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NONLOCAL IN TIME MODEL OF THE LONGITUDINAL VIBRATIONS OF THE HIGH-DAMPING STEEL ROD ELEMENT

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Abstract: The paper is devoted to the modeling of longitudinal vibrations of a 01Yu5T damping steel rod, taking into account the typical features of the material damping. A brief review of the various damping alloys is given, as well as a brief review of the models of frequency-independent and amplitude-dependent internal friction, theoretically applicable to describe the damping capacity of steel 01Y5T. Considered rod is represented in the article as a one-degree-of-freedom system. The model of its longitudinal vibrations, accounting for the internal friction, is based on the principals of nonlocal mechanics: the impact of the previous history of deformation on the current state of the system is taken into account. The IV order Runge-Kutta method was used to solve the equation of motion. The impact of the nonlocal scale parameter on the material damping in terms of the considered model is shown on the basis of the simulation of the rod free oscillation. The calibration of the nonlocal in time model of rod vibrations based on experimental data was performed using the least squares method. The results of the forced vibrations modeling under the stochastic load for an element made of 01Y5T steel, taking into account amplitude-dependent damping, are presented in comparison with the results obtained for a steel with a constant level of internal friction.

Keywords: high-damping alloys, damping steel, rod vibrations, material damping, amplitude-dependent damping, nonlocal mechanics

НЕЛОКАЛЬНАЯ ВО ВРЕМЕНИ МОДЕЛЬ ПРОДОЛЬНЫХ КОЛЕБАНИЙ СТЕРЖНЕВОГО ЭЛЕМЕНТА, ВЫПОЛНЕННОГО ИЗ СТАЛИ СО СВЕРХВЫСОКИМ ДЕМПФИРОВАНИЕМ

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Аннотация: Статья посвящена моделированию продольных колебаний стержня, выполненного из демпфирующей стали 01Yu5T, с учётом характерных свойств внутреннего демпфирования материала. Приведен краткий обзор различных демпфирующих сплавов, а также краткий обзор моделей частотно-независимого и амплитудно-зависимого внутреннего трения, теоретически применимых для описания демпфирующей способности стали 01Ю5Т. Рассматриваемый стержень представлен в статье, как система с одной степенью свободы. Модель для описания его продольных колебаний с учётом внутреннего трения построена на основании положений нелокальной механики: учтено влияние предыдущей истории деформирования на состояние системы в настоящий момент. Для решения уравнения движения использован метод Рунге-Кутта IV порядка. На примере свободных колебаний стержня показано влияние изменения масштабного параметра нелокальной модели на результаты расчета. Калибровка нелокальной во времени модели колебаний стержня выполнена на основании экспериментальных данных методом наименьших квадратов. Приведены результаты моделирования вынужденных колебаний под действием нагрузки, представленной как случайный стационарный процесс, для элемента из стали 01Ю5Т с учетом амплитудно-зависимого демпфирования, в сравнении с результатами, полученными для стали с постоянным уровнем внутреннего трения.

Ключевые слова: сплавы высокого демпфирования, демпфирующая сталь, колебания стержней, внутреннее трение, амплитудно-зависимое демпфирование, нелокальная механика

INTRODUCTION

The protection of the engineering structures against vibrations remains a topical problem due to high level of noise from transport, industrial, and construction facilities. Another key problem of civil engineering consists in the protection of buildings, bridges, and other structures against impact loads including seismic ones. A possible way of solution of such problems consists in the use of specific structural materials with high damping properties. Commonly, one could use layers of polymer materials to decrease the vibration impact on the structure; it should be noted that most polymers have low moduli and strength.

Another promising approach consists in the use of high damping metallic alloys to design functional structural elements [1-4]. Magnesium based alloys show high level of damping properties: 5 times that of iron and 12 times that of aluminum [1]. Moreover, their low mass density makes them useful for various lightweight structures; a good corrosion resistance is also one of the features of Mg-based alloys. At the same time the tensile strength of Mg alloys remains relatively low, thus, they cannot be used in bearing structural elements.

Shape memory alloys, or SMAs, (Copper alloys, Titanium Nickelide, etc.) show also very high damping capacity [2, 5-11]. SMAs of Ti-Ni system having two stable phase constitutions, the low-modulus face-centered martensite stable at lower temperatures and high-modulus volume-centered austenite stable at higher temperatures, provide two possible mechanisms of damping, the martensite inelasticity and pseudoelasticity [5, 9]. The first one consists in structural transitions between twinned and untwinned martensite induced by stress impacts [11, 12]; such strains irreversible in the martensite state could be removed after the temperature-induced martensite-to-austenite phase transform. The second mechanism so-called also “superelasticity” due to high reversible strains consists in the phase transform from austenite into martensite when the stress intensity rises and vice versa when it drops with large hysteretic loops [6, 7, 11]. The maximum damping capacity of TiNi alloy is about 40%; at the same time, it depends strongly on the amplitude. Since the maximum could be reached at strain amplitude level of 10^{-5} and higher, the TiNi damping element is inefficient for low-amplitude vibration suppression but is promising in seismic protection devices [5-7]. On the other hand, the pseudoelastic deforming of SMA damping elements results in the possibility of its shape restauration after unloading; indeed, “…in base isolation of buildings, the superelasticity creates the property of self-centering; the base isolation device acting alone can restore the building to its original position” [6]. Despite these facts the application the high cost of SMAs makes they industrial use in most cases almost impossible, moreover the mechanical behavior of SMAs is very complex, especially in case of thin-walled structures that could buckle under extremely low stresses [13-16].

Iron based high damping alloys have high mechanical strength and also lower cost as compared with shape memory alloys [3]. Fe-Cr alloys were developed and studied e.g. in [17-19]. The researchers were looking for the ways to increase the damping capacity of the material. It is noted in [19] that since the Fe-Cr alloys are ferromagnetic, their high damping capacity is correlated to the magneto-mechanical hysteresis loss. In this regard the iron-based alloys differ from magnesium alloys, where the main cause of increased damping level is dislocation [1].

In [3] the major experimental research of the irons alloyed with rare earth elements is presented. It is shown that the energy loss in the magneto-mechanical hysteresis damping is proportional to the “saturated magnetostriction” [3] and the level of internal stress, i.e. the level of damping in the ferromagnetic alloys depends on the stress rate. The approximation of those correlations for different types of alloys is also provided in the paper.

Steel 01Yu5T [20] that consists 5% of aluminum also has high damping properties due to magnetostriction. The experimental studies of the material have shown that its damping capacity depends on the stress intensity and does not depend on vibration frequency and temperature. The maximum damping capacity of 37% is reached at the 12 MPa stress level. The steel 01Yu5T is strong enough to be used for making of bearing elements and it is way more cost efficient in comparison to the other iron based alloys, especially to the ones that contains rare earth elements.

To unlock the full potential of the high damping steel it is necessary to design structural elements so they work in the stress range where the damping capacity is the highest. The design process requires special damping model, which can describe the behavior of the magnetostriction material. Obviously, this models have to be frequency independent and dependent on stress attitude at the same time.

The frequency independence of internal friction was found out experimentally in the beginning of XX century [21]. The results of that study contradicted to the theory of viscous damping [22] and since then many attempts have been taken by different researchers to develop the uncontroversial mathematical model of the internal damping that does not depend on vibration frequency, but depends on the level of stress.

One of the most well-known frequency independent damping models is the complex stiffness model derived by E.S. Sorokin [23]. This model provides good alignment with an experiment. However, it might be challenging to use it for the analysis of the complicated engineering systems with big amount of degrees of freedom [24].

An interesting concept of the internal damping is proposed in [25]. In this paper a viscous “geometric” damping term is introduced to the equilibrium equation considering an internal shear force to be proportional to the time rate of change of the slope. Even though this model is frequency independent, it has no correlation with the level of stress in the structure, at least in the form proposed in [25].

In [24] three nonlinear models of the frequency independent damping are studied: the hysteretic model, the modified hysteretic model and the quasi-hysteretic model. It is shown on the example of the single-degree-of-freedom system that for the high level of damping properties first two models give significantly different results from the ones for the complex stiffness model and the equivalent viscous damping model. Oppositely, the quasi hysteretic model is holding close to the complex stiffness model and the equivalent viscous damping model for the wide range of damping levels. Such a stability of the quasi hysteretic model is explained by the fact that it deals with the mean values of displacements and velocities, i.e. considers the previous time history of the vibration process.

The model of frequency independent internal damping with the use of the Rzhanitsyn rod approximation is presented in [26]. On each iteration of the vibration process simulation the stress values are determined in every element of the rod structure and according to that the damping constants determined experimentally are entered into the model. This model allows to take into account the dependence of the material damping on the stress level. It is also noted that the history of the rod loading is taken into account.

In [27] the experimentally determined damping constants are implemented in the FEA computational scheme to build up the stress level dependent damping model. It is concluded in the paper that the damping characteristics are integral, i.e. they can be represented as a summation of the instant changing during the vibration process values of damping coefficients. As well as in [26] it is mentioned that such method of damping consideration makes the computational model complex and increases the computing time.

Damping models based on the principals of nonlocal mechanics are flexible enough to be used for the dynamic problems where the classic models do not provide adequate results [28, 29]. Nonlocal in time damping model [30] allows considering the influence of the previous time history of the vibration process on the damping forces in current moment. Moreover, it can be easily incorporated to the FEA algorithm and therefore applied for the modeling of relatively complicated engineering systems. However, the damping described by this model is frequency dependent. Thus it cannot be used for modeling of the dynamics of elements made of the high-damping steel. In [31] the previous history of dynamic deformation of the bending element is considered to affect the elastic forces in the structure. In [32] it is shown with the series of numerical experiments that the model is applicable to simulate frequency independence of damping properties. In addition, the flexibility of the model can be used for modelling of the stress level dependent damping.

MODELLING OF A SINGLE-DEGREE-OF-FREEDOM SYSTEM FREE VIBRATIONS

Consider free oscillation of a rod of constant cross-section area A, referred to the frame $Oxyz$ ($0 \le x \le l$), clamped at a point $x = 0$, with a point mass m at $x = l$. Let the rod material be homogeneous with mass density ρ and Young modulus E, and let us assume $m \gg \rho A l$, therefore the inertial forces in longitudinally deforming rod could be neglected (Figure 1a). Since the stress-strain state of the rod is homogeneous the oscillation of the considered structure could be approximated by the discrete spring-mass system with the stiffness of spring equal to $\frac{EA}{l}$ and the single degree of freedom $y(t) = u(x,t)\big|_{x=l}$, i.e. the longitudinal translation of the mass m (Fig. 1b).

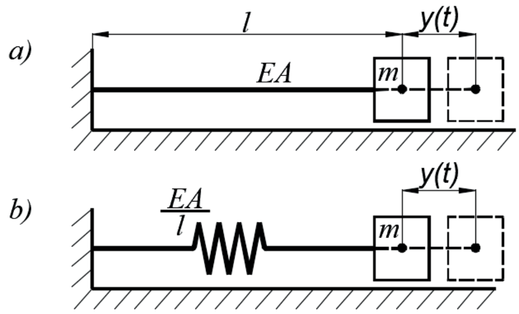

Figure 1 – a) Considered rod with a point mass, subjected to the longitudinal oscillations; b) the spring-mass system equivalent to the considered rod.

Let us assume the damping be viscous; the oscillation of such systems could be defined by the dynamic equation (1) [33]:

$$\ddot{y}(t) + 2n\dot{y}(t) + \omega_0^2 y(t) = 0 \qquad (1)$$

where dots define time derivative, $n = \xi\omega_0$ is the external damping factor, the eigenfrequency ω_0 is determined as follows:

$$\omega_0 = \sqrt{\frac{r_{11}}{m}}, \quad r_{11} = l^{-1}EA \qquad (2)$$

where r_{11} denotes the reaction of the beam on the unit end point translation $y \equiv 1$.

The nonlocal damping model is based on the hypothesis of dependence of the elastic reaction force at the time t as well on the instantaneous deformed state defined here as $y(t)$ on the deforming history, i.e. on the state of the system at any previous time instant denoted hereinafter as t_1. Let us note that the interference between two different states of the system corresponding to t_1 and t vanishes with growing time range $|t - t_1|$ [34, 31]; such effect could be described by the integral operator with the kernel $R(t - t_1)$ normalized as follows:

$$\int_0^t R(t-\tau)\,d\tau = 1 \qquad (3)$$

For instance, the kernel $R(t-t_1)$ could be based on the normalized error function (4) [35]:

$$R(t-\tau) = \frac{2\eta}{\sqrt{\pi}} \cdot e^{-\eta^2(t-\tau)^2}. \qquad (4)$$

Here η denotes the scale parameter that corresponds to the time nonlocality level of the model. Lower η values result in higher nonlocal effects (Fig. 2).

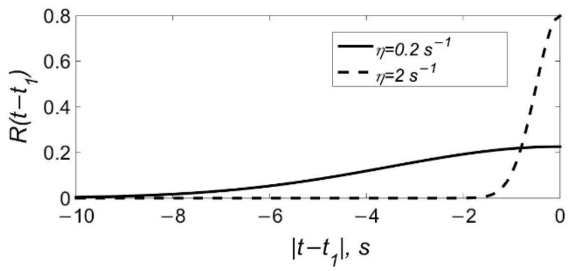

Figure 2 – Kernels $R(t-t_1)$ based on the normalized error function for the different values of the scale parameter η

On the other hand the model reduces to the classical local one when $\eta \to \infty$ and $R(t-t_1) \to \delta(t-t_1)$ where δ is the Dirac delta.

The dynamic equation (1) accounting for nonlocal behavior defined by (3) could be written as follows:

$$\ddot{y}(t) + 2n\dot{y}(t) + \omega_0^2 \int_0^t R(t-t_1) y(t_1) dt_1 = 0. \qquad (5)$$

Let us introduce the following dimensionless variables:

$$\xi(t) = \frac{y(t)}{l}, \tau = t\omega_0, \ \tau_1 = t_1\omega_0 \qquad (6)$$

As a result, we obtain the dimensionless dynamic equation (7):

$$\ddot{\xi}(\tau) + \frac{2n\dot{\xi}(\tau)}{\omega_0} + \frac{1}{\omega_0}\int_0^\tau \frac{2\eta}{\sqrt{\pi}} e^{-\frac{\eta^2}{\omega_0^2}(\tau-\tau_1)^2} \xi(\tau) d\tau_1 = 0. \qquad (7)$$

This equation could be solved numerically using the fourth-order Runge-Kutta algorithm [36].

DETERMINING OF THE SCALE PARAMETER BASED ON THE EXPERIMENTAL DATA

Since the dissipation properties of the investigated 01Yu5T steel depend strictly on the stress amplitude the model calibration should be implemented using the test data [32] and the solution for the damped oscillations of the system (7) excited by the unit initial translation of the mass m .

The impact of the value of the scale parameter η on the oscillations is shown on the Fig. 3:

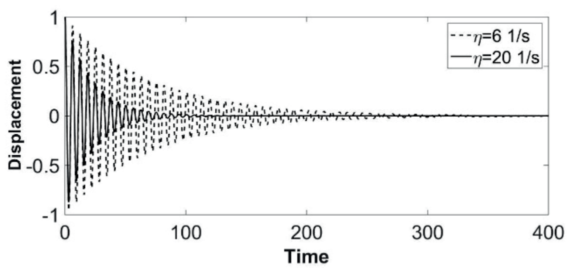

Figure 3 – The impact of the value of the scale parameter η on the oscillations of the rod

It could be seen that the damping in the nonlocal system rises with increasing scale factor η.

To take into account the amplitude dependence of the damping the scale factor η should be computed for a set of different stress amplitudes using the corresponding values of the absorption factor ψ obtained experimentally. The appropriate test data for the 01Yu5T steel [20] is presented on the Figure 4.

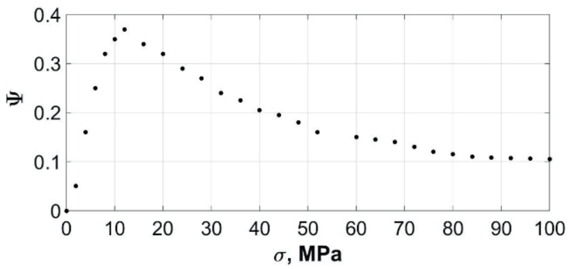

Figure. 4 – Values of the absorption factor ψ for the different stress amplitudes obtained experimentally.

The dependence of the scale parameter η on the active stress amplitude was obtained for the discrete set of points σ^K, $K = 0,1\ldots N$ corresponding to the test data shown on the Fig. 4 by the least square method (Fig. 5). The set of computer simulations was implemented to identify the values of η which provide damping (fig. 3) with the absorption factors ψ matching with the considered σ^K.

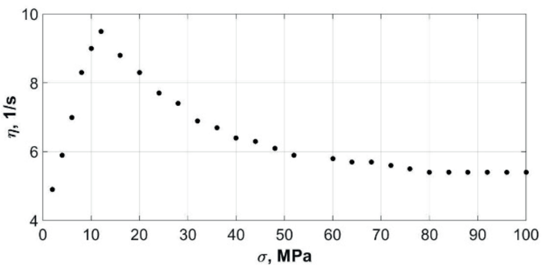

Figure 5 – Values of the scale parameter η for the different stress amplitudes obtained by the least square method

The scale parameter η in the numerical implementation of the proposed nonlocal model could be determined using the simplest linear interpolation:

$$\eta(\sigma)\big|_{\sigma^{K-1}\le\sigma<\sigma^K} \approx \frac{\eta^K-\eta^{K-1}}{\sigma^K-\sigma^{K-1}}\sigma+\eta^K. \qquad (8)$$

Here the normal stress $\sigma(t)$ in the oscillating rod is defined using the formula:

$$\sigma(t) = \frac{Ey(t)}{l} \qquad (9)$$

The integral operator with vanishing memory defined by (4) accounts the stress amplitude history at least near the current time instant. Thus, the nonlocal model presented above allows one to obtain the stress-dependent damping property of a material that could be applied to simulate the dynamics of the 01Yu5T steel structure.

SINGLE-DEGREE-OF-FREEDOM SYSTEM VIBRATIONS UNDER THE STOCHASTIC LOAD

Noise and industrial vibrations generally are stochastic. As soon as the high-damping steel 01Yu5T is expected to be used for protection from those impacts, the oscillations of the rod were simulated under the stochastic load. The equation of motion in this case is:

$$\ddot{\xi}(\tau)+\frac{2n\dot{\xi}(\tau)}{\omega_0}+\frac{1}{\omega_0}\int_0^\tau \frac{2\eta}{\sqrt{\pi}}e^{-\frac{\eta^2}{\omega_0^2}(\tau-\tau_1)^2}\xi(\tau)d\tau_1=\Phi(\tau), \qquad (10)$$

where $\Phi = \frac{\mathrm{F}(\tau)}{\mathrm{m}}$ and F is the dynamic load, modelled as the stochastic stationary process.

The characteristics of a stochastic stationary process do not change over time, so it can be considered as indefinitely long and any point in time can be chosen as the starting point.

In order for the stationary process to proceed uniformly, we require compliance with certain conditions [38]. First, the expected value must be constant (11).

$$m_x(t) = m_x = const \qquad (11)$$

The second condition is the condition of constant variance (12).

$$D_x(t) = D_x = const \qquad (12)$$

The third condition concerns the correlation function of a stationary process. If the stochastic process is stationary, then the correlation moment $K_x(t, t+\tau)$ should depend only on the length of the time interval τ, and not on where exactly this interval is taken on the time axis (13).

$$K_x(t, t+\tau) = k_x(\tau) \qquad (13)$$

In this paper, the method of canonical expansion is used to model a stationary process [38]. In this case a random time function is represented as a sum of elementary random functions:

$$q(t) = \sum_{k=0}^{n} (U_k \cos\omega_k t + V_k \cos\omega_k t) \qquad (14)$$

Here $\omega_k \in (0; \pi)$, U_k, V_k are uncorrelated random variables distributed normally, with mathematical expectations equal to zero and the variances which are the same for each pair of random variables with the same indices *k*. To calculate these variances we select an interval on the ω_k-axis with a total length of *2L* so that the origin is in the middle of this interval. At $|\omega| > L$ the spectral density can be considered equal to zero. We divide the entire selected segment into equal sections of length $\Delta\omega$. Then the variance of the random variables U_k, V_k is calculated as:

$$D_k = 2S(\omega_k) \cdot \Delta\omega \qquad (15)$$

where $S(\omega_k)$ is spectral density of a stationary random function $q(t)$. Since there is a relationship between the spectral density and the correlation function of a random process (16, 17), the spectral decomposition of the random load function is determined by the correlation function [39].

$$S(\omega_k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega t} K_x(\tau) d\tau, \qquad (16)$$

$$K_x(\tau) = \int_{-\infty}^{\infty} e^{-i\omega t} S(\omega_k) d\tau, \qquad (17)$$

The correlation function for the load, which is a Gaussian process, is defined by the expression:

$$K(\tau) = \sigma^2 e^{-\delta|\tau_1 - \tau_2|} \left[\cos\theta(\tau) + \frac{\delta}{\theta}\sin\theta(\tau)\right] \qquad (18)$$

Here σ is the standard deviation of the stationary process, θ is the frequency of the implicit periodicity of the stationary process, δ is a correlation scale parameter of a random function.
With the correlation function (18), the spectral density of a stationary process has the form:

$$S(\omega) = \frac{2\sigma^2 \cdot \delta(\delta^2 + \theta^2)}{\pi[(\omega^2 - \theta^2 - \delta^2)^2 + 4\delta^2 \cdot \omega^2]} \qquad (19)$$

Figure 6 shows one implementation of a random stationary loading. In the considered numerical example the characteristics of spectral density are θ =0.25 and $\delta = 0.05$.

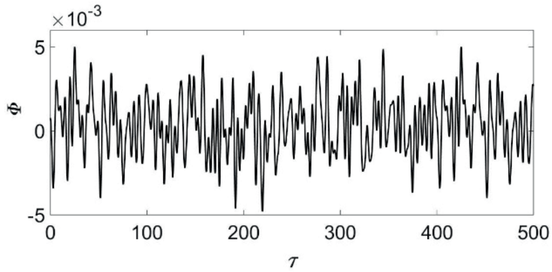

Figure 6 – One implementation of a random stationary loading

The stresses and corresponding scale parameters were determined as it was described in the previous section (8,9). Those values for several time increments of the oscillations under the stochastic load (fig.6) are provided in table 1.

Table 1. Scale parameters η determined for several increments of the oscillation process

τ	σ, MPa	$\eta, 1/s$
15.75	-19.71	8.34
15.8	-13.22	9.29
15.85	-6.80	7.52
15.9	-0.46	4.90
15.95	5.81	6.90
16	12.00	9.50
16.05	18.08	8.54
16.1	24.06	7.70
16.15	29.93	7.16
16.2	35.68	6.72
16.25	41.31	6.37

The results obtained for the 01Yu5T steel with the use of stress-dependent nonlocal model are shown on fig.7 in comparison to the oscillations of the same rod made of the steel St3 which has low and amplitude independent absorption factor $\psi = 0.05$.

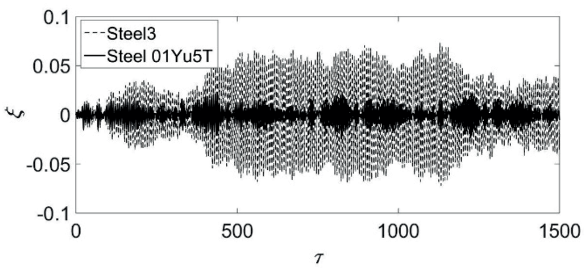

Figure 7 – Comparison of the oscillations of the one-degree-of-freedom element made of 01Yu5T steel and St3

It can be seen from the figure 7 that in comparison to the St3 01Yu5T steel demonstrates stable decrease of the oscillations amplitude.

CONCLUSION

Due to its relatively low cost and high tensile strength high-damping steel 01Yu5T can be effectively applied for vibration protection, particularly as a material for the bearing structural elements.

Modelling of the dynamic behavior of elements made of 01Yu5T steel requires special approaches to simulate its stress dependent damping properties. The application of a non-local approach in describing of the damping properties of a material, depending on the stress level, allows to build a numerical model that is more flexible in comparison to the classic local one.

For now, the stress-dependent nonlocal damping model is derived for the simplest case of longitudinal vibrations, when the stress level is uniform over the entire length of the element and its entire cross-section. To simulate the dynamics of the elements subjected to bending it is necessary to adequately describe the influence of stress distribution on the damping properties.

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