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FILTRATION REGIME AND STABILITY OF THE EARTH DAM DURING RESERVOIR DRAWDOWN

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Abstract. The results of numerical investigations of the filtration regime and stability of an earth dam under reservoir drawdown are considered. The problem of unsteady filtration in a homogeneous earth dam is solved. The parameters of the filtration flow are determined: the position of the depression surface, the values and direction of filtration gradients and flow velocities. The changes in the maximum filtration gradients and the probability of filtration deformations are analyzed. Calculations are made to assess the stability of the upstream slope during the stages of reservoir drawdown with different velocities. It is shown that a dam with a large stability margin when the reservoir is full may lose it and be destroyed in the process of drawdown. Conclusions are drawn about the necessary measures to ensure reliability and safety during operation of the structure.

Keywords: earth dam, reservoir drawdown, unsteady seepage, seepage gradient, slope stability

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ФИЛЬТРАЦИОННЫЙ РЕЖИМ И УСТОЙЧИВОСТЬ ГРУНТОВОЙ ПЛОТИНЫ ПРИ СРАБОТКЕ ВОДОХРАНИЛИЩА

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Аннотация. Рассматриваются результаты численных исследований фильтрационного режима и устойчивости грунтовой плотины при сработке водохранилища. Решается задача неустановившейся фильтрации в однородной земляной плотине. Определяются параметры фильтрационного потока: положение депрессионной поверхности, величины и направление фильтрационных градиентов и скоростей потока. Дается анализ изменения максимальных градиентов фильтрации и вероятность возникновения фильтрационных деформаций. Проведены расчеты по оценке устойчивости верхового откоса на этапах сработки водохранилища с разной скоростью. Показано, что плотина, имеющая большой запас устойчивости при наполненном водохранилище, может потерять его и быть разрушенной в процессе сработки. Сделаны выводы о необходимых мероприятиях по обеспечению надежности и безопасности при эксплуатации сооружения.

Ключевые слова: грунтовая плотина, сработка водохранилища, неустановившаяся фильтрация, градиент фильтрации, устойчивость откоса

Финансирование. Данная работа была поддержана грантом 2024 года на проведение фундаментальных и прикладных научных исследований (НИР/НИОКР) научными коллективами НИУ МГСУ, проект № 28-392/130.

INTRODUCTION

Problems of unsteady filtration are encountered in hydraulic engineering practice during construction and operation of dams, in cases of filling or drawdown of a reservoir, as well as during construction of reclamation dams.

The drawdown of a reservoir may be caused by the technological necessity of operation of a hydroelectric complex, which includes an earth dam, or by an emergency situation. The lowering of the water level upstream of the dam leads to a change in the filtration regime of the ground structure - the position of the depression surface, the values and directions of the gradients and velocities of the filtration flow change. The consequence of this process can be filtration deformations of soils and occurrence of concentrated filtration in the dam body [1-3]. Also in this case, in the upstream slope there is a change in the direction of action of filtration hydrodynamic load towards the upstream, which reduces the stability of the slope of the structure [4-5]. In the practice of hydraulic engineering, there are quite a large number of cases of emergencies associated with the lowering of the reservoir water level [6-7]. The degree of change in the stability of the structure and the filtration regime depends on several factors: the permeability of the soils in the upstream prism of the dam, the magnitude and rate of drawdown, the top slope embedment, and a number of other factors [8-10].

At present, filtration calculations are mandatory in the design of earth dams, on the basis of which the filtration stability of the dam soils, foundation and banks, as well as the stability of the slopes of the structure are assessed (SP39.13330.2012 Dams made of earth materials. Updated version of SNiP2.06.05-84).

The assessment of the stability of the upstream slope in the design justification of the earth dam should be carried out for the case of the maximum possible drawdown of the reservoir water level with the highest possible velocity, where the filtration forces of unsteady filtration are as follows.

This paper investigates the effect of the drawdown rate on the maximum filter gradient and the stability of the upstream slope of a homogeneous earth dam. The changes in these parameters in the process of lowering the upstream water level are considered. Previous studies [10,11] have shown that the maximum filtration gradient can be recorded not at the end of drawdown, but at intermediate stages of water level lowering.

RESEARCH OBJECT

The object of research is a homogeneous earthen dam 24.0m high, located on an impermeable base (Figure 1). The dam slopes are -1:3 for upstream and downstream slopes. At the level mark of the dam foot level equal to 0.00, the upstream water level is 22.0m and the downstream water level is 3.0m. To ensure reliable operation of the dam, a 4m high drainage prism with a 1:3 slope is installed in the downstream prism.

The dam body is composed of clayed sand. The characteristics corresponding to this group of soils, included in filtration, stress-strain and

Table 1. Characteristics of the soil of the dam body under consideration

Type of soil	Particle size, mm	Percentage of soil particles, %	Filtration coefficient, m/day	Cohesion C, kN/m2	Density γ, kN/m3		Strain modulus E, kN/m2	Poisson's ratio ν	Angle of internal friction φ, °
					Saturated condition	Dry condition			
Clayed sand	< 0.002	22	0.10	4.0	19.0	18.0	20000	0.33	30.0
	0.002 - 0.05	25							
	0.05 - 2	53							

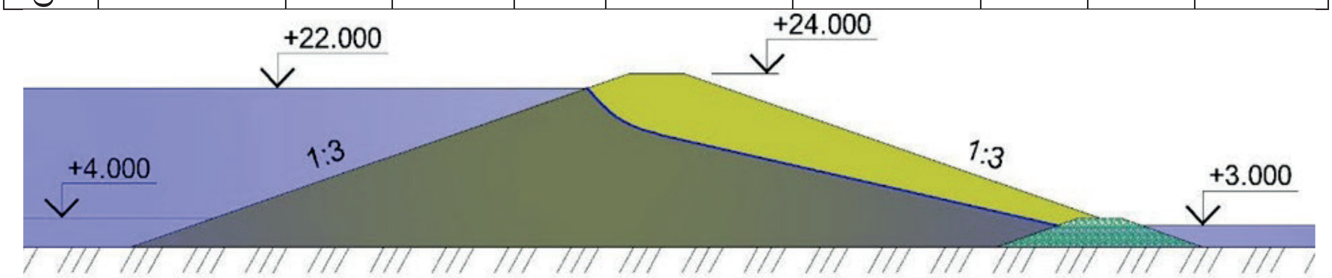

Figure 1. Schematic diagram of a homogeneous earth dam with drainage

stability calculations, are given in Table 1. Characteristics of the drainage prism soil: density in saturated and dry states 19 and 16 kN/m3; modulus of elasticity $E = 30000 kN/m^2$; Poisson's ratio $\nu = 0{,}3$; cohesion $c = 1\, kN/m^2$; angle of internal friction $\varphi = 30°$, filtration coefficient $k = 10 m/day$.

METHODS

Since the middle of the 20th century, due to the difficulty of solving the problem of unsteady filtration, hydraulic methods based on the solution of the Boussinesq equation with some assumptions [12] have become widespread. As the issue of improvement of calculation methods developed, new analytical [13] and hydraulic [14-15] calculation methods appeared.

In modern studies [16-19], the main task is to determine filtration parameters, such as the position of the depression curve, the height of the drawdown gap on the downstream slope of the dam, the filtration flow rate, gradients and filtration velocities. This is achieved using numerical methods of FEM, FVM and BEM.

Modern program complexes (PCs) designed to solve filtration problems are based on the solution of the basic differential equation of filtration theory with known boundary conditions [13]:

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial H}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial H}{\partial z}\right) - \mu \frac{\partial H}{\partial t} = 0, \tag{1}$$

where $H = f\,(x, y, z, t)$ – head function in the considered area, varying along the coordinates x, y, z and in time t; K_x, K_y, K_z– filtration coefficients in axis directions x, y, z; μ – water loss coefficient in the ground.

In such calculations, water movement between particles is assumed when the pores are completely filled with water. Water moves from a higher-pressure region to a lower pressure region. Then, when excluding the finite elements from the computational domain, for which the obtained value of filtration head is less than their altitude position, the position of the depression curve will be determined [18]. Similar problems of soil hydrology were considered in the works of famous foreign [20-23] and Soviet-Russian scientists [24].

When solving filtration problems in soil hydrology, the L.A.Richards equation [25] based on the Darcy equation is used to solve problems of unsteady filtration. One of the modifications of this equation for moisture transport in vertical direction, with gravity pressure gradient equal to one, is presented in the form:

$$\mu\,(\partial \Psi \,/\, \partial t) \;=\; \partial\,(k\,(\partial \Psi / \partial z\, - 1))\,/\partial z, \tag{2}$$

where Ψ – capillary pressure of ground (soil) moisture (cm of water column), k – ground (soil) hydraulic conductivity coefficient (cm/day, is a function of moisture pressure Ψ), μ- coefficient of differential soil water holding capacity (cm of waterstage -1), t - time (days), z – vertical axis coordinate (cm).

When solving such an equation, only numerical methods can be used. Coefficients $\mu = \mu(\Psi)$ and $k = k\,(\Psi)$ are a function of capillary pressure of moisture, but these functions have empirical value [24, 26], which leads to inaccuracy of the obtained results. Value of soil volumetric moisture θ is determined under laboratory conditions and depends on the capillary pressure of moisture Ψ, called the basic hydrophysical characteristic (BHC) [25].

In 1980, Van Genuchten [22], using the Mualem method [21], proposed a model for describing the hydrophysical properties of soils (soils) using expressions of the Mualem-Van Genuchten model. Such a model is widely used in practice nowadays in the numerical modeling of filtration processes [6-7].

RESULTS

The effect of reservoir drawdown on the filtration regime and stability of the structure was studied for a homogeneous earth dam structure under consideration. The problem was solved in Plaxis PC using the finite element method. The computational domain was divided into 1984 elements and 16221 nodes, taking into account the elements and nodes of the impermeable foundation.

Three variants of drawdown were considered depending on its velocity in height with values of 0.25, 1.0, 2.0 s/day. Depletion rate in height was 18 meters: from elevation 22.0 to elevation 4.0 (Figure 1).

During the study, the design calculations were carried out in 3 phases:

Initial Phase - the first phase in which initial conditions of loading by own weight and hydraulic conditions under steady filtration level were set. At this phase depression curve was formed at filled reservoir.

The second phase, in which reservoir drawdown with different rates of water level drawdown was modeled. In this phase, deformations and pore pressure under unsteady filtration are calculated in time.

The third phase, in which the stability of the structure is calculated using the strength reduction method.

At each setback step with a height interval of 1.0 m, the 2nd and 3rd phases of the calculation were solved.

Safety calculation in Plaxis 2D PC is used to determine the calculated actual stability coefficients. During this calculation, the strength characteristics of soils (internal friction angle, cohesion and shear strength) are sequentially reduced until failure. The slope stability coefficient is defined as the ratio of the actual strength to the failure strength. The known criterion of slope stability is used:

$$\gamma_{lc} \ F \leq \ \gamma_c R / \gamma_n \qquad (3)$$

where F – design value of the generalized shear force, determined taking into account the load reliability factor; R – calculated value of the generalized bearing capacity of the system "structure - base", determined taking into account the reliability coefficient for soil, i.e. generalized calculated value of the ultimate shear resistance forces on the considered surface; γ_{lc} – load combination factor, taken as: γ_{lc} = 1,0 – for the main load combination; γ_{lc}= 0,9 – for special load combinations; γ_n – reliability coefficient by the degree of responsibility of structures. For the considered structure we can conditionally assume γ_n =1,20, which corresponds to reliability class II; γ_c – coefficient of working conditions, or taking into account the method of calculation: γ_c=1,0 – for calculation methods taking into account the stress-strain state.

The soil medium was modeled on the basis of the bilinear elastic-plastic Mohr-Coulomb model (Mohr-Coulomb) [20, 21].

Estimation of filtration regime. The solution of the unsteady filtration problem (2nd phase of calculations) allowed us to obtain a picture of changes in the filtration regime of a homogeneous dam at different drawdown rates. Figure 2 shows the position of the depression surface and the distribution of the filtration gradient at the moments of the end of the reservoir drawdown up to the 4.0 mark for the variants of drawdown rates of 0.25 and 2.0 m/day.

It can be noted that for the variant of 0.25 m/day drawdown rate, the depression curve decreased to a greater extent in the upper prism in comparison with the variant with 2.0 m/day drawdown rate (the depression curve mark in the first variant is 12.0 m, in the second - 16.0 m). At elevations below the point of depression curve exit to the upper slope near the slope, zones with increased values of filtration gradients were obtained.

Based on the results obtained, the maximum filtration gradients were plotted (Figure 3) to provide a preliminary assessment of the ground filtration strength and to calculate, if necessary, the parameters of inverse filters. The maximum output gradients occur on the upstream slope of the dam when the upstream slope is drawn down to the marks 14.0 -16.0. The maximum gradients for the 0.25, 1.0, and 2.0 m/day drawdown rates are 0.460, 0.518, and 0.532, respectively.

Estimation of the stability of the upper slope. Based on the results of the third phase calculation, a series of dam stability coefficients for different moments of failure were obtained, as summarized in Table 2. Graphical representation of some of the results is given in Figure 4.

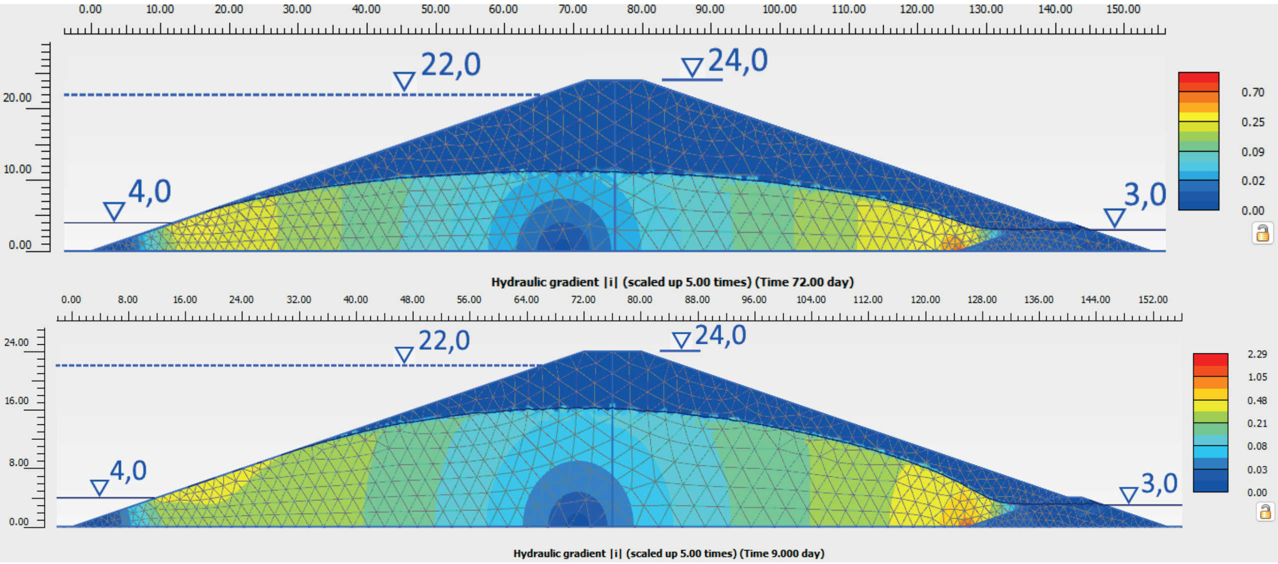

Figure 2. Position of the depression curve at drawdown with velocities of 0.25m/day and 2m/day for the period of the end of reservoir drawdown (isopoles show the distribution of hydraulic gradient)

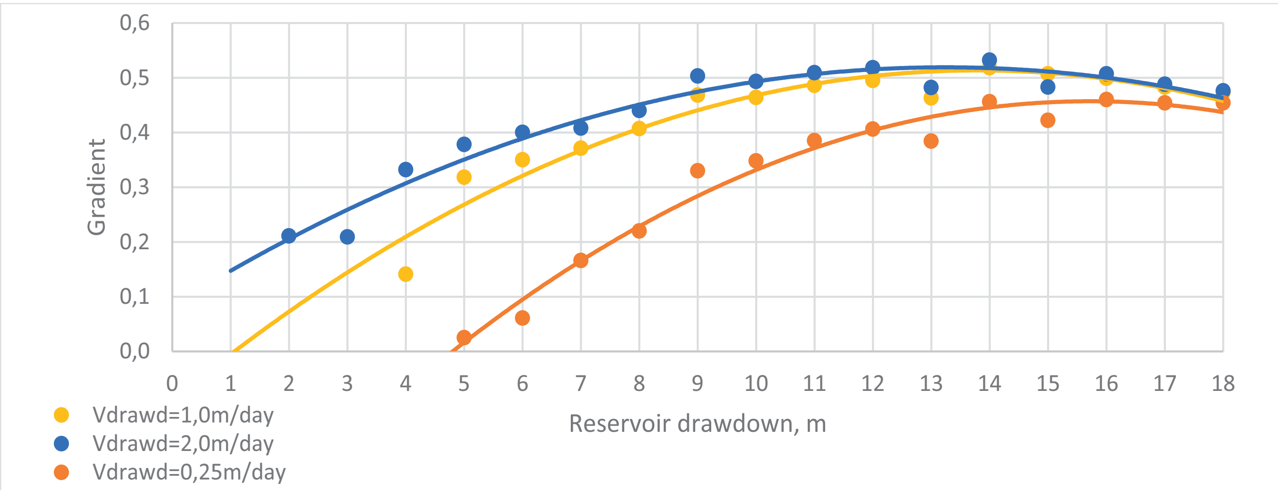

Figure 3. Variation of the maximum filtration gradient as a function of drawdown elevation for variants of drawdown rates

It is possible to note the influence of the displacement rate on the value of the minimum stability coefficients of the top slope (Table 2). As can be seen from the results, the stability coefficient is lower the higher the displacement rate. At the value of stability coefficient Ks =2.127 when the reservoir is filled, this value decreases at drawdown rates of 0.25. 1.0, 2.0 m/day respectively to values 1.360, 1.137 and 1.047.

It can be concluded that at displacement rates higher than 0.5 m/day the slope stability coefficient is less than the normative one (for conditionally assumed 2nd capitalization class K_n =1.2).

Next, the slope stability at different stages of reservoir drawdown was investigated. Step 1m of fixing the stability coefficient was set for cases with drawdown rates of 0.25m/day, 1m/day and 2m/day. At each step, the slope stability coefficient was determined taking into account the filtration hydrodynamic forces determined from the solution of the unsteady filtration problem. According to the results of the numerical experiment, the graphs of changes in the stability coefficient of the upper slope were plotted for the stages of depletion at the values of its velocity equal to 0.25, 1.0 and 2.0 m/day (Figure 5).

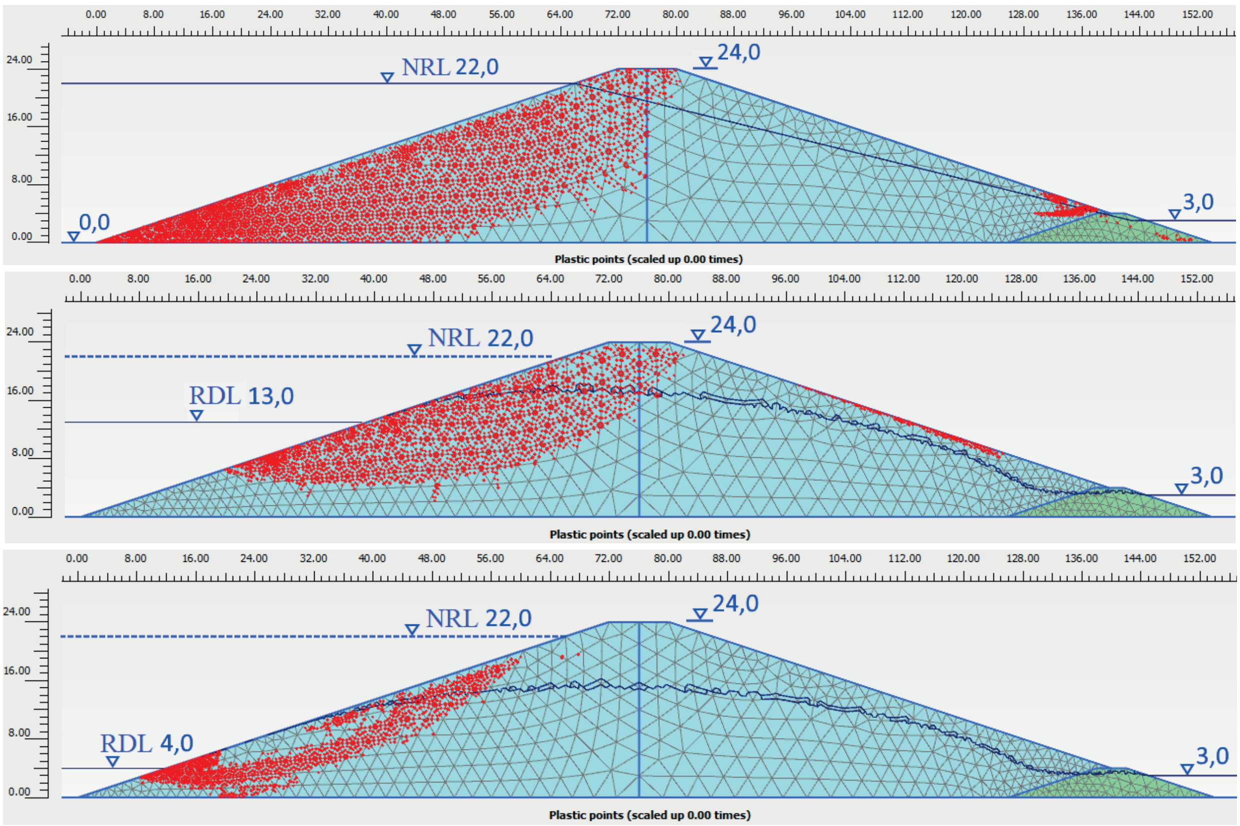

Figure 4. Collapse surfaces with minimum stability coefficients for cases of reservoir drawdown at a rate of 1 m/day: a) at fully filled reservoir; b) at 9.0 m drawdown; c) at 18.0 m drawdown

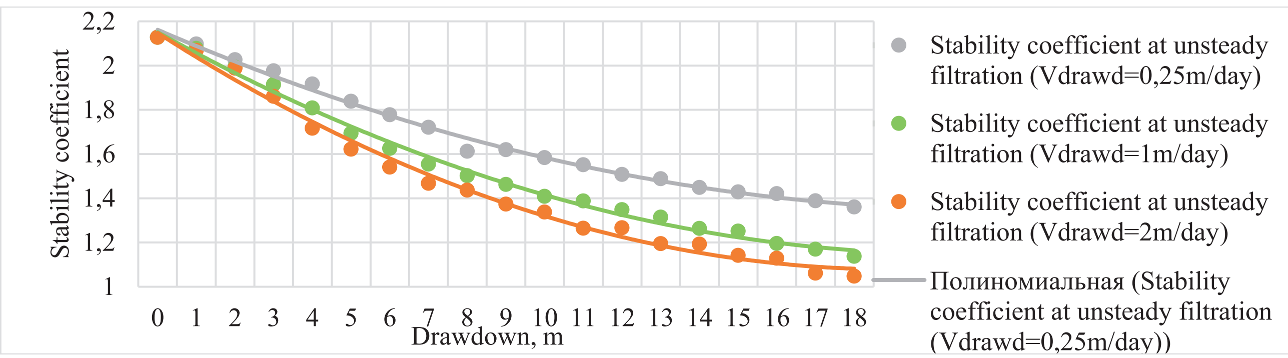

Figure 5. Plots of change in the stability coefficient of the upper slope depending on the degree of displacement and its velocity

Table 2. Calculation results of stability coefficient for different soil variants and displacement rates

Type of soil	Filtration coefficient, m/day	Slope stability coefficient					
		Reservoir drawdown rate, m/day					
		0.10	0,25	0,5	1,0	1,5	2,0
Clayed sand	0.10	1,506	1,360	1,233	1,137	1,072	1,047

As can be seen from the graphs in Figure 5, as the reservoir drawdown proceeds, the stability coefficient decreases significantly. The drawdown rate affects the slope stability: increasing the rate decreases the stability coefficient. At the same time, by the end of drawdown to the 4.0 mark, the minimum stability coefficients were recorded (Table 2, Figure 5).

For the considered variant of the earth dam, the drawdown rates of 1.0 and 2.0 m/day cause the stability coefficient to decrease to values lower than the normative one. Thus, for the rate of 1.0 m/day $K_s \leq K_n$ at reservoir drawdown exceeding 16.0 m from the NRL level, for the rate of 2.0 m/day - exceeding 12.0 m.

CONCLUSIONS

The design of earth dams must take into account the effect of unsteady filtration in the dam body in the event of reservoir drawdown. The degree to which drawdown affects the seepage regime and the stability of the upstream slope depends to a large extent on a number of factors, primarily the rate of water level drawdown.

1. The values of maximum filtration gradients necessary to evaluate the possibility of filtration deformations and to calculate the return filters in case of the top slope fixing device have been obtained for the considered design of the earth dam.

2. The value of the maximum filtration gradient during reservoir drawdown depends on its velocity. The highest gradients at the outlet of filtration flow to the upper reservoir with values 0.46 - 0.53 occur for the considered variant in the lower third of the drawdown interval at grades 13.0 - 16.0 depending on the drawdown rate. The higher is the discharge rate, the greater are the values of output gradients.

3. Reservoir drawdown reduces the stability of the upstream slope. The stability coefficient values are minimal at maximum upstream drawdown. For the considered variant of the dam and the conditions of drawdown at drawdown rates ≥ 1.0 m/day, the value of the slope stability coefficient is less than the normative value K_n=1.2, conventionally accepted for a structure of the 2nd capital class. This relationship occurs at drawdown to elevation 12.0 at 2.0 m/day and to elevation 16.0 at 1.0 m/day.

Further studies may consider the effect of anisotropy of the dam body soils caused by the placement technology and properties of the soils, as well as the effect of drawdown on the performance of earth dams with impervious elements.

REFERENCES

1. **Bellendir E.N., Ivashintsov D.A., Stefanishin D.V.** et al. Probabilistic methods of reliability assessment of ground hydraulic structures // St. Petersburg: VNIIG named after B.E. Vedeneev. B.E. Vedeneev. 2003- 2004. T. 1.
2. **Adamo, Nasrat & Al-Ansari, Nadhir & Sissakian, Varoujan & Laue, Jan & Knutsson, Sven**. Dam Safety Problems Related to Seepage. 2020; 10. 191-239
3. **López-Acosta N.P., Sánchez M.A. Soil Solution, G. Auvinet J.-M. Pereira.** Assessment of exit hydraulic gradients at the toe of levees in water drawdown conditions. Scour and Erosion – Cheng, Draper & An (Eds)©. 2015; Taylor &

Francis Group, London. Available from: https://www.researchgate.net/publication/283129540. DOI: 10.1201/b17703-21.
4. **Fattah M.Y., Hasan A. Omran, Mohammed A. Hassan.** Behavior of an Earth Dam during Rapid Drawdown of Water in Reservoir-Case Study. International Journal of Advanced Research. 2015; Vol. 3, Issue 10, 110 -122.
5. **Upomo T.C., Effendi M.K., Kusumawardani R.** Behaviour of levee on softsoil caused by rapid drawdown. AIP Conference Proceedings. American Institute of Physics Inc. 2018; 1941. DOI: 10.1063/1.5028103.
6. **Stark T.D., Jafari N.H., Zhindon J.S.L., & Baghdady A.** Unsaturated and Transient Seepage Analysis of San Luis Dam. Journal of Geotechnical and Geoenvironmental Engineering. 2017; 143(2), 04016093. DOI: 10.1061/(ASCE)GT.1943-5606.0001602.
7. **Vandenberge D.R.** Total Stress Rapid Drawdown Analysis of the Pilarcitos Dam Failure using the Finite Element Method. Frontiers of Structural and Civil Engineering. 2014; 8(2), 115–123. DOI: 10.1007/s11709-014-0249-7.
8. **Perzlmaier S.** Hydraulic criteria for internal erosion in cohesionless soil. In Internal erosion of dams and their Foundations. Editors R. Fell and J.J. Fry. Taylor & Francis. 2007; 179-190. DOI: 10.1201/9781482266146-20.
9. **Aniskin N.A., Sergeev S.A.** The effect of draw-off on filtration regime of earth-fill dam. International Journal for Computational Civil and Structural Engineering. 2022. 18(1) 40-50. DOI:10.22337/2587-9618-2022-18-1-40-50. Available from: https://ijccse.iasv.ru/index.php/ijccse/article/download/474/283/837.
10. **Aniskin, N.A.; Stupivtsev, A.V.** Influence of the factors on the filtration regime of the earth dam during the upper bank failure // Vestnik of MSCU. 2023. T. 18. Issue. 6. P. 917-926. DOI: 10.22227/1997-0935.2023.6.917-926.
11. **Aniskin N., Stupivtsev A.** Change in filtration gradients during head water drawdown //E3S Web of Conferences. – EDP Sciences, 2023. – T. 457. – P. 02046.
12. Development of research on filtration theory in the USSR // Institute of Hydrodynamics, SB AS USSR, VNIIG named after B.E.Vedeneev. B.E.Vedeneev. VNII of Natural Gases. M. "Nauka.".
13. **Anakhaev, K.N.** About filtration calculation of a cofferdam // Mathematical modeling. 2011. T. 23. № 2. P.148-158.
14. **Bukhartsev V.N., Petrichenko M.R.** Nonsteady filtration in a uniform soil mass. Power Technology and Engineering. 2012; Vol. 46, No. 3. 1-3
15. **Petrichenko M.R., Zaborova D.D., Kotov E.V., Musorina T.A.** Weak solutions of the limit problems of Crocco // Scientific and Technical Bulletins of St. Petersburg State Polytechnic University. Physico-mathematical sciences. 2018. T. 11(3). P.27-38. DOI: 10.18721/JPM.11303.
16. **Lee K.K., Darrell I.** Leap Simulation of a free- surface and seepage face using boundary-fitted coordinate system method. Journal of Hydrology. 1997; Vol.196 (1). pp.297–309. DOI: 10.1016/S0022-1694(96)03246-5.
17. **Billstein, M.** Development of a Numerical Model of Flow through Embankment Dams. Department of Environmental Engineering, Lulea University of Technology. 1998; Lulea, Sweden. P.59.
18. **Hsiao C.C., Chen J.T., Chen K.H.** Application of hypersingular equations to free-surface seepage problems. Computational Methods. 2006; Springer. pp. 73-77.
19. **Aniskin N.A., Antonov A.S**. Development geo-seepage models for solving seepage problems of large dam's foundations, on an example of ANSYS Mechanical APDL. Advanced Materials Research. 2015; Vols. 1079-1080. pp. 198-201. DOI:

10.4028/www.scientific.net/AMR.1079-1080.198.
20. **Burdine, N.T.** Relative permeability calculations from pore size distribution data. Petr. Trans. AIME. 1953; 198, Pp. 71–78. DOI: 10.2118/225-G.
21. **Mualem, Y.** A new model for predicting the hydraulic conductivity of unsaturated porous media. Water Resour. Res. 1976; 12(3), Pp. 513–522. DOI: 10.1029/WR012i003p00513.
22. **Van Genuchten, M.T.** A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. Soil Sci. Soc. Am. J. 1980; 44(5), Pp. 892–898. DOI: 10.2136/sssaj1980.03615995004400050002x.
23. **Kosugi K.** General model for unsaturated hydraulic conductivity for soil with lognormal pore-size distribution. Soil Sci. Soc. Am. J. 1999; 63:Pp. 270-277. DOI: 10.2136/sssaj1999.03615995006300020003x.
24. **Terleev V.V., Mirschel W., Schindler U., Wenkel K.-O.** Estimation of soil water retention curve using some agrophysical characteristics and Voronin's empirical dependence. International Agrophysics. 2010; 24(4):381-387.
25. **Shein E.V.** Course of soil physics // M.: Izd-vo Moscow. unct. 2005. C. 432.
26. **Terleyev V.V.V., Narbut M.A., Topazh A.G., Mirschel V.** Modeling of hydrophysical properties of soil as a capillary-porous body and improvement of the Mualem - Van Genuchten method: theory. // Agrophysics. 2014. №2(14), c. 35 -44.

СПИСОК ЛИТЕРАТУРЫ

1. **Беллендир Е.Н., Ивашинцов Д.А., Стефанишин Д.В. и др**. Вероятностные методы оценки надежности грунтовых гидротехнических сооружений // СПб.: ОАО ВНИИГ им. Б.Е. Веденеева. 2003-2004. Т. 1.
2. **Adamo, Nasrat & Al-Ansari, Nadhir & Sissakian, Varoujan & Laue, Jan & Knutsson, Sven**. Dam Safety Problems Related to Seepage. 2020; 10. 191-239
3. **López-Acosta N.P., Sánchez M.A. Soil Solution, G. Auvinet J.-M. Pereira**. Assessment of exit hydraulic gradients at the toe of levees in water drawdown conditions. Scour and Erosion – Cheng, Draper & An (Eds)©. 2015; Taylor & Francis Group, London. Available from: https://www.researchgate.net/publication/283129540. DOI: 10.1201/b17703-21.
4. **Fattah M.Y., Hasan A. Omran, Mohammed A. Hassan.** Behavior of an Earth Dam during Rapid Drawdown of Water in Reservoir-Case Study. International Journal of Advanced Research. 2015; Vol. 3, Issue 10, 110 -122.
5. **Upomo T.C., Effendi M.K., Kusumawardani R.** Behaviour of levee on softsoil caused by rapid drawdown. AIP Conference Proceedings. American Institute of Physics Inc. 2018; 1941.DOI: 10.1063/1.5028103.
6. **Stark T.D., Jafari N.H., Zhindon J.S.L., & Baghdady A.** Unsaturated and Transient Seepage Analysis of San Luis Dam. Journal of Geotechnical and Geoenvironmental Engineering. 2017; 143(2), 04016093. DOI: 10.1061/(ASCE)GT.1943-5606.0001602.
7. **Vandenberge D.R.** Total Stress Rapid Drawdown Analysis of the Pilarcitos Dam Failure using the Finite Element Method. Frontiers of Structural and Civil Engineering. 2014; 8(2), 115–123. DOI: 10.1007/s11709-014-0249-7.
8. **Perzlmaier S.** Hydraulic criteria for internal erosion in cohesionless soil. In Internal erosion of dams and their Foundations. Editors R. Fell and J.J. Fry. Taylor & Francis. 2007; 179-190. DOI: 10.1201/9781482266146-20.
9. **Aniskin N.A., Sergeev S.A.** The effect of draw-off on filtration regime of earth-fill dam. International Journal for Computational Civil and Structural

Engineering. 2022. 18(1) 40-50. DOI:10.22337/2587-9618-2022-18-1-40-50. Available from: https://ijccse.iasv.ru/index.php/ijccse/article/download/474/283/837.

10. **Анискин Н.А., Ступивцев А.В.** Влияние факторов на фильтрационный режим земляной плотины при сработке верхнего бьефа // Вестник МГСУ. 2023. Т. 18. Вып. 6. С. 917–926. DOI: 10.22227/1997-0935.2023.6.917-926.
11. **Aniskin N., Stupivtsev A.** Change in filtration gradients during head water drawdown //E3S Web of Conferences. – EDP Sciences, 2023. – Т. 457. – С. 02046.
12. Развитие исследований по теории фильтрации в СССР // Институт гидродинамики СО АН СССР, ВНИИГ им. Б.Е.Веденеева. ВНИИ Природных газов. М. “Наука.”.
13. **Анахаев К.Н.** О фильтрационном расчете перемычки // Математическое моделирование. 2011. Т. 23. № 2. С.148–158.
14. **Bukhartsev V.N., Petrichenko M.R**. Nonsteady filtration in a uniform soil mass. Power Technology and Engineering. 2012; Vol. 46, No. 3. 1-3
15. **Петриченко М.Р., Заборова Д.Д., Котов Е.В., Мусорина Т.А.** Слабые решения предельных задач Крокко // Научно-технические ведомости Санкт-Петербургского государственного политехнического университета. Физико-математические науки. 2018. Т. 11(3). С.27-38. DOI: 10.18721/JPM.11303.
16. **Lee K.K., Darrell I.** Leap Simulation of a free- surface and seepage face using boundary-fitted coordinate system method. Journal of Hydrology. 1997; Vol.196 (1). pp.297–309. DOI: 10.1016/S0022-1694(96)03246-5.
17. **Billstein, M.** Development of a Numerical Model of Flow through Embankment Dams. Department of Environmental Engineering, Lulea University of Technology. 1998; Lulea, Sweden. P.59.
18. **Hsiao C.C., Chen J.T., Chen K.H.** Application of hypersingular equations to free-surface seepage problems. Computational Methods. 2006; Springer. pp. 73-77.
19. **Aniskin N.A., Antonov A.S.** Development geo-seepage models for solving seepage problems of large dam’s foundations, on an example of ANSYS Mechanical APDL. Advanced Materials Research. 2015; Vols. 1079-1080. pp. 198-201. DOI: 10.4028/www.scientific.net/AMR.1079-1080.198.
20. **Burdine, N.T.** Relative permeability calculations from pore size distribution data. Petr. Trans. AIME. 1953; 198, Pp. 71–78. DOI: 10.2118/225-G.
21. **Mualem, Y.** A new model for predicting the hydraulic conductivity of unsaturated porous media. Water Resour. Res. 1976; 12(3), Pp. 513–522. DOI: 10.1029/WR012i003p00513.
22. **Van Genuchten, M.T.** A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. Soil Sci. Soc. Am. J. 1980; 44(5), Pp. 892–898. DOI: 10.2136/sssaj1980.03615995004400050002x.
23. **Kosugi K.** General model for unsaturated hydraulic conductivity for soil with lognormal pore-size distribution. Soil Sci. Soc. Am. J. 1999; 63:Pp. 270-277. DOI: 10.2136/sssaj1999.03615995006300020003x.
24. **Terleev V.V., Mirschel W., Schindler U., Wenkel K.-O.** Estimation of soil water retention curve using some agrophysical characteristics and Voronin’s empirical dependence. International Agrophysics. 2010; 24(4):381-387.
25. **Шеин Е.В**. Курс физики почв // М.: Изд-во Моск. ун-та. 2005. С. 432.
26. **Терлеев В.В., Нарбут М.А., Топаж А.Г., Миршель В**. Моделирование гидрофизических свойств почвы как каппилярно-пористого тела и усовершенствование метода Муалема - Ван Генухтена: теория. // Агрофизика. 2014. №2(14), с. 35 -44.

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