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DEVELOPMENT OF COMPUTATIONAL SCHEMES OF GROUP TARGET CONSTRAINTS FOR SOME ELASTIC SYSTEMS PART 2: SAMPLES OF ANALYSIS

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Abstract: For some elastic systems with a finite number of degrees of freedom of masses, in which the directions of mass movement are parallel, methods have been developed for creating additional constraints, the introduction of each of which purposefully increases the value of only one natural frequency to a given value, while not changing any of the other natural frequencies and not one of the natural modes (forms of natural oscillations). If it is necessary to increase the values of several natural frequencies in a targeted manner, then this requirement can be implemented by creating an appropriate number of separate targeted constraints. The computational scheme of each of the individual targeted constraints should include racks installed at the nodes of mass application and directed along the trajectory of their movement. In some cases, individual targeted constraints can be independently installed on the original (initial) system. In most cases, on the basis of individual targeted constraints, a computational scheme of a united group targeted constraint is developed, which increases all the intended frequencies to the set values, without changing any of the other natural frequencies and not one of the natural modes. The distinctive paper is devoted verification of the proposed algorithm for the development of group targeted constraint on SCAD and “Lira Software” products.

Keywords: natural frequency, natural mode, targeted constraint, group targeted constraint, |additional stiffness coefficients, algorithm

О ФОРМИРОВАНИИ РАСЧЕТНЫХ СХЕМ ГРУППОВЫХ ПРИЦЕЛЬНЫХ СВЯЗЕЙ ДЛЯ НЕКОТОРЫХ УПРУГИХ СИСТЕМ ЧАСТЬ 2: ПРИМЕРЫ РАСЧЕТОВ

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Аннотация: Для некоторых упругих систем с конечным числом степеней свободы масс, у которых направления движения масс параллельны, разработаны методы создания дополнительных связей, введение каждой из которых прицельно увеличивает величину только одной собственной частоты до заданного значения, не изменяет при этом ни одну из остальных собственных частот и не одну из форм собственных колебаний. Если необходимо прицельно увеличить величины нескольких собственных частот, то это требование можно реализовать созданием соответствующего количества отдельных прицельных связей. Расчётная схема каждой из отдельных прицельных связей должна включать стойки, установленные в узлах приложения масс и направленные по траектории их движения. В некоторых случаях отдельные прицельные связи могут автономно устанавливаться на исходной системе. В большинстве случаев на основе отдельных прицельных связей формируется расчётная схема единой групповой прицельной связи, которая увеличивает все намеченные частоты до заданных значений, не изменяя при этом ни одну из остальных собственных частот и не одну из форм собственных колебаний. В данной статье рассмотрен способ формирования матрицы дополнительных жесткостей, которой соответствует групповая прицельная связь. Сформулированы требования к тем отдельным прицельным связям, на основе которых формируется групповая прицельная связь. Рассматривается верификация алгоритма формирования групповых прицельных связей с учётом сформулированных требований на основе решения тестовых задач с использованием программных продуктов “SCAD” и “Лира”.

Ключевые слова: частота собственных колебаний, форма собственных колебаний, отдельная прицельная связь, групповая прицельная связь, коэффициенты дополнительных жесткостей

Let us demonstrate the implementation of the algorithm using examples from [4, 5, 18-20]. Figure 1*a* shows the original (initial) system.

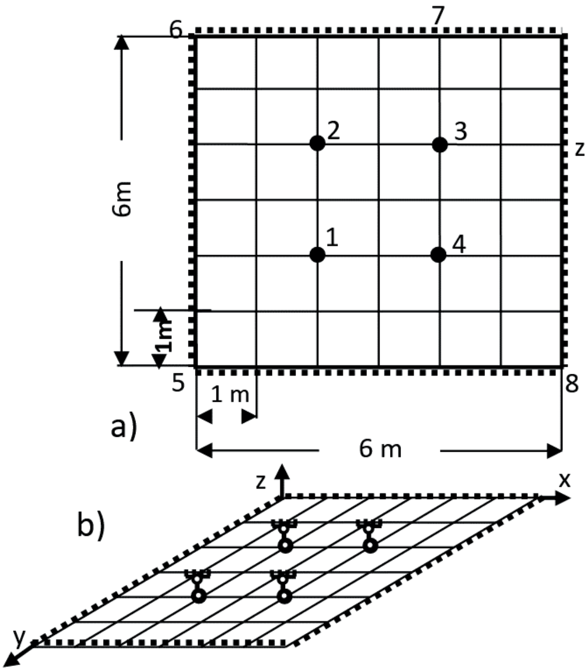

Figure 1. Considering problem

The plate is hinged along the contour. The plate thickness is equal to 0.12 *m*. The modulus of elasticity of the plate material is equal to E = 24000000000 N/m^2 Poisson's ratio is equal to $\nu_0 = 0.2$.

Let the mass of the plate be extremely small (0.00000001 kg/m^3).

The masses

$$m[1] = 1000\,kg,\quad m[2] = 1100\,kg,$$
$$m[3] = 1150\,kg,\quad m[4] = 1200\,kg$$

are located at the nodes of the plate. Figure 1*b* shows the basic system of the displacement method [1-3, 6, 7, 16, 17].

The coefficients of the equations of the displacement method and the values of the nodal masses form the matrices $A = \|r[i,k]\|$ and $M = \|m[i]\|$.

The values of the coefficients of the displacement method equations are given in Table 1.

The roots of the equation

$$\left|A - \omega^2 M\right| = 0 \qquad (2)$$

determine the frequency spectrum of the natural oscillations of the system. The natural frequencies and natural modes (forms of the natural oscillations) of the plate are presented in Table 2.

It is required to increase the third natural frequency to 220 sec^{-1} and the fourth to 230 sec^{-1} by introducing targeted constraints.

Thus, for further implementation of the first action of the algorithm for the example under consideration, it is necessary to create two separate targeted constraints, one of which will increase the third natural frequency to 220 sec^{-1}, and the other the fourth to 230 sec^{-1}.

In [4, 5, 18-20] it is shown that the development of targeted generalized constraint is based on the formation of a matrix of additional stiffness coefficients

$$A_0 = \|a_0[i,k]\|_{i,k=1}^{n} \qquad (2)$$

and the procedure for their determination is given. The values of the coefficients of the additional stiffness matrix, increasing the third natural frequency to 220 sec^{-1}, are presented in Table 3.

The roots of equation (3) determine the spectrum of frequencies and modes of natural oscillations of the system, reinforced by a targeted constraint, increasing the third natural frequency to 220 sec^{-1}

$$\left|(A + A_0) - \omega^2 M\right| = 0 . \qquad (3)$$

Table 1. The values of the coefficients of the displacement method equations.

i, k	1	2	3	4
1	22758257.28	-9604812.96	585308.28	-9604812.96
2	-9604812.96	22758257.28	-9604812.96	585308.28
3	585308.28	-9604812.96	22758257.28	-9604812.96
4	-9604812.96	585308.28	-9604812.96	22758257.28

Table 2. The natural frequencies and natural modes of the plate.

ω	60,932	138,865	143,624	196,414
1	0.4905	0.0001	0.7047	-0.5934
2	0.4966	-0.7075	0.0945	0.5167
3	0.5058	-0.0711	-0.7029	-0.4387
4	0.5069	0.7032	0.0190	0.4341

Table 3. The values of the coefficients of the additional stiffness matrix, increasing the third natural frequency to 220 sec^{-1}.

i, k	1	2	3	4
1	12829437.77	1892709.01	-14715733.27	414235.72
2	1892709.01	279228.71	-2170991.55	61111.62
3	-14715733.27	-2170991.55	16879368.34	-475140.26
4	414235.72	61111.62	-475140.26	13374.81

Table 4. The natural frequencies and natural modes of the plate.

ω	60,932	138,865	196,414	220
1	0.4905	0.0001	-0.5934	-0.5934
2	0.4966	-0.7075	0.5167	0.5167
3	0.5058	-0.0711	-0.4387	-0.4387
4	0.5069	0.7032	0.4341	0.4341

The frequencies and coordinates of the natural modes are presented in Table 4.

Within development of computational scheme of this targeted constraint, the length of the first rack was taken to be equal to $l_{st}[1] = 0.4$ *m*, and the cross-sectional area of the rack rods is equal to $F_{st} = 0.004$ m^2, corresponding diameter is equal to $D_{st} = 0.07136496$ *m*. The cross-sectional areas of the belt rods were taken to be the same, and their value was determined in the process of development of constraint [4, 5, 18-20].

Parameters of cross-sectional area of the belt rods are equal to $F_p = 0.00684$ m^2, $D_p = 0.0933$ *m*.

We have the following lengths of racks of the constraint:

$$l_{st}[1] = 0.4\,m, \quad l_{st}[2] = 0.09782\,m,$$
$$l_{st}[3] = -0.26064\,m, \quad l_{st}[4] = 0.05770\,m.$$

The modulus of elasticity of the material of the rods of constraint is equal to

$$E_{sp} = 206000000000\ N/m^2 .$$

The lengths of the racks turned out to be of different signs. Therefore, the rods of the belts connecting the tops of racks 2 – 3 and 3 - 4 must pass "through" the plate. Structurally, such a scheme requires an ideally free "passage" of part of the rods of the targeted constraint "through" the original system, which is almost impossible to implement.
The general appearance of this targeted constraint is shown in Figure 2.

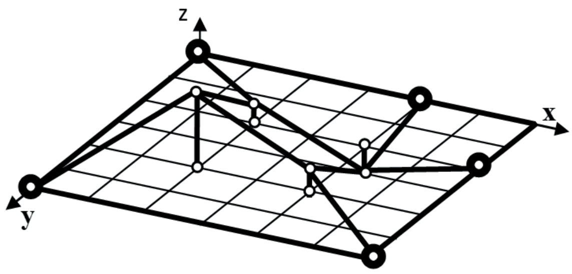

Figure 2. The general appearance of targeted constraint

In these cases, the targeted constraint should be shifted in the direction of mass movement in a positive or negative direction by an amount at which the values of all the lengths of the main racks will be of the same sign.
When choosing the shift values, it is necessary to take into account the operating conditions and design requirements within development of a group targeted constraint.
In this example, the value of the shift of constraint is equal to 0.3 *m*. Accordingly, the length of the first rack will now be equal to 0.7 *m*, and the cross-sectional areas of the racks are equal to $F_{st} = 0.004\ m^2$.
After the shift, the lengths of the racks changed:

$$l_{st}[1] = 0.7\,m,\quad l_{st}[2] = 0.39782\,m,$$
$$l_{st}[3] = 0.03945\,m,\quad l_{st}[4] = 0.35770\,m.$$

The general appearance of this modified targeted constraint is shown in Figure 3.
A separate targeted constraint, increasing the fourth natural frequency to 230 sec^{-1}, is developed similarly to the previous constraint. The values of the coefficients of the additional stiffness matrix are presented in Table 5.
The length of the first rack is set at 0.4 *m*, and the cross-sectional areas of the racks are taken as $F_{st} = 0.004\ m^2$, that is, the same as when creating the previous targeted constraint.

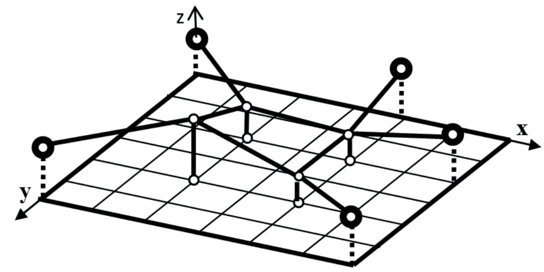

Figure 3. The general appearance of modified targeted constraint

The cross-sectional areas of the belt rods are taken to be the same, and their value was determined in the process of development of the constraint [4, 5, 18-20]. The frequencies and coordinates of the natural modes are presented in Table 6.
Parameters of the sections of the belt rods are equal to $F_p = 0.000697\ m^2$, $D_p = 0.02988\ m$.
The lengths of the racks of this constraint are equal to

$$l_{st}[1] = 0.4\,m,\quad l_{st}[2] = -0.43643\,m,$$
$$l_{st}[3] = 0.22436\,m,\quad l_{st}[4] = -0.38251\,m.$$

The lengths of the racks here also turned out to be of different signs. The general appearance of this targeted constraint is shown in Figure 4.

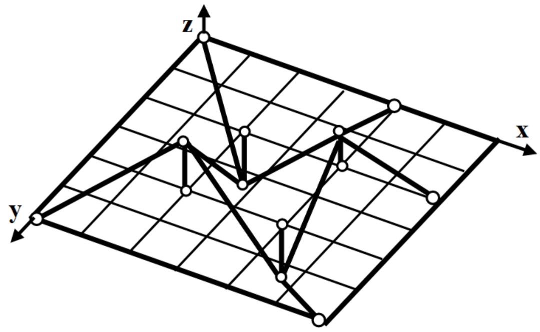

Figure 4. The general appearance of targeted constraint.

Therefore, the targeted constraint should be shifted in the direction of the mass movement in a positive or negative direction by an amount at which the values of all the lengths of the main racks will be of the same sign.

Table 5. The values of the coefficients of the additional stiffness matrix, increasing the fourth natural frequency to 230 sec^{-1}.

i, k	1	2	3	4
1	4613401.93	-4418412.37	3922001.78	-4049185.23
2	-4418412.37	4231664.23	-3756234.88	3878042.80
3	3922001.78	-3756234.88	3334220.22	-3442342.97
4	-4049185.23	3878042.80	-3442342.97	3553971.95

Table 6. The natural frequencies and natural modes of the plate.

ω	60,932	138,865	143,624	230
1	0.4905	0.0001	0.7047	-0.5934
2	0.4966	-0.7075	0.0945	0.5167
3	0.5058	-0.0711	-0.7029	-0.4387
4	0.5069	0.7032	0.0190	0.4341

Table 7. The values of the coefficients of the group matrix of additional stiffness.

i, k	1	2	3	4
1	4613401.93	-4418412.37	3922001.78	-4049185.23
2	-4418412.37	4231664.23	-3756234.88	3878042.80
3	3922001.78	-3756234.88	3334220.22	-3442342.97
4	-4049185.23	3878042.80	-3442342.97	3553971.95

The general appearance of this modified targeted constraint is shown in Figure 5.

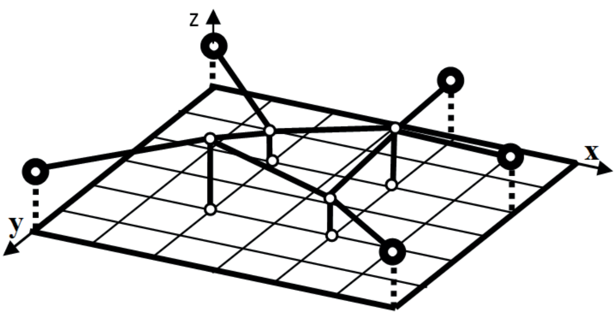

Figure 5. The general appearance of modified targeted constraint.

Let us assume that in accordance with the design requirements the belt rods do not intersect in the computational scheme of the group targeted constraint. Therefore, in the considering case, the value of the constraint shift is adopted at 0.85 *m*. Thus, we have

$$l_{st}[1] = 1.25\,m,\quad l_{st}[2] = 0.4136\,m,$$
$$l_{st}[3] = 1.0744\,m,\quad l_{st}[4] = 0.4674\,m.$$

The general appearance of this targeted constraint is shown in Figure 5.

After forming all the individual targeted constraints, which are required for solution of the problem, it is possible to form a group matrix of additional stiffness coefficients. In order to do this, it is sufficient to sum up the coefficients of the additional stiffness matrices of individual targeted constraints.

In the considering sample the corresponding coefficients of Tables 3 and 5 are summed up. The coefficients of the group matrix of additional stiffness are given in Table 7.

If we denote the group matrix of additional stiffness, then the roots of the equation

$$\left|(A+\overline{A}_0)-\omega^2 M\right|=0 \quad (4)$$

determine the spectrum of frequencies and forms of natural oscillations of the system, enhanced by group targeted constraint, increasing the third frequency of natural oscillations to 220 sec^{-1}, and the fourth to 230 sec^{-1}. The natural frequencies and coordinates of the natural modes are presented in Table 8.

Table 8. The natural frequencies and natural modes of the plate

ω	60,932	138,865	220	230
1	0.4905	0.0001	0.7047	-0.5934
2	0.4966	-0.7075	0.0945	0.5167
3	0.5058	-0.0711	-0.7029	-0.4387
4	0.5069	0.7032	0.0190	0.4341

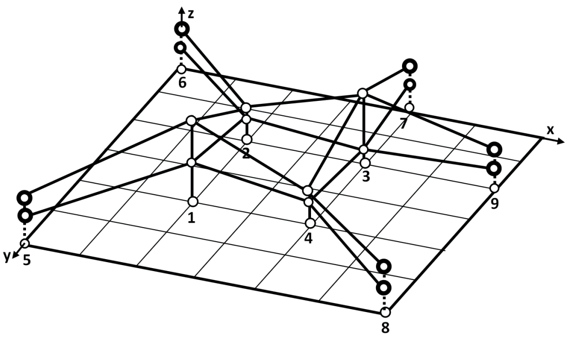

Figure 6. The general appearance of the group targeted constraint

By implementing the second, the third and the fourth steps of the above algorithm, we form a group targeted constraint. The general appearance of this constraint is shown in Figure 6.
The plan view of this constraint and the numbering of the nodes are shown in Figure 7.
The z coordinates of the nodes of the group targeted constraint are given in Table 9.
When implementing the second step of the algorithm of development of a group targeted constraint, individual targeted constraints are placed in the nodes of the plate. In this case, on the lower sections of the constraint, the racks are combined and, of these, on each of these sections, only one is left.
Within development of individual targeted constraints the stiffnesses of the sections of the rods of the racks were set so that they would coincide with each other in each node.

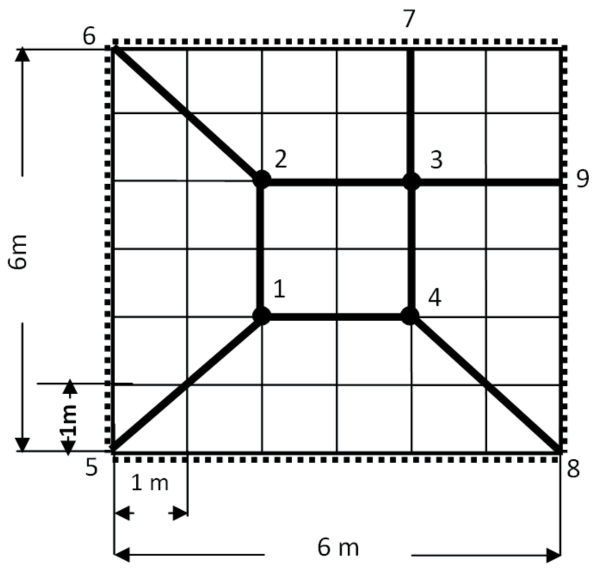

Figure 7. The plan view of the targeted constraint and the numbering of the nodes

Table 10. The natural frequencies and natural modes of the plate

ω	60,341	137,618	142,334	194,681
1	0.4903	0.0012	0.7213	-0.5747
2	0.4968	-0.7042	0.0805	0.5212
3	0.5056	0.0721	-0.6879	-0.4586
4	0.5071	0.7063	0.0072	0.4333

Table 11. The natural frequencies and natural modes of the plate

ω	60,341	137,618	219,151	227,946
1	0.4903	0.0012	0.7213	-0.5747
2	0.4968	-0.7042	0.0805	0.5212
3	0.5056	0.0721	-0.6879	-0.4586
4	0.5071	0.7063	0.0072	0.4333

Table 9. The z coordinates of the nodes of the group targeted constraint

Number of node	Coordinates z	
	top	bottom
1	1.25	0.7
2	0.4136	0.3978
3	1.0744	0.03945
4	0.4764	0.3577
5	0.85	0.3
6	0.85	0.3
7	0.85	0.3
8	0.85	0.3
9	0.85	0.3

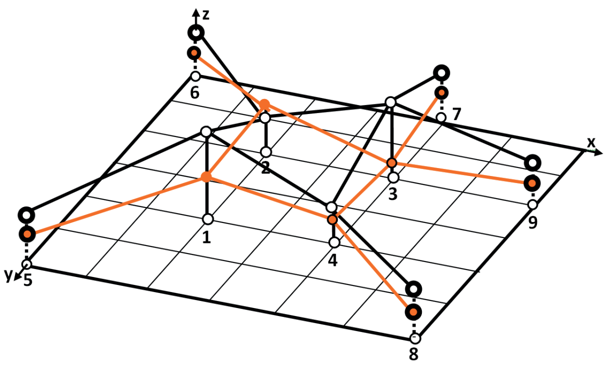

Figure 8. The general appearance of the group targeted constraint

It is this circumstance that ensures the adequacy of the parameters of the main racks of individual targeted constraints to their parameters in the group constraint.

Let us test this group targeted constraint with the use of "SCAD" and "Lira" software products [8-15]. First, we will determine the natural frequencies and coordinates of the natural modes of the original plate. The test results presented in Table 10 are quite close (differences of about one percent) to the results given in Table 2, which confirms the reliability of the initial data. The natural frequencies and coordinates of the natural modes presented in Table 8 were obtained as the roots of equation (4) based on the use of a group matrix of additional stiffness coefficients $\overline{A}_0$ before development the computational scheme of the group targeted constraint.

The natural frequencies and coordinates of the natural modes of computational scheme of the group targeted constraint obtained by the test are presented in Table 11. These results were obtained by testing the computational scheme of the group targeted constraint. Comparison of the results given in Tables 10 and 11 shows that only those frequencies have changed that the group constraint was targeted at, while the coordinates of the natural modes and other natural frequencies have remained the same. These test results confirm the "targeting" of the group constraint with sufficient accuracy.

The closeness of the results given in Tables 8 and 11 confirms the reliability of the proposed approach.

Let us consider the case when the operating conditions require that the lengths of the individual targeted constraints be greater than 0.08 *m*. The individual targeted constraint racks that increase the fourth natural frequency meet this condition. Among the individual targeted constraint racks that increase the third natural frequency we have

$$l_{st}[3] = 0.03945\,m < 0.08\,m\,.$$

It doesn't meet the stated condition. Therefore, the shift of this initially created targeted constraint link must be increased, for example, from 0.3 *m* to 0.36 *m*. Thus, we have the following modified parameters:

$$l_{st}[1]=0.76\,m,\quad l_{st}[2]=0.4578\,m,$$
$$l_{st}[3]=0.0994\,m,\quad l_{st}[4]=0.4177\,m.$$

The development of the group targeted constraint in this case is implemented in the same way as in the previous one, but taking into account the changes in the parameters of an individual targeted constraint, increasing the third natural frequency. The z coordinates of the nodes of the group targeted constraint are given in Table 12. The general view of this group targeted constraint is shown in Figure 8.

In this group constraint, some of the rods of the belts intersect (in spans “1 – 2”, “2 – 6”, “2 – 3”). The nodes in which the rods intersect are structurally feasible, but their specificity is not discussed in this paper.

A test check of the computational scheme of this group constraint confirmed its targeting.

So, this paper considers a verification of method of forming a matrix of additional stiffness, which corresponds to a group targeted constraint. Requirements are formulated for those individual targeted constraints, on the basis of which the group targeted constraint is formed.

Table 12. The z coordinates of the nodes of the group targeted constraint

Number of node	Coordinates z	
	black	red
1	1.25	0.76
2	0.4136	0.4578
3	1.0744	0.0994
4	0.4764	0.4177
5	0.85	0.36
6	0.85	0.36
7	0.85	0.36
8	0.85	0.36
9	0.85	0.36

An algorithm for development of group targeted constraint is proposed with allowance for formulated requirements. The proposed algorithm was tested with the use of “SCAD” and “Lira” software products [8-15].

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