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ASSESSMENT THE PROBABILITY OF FAILURE OF A REINFORCED CONCRETE FRAME UNDER A COLUMN REMOVAL SCENARIO USING THE FIRST-ORDER RELIABILITY METHOD

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Abstract: The article considers issues related to the assessment of survivability of structural reinforced concrete systems in special limit states under the scenario of removal of one of the load-bearing columns within the framework of the probabilistic approach. Probabilistic models of basic (stochastic) variables included in the load and resistance functions are presented. Statistical parameters of the uncertainty of the resistance model and the model of the effect of external actions are obtained and integrated as a basic variable in probabilistic modeling. The functions of the limit state of the structural system in a special calculated situation with a sudden removal of the central column of the first floor are determined. Probabilistic modeling is performed using the first-order reliability method (FORM) for structural reinforced concrete systems designed according to the current standards of the Russian Federation. As a result, the values of the failure probabilities and the corresponding reliability indices are obtained for the considered structural systems.

Keywords: Progressive collapse, probability, structural reliability, first order reliability method, robustness, column removal scenarios, reinforced concrete structures

ОЦЕНКА ВЕРОЯТНОСТИ ОТКАЗА ЖЕЛЕЗОБЕТОННОЙ РАМЫ ПРИ СЦЕНАРИИ УДАЛЕНИЯ КОЛОННЫ МЕТОДОМ НАДЕЖНОСТИ ПЕРВОГО ПОРЯДКА

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Аннотация: В рамках вероятностного подхода рассмотрены вопросы связанные с оценкой живучести конструктивных железобетонных систем в особых предельных состояниях при сценарии удалении одной из несущих колонн. Представлены вероятностные модели базисных (стохастических) переменных, входящих в функции нагрузок и сопротивлений. Получены и интегрированы в виде базисной переменной при вероятностном моделировании статистические параметры неопределенности модели сопротивления и модели эффекта от внешних воздействий. Определены функции предельного состояния конструктивной системы в особой расчетной ситуации при внезапном удалении центральной колонны первого этажа. Выполнено вероятностное моделирование по методу расчета надежности первого порядка (FORM) конструктивных железобетонных систем железобетона, запроектированных по действующим нормам РФ. В результате для рассматриваемых конструктивных систем получены значения вероятностей отказа и соответствующих индексов надежности.

Ключевые слова: Прогрессирующее обрушение, вероятность, надежность конструкций, метод надежности первого порядка, прочность, сценарии удаления колонн, железобетонные конструкции

1. INTRODUCTION

Progressive collapse is a catastrophic partial or complete failure of a structure caused by local structural damage [1]. Local failure initiates a chain reaction of failures that propagates vertically or horizontally through the structural system, resulting in widespread partial or complete collapse. The key feature of progressive collapse is that the resulting ultimate damage is disproportionate to the local damage caused by the initiating event. Such collapses may be initiated by specific beyond design basis events (e.g. gas explosions, vehicle collisions, and sabotage, fires, extreme environmental impacts, and human errors in design and construction) [2].

Progressive collapse and stability of structures under such special beyond design loads became an important factor in research and design after the collapse of the Ronan Point apartment tower in 1968. Interest in this topic has recently increased as a result of the terrorist attack on the Murrah Federal Building in Oklahoma City in 1995 and the attack on the World Trade Center in New York in 2001 [3].

At present, although the stability of structures against progressive collapse under special beyond design basis events has become a worldwide research topic, there is no unified theory for assessing structural stability, nor a general methodology for quantitatively assessing the progressive collapse resistance of real complex structures. Below are presented some notable research directions proposed for assessing the stability of reinforced concrete structures against progressive collapse.

One of the main areas of research in assessing the response of structures to beyond design actions is the conduct of experiments on structural models. A number of quasi-static tests have been conducted on various structural models, such as a beam-column and beam-slab substructure, flat frame structures and large-scale buildings, ... [4]. Thanks to these experiments, the load-bearing capacity, as well as the deformation and failure characteristics of the structure, have been studied quite thoroughly. During the deformation of the substructure, various mechanisms of resistance to progressive collapse were discovered with an increase in the quasi-static concentrated load above the removed column.

At early stages, when the applied force is still small, the bending moment in dangerous sections is less than the bearing capacity M_{ult}, the structure operates according to the bending mechanism. The internal forces in the beam are only the transverse force and the bending moment. The longitudinal force is practically equal to zero [5].

When the quasi-static external force applied above the removed column continues to increase, the bending moment reaches the limiting value M_{ult} in the sections near the side columns. At the same time, as the external load increases, the value of the bending moment in these sections no longer increases. The stresses in the reinforcement in the tension zone of the dangerous sections reach the yield point, and the plastic deformation in them increases significantly. The compression zone height of concrete gradually decreases, and numerous cracks appear in the tension zone of concrete. At this stage, it can be observed that the compressed concrete strip forms an arc (arch). The horizontal displacement at the base of the formed arch is limited by the adjacent structures. Due to the work of the compressive concrete arch, the bearing capacity of the system increases significantly. The structure enters the stage according to the arch mechanism [6]. It should be noted that when calculating reinforced concrete structures under the scenario of removing a load-bearing column, the arch mechanism cannot be modeled using bar and plate elements in traditional finite element analysis.

As the quasi-static external force applied above the removed column increases, the displacement of the structure also increases. The height of the compressive concrete arch gradually decreases to zero. At this stage, the arch mechanism completely disappears and is replaced by a catenary mechanism [7]. All reinforcement in dangerous sections is subjected to tension. This also means that the longitudinal force in the structure begins to manifest itself and increases to a limiting value when the stresses in the reinforcement reach the tensile strength. The structure completely collapses when the longitudinal reinforcement ruptures.

It should be emphasized that the above-mentioned deformation and failure characteristics were discovered as a result of quasi-static experiments. Therefore, it does not fully reflect the dynamic nature of the process of progressive collapse and does not explain the change in the dynamic strength of the material under high-strain-rate conditions. Several dynamic experiments have been conducted so far to overcome the shortcomings of quasi-static experiments [8,9]. However, these experiments are very complex and require expensive equipment.

In parallel with the experiments, many scientists choose the method of predicting the bearing capacity, as well as the deformability and failure characteristics of structures using numerical methods, such as the finite element method [10]. However, the output results of the numerical analysis largely depend on the correctness of the input data, the suitability of the material model, the types of finite elements, as well as the perfection of the calculation algorithm and the skills in using specialized software. It should be noted that the numerical analysis here can be either nonlinear dynamic or nonlinear static. In the case of nonlinear static analysis, a dynamic amplification factor is applied so that the structural system provides equivalent dynamic behavior in the scenario of sudden column failure [11].

Another approach proposed for assessing the dynamic response of structures is the energy approach. Based on the nonlinear relationship between load and displacement in a quasi-static test and assuming that the work of the external force is balanced by the energy stored inside the structure when the column fails, we can find the greatest displacement that the system can achieve [12]. The energy approach is relatively simple, but it does not allow us to assess the effect of strain rate on the dynamic strength of materials in reinforced concrete structures.

Some researchers use the tools of structural dynamics to solve the problem of dynamic behavior of structures under beyond-design actions in the form of sudden failure of load-bearing columns. Jian Weng and his colleagues proposed a linear model with single-degree-of-freedom, which takes into account a fixed damping coefficient and failure time of the column [13]. Jun Yu and his colleagues proposed a nonlinear model with single-degree-of-freedom, in which the system stiffness is described by a trilinear function, which is able to describe various mechanisms of resistance to progressive collapse of the structure [14,15].

Another approach to studying the resistance of building structures to progressive collapse is to use reliability/risk indices [16,17]. The ability of a structure to withstand damage can be determined based on the reliability analysis of the structure and probability assessment. Probabilistic analysis will allow design engineers to decide whether further protective measures are required to prevent the risk of collapse. Izzuddin A. et al. [18] proposed a probabilistic risk assessment system for multi-story steel buildings under beyond design actions. In their work, a simple probabilistic scheme was used, including the basic requirements for describing the uncertainty of hazards, the local damage associated with them, and the consequences of global failure. Xu and Ellingwood assessed the resistance of steel frames to progressive collapse by developing a probabilistic model of failure of beam-column connections [19]. In the proposed model, the load values as well as the load-bearing capacity of the structure are uncertain quantities. Brunesi and Parisi used Monte Carlo simulation to create two-dimensional (2D) and three-dimensional (3D) stochastic realizations of structural models from which an analytically model of progressive failure of reinforced concrete frame structures could be derived [20].

The main objective of this study is to develop an assessment method for predicting the resistance to progressive collapse of a reinforced concrete frame structure based on the probability principle. In the reliability analysis, a global limit state equation for the bearing capacity of the structural system is first formulated, in which the safety factor is the difference between the ultimate bearing capacity of the structural system and the effects of external loads. An analytical approach (the first-order reliability method (FORM)) is applied to analyze the reliability of multi-story reinforced concrete frame structures subject to column failure based on the structural reliability index.

2. METHODS

Fundamentals of a progressive collapse risk management strategy

In general, the strategy for managing the risks of progressive collapse focuses on calculation methods that assess the ability of a damaged structural system to maintain survivability after a special event associated with the occurrence of a special action.

If each of the threats discussed earlier is represented as a random event, then the total probability of collapse of the structural system can be written as follows:

$$P(F)=\underbrace{\overbrace{\overbrace{P(F|DH)}^{\text{robustness}}\overbrace{P(DH|H)}^{\text{element behaviour}}}^{\text{collapse resistance}}}_{\text{vulnerability}}\underbrace{\overbrace{P(H)}^{\text{event control}}}_{\text{hazard}} \quad \begin{matrix}\}\text{maximise}\\ \}\text{minimise,}\end{matrix} \tag{1}$$

where F is an event defined as a disproportionate or progressive collapse of a structural system;

$P(H)$ is the probability of occurrence of a special event associated with the threat H;

$P(DH|H)$ is the conditional probability of local failure of an individual structural element D as a result of the realization of the threat H;

$P(F|DH)$ is the conditional probability of the occurrence of progressive collapse of a structural system F under the condition that local failure of an individual element D occurs upon the realization of a special event H.

In the case of a progressive collapse, the main consequence of which is loss of life, acceptable reliability $P(F)$ will be achieved if the following condition is met:

$$P(F)\le p_{tag}. \tag{2}$$

The basic question in the approaches adopted by almost all regulatory documents containing requirements for calculations for special actions is the following: what level of probability of collapse of a structural system should be considered acceptable?

Within the framework of the generally accepted concept, it is considered that the central criterion value of the probability of collapse of a structural system is $P_{tag}=10^{-7}/год$ for a reference period of 1 year.

It is easy to see that the probability of occurrence of disproportionate progressive collapse of the structural system can be reduced by reducing either each individual or all three probabilities included in (1). In this case, the probability of occurrence of a special event $P(H)$ associated with a threat H is independent during design. It can be controlled by the space-planning solution or placement of the building, reduction of possible risks inside the structure with organized security measures, personnel training, etc. When implementing such measures, many risks can be effectively prevented (e.g., terrorist attacks, improper operation of the building and structure, ...). In addition, these measures include quality control during design to prevent human errors and recommendations for technical maintenance.

The design strategy of key elements aimed at ensuring resistance to local failure is reduced to minimizing the probability $P\left(DH\middle|H\right)$. Within the framework of this strategy, a key element is understood to be the load-bearing elements and connections that are necessary to ensure the resistance of the original structural system as a whole. This strategy may be difficult to implement (due to the uncertainty of the magnitude of special actions), contain significant risks, or initially give uneconomical results.

In the case where the alternate load path method is used in assessing survivability, the conditional probability of a local failure of a key element when a special event H occurs is conservatively taken as $P\left(DH\middle|H\right)$ = 1 (the failure of a key element occurs when a special event occurs, although in the general case a structural element designed in a design traditional calculated situation will have some reliability under the action of special loads). Then expression (1) can be written in the following form:

$$P\left(F\right)=P\left(F\middle|DH\right)P\left(H\right). \quad (3)$$

Thus, the design task in a special calculated situation is reduced mainly to minimizing the probability $P\left(F\middle|DH\right)$. This strategy should be implemented in a wide range: from design measures aimed at creating the continuity and structural integrity of the system to a complete calculation of damage to the structural system taking into account various mechanisms of resistance to progressive collapse, which are not taken into account in traditional design (for example, compressive arch and catenary mechanisms, the effect of high-speed deformation of materials, large deformations and displacements, physical and geometric nonlinearity, etc.).

Taking into account that the occurrence of a special event H can be modeled by a Poisson process with an annual average occurrence frequency λ_H, the probability of occurrence of this event during some estimated period T (service life) for small λ_H is approximately $P\left(H\right)=\lambda_H T$. From which it follows that:

$$P\left(F\right)=P\left(F\middle|DH\right)\lambda_H T. \quad (4)$$

Thus, the permissible probability of progressive collapse of the structural system is determined by the formula

$$P\left(F\middle|DH\right)\le\frac{p_{tag}}{\lambda_H T}. \quad (5)$$

The value of λ_H is determined by statistical assessment of threats and can be taken in the range of 10^{-6}/year…10^{-5}/year depending on the type of threat. Thus, the target average annual conditional probability of progressive collapse of a damaged structural system can be taken equal to

$$P\left(F\middle|DH\right)\le\frac{\dfrac{10^{-7}}{год}}{\left(\dfrac{10^{-6}}{\text{year}}\dots\dfrac{10^{-5}}{\text{year}}\right)\text{year}}=\frac{10^{-2}}{\text{year}}\dots\frac{10^{-1}}{\text{year}}. \quad (6)$$

Limit state function

In order to determine the probability $P\left(F\middle|DH\right)$, one must first postulate a mathematical model (limit state function) *G(X)* in accordance with the principles established in reliability theory [21].

From the point of view of reliability theory, the behavior of a structural system or its element at an arbitrary moment in time can be described using a set of inherent limit states that designate the boundary between safe and unsafe states, the transition through which is considered a failure.

The model describing the limit state is represented by the so-called state function *G( )*, which includes all the basic variables *X* necessary for design, describing the effects of external loads *E(X)* and the resistance of the system itself *R(X)*:

$$G(X)=R(X)-E(X)=R(X_1, X_2, \dots, X_n) - \quad (7)$$

$$E(X_1, X_2, \dots, X_n),$$

where *E(X)* is a function representing the model of the effect of external loads; R(X) is a function representing the model of the element's resistance; *X* is a vector of basic (stochastic) variables, which are the parameters of the calculation model *E(•)* and *R(•)*.

For specific basic variables *X*, the function *G(X)* shows the state of the structural element: safe (operational) when *G(X)* ≥ 0, or in a state of failure when *G(X)*<0. Progressive collapse of a structural system is a probabilistic event, and the probability of its occurrence $P\left(F\left|DH\right.\right)$ is determined as follows:

$$P\left(F\left|DH\right.\right) = Prob\left\{G\left(X\right) < 0\right\}. \quad (8)$$

Reliability index β

The basic stochastic variable has two characteristic numerical values: the mathematical expectation (mean value) and the standard deviation. Let μ_G be the mean value of the limit state function *G( )*. σ_G is the standard deviation, then σ_G^2 is the variance of the function *G( )*. The reliability index β is defined as the ratio

$$\beta = \mu_G / \sigma_G. \quad (9)$$

If *R(X)* and *E(X)* are normally distributed, independent and uncorrelated, we have

$$\beta = \frac{\mu_R - \mu_E}{\sqrt{\sigma_R^2 + \sigma_E^2}}. \quad (10)$$

Let us consider the case when *R(X)* and *E(X)* are linear functions of the basic stochastic variables X_i:

$$R = \sum_{i=1}^{n} a_i X_i + a_0; E = \sum_{k=1}^{n} b_k X_k + b_0, \quad (11)$$

where a_i and b_k are constants. Then *G(X)=R(X)-E(X)* is also a linear function of these variables X_i:

$$G\left(X\right) = \sum_{i=1}^{n} a_i X_i - \sum_{k=1}^{n} b_k X_k + \left(a_0 - b_0\right). \quad (12)$$

Thus, the mean value μ_G of the limit state function has the form

$$\mu_G = \sum_{i=1}^{n} a_i \mu_{X_i} - \sum_{k=1}^{n} b_k \mu_{X_k} + \left(a_0 - b_0\right), \quad (13)$$

and the variance σ_G^2 of the function has the form

$$\sigma_G^2 = \sum_{i=1}^{n} a_i \sigma_{X_i}^2 - \sum_{k=1}^{n} b_k \sigma_{X_k}^2. \quad (14)$$

Then, determining the reliability index β in this case is quite simple, based on formula (9).

If *R(X)* and *E(X)* are nonlinear functions of the basic stochastic variables X_i, approximate methods such as linearization methods (e.g., FORM, SORM, FOSM) or the iterative method can be used to find the reliability index β.

Theoretical and experimental studies [22] have shown that the values of the physical and mechanical properties of materials, the geometric dimensions of the structure, and external loads have the properties of random variables and have a normal distribution law. In addition, when calculating the structures of buildings and structures for non-bearing capacity, the limit state function is established using analytical formulas. These are favorable conditions for applying the first-order reliability method (FORM) to determine the probability of progressive collapse of a building or structure under special beyond-design actions.

First-order reliability method (FORM)

To determine the reliability index β, it is necessary to know the analytical expression *G( )* for the basic stochastic variables X_i and determine the mean value μ_G, the standard deviation σ_G of the limit state function *G( )* for these variables. The mean value μ_G is determined by replacing the values of these variables X_i with their average values μ_{X_i} in the expression *G(X)*. Determining the standard deviation σ_G requires more complex calculations.

If *G(X)* is a linear function of the basic stochastic variables X_i, then σ_G is calculated using formula (14).

If *G(X)* is a nonlinear function of the basic stochastic variables X_i, then we perform a Taylor expansion of the function *G(X)* around the mean value of the variables. Ignoring the higher-order components in this expansion, we can determine the approximate value of *G(X)* as a linear function of the basic stochastic variables X_i.

For example, the Taylor expansion of the function *G(X)* around the mean value has the form

$$G = G\left(\mu_{X_i}\right) + \sum_{i=1}^{n} \left.\frac{\partial G}{\partial X_i}\right|_{\mu_{\bar{X}}} \left(X_i - \mu_{X_i}\right) + \frac{1}{2}\sum\sum \left.\frac{\partial^2 g}{\partial X_i \partial X_j}\right|_{\mu_{\bar{X}}} \left(X_i - \mu_{X_i}\right)\left(X_j - \mu_{X_j}\right) + ..., \tag{15}$$

where $\left.\frac{\partial G}{\partial X_i}\right|_{\mu_{\bar{X}}}$ is the partial derivative of *G(X)* with respect to X_i at μ_{X_i}. If only first-order terms are retained, *G(X)* is approximated as a linear function of the basis stochastic variables.

$$G \approx G\left(\mu_{\bar{X}}\right) + \sum_{i=1}^{n} \left.\frac{\partial G}{\partial X_i}\right|_{\mu_{\bar{X}}} \left(X_i - \mu_{X_i}\right). \tag{16}$$

By replacing (16) the mean value of the variables X_i, we obtain the average value of the limit state function *G(X)*.

$$\mu_G \approx G\left(\mu_{X_i}, \mu_{X_2}, ..., \mu_{X_n}\right), \tag{17}$$

and variance:

$$\sigma_G^2 = \sum_{i=1}^{n} \left(\frac{\partial G}{\partial X_i}\right)^2 \operatorname{var}\left(X_i\right). \tag{18}$$

Knowing the mean value μ_G and the variance σ_G^2, we can determine the reliability index β using formula (9). Using the table of normalized distribution functions *Φ*, we find $P\left(F\middle|DH\right)$.

$$P\left(F\middle|DH\right) = \Phi(-\beta) = 1 - \Phi(\beta) \tag{19}$$

In the case of using a normal (or lognormal) distribution, the probability of progressive collapse of a damaged structural system is related to the reliability index β by means of the Laplace function:

$$\beta = -\Phi^{-1}\left[P\left(F\middle|DH\right)\right] \tag{20}$$

where $\Phi^{-1}(\,)$ is the inverse function of the standard normal distribution function.

The limiting value of $P\left(F\middle|DH\right)$ is in the range of 10^{-2}/year … 10^{-1}/year, which means that the limiting value of the reliability index will be in the range:

$$\beta_{tag} = \frac{1{,}28}{\text{year}} ... \frac{2{,}33}{\text{year}}. \tag{21}$$

3. RESULTS AND DISCUSSION

To illustrate the proposed calculation method, a multi-story building frame made of monolithic reinforced concrete is used as a visual example (Fig. 1a). A scenario of a special action in the form of a sudden removal of the load-bearing middle column of the first floor is considered. As a result of recent studies by many scientists, it has been shown that the area where progressive collapse can occur is the part of the structure above the damaged column (the local failure zone). That is, the failure of the structural elements can spread vertically, while adjacent horizontal structures are usually not strongly exposed to the special actions mentioned above. Therefore, the calculation of the stability of the entire building against progressive collapse can be simplified by studying the stress-strain states in time of the local failure zone.

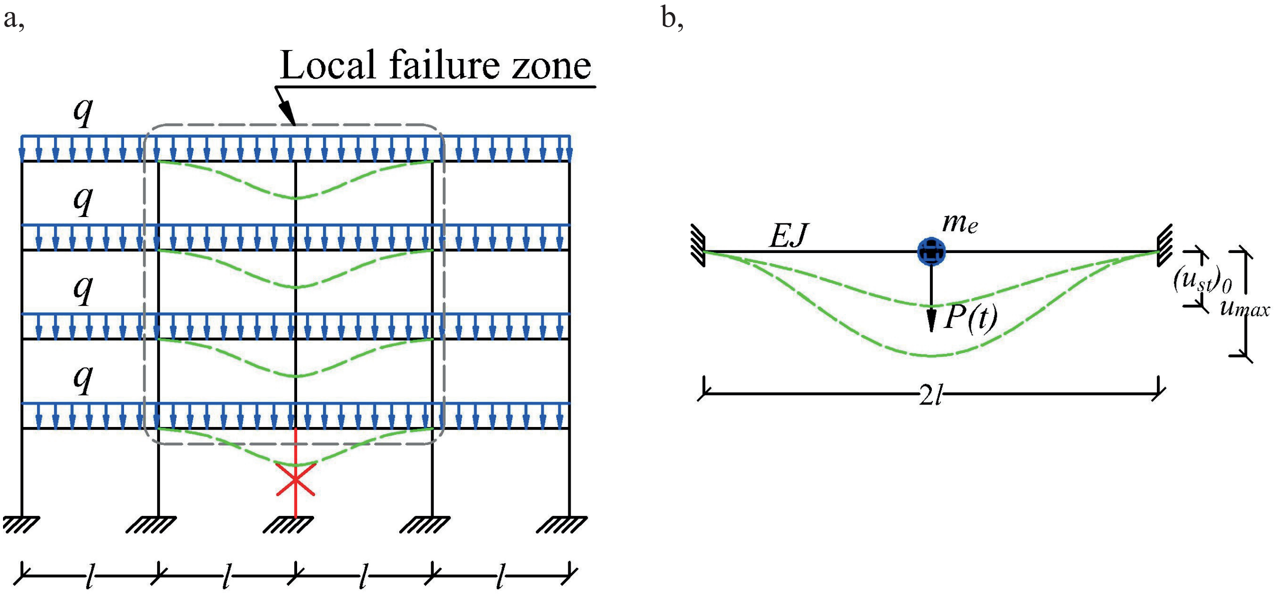


Figure 1. Monolithic reinforced concrete frame of the building in the scenario of sudden removal of the middle column (a) and the calculated scheme of a typical floor (b)

If we assume that the floors above removed supporting column are typical, their design characteristics are the same and under the same operational load, then we can see that their deformations, displacements, vibrations and failure are practically synchronous with each other.

This is the basis for further simplification of the calculated scheme. The study of the stress-strain state in time of the local failure zone is replaced by consideration of only the stress-strain state of one floor in the form of a fixed-ended beam of length *2l*. Fig. 1b shows the calculated scheme of the specified beam.

Figure 2 shows the diagram of bending moments M_{n-1}^{st} in the case of slow column failure (without taking into account the inertia force), and the diagram of bending moments M_{n-1}^{d} in the case of rapid column removal in a short time t_r. In section 1-1, at both ends of the beam, the bending moment has the greatest value. Therefore, it can be concluded that progressive collapse occurs when the moment at the end of the beam exceeds its ultimate value. Reliability calculations in these sections will give us the probability of occurrence or absence of progressive collapse.

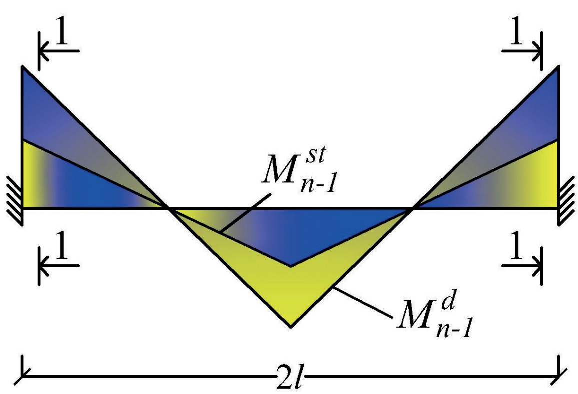


Figure 2. Bending moment diagrams in a beam with slow (M_{n-1}^{st}) and fast (M_{n-1}^{d}) removal of the middle column

As proven in [23], the maximum value of the bending moment in section 1-1 is achieved at time $t_0=t_r/2+T/2$. Where T is the period of free oscillations of the fixed-ended beam, representing the work of a typical floor. This value can be determined based on the maximum displacement of the beam point above the removed column as follows.

$$M_{1-1}^{\max} = -\frac{ql^2}{48} - \frac{24EJ}{l^2}u_{\max}, \tag{22}$$

where q is the calculated operational load in the form of a distributed force, EJ is the flexural rigidity of the beam, l is the distance between the supporting columns, u_{max} is the maximum displacement that the beam system can achieve at a point above the removed column during the oscillation process.

If the energy absorption capacity of the system is neglected, the maximum displacement u_{max} can be determined by the following formula:

$$u_{\max} = \left(u_{st}\right)_0\left(1+\frac{\sin\pi\frac{t_r}{T_n}}{\pi\frac{t_r}{T_n}}\right) = \left(u_{st}\right)_0\left(1+\frac{\sin\frac{\omega_n t_r}{2}}{\frac{\omega_n t_r}{2}}\right), \tag{23}$$

where $\left(u_{st}\right)_0 = \frac{ql^4}{384EJ}$ is the static displacement at the midpoint of the beam during slow failure of the column ($t_r \to \infty$), $T_n=2\pi/\omega_n$ and $\omega_n = \frac{1}{l^2}\sqrt{\frac{512EJ}{m}}$ is the period and natural frequency of oscillations of the damaged structural system, $J = bh^3/12$ is the moment of inertia of the rectangular cross-section of the beam.

Substituting the formulas of the given parameters into (23) and substituting (23) into (22), we obtain the final analytical expression for the maximum bending moment in section 1-1 of the structural system depending on the basic variables q, l, t_r, EJ, m.

$$M_{1-1}^{\max} = -\frac{ql^2}{48} - \frac{ql^2}{16}\left(1+\frac{\sin\frac{t_r}{2l^2}\sqrt{\frac{512EJ}{m}}}{\frac{t_r}{2l^2}\sqrt{\frac{512EJ}{m}}}\right). \tag{24}$$

Determination of the total variance of the maximum displacement of a structural system

To determine the influence of the basic variables on the maximum value of the bending moment $M_{1-1}^{\max}$, we determine the partial derivative of the function $M_{1-1}^{\max}$ with respect to each variable.

$$\begin{aligned}
\frac{\partial M_{1-1}^{\max}}{\partial q} &= \frac{\sqrt{2}l^4}{256t_r}\sqrt{\frac{m}{EJ}}\sin\left(\frac{8\sqrt{2}t_r}{l^2}\sqrt{\frac{EJ}{m}}\right)+\frac{l^2}{24},\\
\frac{\partial M_{1-1}^{\max}}{\partial l} &= \frac{\sqrt{2}l^3 q}{64t_r}\sqrt{\frac{m}{EJ}}\sin\left(\frac{8\sqrt{2}t_r}{l^2}\sqrt{\frac{EJ}{m}}\right)+\frac{lq}{12}-\frac{lq}{8}\cos\left(\frac{8\sqrt{2}t_r}{l^2}\sqrt{\frac{EJ}{m}}\right),\\
\frac{\partial M_{1-1}^{\max}}{\partial t_r} &= -\frac{\sqrt{2}l^4 q}{256{t_r}^2}\sqrt{\frac{m}{EJ}}\sin\left(\frac{8\sqrt{2}t_r}{l^2}\sqrt{\frac{EJ}{m}}\right)+\frac{l^2 q}{32EJ}\cos\left(\frac{8\sqrt{2}t_r}{l^2}\sqrt{\frac{EJ}{m}}\right),\\
\frac{\partial M_{1-1}^{\max}}{\partial EJ} &= \frac{\sqrt{2}l^4 q}{512t_r EJ}\sqrt{\frac{m}{EJ}}\sin\left(\frac{8\sqrt{2}t_r}{l^2}\sqrt{\frac{EJ}{m}}\right)+\frac{l^2 q}{32EJ}\cos\left(\frac{8\sqrt{2}t_r}{l^2}\sqrt{\frac{EJ}{m}}\right),\\
\frac{\partial M_{1-1}^{\max}}{\partial m} &= \frac{\sqrt{2}l^4 q}{512t_r m}\sqrt{\frac{m}{EJ}}\sin\left(\frac{8\sqrt{2}t_r}{l^2}\sqrt{\frac{EJ}{m}}\right)-\frac{l^2 q}{32m}\cos\left(\frac{8\sqrt{2}t_r}{l^2}\sqrt{\frac{EJ}{m}}\right).
\end{aligned} \tag{25}$$

The standard deviation of the maximum bending moment depending on each variable separately is determined by the formula:

$$\sigma'_q = \left|\frac{\partial M_{1-1}^{\max}}{\partial q}\right|\sigma_q, \sigma'_l = \left|\frac{\partial M_{1-1}^{\max}}{\partial l}\right|\sigma_l,$$
$$\sigma'_{t_r} = \left|\frac{\partial M_{1-1}^{\max}}{\partial t_r}\right|\sigma_{t_r}, \sigma'_{EJ} = \frac{\partial M_{1-1}^{\max}}{\partial EJ}\sigma_{EJ}, \qquad (26)$$
$$\sigma'_m = \left|\frac{\partial M_{1-1}^{\max}}{\partial m}\right|\sigma_m.$$

where σ_q, σ_l, σ_{t_r}, σ_{EJ}, σ_m is the standard deviation of the variables, determined from the statistical data of actual measurements or experimental results.

Using the property of independence of variables, we determine the total variance of the maximum bending moment:

$$\sigma^2_{M_{1-1}^{\max}} = \left(\sigma'_q\right)^2 + \left(\sigma'_l\right)^2 + \left(\sigma'_{t_r}\right)^2 + \left(\sigma'_{EJ}\right)^2 + \left(\sigma'_m\right)^2. \qquad (27)$$

Similar to the calculation process described above, we can determine the total variance of the ultimate bending moment of reinforced concrete elements during bending. Depending on the location of the longitudinal reinforcement, two cases are distinguished: bending reinforced concrete elements with single and double reinforcement.

Determination of the total variance of the ultimate bending moment of reinforced concrete elements with single reinforcement

The stress diagram of a rectangular reinforced concrete section with single reinforcement is shown in Figure 3.

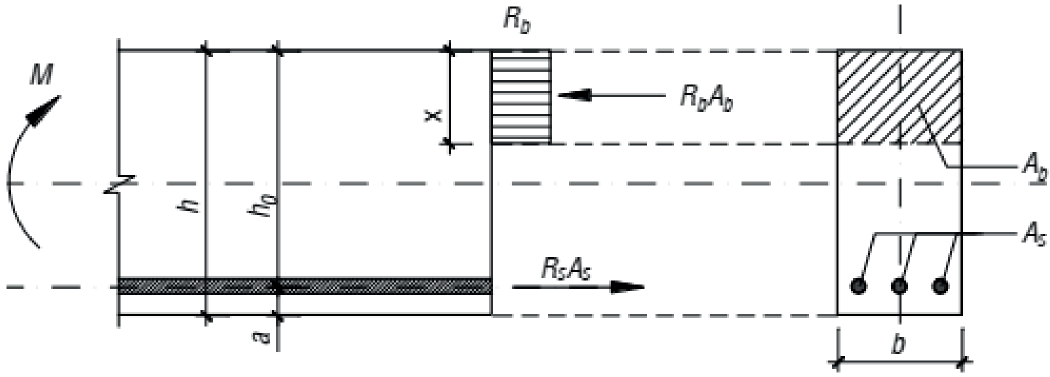


Figure 3. Internal force diagram and stress diagram of a rectangular section with single reinforcement

The calculation of bending concrete elements should be made based on the strength condition:

$$M \le M_{ult} \rightarrow M \le R_b bx\left(h_0 - \frac{x}{2}\right). \qquad (28)$$

From the condition of equilibrium of the projections of internal forces in the section on the beam axis, it is possible to determine the compression zone height of concrete:

$$R_b bx = R_s A_s \Rightarrow x = \frac{R_s A_s}{R_b b}. \qquad (29)$$

From (28) and (29) we can rewrite the expression for the ultimate bending moment:

$$M_{ult} = R_b b\frac{R_s A_s}{R_b b}\left(h_0 - \frac{R_s A_s}{2R_b b}\right) = R_s A_s h_0 - \frac{R_s^2 A_s^2}{2R_b b} \qquad (30)$$

Let us determine the partial variance of the ultimate bending moment for each basic variable:

$$\sigma'_{R_b} = \left|\frac{\partial M_{ult}}{\partial R_b}\right|\sigma_{R_b} = \left|\frac{R_s^2 A_s^2}{2R_b^2 b}\right|\sigma_{R_b};$$

$$\sigma'_{R_s} = \left|\frac{\partial M_{ult}}{\partial R_s}\right|\sigma_{R_s} = \left|A_s h_0 - \frac{R_s A_s^2}{R_b b}\right|\sigma_{R_s};$$

$$\sigma'_{A_s} = \left|\frac{\partial M_{ult}}{\partial R_s}\right|\sigma_{A_s} = \left|R_s h_0 - \frac{R_s A_s^2}{R_b b}\right|\sigma_{A_s}; \quad (31)$$

$$\sigma'_{h_0} = \left|\frac{\partial M_{ult}}{\partial h_0}\right|\sigma_{h_0} = \left|R_s A_s\right|\sigma_{h_0};$$

$$\sigma'_{b} = \left|\frac{\partial M_{ult}}{\partial b}\right|\sigma_{b} = \left|\frac{R_s^2 A_s^2}{2R_b b^2}\right|\sigma_{b},$$

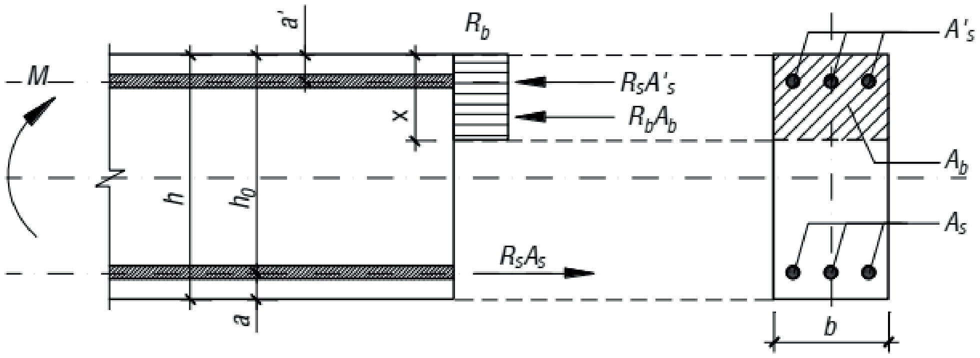
where σ_{R_b}, σ_{R_s}, σ_{A_s}, σ_{h_0}, σ_b is the standard deviation of the variables, determined from the statistical data of actual measurements or experimental results.

The total variance is equal to the following sum

$$\sigma^2_{M_{ult}} = \left(\sigma'_{R_b}\right)^2 + \left(\sigma'_{R_s}\right)^2 + \left(\sigma'_{A_s}\right)^2 + \left(\sigma'_{h_0}\right)^2 + \left(\sigma'_{b}\right)^2. \quad (32)$$

Determination of the total variance of the ultimate bending moment of reinforced concrete elements with double reinforcement

The stress diagram of a reinforced concrete rectangular section with double reinforcement is shown in Figure 4.



<u>Figure 4.</u> Internal force diagram and stress diagram of a rectangular cross-section with double reinforcement

The calculation of bending concrete elements should be made based on the strength condition:

$$M \le M_{ult} \rightarrow M \le R_b b x\left(h_0 - \frac{x}{2}\right) + R_{sc}A'_s\left(h_0 - a'\right). \quad (33)$$

From the condition of equilibrium of the projections of internal forces in the section on the beam axis, it is possible to determine the compression zone height of concrete:

$$R_b b x + R_{sc}A'_s = R_s A_s \Rightarrow x = \frac{R_s A_s}{R_b b} - \frac{R_{sc}A'_s}{R_b b} \quad (34)$$

Substituting the value of x into (33) and performing the transformation, we obtain:

$$M_{ult} = R_b b\frac{R_s A_s}{R_b b}\left(h_0 - \frac{R_s A_s}{2R_b b}\right) =$$
$$R_s A_s h_0 - \frac{R_s^2 A_s^2}{2R_b b} - \frac{R_{sc}^2 A_s'^2}{2R_b b} + \frac{R_s A_s R_{sc} A'_s}{R_b b} - R_{sc}A'_s a'. \quad (35)$$

To determine the influence of individual variables on the overall standard deviation of M_{ult}, we determine the partial derivative of M_{ult} with respect to each basis variable. From (35) we have

$$\frac{\partial M_{ult}}{\partial R_s} = A_s h_0 - \frac{R_s A_s^2}{R_b b} + \frac{A_s R_{sc} A'_s}{R_b b};$$
$$\frac{\partial M_{ult}}{\partial A_s} = R_s h_0 - \frac{R_s A_s^2}{R_b b} + \frac{R_s R_{sc} A'_s}{R_b b}; \quad (36)$$

$$\frac{\partial M_{ult}}{\partial h_0} = R_s A_s;$$
$$\frac{\partial M_{ult}}{\partial R_{sc}} = -\frac{R_{sc} A_s^{'2}}{R_b b} + \frac{R_s A_s A_s^{'}}{R_b b} - A_s^{'} a^{'};$$
$$\frac{\partial M_{ult}}{\partial a^{'}} = -R_{sc} A_s^{'};$$
$$\frac{\partial M_{ult}}{\partial R_b} = \frac{R_s^2 A_s^2}{2R_b^2 b} + \frac{R_{sc}^2 A_s^{'2}}{2R_b^2 b} - \frac{R_s A_s R_{sc} A_s^{'}}{R_b^2 b};$$
$$\frac{\partial M_{ult}}{\partial b} = \frac{R_s^2 A_s^2}{2R_b b^2} + \frac{R_{sc}^2 A_s^{'2}}{2R_b b^2} - \frac{R_s A_s R_{sc} A_s^{'}}{R_b b^2};$$
$$\frac{\partial M_{ult}}{\partial R_{sc}} = -\frac{R_{sc} A_s^{'2}}{R_b b} + \frac{R_s A_s A_s^{'}}{R_b b} - A_s^{'} a^{'};$$
$$\frac{\partial M_{ult}}{\partial A_s^{'}} = -\frac{R_{sc}^2 A_s^{'}}{R_b b} + \frac{R_s A_s R_{sc}}{R_b b} - R_{sc} a^{'};$$

Based on the measurement data and experimental studies, the standard deviation of the parameters R_s, A_s, R_{sc}, A_s', R_b, b, h_0, a' can be determined as σ'_{R_s}, σ'_{A_s}, $\sigma'_{R_{sc}}$, $\sigma'_{A_s^{'}}$, σ'_{R_b}, σ'_b, σ'_{h_0}, $\sigma'_{a'}$.

$$\begin{aligned}
\sigma'_{R_s} &= \left|\frac{\partial M_{ult}}{\partial R_s}\right| \sigma_{R_s}; \sigma'_{A_s} = \left|\frac{\partial M_{ult}}{\partial A_s}\right| \sigma_{A_s};\\
\sigma'_{R_b} &= \left|\frac{\partial M_{ult}}{\partial R_b}\right| \sigma_{R_b}; \sigma'_{R_{sc}} = \left|\frac{\partial M_{ult}}{\partial R_{sc}}\right| \sigma_{R_{sc}};\\
\sigma'_{A_S'} &= \left|\frac{\partial M_{ult}}{\partial A_S^{'}}\right| \sigma_{A_S'}; \sigma'_b = \left|\frac{\partial M_{ult}}{\partial b}\right| \sigma_b;\\
\sigma'_{h_0} &= \left|\frac{\partial M_{ult}}{\partial h_0}\right| \sigma_{h_0}; \sigma'_{a'} = \left|\frac{\partial M_{ult}}{\partial a^{'}}\right| \sigma_{a'}.
\end{aligned} \tag{37}$$

The total variance of the ultimate bending moment of a reinforced concrete section is determined by the formula:

$$\begin{aligned}
\sigma_{M_{ult}}^2 &= \left(\sigma'_{R_s}\right)^2 + \left(\sigma'_{A_s}\right)^2 + \left(\sigma'_{R_{sc}}\right)^2 +\\
&+ \left(\sigma'_{A_s'}\right)^2 + \left(\sigma'_{R_b}\right)^2 + \left(\sigma'_{h_0}\right)^2 + \left(\sigma'_b\right)^2 + \left(\sigma'_{a'}\right)^2.
\end{aligned} \tag{38}$$

After determining the total variance of M_{ult} and $M_{1-1}^{\max}$, we can determine the reliability index using the following formula:

$$\beta = \frac{\mu_{M_{ult}} - \mu_{M_{1-1}^{\max}}}{\sqrt{\sigma_{M_{ult}}^2 + \sigma_{M_{1-1}^{\max}}^2}} \tag{39}$$

By comparing the value of the calculated reliability index with its limit value according to formula (21), we can draw conclusions about the reliability of monolithic reinforced concrete structures in the scenario of sudden removal of load-bearing structural elements.

5. CONCLUSIONS

In this article, a quantitative assessment of the stability of monolithic reinforced concrete frame structures against progressive collapse is presented using a probabilistic approach. The distinctive feature of the proposed approach is that the probabilistic models take into account the basic variable loads and resistances of factors caused by the phenomenon of progressive collapse. Using the first-order reliability method (FORM), the conditional probabilities of progressive collapse and reliability index of the monolithic structural system are obtained under the scenario of removing one of the load-bearing structural elements.

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