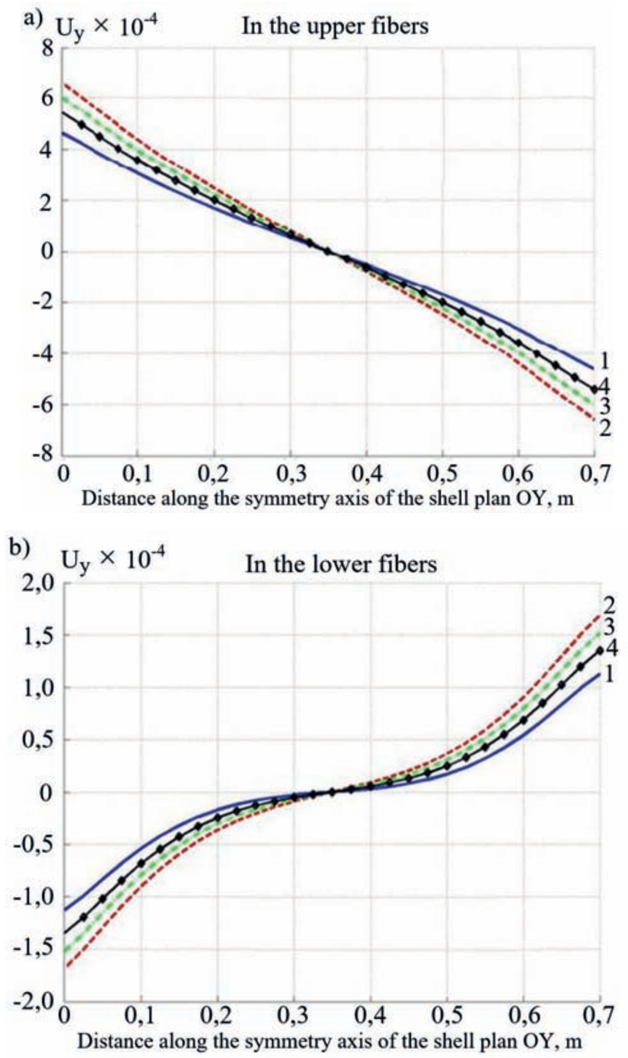


Figure 7. Distribution of displacements U_Y along the line A-B, m: in the upper fibers (a); in the lower fibers (b)

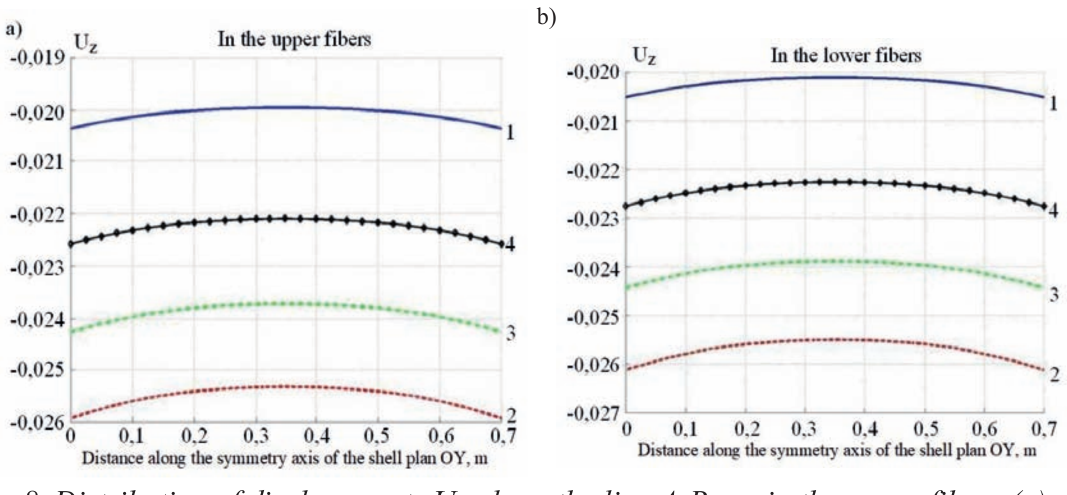


Figure 8. Distribution of displacements U_Z along the line A-B, m: in the upper fibers (a); in the lower fibers (b)

In Fig. 4 – 8 the introduced digital designations of the graphs of changes in the calculated values, refer to solutions obtained on the basis of various models of constitutive relations: 1 – author's model; 2 – model based on the classical theory of deformation of orthotropic materials using the generalized Hooke's law [18] and using the medium mechanical characteristics of the material; 3 – model by S.A. Ambartsumyan [14]; 4 – model by R.M.Jones – D.A.R.Nelson [8, 9, 11, 12].

4. ANALYSIS OF THE RESULTS OBTAINED

Analyzing the results of calculations of cylindrical shells with structural orthotropy and made of ATJ-S composite, which are presented in Fig. 4 – 8, important theoretical features can be noted. These features are concluded in the fact that traditional mathematical models, which are based on the generalized Hooke's law, which does not take into account the dependence of the rigidity properties of the material on the kind of stress state, lead to serious errors in determining stresses up to 58% – 78%. In this case, errors in determining the points of the cylindrical shell can reach 35%. If the R.M.Jones – D.A.R.Nelson model is used for shell calculations [8, 9, 11, 12], then the results will differ from the theoretical data using the generalized Hooke's law somewhat less and reach 16–36% and 18%, respectively. An explanation for this may be the not entirely correct consideration of the effect of a complex stress state on the change in the components of the compliance tensors, which are used by models [8, 9, 11, 12] and [14] in comparison with equations obtained using normalized tensor stress spaces [1 – 5].

5. CONCLUSION

The results of the calculations performed and the analysis of the results obtained demonstrate and confirm the fact that the design of elements of spatial structures with a curvilinear outline such as cylindrical shells made of graphite-based composite materials should be carried out most carefully. In this case, it is imperative to take into account the peculiarities of the mechanical behavior of such composites. In addition, when performing calculations of stresses and strains of shells, special attention should be paid to the selection of mathematical models of constitutive relations that do not conflict with known experimental data and are most adequate to the latter when changing complex kinds of stress states.

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