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SOLUTION OF THE PROBLEM OF TWO-DIMENSIONAL THEORY OF ELASTICITY WITH THE USE OF DISCRETE-CONTINUAL FINITE ELEMENT METHOD IN THE PRESENCE OF A CRACK

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Abstract: Solution of the problem of two-dimensional theory of elasticity with the use of discrete-continual finite element method in the presence of crack is under consideration in the distinctive paper. The original operational continual and discrete-continual formulations of the problem are given, an example of analysis are presented.

Keywords: discrete-continual finite element method, numerical solution, two-dimensional theory of elasticity, structural analysis

РЕШЕНИЕ ДВУМЕРНОЙ ЗАДАЧИ ТЕОРИИ УПРУГОСТИ С ИСПОЛЬЗОВАНИЕМ ДИСКРЕТНО-КОНТИНУАЛЬНОГО МЕТОДА КОНЕЧНЫХ ЭЛЕМЕНТОВ ПРИ НАЛИЧИИ ЩЕЛИ

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Аннотация: В настоящей статье рассматривается решение двумерной задачи теории упругости с использованием дискретно-континуального метода конечных элементов при наличии щели. Приведены исходные операторные континуальная и дискретно-континуальная постановки задачи, представлен пример расчета.

Ключевые слова: дискретно-континуальный метод конечных элементов, численное решение, двумерная задача теории упругости, расчеты конструкций

INTRODUCTION

Numerical solution of the problem of two-dimensional theory of elasticity with the use of discrete-continual finite element method (DCFEM) has been already considered in [1, 2, 4-8, 12]. Systematic literature review has been presented in [1, 2, 4-8, 12] as well. In this regard, let us immediately move on to the main material of the distinctive paper in case of presence of a crack.

1. FORMULATIONS OF THE PROBLEM

Let the constancy of the parameters of the problem be in the direction corresponding to x_2 (main direction). The operational formulation of the problem with the use of so-called method of extended domain [5], taking into account the selection of the main direction, is determined by the equation [6]:

$$L\,\bar{u} = \bar{\bar{F}}\,, \quad 0 \le x_1 \le \ell_1\,,\; 0 \le x_2 \le \ell_2\,, \qquad (1.1)$$

where we have

$$L = -L_{vv}\,\partial_2^2 + L_{uv}\,\partial_2 + L_{uu}\,; \qquad (1.2)$$

$$L_{uu} = \partial_1^* D_1^T C D_1 \partial_1\,; \qquad (1.3)$$

$$L_{uv} = \partial_1^* D_1^T C D_2 - D_2^T C D_1 \partial_1\,; \qquad (1.4)$$

$$L_{vv} = D_2^T C D_2\,; \qquad (1.5)$$

$$D_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}; \quad D_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}; \tag{1.6}$$

$$C = \begin{bmatrix} 2\mu+\lambda & \lambda & 0 \\ \lambda & 2\mu+\lambda & 0 \\ 0 & 0 & \mu \end{bmatrix}; \tag{1.7}$$

ℓ_1, ℓ_2 are corresponding dimensions of extended domain (linear dimensions of considering structure) [3]; $x = (x_1,\, x_2)$; $x_1,\, x_2$ are Cartesian coordinates; μ and λ are Lame parameters; Ω is the domain, occupied by structure;

$$\Omega = \{\, (x_1,\, x_2): \quad 0 < x_1 < \ell_1; \quad 0 < x_2 < \ell_2 \,\}; \tag{1.8}$$

$\theta(x_1, x_2)$ is characteristic function of domain Ω; $\delta_\Gamma = \delta_\Gamma(x_1, x_2)$ is the delta function of boundary $\Gamma = \partial\Omega$; $\bar{n} = [\, n_1 \;\; n_2 \,]^T$ is boundary normal vector; $\bar{F}$ is the load vector in domain Ω; $\bar{f}$ is the corresponding boundary load vector; $\bar{u}$ is the vector of displacements (unknown vector function),

$$\bar{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}; \tag{1.9}$$

$\partial_s = \partial / \partial x_s$, $s = 1, 2$; $\partial_s^* = -\partial / \partial x_s$, $s = 1, 2$.

Let us introduce the following notations

$$\bar{v} = \partial_2 \bar{u} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}; \tag{1.10}$$

$$\bar{u}' = \partial_2 \bar{u}; \quad \bar{v}' = \partial_2 \bar{v}. \tag{1.11}$$

Thus, we can rewrite (1.1):

$$L_{uu}\, \bar{u} + L_{uv}\bar{v} - L_{vv}\partial_2\bar{v} = \bar{\bar{F}}. \tag{1.12}$$

Finally we obtain system of differential equations with operational coefficients:

$$\begin{cases} \bar{u}' = \bar{v} \\ \bar{v}' = L_{vv}^{-1} L_{uu}\bar{u} + L_{vv}^{-1} L_{uv}\bar{v} - L_{vv}^{-1}\bar{\bar{F}} \end{cases} \tag{1.13}$$

or

$$\bar{z}' = \widetilde{L}\bar{z} + \bar{\bar{F}}, \tag{1.14}$$

where

$$\widetilde{L} = \begin{bmatrix} 0 & E \\ L_{vv}^{-1} L_{uu} & L_{vv}^{-1} L_{uv} \end{bmatrix}; \quad \bar{\bar{F}} = \begin{bmatrix} 0 \\ -L_{vv}^{-1}\bar{\bar{F}} \end{bmatrix}; \tag{1.15}$$

$$\bar{z} = \begin{bmatrix} \bar{u} \\ \bar{v} \end{bmatrix}; \quad E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{1.16}$$

The system of equations (1.14) is supplemented by boundary conditions, which are set in sections with coordinates $x_2^1 = 0$ and $x_2^2 = \ell_2$.

The discrete component of the numerical solution is represented by direction along the axe x_1.

2. EXAMPLE OF ANALYSIS

2.1. Formulation of the problem.

As a model example, let us consider the determination of the displacements of a beam-wall, fixed along the side faces in three directions, under the influence of a load concentrated in the center (Figure 2.1).

Let us consider the following geometric parameters:

$$\ell_1 = 1 \text{ m}; \quad \ell_2 = 1 \text{ m}.$$

Let us consider the following design parameters of material of the beam: coefficient of elasticity $E = 2650 \cdot 10^4$ kN/m^2; Poisson's ratio $\nu = 0.15$.

Let external load parameter be equal to $P = 100$ kN.

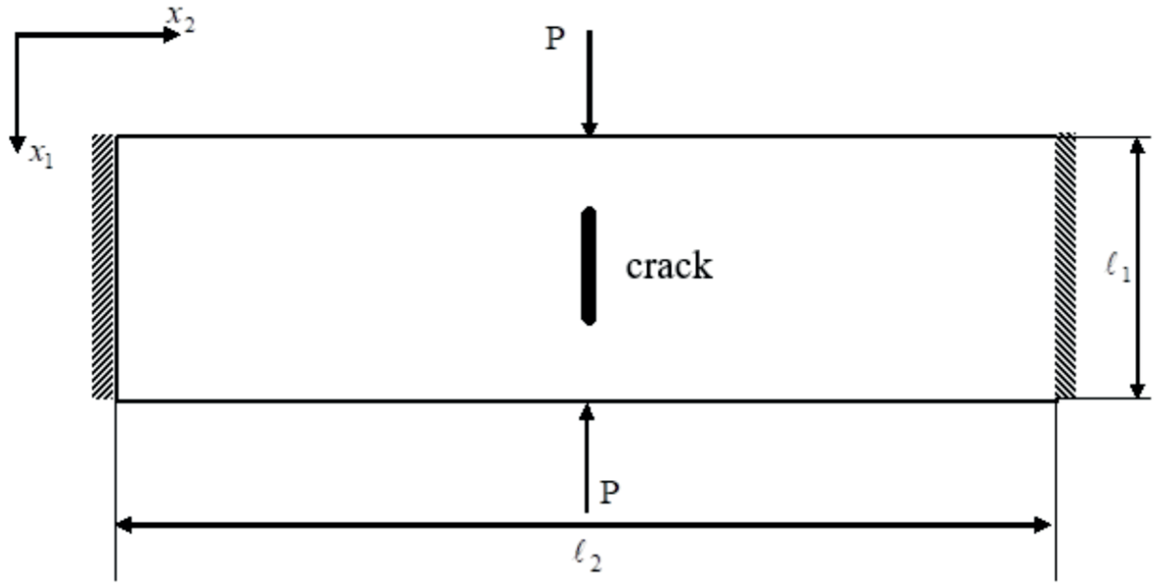


Figure 2.1. Example of analysis

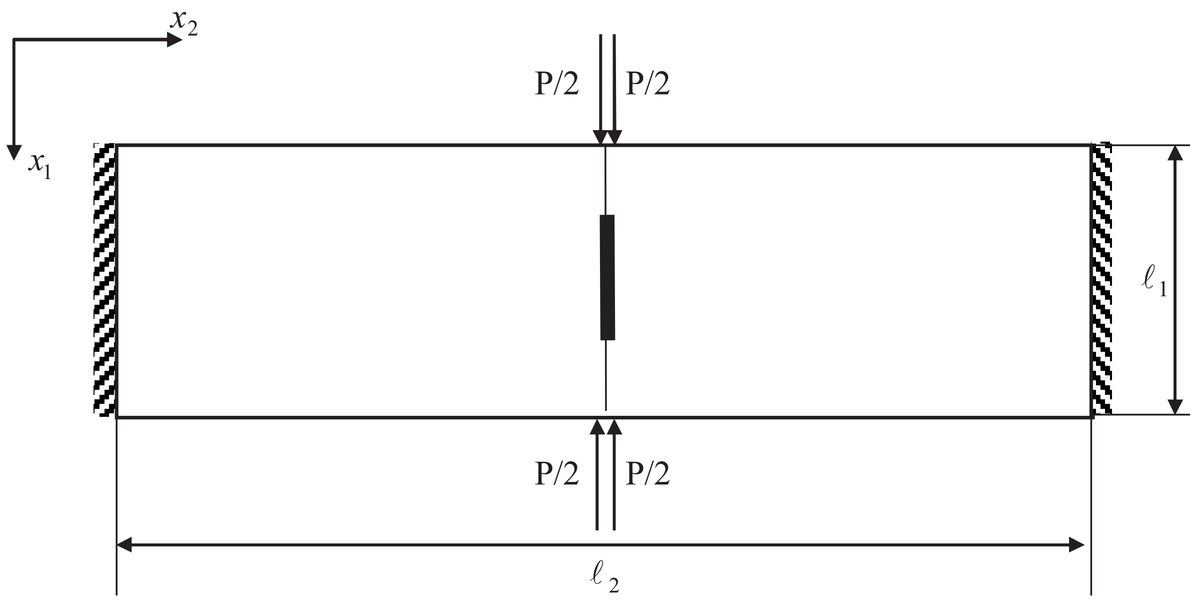
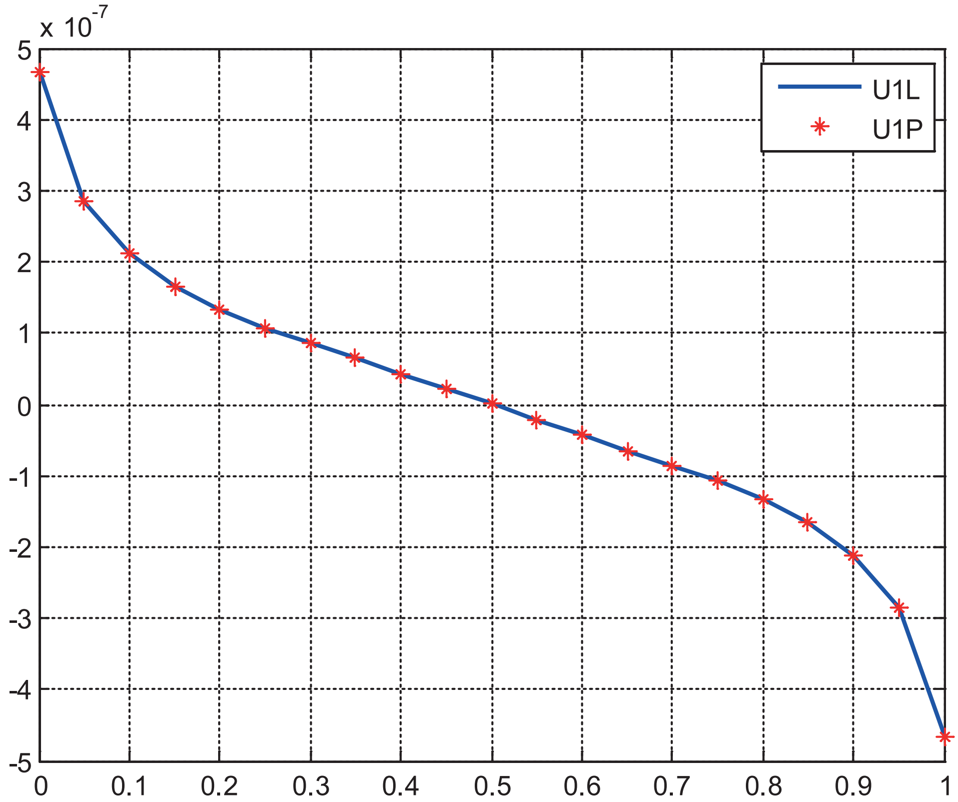


Figure 2.2. Design model

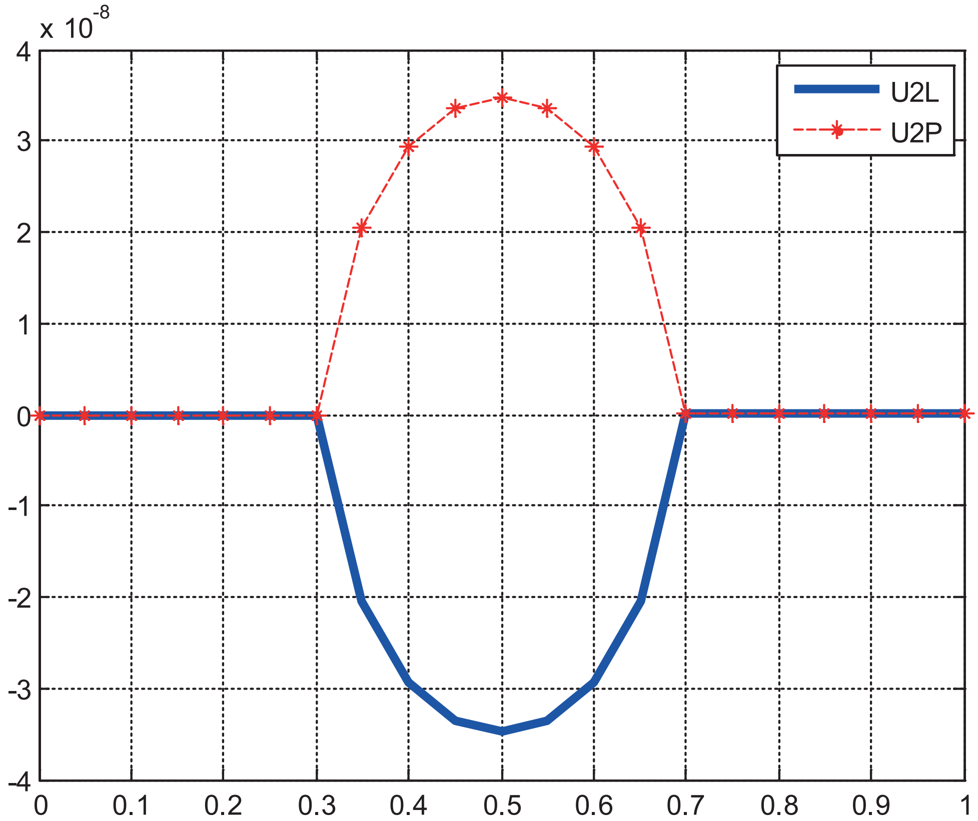
Let us consider the initial problem in the form of a combination of two boundary problems (Figure 2.2).

The contact boundary runs vertically, including the crack. In this case, the contact boundary equations at nodes outside the crack represent the conditions of ideal contact. The equations at the nodes that fall within the crack zone represent the free edge conditions. The load is specified on both sides of the contact boundary at a distance of $2 \cdot 10^{-7}$ m. The discrete component includes $n = 21$ nodes (finite elements): 20 upper and 7 lower nodes for ideal contact equations; internal 7 nodes for equations of free edges on the left side and right side.

Figures 2.3-2.6 are graphical visualizations of the numerical solution (displacements, stresses) at the contact nodes, while "*L*" means the solution for the boundary value problem on the left side, and "*P*" means the solution for the boundary value problem on the right side.



<u>Figure 2.3.</u> Displacements U1L and U1P



<u>Figure 2.4.</u> Displacements U2L and U2P

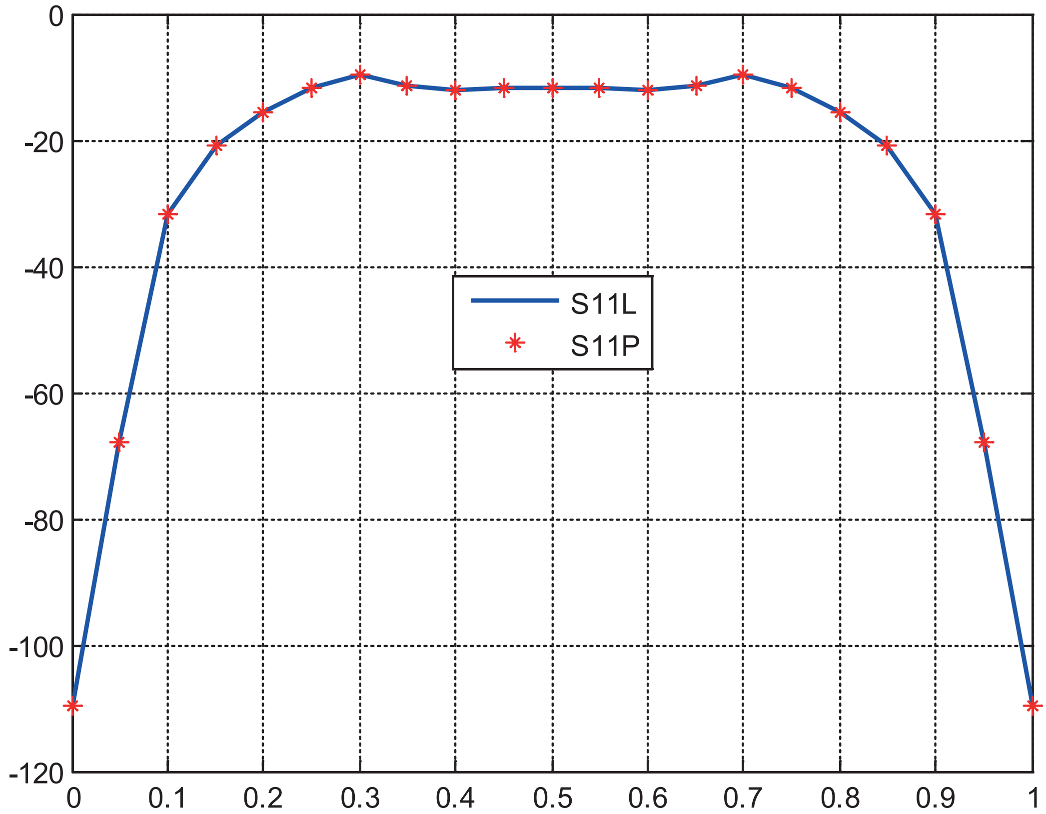


Figure 2.5. Stresses $\sigma 11L$ and $\sigma 11P$

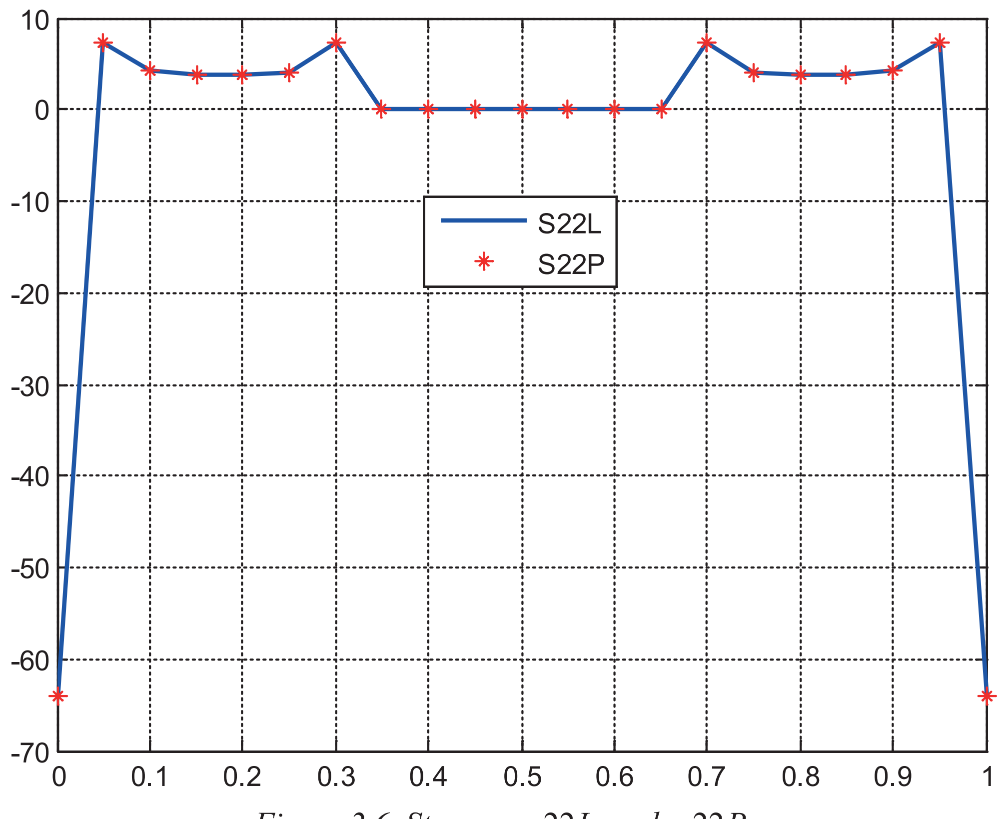


Figure 2.6. Stresses $\sigma 22L$ and $\sigma 22P$.

The presented sample of analysis illustrates the effectiveness of using the discrete-continuous finite element method, since the length significantly exceeds the width of the beam-wall, and the traditional finite element method [9-11] would lead to a significant increase in the nodes of the mesh region.

REFERENCES

1. **Akimov P.A.** Correct Direct Discrete-Continual Boundary Element Method of Structural Analysis. // Applied Mechanics and Materials Vols. 395-396 (2013), pp. 529-532.
2. **Akimov P.A., Sidorov V.N.** Correct Method of Analytical Solution of Multipoint Boundary Problems of Structural Analysis for Systems of Ordinary Differential Equations with Piecewise Constant Coefficients. // Advanced Materials Research Vols. 250-253, 2011, pp. 3652-3655.
3. **Akimov P.A., Mozgaleva M.L.** Method of Extended Domain and General Principles of Mesh Approximation for Boundary Problems of Structural Analysis. // Applied Mechanics and Materials, 2014, Vols. 580-583, pp. 2898-2902.
4. **Akimov P.A., Mozgaleva M.L.** Wavelet-based Multilevel Discrete-Continual Finite Element Method for Local Deep Beam Analysis. // Applied Mechanics and Materials Vols. 405-408 (2013), pp. 3165-3168.
5. **Akimov P.A., Mozgaleva M.L., Mojtaba Aslami, Negrozov O.A.** About Verification of Discrete-Continual Finite Element Method of Structural Analysis. Part 1: Two-Dimensional Problems // Procedia Engineering, Vol. 91 (2014), pp. 2-7.
6. **Akimov P.A., Mozgaleva M.L., Mojtaba Aslami, Negrozov O.A.** Wavelet-Based Discrete-Continual Finite Element Method of Local Structural Analysis for Two-Dimensional Problems // Procedia Engineering, Vol. 91 (2014), pp.8-13.
7. **Akimov P.A., Mozgaleva M.L., Negrozov O.A.** About Verification of Discrete-Continual Finite Element Method for Two-Dimensional Problems of Structural Analysis. Part 1: Deep Beam with Constant Physical and Geometrical Parameters Along Basic Direction. // Advanced Materials Research, Vols. 1025-1026 (2014), pp. 89-94.
8. **Akimov P.A., Mozgaleva M.L., Negrozov O.A.** About Verification of Discrete-Continual Finite Element Methodfor Two-Dimensional Problems of Structural Analysis. Part 2: Deep Beam with Piecewise Constant Physical and Geometrical Parameters Along Basic Direction. // Advanced Materials Research, Vols. 1025-1026 (2014), pp. 95-103.
9. **Zienkiewicz O.C., Morgan K.** Finite Elements and Approximation. Dover Publications, 2006, 352 pages.
10. **Zienkiewicz O.C., Taylor R.L., Zhu J.Z.** The Finite Element Method: Its Basis and Fundamentals. Butterworth-Heinemann, Sixth edition, 2005, 752 pages.
11. **Zienkiewicz O.C., Taylor R.L.** The Finite Element Method for Solid and Structural Mechanics. Volume 2. Butterworth-Heinemann, Sixth Edition, 2005, 736 pages.
12. **Zolotov A.B., Akimov P.A.** Semianalytical Finite Element Method for Two-dimensional and Three-dimensional Problems of Structural Analysis. // Proceedings of the International Symposium LSCE 2002 organized by Polish Chapter of IASS, Warsaw, Poland, 2002, pp. 431-440.
13. **Lur'e A.I.** Teorija uprugosti [Theory of Elasticity]. Moscow, Nauka, 1970, 939 pages (in Russian).

СПИСОК ЛИТЕРАТУРЫ

1. **Akimov P.A.** Correct Direct Discrete-Continual Boundary Element Method of Structural Analysis. // Applied Mechanics and Materials Vols. 395-396 (2013), pp. 529-532.
2. **Akimov P.A., Sidorov V.N.** Correct Method of Analytical Solution of Multipoint

Boundary Problems of Structural Analysis for Systems of Ordinary Differential Equations with Piecewise Constant Coefficients. // Advanced Materials Research Vols. 250-253, 2011, pp. 3652-3655.
3. **Akimov P.A., Mozgaleva M.L.** Method of Extended Domain and General Principles of Mesh Approximation for Boundary Problems of Structural Analysis. // Applied Mechanics and Materials, 2014, Vols. 580-583, pp. 2898-2902.
4. **Akimov P.A., Mozgaleva M.L.** Wavelet-based Multilevel Discrete-Continual Finite Element Method for Local Deep Beam Analysis. // Applied Mechanics and Materials Vols. 405-408 (2013), pp. 3165-3168.
5. **Akimov P.A., Mozgaleva M.L., Mojtaba Aslami, Negrozov O.A.** About Verification of Discrete-Continual Finite Element Method of Structural Analysis. Part 1: Two-Dimensional Problems // Procedia Engineering, Vol. 91 (2014), pp. 2-7.
6. **Akimov P.A., Mozgaleva M.L., Mojtaba Aslami, Negrozov O.A.** Wavelet-Based Discrete-Continual Finite Element Method of Local Structural Analysis for Two-Dimensional Problems // Procedia Engineering, Vol. 91 (2014), pp.8-13.
7. **Akimov P.A., Mozgaleva M.L., Negrozov O.A.** About Verification of Discrete-Continual Finite Element Method for Two-Dimensional Problems of Structural Analysis. Part 1: Deep Beam with Constant Physical and Geometrical Parameters Along Basic Direction. // Advanced Materials Research, Vols. 1025-1026 (2014), pp. 89-94.
8. **Akimov P.A., Mozgaleva M.L., Negrozov O.A.** About Verification of Discrete-Continual Finite Element Methodfor Two-Dimensional Problems of Structural Analysis. Part 2: Deep Beam with Piecewise Constant Physical and Geometrical Parameters Along Basic Direction. // Advanced Materials Research, Vols. 1025-1026 (2014), pp. 95-103.
9. **Zienkiewicz O.C., Morgan K.** Finite Elements and Approximation. Dover Publications, 2006, 352 pages.
10. **Zienkiewicz O.C., Taylor R.L., Zhu J.Z.** The Finite Element Method: Its Basis and Fundamentals. Butterworth-Heinemann, Sixth edition, 2005, 752 pages.
11. **Zienkiewicz O.C., Taylor R.L.** The Finite Element Method for Solid and Structural Mechanics. Volume 2. Butterworth-Heinemann, Sixth Edition, 2005, 736 pages.
12. **Zolotov A.B., Akimov P.A.** Semianalytical Finite Element Method for Two-dimensional and Three-dimensional Problems of Structural Analysis. // Proceedings of the International Symposium LSCE 2002 organized by Polish Chapter of IASS, Warsaw, Poland, 2002, pp. 431-440.
13. **Лурье А.И.** Теория упругости. – М.: Наука, 1970. – 939 с.

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