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CONCRETE DELAYED FAILURE TIME

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Abstract: At a single action on a concrete and reinforced concrete structure short-term dynamic load, the value of which may exceed the value of the static load-bearing capacity of the structure, it was observed that failure will occur not immediately, but after a certain time (delayed failure time t_d). If the dynamic load action is stopped before the moment t_d, the structure will not collapse. Therefore, the accurate determination of the delayed failure time of concrete is an important and relevant problem. To solve this problem, the paper presents a visco-elastic-plastic model to describe the stress-strain state of concrete under dynamic loading. This model consists of 2 elements: a nonlinear spring A and a piston B connected in parallel. Element A describes the nonlinear elastic-plastic properties of concrete, and element B takes into account the high-strain-rate effect of concrete. Under the action of sudden dynamic loads with an intensity greater than the static bearing capacity of the concrete, piston B helps to inhibit the development of deformations initiated in element A. Based on the proposed model, the delayed failure time is defined by the time interval required for the deformation of concrete to reach its ultimate value. The main factors affecting the deformation and failure of concrete such as concrete compressive strength, overload level, and viscosity are also investigated. Specifically, as follows: The higher the static compressive strength of concrete, the lower the delayed failure time. When a dynamic force of greater intensity is applied compared to the bearing capacity of the concrete, the faster the specimen will destroy. In addition, the viscosity coefficient significantly reduces the strain rate of concrete and the corresponding delayed failure time increases as the viscosity increases.

Keywords: Dynamic increase factor, delayed failure time, visco-elastic-plastic model, concrete, strain rate effect, viscosity, Kelvin–Voigt model

ВРЕМЯ ЗАДЕРЖКИ РАЗРУШЕНИЯ БЕТОНА

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Аннотация: При однократном действии на бетонное и железобетонное сооружение кратковременной динамической нагрузки аварийного типа, величина которой может превышать величину статической несущей способности конструкции наблюдалось что разрушение произойдет не сразу, а по истечении определенного времени (время задержки разрушения t_d). Если действие динамической нагрузки прекратить до момента t_d, то конструкция не разрушится. Поэтому точного определение времени задержки разрушения бетона является важной и актуальной проблемой. Для решения данной проблемы, в статье представлена вязко-упруго-пластическая модель, позволяющая описывать напряженно-деформированное состояние бетона при динамическом нагружении. Эта модель состоит из 2-х элементов: нелинейная пружина А и поршень В, соединенных между собой параллельно. Элемент А описывает нелинейные упругопластические свойства бетона, а элемент В учитывает влияние скоростного деформирования бетона. Под действием внезапных динамических нагрузок с интенсивностью большей, чем статическая несущая способность бетона вязкий элемент В способствует торможению развития деформаций, инициируемых в элементе А. На основе предлагаемой модели время задержки разрушения определяется отрезком времени, необходимым для того, чтобы деформация бетона достигла своего предельного значения. Также доказаны основные факторы, влияющие на деформацию и разрушение бетона, такие как бетона по прочности на сжатие, уровень перенагружения и коэффициент вязкого сопротивления.

Ключевые слова: Коэффициент динамического упрочнения, время задержки разрушения, вязко-упруго-пластическая модель, бетон, скорость деформирования, модуль вязкого сопротивления, модель Кельвина-Фойгта

1. INTRODUCTION

In practical structural calculations, an important independent variable, time, is excluded from concrete deformation models. Such models adequately describe the inelastic behavior of a material only in the case of loading with low speeds, when the effect of strain lag can be neglected (quasi-static loading regime). Under high-speed loading, the deformation and fracture patterns of concrete specimens are very different from those under static loading . A large number of experimental studies have been devoted to the study of dynamic mechanical characteristics of concrete, conducted mainly under uniaxial loading [1–3]. Based on experimental results in uniaxial compression as well as in tension it was shown that with increasing strain rate there is an increase in concrete strength and modulus of elasticity [4] [5] [6]. In addition, the strain rate effect on the increase of concrete strength becomes significant when the strain rate is higher than a certain value, which is estimated differently in different scientific publications [7–9].

It has also been shown that the dynamic strength of concrete depends on the properties of the materials used, the characteristics of the concrete structure, and the moisture content [10–12]. The inhomogeneity of concrete microstructure and the presence of initial defects (especially large defects) affect the reduction of the dynamic strength of concrete to a greater extent than the static strength [13–15]. The dynamic strength of concrete increases when saturated with water [16,17].

One of the main criteria determining the behavior of concrete under dynamic loading is the dynamic increase factor (φ_d), i.e. the ratio of the strength of concrete under dynamic loading σ_d to its static strength R_b. To obtain the average dependence of the dynamic increase factor for concrete in compression and tension on the duration of increasing load intensity (τ) Y.M. Bazhenov tested 500 specimens of various concretes [1]. During this time, the load intensity is gradually increased from 0 to the maximum value σ_d.

The obtained empirical dependences of the dynamic increase factor by Y.M. Bazhenov for heavy concrete are presented in the form of:

In compression:

$$\varphi_d = 1.58 - 0.35\lg\tau + 0.07\left(\lg\tau\right)^2 \qquad (1)$$

In tension:

$$\varphi_{dt} = 1.42 - 0.15\lg\tau + 0.01\left(\lg\tau\right)^2 \qquad (1)$$

where the duration of increasing load intensity is expressed in milliseconds and is limited in the interval 1-2000 msec.

These results show that as the loading time decreases, i.e. as the strain rate increases, concrete hardening occurs. All these facts suggest that the basic physical law linking stresses to strains must contain time.

Further in [18] Y.M. Bazhenov, V.S. Udaltsov tested concrete specimens under sudden application of a constant load, the value of which is greater than the static compressive strength ($\sigma_d = const > R_b$). It has been found that failure will not occur immediately, but after a certain amount of time, and this will no longer require an increase in load. That is, a lag of deformations is observed. In scientific publications, the value of t_d is referred to as the delayed failure time (i.e., the time interval from the beginning of the loading moment until the element fails). The value of t_d depends mainly on the overload level, i.e., on $\varphi_d = \sigma_d / R_b$, and can be determined by the empirical formula applied by V.S. Udaltsov [18]:

$$\lg t_d = 7.55 - 4.88\varphi_d \qquad (3)$$

where t_d is expressed in milliseconds.

If the load action is stopped before the moment t_d that the strain in the concrete does not have time to reach its ultimate value, the concrete will not fail. It should be noted that the values of t_d and τ are not identical due to different loading regimes in studies [1] and [18].

Similar conclusions are made in the work of V.A. Rakhmanov [19]. In addition, V.A. Rakhmanov also observed that the lag of transverse deformation is much larger than that of longitudinal deformation.

In recent years, numerous experiments have been conducted on reinforced concrete frames subjected to sudden column failure scenarios [20,21]. When observing the deformation and failure process of the frame under the slow-motion camera, it is seen that after the removal of the load-bearing column, progressive failure in characteristic cross-sections occurs only after a certain time interval t_d [22]. Therefore, the determination of the concrete delayed failure time is very important in calculations for the progressive collapse of buildings and structures.

It should be emphasized that the studies conducted by the above scientists are descriptive and will not explain the nature of concrete behavior under high-speed loading.

In parallel with the experimental studies, many theoretical studies on the establishment of analytical laws of deformation of concrete take into account the influence of strain rate. The complexity of this problem is because the concrete in its structure is an inhomogeneous material, which consists of cement stone forming a spatial lattice, aggregate of crushed stone and sand, and a large number of micropores and capillaries, containing water, water vapor, and air. The simultaneous presence of solid, liquid, and gaseous phases in concrete determines a complex picture of concrete behavior under loading.

Speed deformation of concrete structure causes the redistribution of liquid-gas phase in pores and capillaries and disturbs internal equilibrium between phases of the concrete. In this case, the greater the rate of deformation, the stronger the resistance of the liquid-gas medium. This explains the fact that the resistance of concrete increases under rapidly increasing loads. Taking into account all these factors inevitably leads to cumbersome dependencies that are of little use as a basis for practical methods of calculation of reinforced concrete and concrete structures.

To obtain sufficiently simple structure calculation formulas and dependencies between the level of overloading of concrete ($\varphi_d = T / T_s$) and the delayed failure time t_d, G.A. Geniev considered concrete as a two-phase medium consisting of solid and liquid-gas phases [23]. This allows us to explain many processes occurring in concrete, including the peculiarities of its behavior under short-term loading. Taking that the tangential stress intensity and shear strain intensity of the solid phase (element A of the model) are related by a nonlinear law $T_A = 2T_s\left(1 - \Gamma / \Gamma_s\right)\Gamma / \Gamma_s$, and the stress state of the liquid phase (viscous element B of the model) depend on the strain rate and change according to the law of ideally viscous fluid $T_Б = Kd\Gamma / dt$, we obtained a model of elastic-plastic-viscous body (K- viscosity). When stress intensity $T = T_A + T_Б = const$ is applied to a structural element, shear strain intensity develops and reaches its maximum value $\Gamma_s = 2T_s / G_0$ within a very small time interval t_d (delayed failure time). During this time interval, the liquid phase contributes to the inhibition of the development of deformations initiated in the solid phase.

Based on the proposed model, A.G. Geniev established a relationship between the delayed failure time and the level of overloading of concrete φ_d:

$$t_d \omega = \frac{2arccot\sqrt{\varphi_d - 1}}{\sqrt{\varphi_d - 1}} \tag{4}$$

It can be said that this model is the simplest and most successful for explaining the dynamic strength under the action of external dynamic (impact) loads in a short time. On the other hand, in the considered model, the ultimate deformation of concrete under uniaxial compression can be calculated by the formula $\varepsilon_{ult} = 2R_b / E_b$. This value is approximately 1.75

times less than the ultimate strain of concrete specified in the current normative documents ($\varepsilon_{b2} = 0.0035$). Therefore, the delayed failure time calculated from the ultimate strain $\varepsilon_{ult} = 2R_b / E_b$, will not be estimated accurately and is many times less than the actual value.

The purpose of the present work is to eliminate the above-mentioned shortcomings of A.G. Geniev's model to describe the nonlinear dynamic behavior of concrete under external (impact) loads. By correcting the solid phase deformation diagram, the delayed failure time of concrete is more accurately specified.

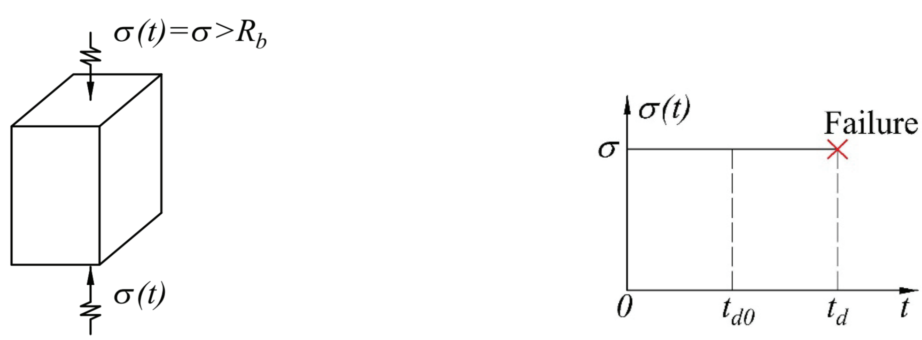
2. METHODS

Consider a specimen of heavy concrete with axial uniaxial compressive strength R_b and initial modulus of elasticity E_b. For simplicity, we will take only the case of the uniaxial stress state, in which it is easier to understand the essence of the phenomenon. At the initial moment of time $t = 0$, we suddenly apply a dynamic load with a constant value ($P(t) = P = const$) to this specimen (see Fig. 1a). Thus, the uniaxial stress arising in the concrete specimen under the action of an external dynamic force P can be determined by the formula

$$\sigma(t) = \frac{P(t)}{F} = \sigma = const \tag{5}$$

where F is the cross-section area of the concrete specimen

When the stress in concrete exceeds the value of concrete compressive strength ($\sigma > R_b$) the above experimental studies show that concrete does not fail immediately at time $t = 0$, but deforms and then fails after a certain short period (delayed failure time t_d) (see Fig. 1a).



a) Concrete specimen loading scheme *b) graph of stress variation as a function of time*

Figure 1. Concrete specimen loading scheme and graph of stress variation as a function of time

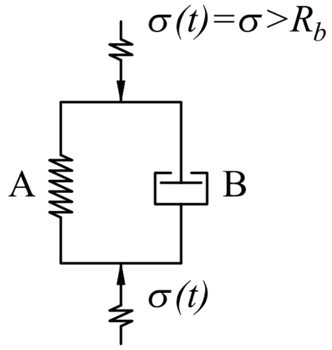


Figure 2. Model describing the behavior of concrete under dynamic loading

In this paper, we will determine the delayed failure time td of concrete by analytical methods based on the model of G.A. Geniev. This model includes two elements A and B connected in parallel and under stress $\sigma(t) = \sigma > R_b$ (see Fig. 2).

In this case, element A represents the nonlinear behavior of concrete under static loading and is depicted as a nonlinear spring (see Fig. 3). Under axial compression, we accept a nonlinear concrete deformation diagram according to the formula:

$$\sigma_A = \begin{cases} E_0\left(1-\dfrac{\varepsilon_A}{2\varepsilon_{b0}}\right)\varepsilon_A \;;\; 0 \le \varepsilon_A \le \varepsilon_{b0} \\ R_b \;;\; \varepsilon_{b0} \le \varepsilon_A \le \varepsilon_{b2} \end{cases} \quad (6)$$

Where σ_A and ε_A - stress and relative strain of element A, respectively; $\varepsilon_{b0} = 2R_b / E_b$ - relative strain of element A at the point corresponding to the maximum value of stress on the diagram σ_A - ε_A; ε_{b2} - limiting relative strain of element A.
The stress-strain diagram of spring A is shown graphically in Fig. 3.

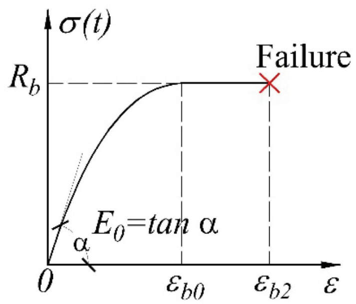


Figure 3. Concrete deformation diagram under uniaxial compression

Thus, according to Fig. 3, the concrete specimen is considered to be ruined under the action of sudden constant stress when the deformation in it reaches the value of ε_{b2}. According to the Russian standard for the design of concrete and reinforced concrete structures SP 63.13330.2012, this value is assumed to be 0.0035 for concrete of compressive strength class B60 and below.
Element B describes the viscosity of concrete and is represented as a piston as shown in Figure 2. The simplest law of deformation in time of piston B is established for an ideal fluid in which stresses are proportional to the strain rate

$$\sigma_B = K\frac{d\varepsilon_B}{dt} \quad (7)$$

where the value of K is the viscosity.

According to the adopted model, the relative deformation of element A and element B is equal

$$\varepsilon_A = \varepsilon_B = \varepsilon \quad (2)$$

The total stress in element A and element B is equal to the stress $\sigma(t) = \sigma > R_b$ caused by the external force:

$$\sigma_A + \sigma_B = \sigma \quad (9)$$

Let's consider two stages of concrete deformation: when the relative deformation of concrete is less than and $\varepsilon_{b0} = 2R_b / E_b$ when it is within the $[\varepsilon_{b0}, \varepsilon_{b2}]$.

Stage 1: $0 \le \varepsilon \le \varepsilon_{b0}$ - By substituting Eq. (6) and Eq. (7) into Eq. (9), we have

$$E_0\left(1-\frac{\varepsilon}{2\varepsilon_{b0}}\right)\varepsilon + K\frac{d\varepsilon}{dt} = \sigma \quad (10)$$

Eq. (10) is a first-order differential equation with separating variables. Using basic transformations, Eq. (10) takes the following form

$$\frac{d\varepsilon}{\dfrac{E_0}{2\varepsilon_{b0}}\varepsilon^2 - E_0\varepsilon + \sigma} = \frac{dt}{K} \quad (11)$$

To obtain the solution of the initial differential equation, it is necessary to integrate both parts of Eq. (11)

$$\int \frac{d\varepsilon}{\dfrac{E_0}{2\varepsilon_{b0}}\varepsilon^2 - E_0\varepsilon + \sigma} = \frac{t}{K} + C_1 \quad (12)$$

where C_1 - integration constant.

From Eq. (12) it is necessary to calculate the integral

$$f(\varepsilon)=\int\frac{d\varepsilon}{a\varepsilon^2+b\varepsilon+c} \tag{13}$$

The solution of the integral (13) depends on the sign of the discriminant

$$\begin{aligned}\Delta&=b^2-4ac=\left(-E_0\right)^2-4\frac{E_0}{2\varepsilon_{b0}}\sigma\\&=\left(-E_0\right)^2-4\frac{E_0}{2\frac{2R_b}{E_0}}\sigma=E_0^2-E_0^2\frac{\sigma}{R_b}\\&=E_0^2\left(1-\varphi_d\right)\end{aligned} \tag{14}$$

where $\varphi_d=\sigma/R_b$ - is called the level of concrete overload. According to the condition of the problem $\sigma>R_b$ it should be deduced that $\varphi_d>1$ and corresponds to the value $\Delta<0$. Thus, the solution of the integral is written in the form

$$\begin{aligned}f(\varepsilon)&=\int\frac{d\varepsilon}{a\varepsilon^2+b\varepsilon+c}\\&=\frac{2}{\sqrt{-\Delta}}\arctan\left(\frac{2a\varepsilon+b}{\sqrt{-\Delta}}\right)+C_1\end{aligned} \tag{15}$$

or

$$\begin{aligned}f(\varepsilon)&=\frac{2}{\sqrt{E_0^2\left(\varphi_d-1\right)}}\arctan\left(\frac{2\frac{E_0}{2\varepsilon_{b0}}\varepsilon-E_0}{\sqrt{E_0^2\left(\varphi_d-1\right)}}\right)\\&+C_1=\frac{2}{E_0\sqrt{\varphi_d-1}}\arctan\left(\frac{\frac{\varepsilon}{\varepsilon_{b0}}-1}{\sqrt{\varphi_d-1}}\right)+C_1\end{aligned} \tag{16}$$

Substituting Eq. (16) into Eq. (12) we obtain

$$\frac{2}{E_0\sqrt{\varphi_d-1}}\arctan\left(\frac{\frac{\varepsilon}{\varepsilon_{b0}}-1}{\sqrt{\varphi_d-1}}\right)=\frac{t}{K}+C_1 \tag{17}$$

To determine the value of the integration constant C_1, let us use the initial conditions of the problem: At time t=0, the relative deformation of the concrete specimen is 0 (ε=0). Substituting them into Eq. (17) we find the value of

$$C_1=\frac{2}{E_0\sqrt{\varphi_d-1}}\arctan\frac{-1}{\sqrt{\varphi_d-1}} \tag{18}$$

Thus, the partial solution of Eq. (9) satisfying the initial conditions has the form

$$\begin{aligned}&\frac{2}{E_0\sqrt{\varphi_d-1}}\arctan\left(\frac{\frac{\varepsilon}{\varepsilon_{b0}}-1}{\sqrt{\varphi_d-1}}\right)\\&=\frac{t}{K}+\frac{2}{E_0\sqrt{\varphi_d-1}}\arctan\frac{-1}{\sqrt{\varphi_d-1}}\end{aligned} \tag{19}$$

From Eq. (19), through some simple transformations, we obtain a formula representing the value of the relative strain of concrete over time

$$\varepsilon=\frac{\varepsilon_{b0}\cdot\varphi_d\cdot\tan\left(\frac{E_0\sqrt{\varphi_d-1}}{2K}t\right)}{tg\left(\frac{E_0\sqrt{\varphi_d-1}}{2K}t\right)+\sqrt{\varphi_d-1}}; \tag{20}$$

It is easy to see that Eq. (20) describes the change in the relative strain of concrete in the range $\varepsilon\in\left[0,\varepsilon_{b0}\right]$ with time *t*. At time *t*=0, the concrete is not deformed. At time $t=t_{d0}$, the concrete deformation reaches the value $\varepsilon_{b0}=2R_b/E_b$. We find the value of t_{d0} by substituting $\varepsilon=\varepsilon_{b0}$ into Eq. (20). We have

$$t_{d0}=\frac{2K}{E_0\sqrt{\varphi_d-1}}\arctan\frac{1}{\sqrt{\varphi_d-1}} \tag{21}$$

Taking the derivative of both parts of Eq. (20), we obtain the formula for the strain rate in time in the interval $t\in\left[0,t_{d0}\right]$

$$\dot{\varepsilon}=\frac{E_0\varepsilon_{b0}\varphi_d\left(\varphi_d-1\right)}{K\left[2\sqrt{\varphi_d-1}\cdot\sin\left(\frac{E_0\sqrt{\varphi_d-1}}{K}t\right)+\left(\varphi_d-2\right)\cos\left(\frac{E_0\sqrt{\varphi_d-1}}{K}t\right)+\varphi_d\right]} \quad (22)$$

Stage 2: $\varepsilon_{b0}\le\varepsilon\le\varepsilon_{b2}$ - then the stress of the non-linear spring A has a constant value and is equal to $\sigma_A=R_b$. Substituting Eq. (6) and Eq. (7) into Eq. (9), we have

$$R_b+K\frac{d\varepsilon}{dt}=\sigma \quad (23)$$

From Eq. (23) we obtain the formula for determining the strain rate

$$\dot{\varepsilon}=\frac{\sigma-R_b}{K} \quad (24)$$

Solving the differential Eq. (24), we find an expression for the relative strain of concrete as a function of time

$$\varepsilon=\frac{\sigma-R_b}{K}t+C_2 \quad (25)$$

The integration constant C_2 is determined from the initial condition as follows: at time $t=t_{b0}$ the relative strain of concrete reaches the value $\varepsilon=\varepsilon_{b0}$. We have

$$\varepsilon_{b0}=\frac{\sigma-R_b}{K}t_{b0}+C_2 \quad (26)$$

or

$$C_2=\varepsilon_{b0}-\frac{\sigma-R_b}{K}t_{b0} \quad (27)$$

Substituting the found value C_2 into Eq. (25), we have

$$\varepsilon=\frac{\sigma-R_b}{K}\left(t-t_{b0}\right)+\varepsilon_{b0}\ ;\ \varepsilon\ge\varepsilon_{b0} \quad (28)$$

Note that Eq. (28) describes the change in the relative strain of concrete in the range $\varepsilon\in\left[\varepsilon_{b0},\varepsilon_{b2}\right]$ with time *t*. When the relative strain reaches the limit value $\varepsilon=\varepsilon_{b2}$, the concrete specimen is considered to have failed. By substituting $\varepsilon=\varepsilon_{b2}$ into Eq. (28), we can find the time required for the concrete specimen to fail when subjected to a sudden dynamic load $\sigma\left(t\right)=\sigma>R_b$.

$$t_d=\frac{2K}{E_0\sqrt{\varphi_d-1}}\arctan\frac{1}{\sqrt{\varphi_d-1}}+\frac{\left(\varepsilon_{b2}-\varepsilon_{b0}\right)K}{R_b\left(\varphi_d-1\right)} \quad (29)$$

The values of t_{d0} and t_d in Eqs. (21)-(29) can be expressed as a function depending on the level of concrete overload, as shown in the following Fig. 4.

From Fig. 4, we see that the diagram t_d - φ_d can be obtained by moving the diagram t_{d0}-φ_d to the right - along the time axis *t* by a distance equal to $\frac{\left(\varepsilon_{b2}-\varepsilon_{b0}\right)K}{R_b\left(\varphi_d-1\right)}$.

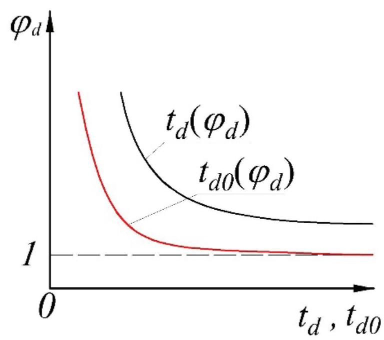


Figure 4. Dependence of t_{d0} and t_d on the level of overload φ_d

Generalizing the above two cases, we have a formula for determining the relative strain of concrete over time as follows:

$$\varepsilon = \begin{cases} \dfrac{\varepsilon_{b0} \cdot \varphi_d \cdot tg\left(\dfrac{E_0\sqrt{\varphi_d - 1}}{2K}t\right)}{tg\left(\dfrac{E_0\sqrt{\varphi_d - 1}}{2K}t\right) + \sqrt{\varphi_d - 1}}; 0 \le \varepsilon \le \varepsilon_{b0} \\ \dfrac{R_b(\varphi_d - 1)}{K}(t - t_{b0}) + \varepsilon_{b0} \; ; \varepsilon_{b0} \le \varepsilon \le \varepsilon_{b2} \end{cases} \quad (30)$$

and a formula for determining the strain rate of concrete over time:

$$\dot{\varepsilon} = \begin{cases} \dfrac{E_0\varepsilon_{b0}\varphi_d(\varphi_d - 1)}{K\left[2\sqrt{\varphi_d - 1} \cdot \sin\left(\dfrac{E_0\sqrt{\varphi_d - 1}}{K}t\right) + (\varphi_d - 2)\cos\left(\dfrac{E_0\sqrt{\varphi_d - 1}}{K}t\right) + \varphi_d\right]} \; ; 0 \le \varepsilon \le \varepsilon_{b0} \\ \dfrac{R_b(\varphi_d - 1)}{K} \; ; \varepsilon_{b0} \le \varepsilon \le \varepsilon_{b2} \end{cases} \quad (31)$$

From Eqs. (30)-(31), we can plot the strain and strain rate versus time (see Fig. 5 and Fig. 6).

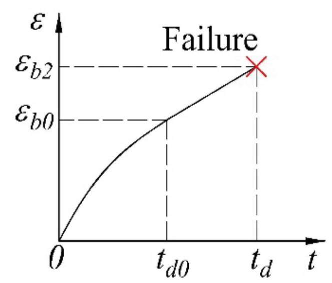


Figure 5. Dependence of $\varepsilon - t$

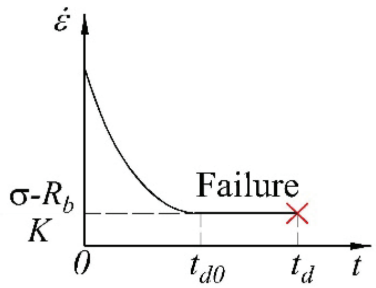


Figure 6. Dependence of $\dot{\varepsilon} - t$

3. RESULTS AND DISCUSSION

3.1 Effect of strength class on deformation and failure of concrete over time

Based on the proposed model, the influence of concrete compressive strength class R_b on the strain state, as well as the process of concrete failure over time under the action of an external load, the value of which exceeds the load-bearing capacity of concrete ($\sigma > R_b$), is considered. Mechanical parameters of concrete of different classes according to Russian standard SP 63.13330.2018 are presented in Table 1.

Table 1. Mechanical parameters of concrete of different compressive strength classes

	B20	B30	B40	B50
Rb(MPa)	15	22	29	36
$E_b \cdot 0{,}001$ (MPa)	27	32,5	36	39
$\varepsilon_{b0} = 2\dfrac{R_b}{E_b}$	0,0011	0,0014	0,0016	0,0018
ε_{b2}	0,0035			

Based on Eq. (6) and input data in Table 1, we can construct the stress-strain curves of concrete with different compressive strength classes when subjected to axial static force (see Fig. 7). It is noticed that as the compressive strength classes increase, the ultimate compressive strength of concrete and strain at that strength value become larger.
To facilitate the study of the influence of concrete compressive strength class on the delayed failure time, we assume the value of viscosity

K=400 $\text{MPa}\cdot\text{sec}$, and the level of overload - $\varphi_d = \sigma / R_b = 1.4$. Then based on Eqs. (21)-(29) it is possible to plot the relationship between the delayed failure time and the compressive strength class of concrete (see Fig. 8).

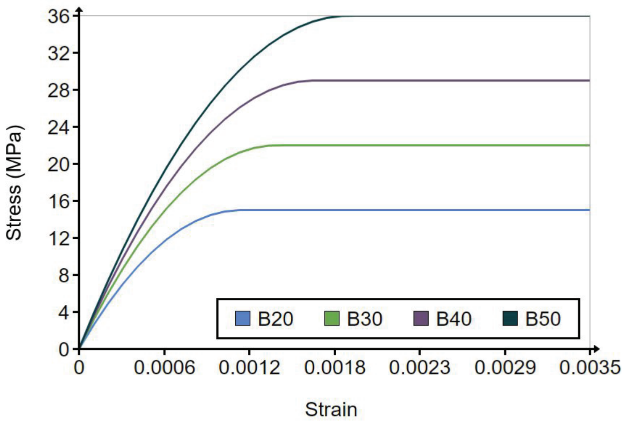


Figure 7. Stress-strain curves of concrete of different compressive strength classes under the static uniaxial compressive loading

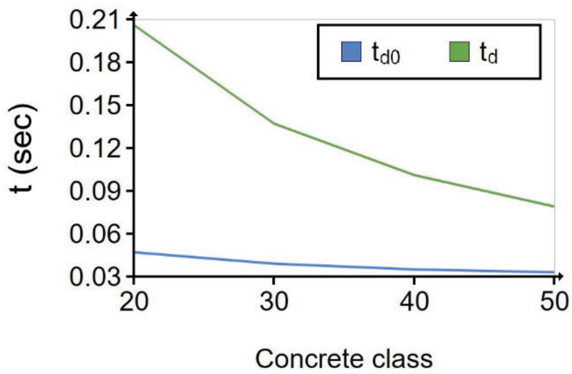


Figure 8. Relationship between concrete compressive strength class and delayed times

It is observed that for concrete B20 when the external dynamic load exceeds the bearing capacity 1.4 times, the relative strain reaches the value $\varepsilon_{b0} = 0.0011$ after a period of t_{d0}=0.047 sec. Concrete is considered to fail after a time interval t_d = 0.206 sec when the relative strain in it reaches the limit value $\varepsilon_{b2} = 0.0035$. When increasing the concrete class, the delayed failure time decreases significantly. In particular, when the concrete class is increased by 2.5 times (from concrete B20 to concrete B50), the time t_{d0} decreases by 1.4 times, and the delayed failure time t_d by 2.6 times. Thus, when increasing the concrete class, the delayed failure time will decrease faster than the time for the concrete to deform to the value ε_{b0}.This can be explained as follows. The compressive strength class of concrete is closely related to its brittleness. The higher the class of concrete, the more brittle it is. Consequently, when subjected to impact loads exceeding its bearing capacity, concrete with a high compressive strength class will undergo brittle failure, and the delayed failure time will be shorter than that of concrete with a low strength class.

3.2 Effect of level of overloading on deformation and failure of concrete over time

Based on Eqs. (21)-(29), it is possible to express the effect of the level of overloading on the time of delayed fracture in the case of B20 concrete with viscosity taken K=400 $MPa\cdot\text{sec}$ (see Fig. 9).

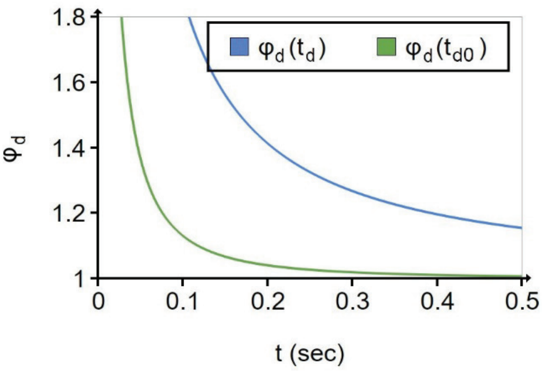


Figure 9. Dependence of delayed times on the level of overloading

From Fig. 9 we see that at sudden application of dynamic load with intensity exceeding the bearing capacity of the concrete specimen more than 1.1 times, the time required to reach the value of strain $\varepsilon_{b0} = 0.0011$, is t_{d0}= 0.118 c. And the concrete failure occurs at time t_d=0.756 c. Thus, the time t_d is 6.4 times longer than t_{d0}.

When the level of overload increases, the delay times will decrease dramatically. For example, when φ_d increases 1.6 times from 1.1 to 1.8, t_{d0} will decrease 4.2 times. The delayed failure time will accordingly be reduced by 7.1 times.

3.3 The influence of the viscosity on the deformation and failure of concrete over time

To study the effect of the viscosity on the deformation and failure of concrete over time, consider the example of concrete of class B20 with different viscosities (*K* = 200, 500, and 800 MPa/s). In practice, the viscosity can be adjusted by changing the porosity value or changing the water-cement ratio of the concrete mixture. Based on Eq. (29), we find out the effect of viscosity on the graph of "overload level - delayed failure time " (see Fig. 10). It is found that when the viscosity increases, the delayed failure time increases significantly. At an overload level of 1.4 and a viscosity of 200 MPa/s, the delayed failure time reaches a value of 0.103 s. When the viscosity increases by 2.5 times (*K*=500 MPa/s), the delayed failure time increases by 2.5 times (t_d=0.258 s).

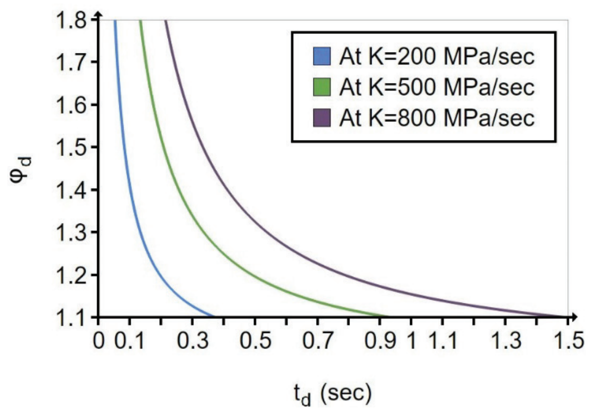


Figure 10. Effect of viscosity on delayed failure time

Eq. (30) gives us the characteristics of relative strain variation in time. Let's draw graphs "$\varepsilon - t$" for 3 cases (at *K*=200, 500, and 800 MPa/s) (see Fig. 11). It is easy to see that when *t* is in the range of $[0, t_{d0}]$, the graph has a curvilinear shape. When t is in the range of $[t_{d0}, t_d]$, the relative strain depends on time in a linear relationship. At any instant of time (e.g., t=0.05 s), the relative strain achieved is different in cases of different viscosity. At *K*=200 MPa/s, the relative strain at this moment is 0.002. At *K*=500 MPa/s the attainable strain is 2 times lower (ε=0.001). At *K*=800 MPa/s, the relative strain reaches 0.00075. Thus, it can be concluded that the relative strain of concrete at any moment in time depends on the value of viscosity.

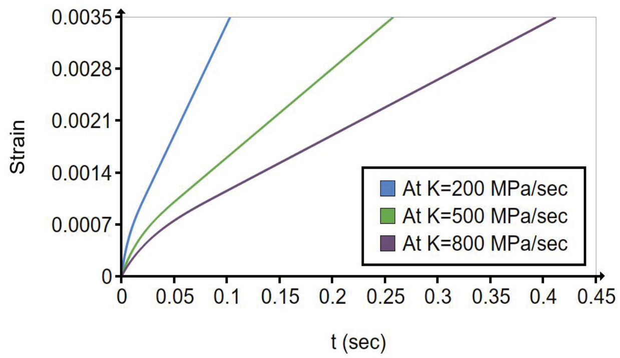


Figure 11. Effect of viscosity on $\varepsilon - t$ curves

Based on Eq. (31), we investigate the change of strain rate in time in cases of different *K* coefficients (see Fig. 12).

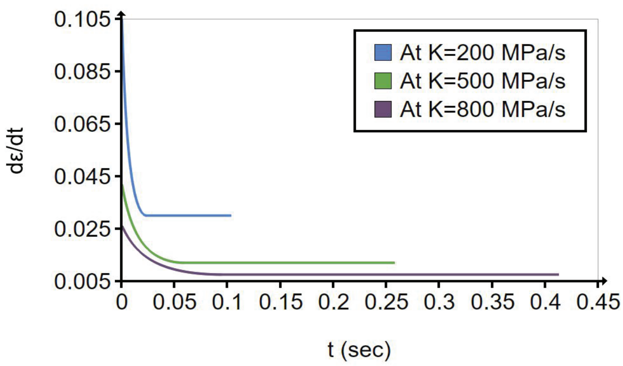


Figure 12. Effect of viscosity on the strain rate of concrete

It can be seen that when a sudden dynamic force with intensity $\sigma > R_b$ is applied to the concrete specimen, the strain rate in the concrete at the time of applying the force reaches its maximum value. This maximum value can be determined by substituting t=0 into Eq. (22):

$$\dot{\varepsilon}_{\max} = \frac{E_0 \varepsilon_{b0} \varphi_d}{2K} \tag{32}$$

Eq. (32) shows that the maximum strain rate is proportional to the overload level and inversely proportional to the viscosity of the concrete. The strain rate decreases very rapidly over time in the range $\left[0, t_{d0}\right]$ and keeps a constant value in the range $\left[t_{d0}, t_d\right]$.

5. CONCLUSIONS

This paper presents a model that explains the phenomenon of delayed failure of concrete when a sudden dynamic load is applied to a structure, the intensity of which exceeds the bearing capacity of concrete. Using the analytical method, we can accurately determine the delayed failure time, relative strain, and strain rate at each time t. The main factors affecting the deformation and failure of concrete, such as concrete compressive strength, overload level, and viscosity, are thoroughly studied. The results of this study can be used in dynamic problems of reinforced concrete structures, for example, in the problem of calculating structural stability against progressive collapse of buildings and structures under the scenario of sudden removal of load-bearing elements.

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