

RATING THE LIMITING SHIFT OF A STORY AS A CRITERIA FOR A SPECIAL LIMITING STATE

Oleg V. Mkrtychev, Ekaterina M. Lokhova, Timur A. Tagirov

National Research Moscow State University of Civil Engineering, Moscow, RUSSIA

Abstract: The article presents a verification of one of the existing seismic resistance criteria found in domestic and international standards. The study examines the limit value of relative displacement of the first-floor column in a reinforced concrete building under seismic loading using two different material models in a nonlinear dynamic framework. The article introduces a modeling approach that allows for the assessment of the seismic resistance of reinforced concrete buildings and structures in a nonlinear dynamic framework. The objective of the work is to verify the standardized limit drift value for a specific structural scheme when modeling the column with volumetric elements including direct reinforcement.

Two different material models, implemented in the LS-DYNA software, were used in the column modeling. The first material model is the Karagozian & Case (K&C) Concrete model. This material model represents a three-invariant model with multiple strength and yield surfaces. The model's input parameters include various factors, including empirical ones, such as yield strength, ultimate strength, and residual strength, which are key parameters for the damage accumulation function. The second material model is the Continuous Surface Cap Model, represented by a closed surface (a cap model with a continuous yield surface). This model also accounts for damage accumulation and the degradation of stiffness and strength under cyclic loading.

The results include isocontours of the damage accumulation function in concrete, isocontours of strain and stress intensities in concrete and reinforcement at various time steps, as well as graphs of the damage accumulation functions. The findings confirm the accuracy of the numerically determined limit displacement value and identify which material model most precisely represents the failure mechanisms of concrete, thus indicating its suitability for further in-depth research.

Keywords: seismic resistance, accumulation of damage, calculation in nonlinear dynamic setting, material models, criteria for seismic resistance

НОРМИРОВАНИЕ ПРЕДЕЛЬНОГО СДВИГА ЭТАЖА КАК КРИТЕРИЯ ОСОБОГО ПРЕДЕЛЬНОГО СОСТОЯНИЯ

О.В. Мкртычев, Е.М. Лохова, Т.А. Тагиров

Национальный исследовательский Московский государственный строительный университет, г. Москва, РОССИЯ

Аннотация: В статье представлена верификация одного из существующих в отечественной и зарубежной нормативной документации критерия сейсмостойкости. Рассматривается предельное относительное перемещение колонны первого этажа железобетонного здания при сейсмическом воздействии. Верификация производится с использованием двух различных моделей материалов в нелинейной динамической постановке. В статье приведен подход моделирования, позволяющий производить оценку сейсмостойкости конструкции железобетонных зданий и сооружений в нелинейной динамической постановке. Целью работы является верификация нормируемого предельного значения перегиба для конкретной конструктивной схемы при моделировании колонны объемными элементами с непосредственным армированием. При моделировании колонны были использованы две различные модели материалов, реализованные в программном комплексе LS-DYNA. Первая модель материала - Karagozian & Case (K&C) Concrete. Данная модель материала представляет трехинвариантную модель с множественными поверхностями прочностями и текучести. Исходными параметрами модели являются различные параметры, в том числе и эмпирические, в частности предел текучести, предельная прочность и остаточная прочность, которые являются определяющими параметрами для функции накопления повреждений. Вторая модель материала - Continuous Surface Cap Model представлена замкнутой поверхностью (шатровая модель с непрерывной

предельной поверхностью). Данная модель также учитывает накопление повреждений и деградацию жесткости и прочности при циклических нагрузках.

В качестве результатов представлены изополя функции накопления повреждений в бетоне, изополя интенсивности деформаций и напряжений в бетоне и арматуре в различные моменты времени, а также графики функций накопления повреждений. Результаты исследований свидетельствуют об обоснованности численного значения предельного перемещения, а также позволяют сделать вывод о том, какая из моделей материала наиболее точно описывает характер разрушения бетона и может быть использована для дальнейших исследований.

Ключевые слова: сейсмостойкость, накопление повреждений, расчеты в нелинейной динамической постановке, модели материалов, критерии сейсмостойкости

INTRODUCTION

Existing domestic and international seismic design standards commonly utilize maximum allowable relative displacement, or inter-story drift, as one of the criteria for assessing seismic resilience. The maximum displacement can be taken equal to $h/150$ (h – floor height). [1] However, this criterion has yet to be substantiated through rigorous nonlinear dynamic analysis. [2,3] The article presents a detailed examination of this specific limit state criterion, including an analysis of the seismic resistance of first-floor columns in a reinforced concrete structure. The study employs two distinct material models within a nonlinear dynamic framework, implemented using the LS-DYNA software package.

The primary objective of this study is to verify the standardized allowable drift criterion for a specific structural configuration. This is achieved by modeling the column with volumetric elements and detailed reinforcement, utilizing material models that incorporate effects of degradation and damage accumulation.

Methods

The study examines a first-floor column (Figure 1). The concrete used is of strength class B25, with a column height of 6 meters and a cross-section of 400x400 mm. The longitudinal reinforcement consists of four rebars with a diameter of 28 mm (A500C). The transverse(shear) reinforcement is provided by hoops by 10 mm diameter ties (A500C) in the central part of the column and 14 mm diameter ties (A500C) at the ends,

spaced 200 mm apart. The concrete was modeled using solid elements, while the reinforcement bars were modeled using beam elements.

The finite element mesh size for both the reinforcement and concrete was set at 12.5 mm in all directions. Column base is fixed in three linear directions. The top of the column is subjected to a forced horizontal displacement of 2 cm along the X-axis (varying sinusoidally), along with a vertical load of 3 tons applied at each node. The model consists of 248521 finite elements. Several material models were used: Piecewise Linear Isotropic Plasticity for the reinforcement, and both the Continuous Surface Cap Model and the Karagozian & Case (K&C) Concrete model for the concrete.

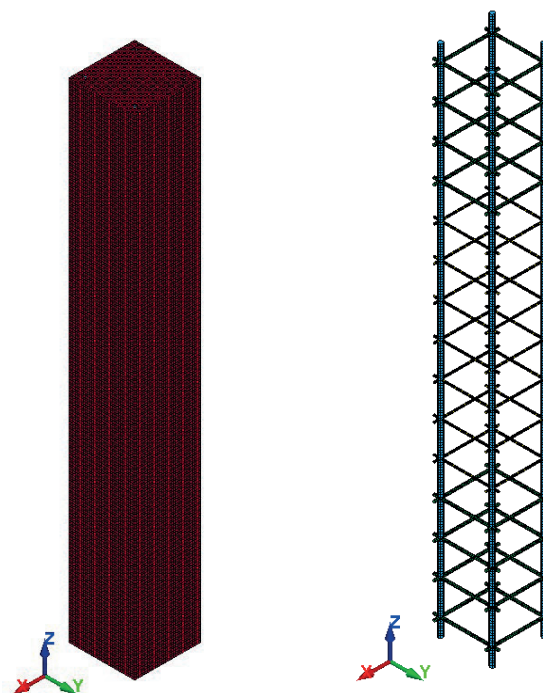


Figure 1. Finite element model used

Considered mathematical models of materials are presented below.

KARAGOZIAN & CASE (K&C) CONCRETE MAT072R3

This material model is primarily designed for simulating the dynamic responses of reinforced concrete structures. It accounts for various concrete properties such as hardening, softening, strain rate effects, compaction, and shear dilatancy. The model is a three-invariant formulation featuring multiple yield and strength surfaces. The initial parameters of the model are various parameters, including empirical ones. In particular, the yield strength, ultimate strength and residual strength of the material, determined as functions of damage accumulation during compression and tension.

The model decomposes the stress into hydrostatic pressure and deviatoric stress components:

$$\sigma_{ij} = s_{ij} + \frac{1}{3}\sigma_{ii}\delta_{ij}, \quad (1)$$

where:

σ_{ij} - is the stress tensor;

s_{ij} - is the deviatoric stress tensor;

σ_{ii} - is the hydrostatic pressure.

Hydrostatic pressure is related to the material's volumetric strain, while deviatoric stress is associated with its shear resistance. The deviatoric stress is usually expressed through the second invariant of the deviatoric stress tensor:

$$J_2 = \frac{1}{2}s_{ij}s_{ji} = \frac{s_1^2 + s_2^2 + s_3^2}{2} = \frac{1}{2}\sigma' : \sigma' \quad (2)$$

The model features three independent strength surfaces: the maximum strength surface, the yield surface, and the residual strength surface, as illustrated in Figure 2.

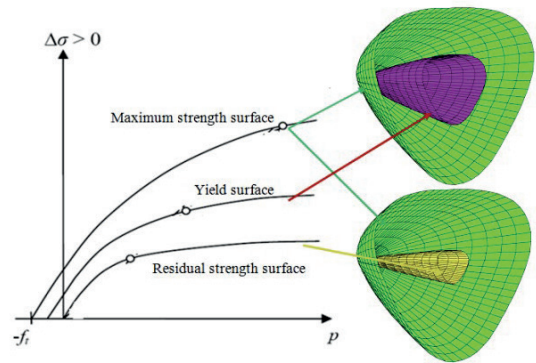


Figure 2. Strength surfaces for the Karagozian and Case concrete model (KCC) material model [4]

The general expression for the strength surfaces can be written as:

$$\Delta\sigma = \sqrt{3J_2} = f(p, J_2), \quad (3)$$

where: $\Delta\sigma$ - is the difference between the principal stresses; p - is the hydrostatic pressure.

Typically, equation (3) refers to the compressive meridian. The whole failure curve can be obtained through rotation of the compressive meridian around the hydrostatic pressure axis by multiplying Θ_L [5] (4-5):

$$\Theta_L = \frac{r}{r_c} = \frac{2(1-\psi^2)\cos\theta_L + (2\psi-1)\sqrt{4(1-\psi^2)\cos^2\theta_L + 5\psi^2 - 4\psi}}{4(1-\psi^2)\cos^2\theta_L + (1-2\psi)^2}$$

$$\Delta\sigma = r_3\Theta_L \cdot \sqrt{3J_2} = f(p, J_2, J_3); \quad (4)$$

$$\theta = \cos^{-1}\left(\frac{3\sqrt{3}}{2} \frac{J_3}{J_2^{3/2}}\right) / 3$$

$$\psi = \begin{cases} \frac{1}{2} & \text{if } p \leq 0 \\ \frac{1}{2} + \frac{3f_t}{2f'_c} & \text{if } p = \frac{f'_c}{3} \\ \frac{\alpha f'_c}{a_0 m + \frac{2\alpha f'_c}{3a_1 m + 2a_2 m \alpha f'_c}} & \text{if } p = \frac{2\alpha f'_c}{3}, \alpha \approx 1.15 \\ 0.753 & \text{if } p = 3f'_c \\ 1.000 & \text{if } p \geq 8.45f'_c \end{cases} \quad (5)$$

where:

ψ - is the ratio between the tensile and compressive meridians. This ratio is a function of pressure and can be calculated using the Malvar method [5].

f'_c - is the compressive strength;

f'_t - is the tensile strength.

At the initial increase in hydrostatic pressure, deviatoric stresses remain within the elastic range until the yield surface is reached. Deviatoric stresses can continue to evolve until the ultimate strength surface is attained, at which point the material begins to fail. Following the onset of failure, the material progressively loses its load-bearing capacity and eventually reaches the residual strength surface. The equations for these three surfaces are provided in the following equations [4]:

Yield surface:

$$\Delta\sigma_y = a_{0y} + \frac{p}{a_{1y} + a_{2y}p}. \quad (6)$$

Maximum strength surface:

$$\Delta\sigma_m = a_0 + \frac{p}{a_1 + a_2p}. \quad (7)$$

Residual strength surface:

$$\Delta\sigma_r = \frac{p}{a_{1f} + a_{2f}p}. \quad (8)$$

The parameters $a_0, a_1, a_2, a_{1f}, a_{2f}, a_{0y}, a_{1y}, a_{2y}$, used for these three surfaces, can be determined based on the results of experimental tests, such as triaxial compression, biaxial compression, or uniaxial tension/compression.[5]

After reaching the initial yield surface but before attaining the ultimate strength surface, the current surface can be obtained through linear interpolation between the yield surface and the ultimate strength surface.

Once the maximum strength surface is reached, ongoing failure is interpolated between the max-

imum strength surface and the residual strength surface. The dynamic form of the failure surface is implemented by interpolating between pairs of fixed strength surfaces based on the damage parameter λ . The evolution of this parameter is defined as a function of the plastic strain rate tensor $\hat{\epsilon}^p$:

$$\dot{\lambda} = h(p)\hat{\epsilon}^p; \quad (9)$$

$$\hat{\epsilon}^p = \sqrt{(\dot{\epsilon}^p : \dot{\epsilon}^p) \frac{2}{3}}. \quad (10)$$

The function $h(p)$ describes the dependency between pressure and the rate of change of damage accumulation.

$$h(p) = \begin{cases} \left[1 + \frac{p}{(r_f f_t)} \right]^{-b_1} / r_f, & p \geq 0; \\ \left[1 + \frac{p}{(r_f f_t)} \right]^{-b_2} / r_f, & p < 0. \end{cases} \quad (11)$$

where :

r_f - is the Dynamic Increase Factor(DIF).

b_1, b_2 – are experimentally determined parameters.

The concrete model also uses the parameter η for interpolation between different strength surfaces. This parameter varies from zero to one, depending on the accumulated effective plastic strain. Specifically:

1. As λ begins to increase, η increases from 0 to 1.
2. When λ reaches the value λ_m (plastic strain at maximum strength) η becomes 1.

3. η then decreases back to 0 as λ continues to increase ($1 < \lambda < 2$). indicating softening and progressive failure. The material continues to degrade after reaching its maximum strength.

The parameter η aids in modeling the transition between the ultimate strength surface, the yield surface, and the residual strength surface, depending on the degree of material damage.

Theoretically, each of the three strength surfaces is described by equations, with nine parameters calibrated based on experimental data. Howev-

er, due to limited data, it may be impossible to calibrate the parameters across the entire range of pressures. As a result of this scarcity of experimental data, several assumptions have been incorporated into the model when using the strength surfaces.[6]

Firstly, experimental data for calibrating the ultimate strength surface at pressures below $f'_c/3$, are likely unavailable, as damage does not occur within this range. Consequently, in the model implementation, the ultimate strength surface is predefined as:

$$\hat{\sigma}_m = \begin{cases} a_{0m} + \frac{p}{a_{1m} + a_{2m}p} & p \geq \frac{f'_c}{3} \\ \frac{3}{2\psi}(p + f_t) & 0 \leq p \leq \frac{f'_c}{3} \text{ or } \lambda \leq \lambda_m \\ -f_t \leq p \leq 0 \\ 3\left(\frac{p}{\eta} + f_t\right) & p \leq 0 \text{ and } \lambda > \lambda_m \end{cases} \quad (12)$$

Similarly, the yield surface cannot be calibrated for pressures between 0 and $0.8 f'_c/3$ ($f'_c/3$ — is the yield limit), as yielding does not occur within this pressure range. Therefore, the yield surface is modified as follows:

$$\hat{\sigma}_y = \begin{cases} a_{0y} + \frac{p}{a_{1y} + a_{2y}p} & p \geq \frac{f'_{yc}}{3} \\ 1.35f_t + 3p\left(1 - \frac{1.35f_t}{f'_{yc}}\right) & 0 \leq p \leq \frac{f'_{yc}}{3} \\ 1.35(p + f_t) & p \leq 0 \end{cases} \quad ((13))$$

The residual strength surface (Figure 2) models the transition from brittle to ductile behavior in concrete as lateral pressure increases. At very high lateral pressures, the concrete may exhibit strain-hardening behavior. The model accounts for this by defining the residual surface so that, at high pressures, it equals or exceeds the ultimate strength surface.

CONTINUOUS SURFACE CAP MODEL

This model also incorporates damage accumulation, with the damage accumulation function accounting for both the reduction in material strength and the degradation of the elastic modulus. [7]

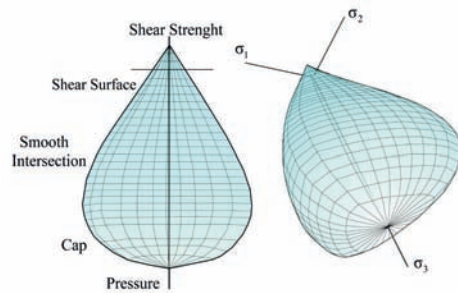


Figure 3. Continuous surface cap model CSCM

The damage accumulation is mathematically expressed as follows:

$$\sigma_{ij}^d = (1 - d) \sigma_{ij}^{vp}. \quad (14)$$

Here d — is a scalar damage parameter that transforms the undamaged stress tensor, denoted as σ^{vp} , into the damaged stress tensor, denoted as σ^d . The damage parameter d varies from 0 (indicating no damage) to 1 (indicating complete loss of strength). The effect of this parameter also includes the proportional reduction of both the bulk modulus and the shear modulus simultaneously.[8]

Brittle damage accumulates under tension. The accumulation of brittle damage depends on the maximum principal strain ϵ_{max} :

$$\tau_b = \sqrt{E \epsilon_{max}^2}. \quad (15)$$

where τ_b — is an energy-type parameter. Brittle damage initiates when τ_b exceeds the initial threshold r_{ob} .

Plastic damage accumulates under compressive loading and is dependent on the components of the total strain ϵ_{ij} :

$$\tau_d = \sqrt{\frac{1}{2} \sigma_{ij} \varepsilon_{ij}}, \quad (16)$$

where τ_d - is an energy-based parameter. Plastic damage begins to accumulate when τ_d exceeds the initial threshold r_{0d} .

RESULTS AND DISCUSSION

The key results are summarized as follows:

Fig. 4 shows the calculation scheme with iso-fields of the damage accumulation function, and also highlights the fragment for which the results will be presented.

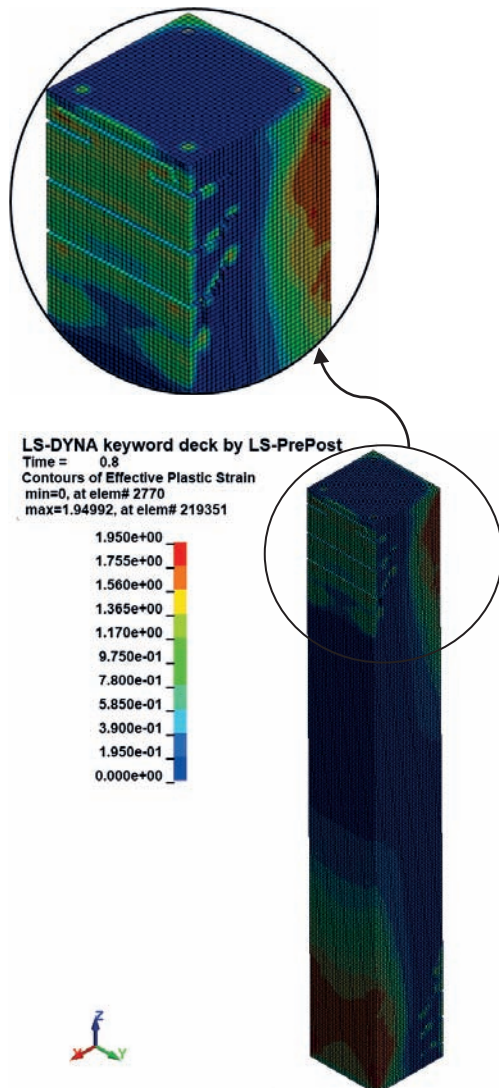


Figure 4. Calculation scheme with iso-fields of the damage accumulation function ($t=0,8$ s)

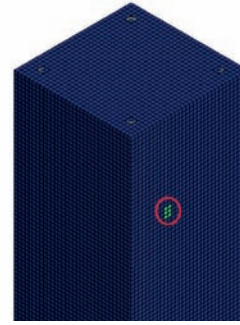


Figure 5. The area under consideration

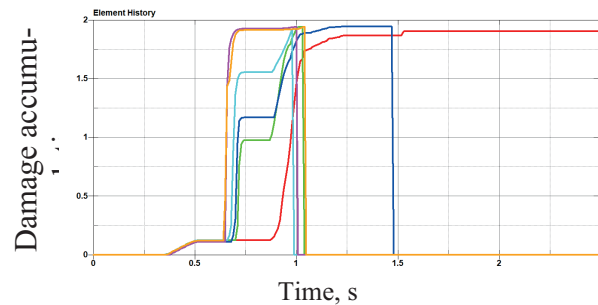


Figure 6. Damage accumulation function in concrete element

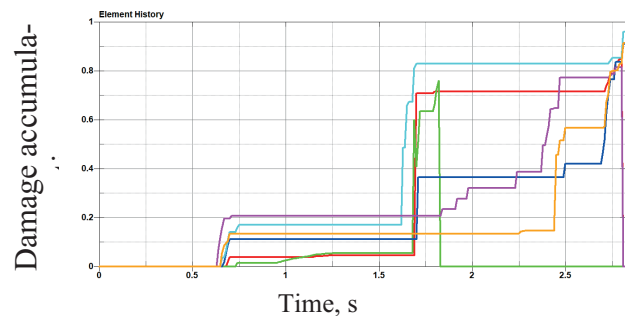
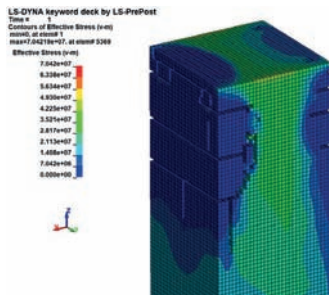


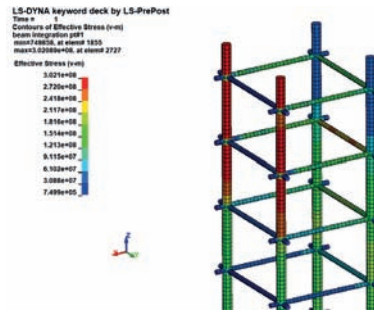
Figure 7. Damage accumulation function in concrete element

Figures 8-11 depict the isopleths of stress intensity in concrete at different time steps. Figures 12-15 depict the isopleths of stress intensity in reinforcement at different time steps.

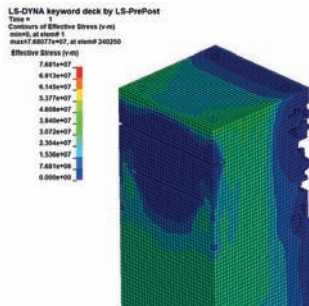
Figures 16-19 depict the isopleths of strain intensity in concrete at different time steps. Figures 20-23 depict the isopleths of strain intensity in reinforcement at different time steps.



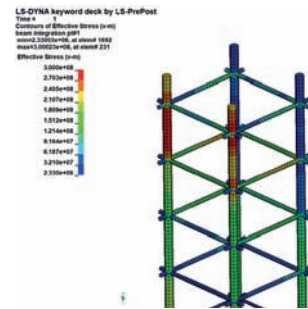
*Figure 8. Stress intensity in concrete
($t=1,0$ s), Pa
(Karagozian & Case (K&C) Concrete model)*



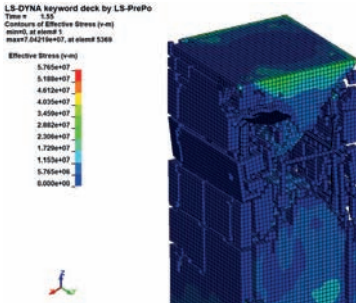
*Figure 12. Stress intensity in in reinforcement
($t=1,0$ s), Pa
(Karagozian & Case (K&C) Concrete model)*



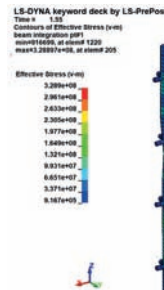
*Figure 9. Stress intensity in concrete
($t=1,0$ s), Pa
(Continuous Surface Cap Model)*



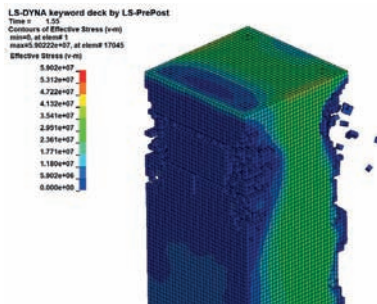
*Figure 13. Stress intensity in in reinforcement
($t=1,0$ s), Pa
(Continuous Surface Cap Model)*



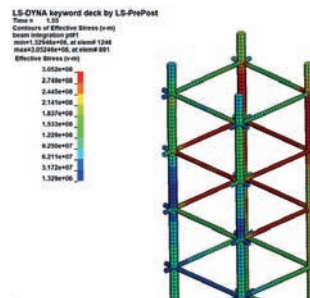
*Figure 10. Stress intensity in concrete
($t=1,55$ s), Pa
(Karagozian & Case (K&C) Concrete model)*



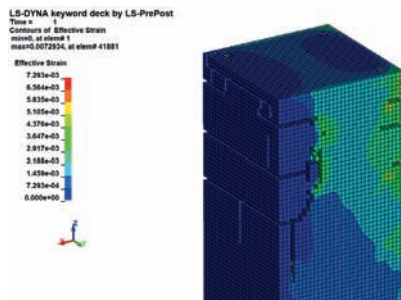
*Figure 14. Stress intensity in reinforcement
($t=1,55$ s), Pa
(Karagozian & Case (K&C) Concrete model)*



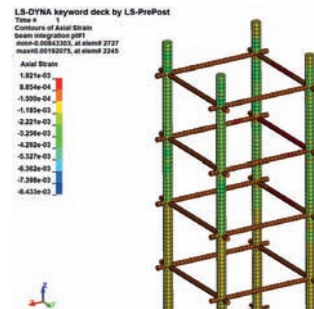
*Figure 11. Stress intensity in concrete
($t=1,55$ s), Pa
(Continuous Surface Cap Model)*



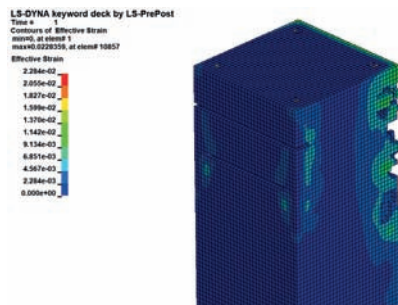
*Figure 15. Stress intensity in reinforcement
($t=1,55$ s), Pa
(Continuous Surface Cap Model)*



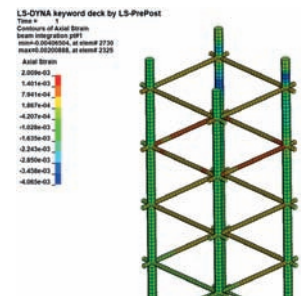
*Figure 16. Intensity of deformations in concrete ($t=1,0$ s)
(Karagozian & Case (K&C) Concrete model)*



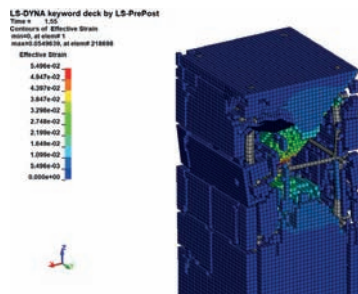
*Figure 20. Intensity of deformations in reinforcement ($t=1,0$ s)
(Karagozian & Case (K&C) Concrete model)*



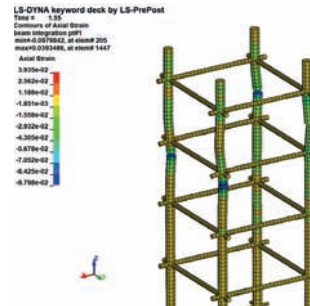
*Figure 17. Intensity of deformations in concrete ($t=1,0$ s)
(Continuous Surface Cap Model)*



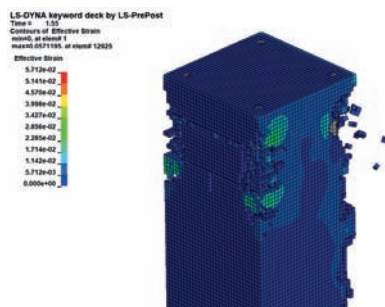
*Figure 21. Intensity of deformations in reinforcement ($t=1,0$ s)
(Continuous Surface Cap Model)*



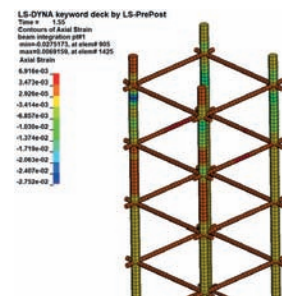
*Figure 18. Intensity of deformations in concrete ($t=1,55$ s)
(Karagozian & Case (K&C) Concrete model)*



*Figure 22. Intensity of deformations in reinforcement ($t=1,55$ s)
(Karagozian & Case (K&C) Concrete model)*



*Figure 19. Intensity of deformations in concrete ($t=1,55$ s)
(Continuous Surface Cap Model)*



*Figure 23. Intensity of deformations in reinforcement ($t=1,55$ s)
(Continuous Surface Cap Model)*

Figures 24-28 present the isopleths of the damage accumulation function at various time intervals.

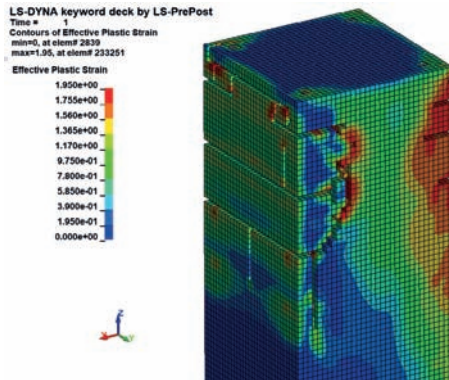


Figure 24. Damage accumulation function in concrete ($t=1$ s) (Karagozian & Case (K&C) Concrete model)

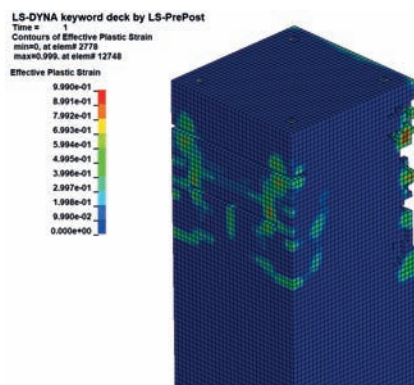


Figure 25. Damage accumulation function in concrete ($t=1$ s) (Continuous Surface Cap Model)

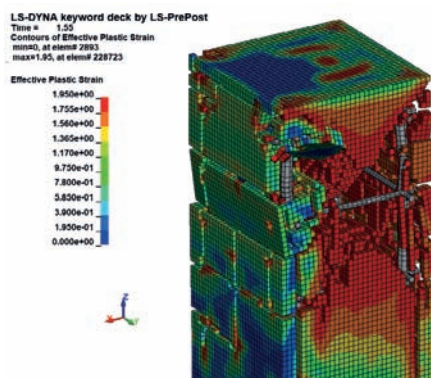


Figure 26. Damage accumulation function in concrete ($t=1.55$ s) (Karagozian & Case (K&C) Concrete model)

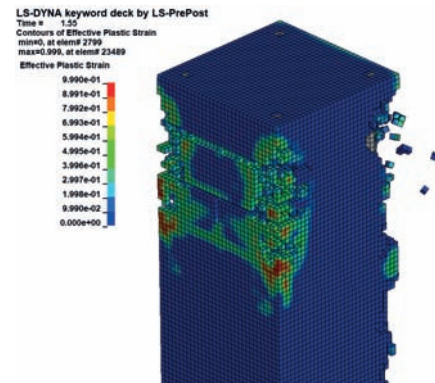


Figure 27. Damage accumulation function in concrete ($t=1.55$ s) (Continuous Surface Cap Model)

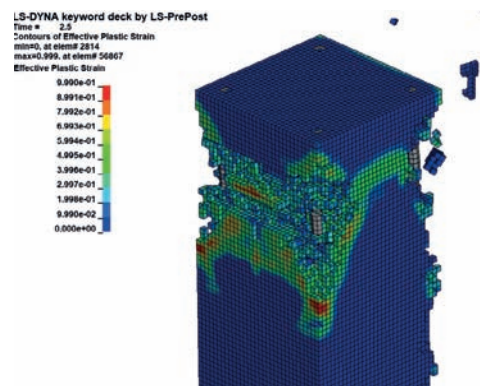


Figure 28. Damage accumulation function in concrete ($t=2.60$ s) (Continuous Surface Cap Model)

CONCLUSION

1. The research results indicate the adequacy and validity of the limit value for the inter-story drift for this structural scheme, which is numerically set at 2 cm.
2. The approach to modeling proposed in this study enables the evaluation of the seismic resistance of reinforced concrete buildings and structures in a nonlinear dynamic framework. This approach accounts for the degradation of strength and softening of the structural system, as well as the accumulation of damage under cyclic loading.
3. Based on the results of the study, it can be concluded that the Karagozian & Case (K&C) Concrete material model more adequately de-

scribes the nature of concrete destruction and can be utilized in further research.

ACKNOWLEDGEMENTS

The research was funded by the National Research Moscow State University of Civil Engineering (grant for fundamental and applied scientific research, project No. 32-392/130.

REFERENCES

1. NEHRP recommended provisions for seismic regulations For new buildings and other structures (2004) *Building Seismic Safety Council for the Federal Emergency Management Agency*.
2. **Mkrtychev O.V., Lokhova E.M.** (2024) Proektirovanie sejsmostojkih zdaniy. Normy novogo pokoleniya [Design of earthquake-resistant buildings. New generation standards]. *Zhilishchnoe Stroitel'stvo* [Housing Construction]. 2023. No. 4, pp. 42–46. DOI: <https://doi.org/10.31659/0044-4472-2024-4-42-46>.
3. **Dorozhinsky V.B.** (2022) Nelinejnye metody raschetov pri provedenii nauchno-tehnicheskogo soprovozhdeniya [Nonlinear calculation methods for scientific and technical support] *Promyshlennoe i grazhdanskoe stroitel'stvo*. No. 10, pp. 32–36. DOI: 10.33622/0869-7019.2022.10.32-36.
4. **Jun Wu, Liang Li, Xiuli Du and Xuemei Liu** (2017) Numerical Study on the Asphalt Concrete Structure for Blast and Impact Load Using the Karagozian and Case Concrete Model, *Advanced Asphalt Materials and Paving Technologies*, vol. 7, no. 202.
5. **Malvar L. J., Crawford J. E., Wesevich, J. W., and Simons, D.** (1997) A plasticity concrete material model for DYNA3D. *Int. J. Impact*, 19(9–10), pp. 847–873.
6. **Youcai Wu and John E. Crawford** (2015) Numerical modeling of concrete using a partially associative plasticity model, *Amer-*

- ican *Society of Civil Engineers*, vol.141, no.12. DOI: 10.1061/(ASCE)EM.1943-7889.0000952
7. **Nemecek J., Padevet P., Bittnar Z.** Effect of stirrups on behavior of normal and high strength concrete columns]. *Acta Polytechnica*, 2004, vol. 44(5-6), pp. 158-164.
8. **Murray Y.D.** Users manual for LS-DYNA. Concrete material model 159-HRT-05-062. 2007. 77 p.

СПИСОК ЛИТЕРАТУРЫ

1. NEHRP recommended provisions for seismic regulations For new buildings and other structures (2004) *Building Seismic Safety Council for the Federal Emergency Management Agency*.
2. **Мкртычев О.В., Лохова Е.М.** Проектирование сейсмостойких зданий. Нормы нового поколения // *Жилищное строительство*. 2024. № 4. С. 42–46. DOI: <https://doi.org/10.31659/0044-4472-2024-4-42-46>.
3. **Дорожинский В.Б.** Нелинейные методы расчетов при проведении научно-технического сопровождения // *Промышленное и гражданское строительство*. 2022. № 10. С. 32–36. DOI: 10.33622/0869-7019.2022.10.32-36.
4. **Jun Wu, Liang Li, Xiuli Du and Xuemei Liu** (2017) Numerical Study on the Asphalt Concrete Structure for Blast and Impact Load Using the Karagozian and Case Concrete Model, *Advanced Asphalt Materials and Paving Technologies*, vol. 7, no. 202.
5. **Malvar L. J., Crawford J. E., Wesevich, J. W., and Simons, D.** (1997) A plasticity concrete material model for DYNA3D. *Int. J. Impact*, 19(9–10), pp. 847–873.
6. **Youcai Wu and John E. Crawford** (2015) Numerical modeling of concrete using a partially associative plasticity model, *American Society of Civil Engineers*, vol.141, no.12. DOI: 10.1061/(ASCE)EM.1943-7889.0000952

7. **Nemecek J., Padevet P., Bittnar Z.** Effect of stirrups on behavior of normal and high strength concrete columns]. *Acta Polytechnica*, 2004, vol. 44(5-6), pp. 158-164.
8. **Murray Y.D.** Users manual for LS-DYNA. Concrete material model 159-HRT-05-062. 2007. 77 p.

Mkrtychev Oleg Vartanovich, D.Sc., Professor, Head of the Department of Strength of Materials, Federal State Budget Educational Institution of Higher Education «Moscow State University of Civil Engineering (National Research University) (MSUCE), 26, Yaroslavskoe shosse, Central Federal District, Moscow, 129337, Russia, e-mail: mkrtychev@yandex.ru

Lokhova Ekaterina Mikhailovna, Senior Lecturer, Department of Strength of Materials, Federal State Budget Educational Institution of Higher Education «Moscow State University of Civil Engineering (National Research University) (MSUCE), 26, Yaroslavskoe shosse, Central Federal District, Moscow, 129337, Russia, e-mail: elm97@mail.ru

Tagirov Timur Alikhanovich, postgraduate student of the Department of Strength of Materials, engineer of the Research Center, Federal State Budget Educational Institution of Higher Education «Moscow State University of Civil Engineering (National Research University) (MSUCE), 26, Yaroslavskoe shosse, Central Federal District, Moscow, 129337, Russia, e-mail: tagirovv_timur@mail.ru

Мкртычев Олег Вартанович, д.т.н, профессор, заведующий кафедрой «Соппротивление материалов», Федеральное государственное бюджетное образовательное учреждение высшего образования «Национальный исследовательский Московский государственный строительный университет» (НИУ МГСУ), д.26, Ярославское шоссе, Центральный федеральный округ, г. Москва, 129337, Россия. Эл. почта: mkrtychev@yandex.ru

Лохова Екатерина Михайловна, старший преподаватель кафедры «Соппротивление материалов», Федеральное государственное бюджетное образовательное учреждение высшего образования «Национальный исследовательский Московский государственный строительный университет» (НИУ МГСУ), д.26, Ярославское шоссе, Центральный федеральный округ, г. Москва, 129337, Россия. Эл. почта: elm97@mail.ru

Тагиров Тимур Алиханович, аспирант кафедры «Соппротивление материалов», инженер НИЦ «Надежность и сейсмостойкость сооружений», Федеральное государственное бюджетное образовательное учреждение высшего образования «Национальный исследовательский Московский государственный строительный университет» (НИУ МГСУ), д.26, Ярославское шоссе, Центральный федеральный округ, г. Москва, 129337, Россия. Эл. почта: tagirovv_timur@mail.ru