

WAYS OF INCREASING OF LOADING CAPACITY OF THE REINFORCED CONCRETE SHALLOW SHELLS WITH A FLAT RECTANGULAR CONTOUR

Khanlar K. Seyfullayev¹, Gulnara Kh. Jabrayilova²

¹ Azerbaijan Scientific-Research Institut of Construction and Architecture. Baku, AZERBAIJAN

² Azerbaijan University of Architecture and Construction. Baku, AZERBAIJAN

Abstract: Shallow shells with variable curvature, prestressing contour and fixed above by horizontal reinforcing of two opposite contour elements are considered in the given article. These two offered ways practically produces non-moment state and increase loading capacity of the shallow shells several times. Method of definition of loading capacity of the shallow shells with a flat rectangular contour was worked out, and it's aim consists of that initial moment state before loss of loading capacity is described by nonlinear differential equations of the theory of the shallow shells. And then accentuating the main part of the solution of equations we receive the linear system of homogeneous differential equations with variable coefficients. The solution of the linearized equations enables to define the forms of losing of loading capacity of shallow shells.

Key words: capacity, reinforced concrete, shallow shells, flat rectangular contour, non-moment, moment state, moment and to linear state, nonlinear and states moment state

СПОСОБЫ ПОВЫШЕНИЯ НЕСУЩЕЙ СПОСОБНОСТИ ПОЛОГИХ ЖЕЛЕЗОБЕТОННЫХ ОБОЛОЧЕК С ПРЯМОУГОЛЬНЫМ КОНТУРОМ

Х.К. Сейфуллаев¹, Г.Х. Джебраилова²

¹ Азербайджанский научно-исследовательский институт строительства и архитектуры, г. Баку, АЗЕРБАЙДЖАН

² Азербайджанский Архитектурный и Строительный Университет, г. Баку, АЗЕРБАЙДЖАН

Аннотация: В работе рассматриваются способы повышения несущей способности пологих железобетонных оболочек переменной кривизны с предварительно-напряженным контуром и закрепленных сверху горизонтальными арматурами двух противоположных контурных элементов. Эти два предложенных способа практически создают безмоментное состояние и повышают несущую способность пологих оболочек в несколько раз.

Ключевые слова: несущая способность, железобетон, пологие оболочки, плоский прямоугольный контур, безмоментное, моментное состояние, моментное и линейное состояние, нелинейное и моментное состояние

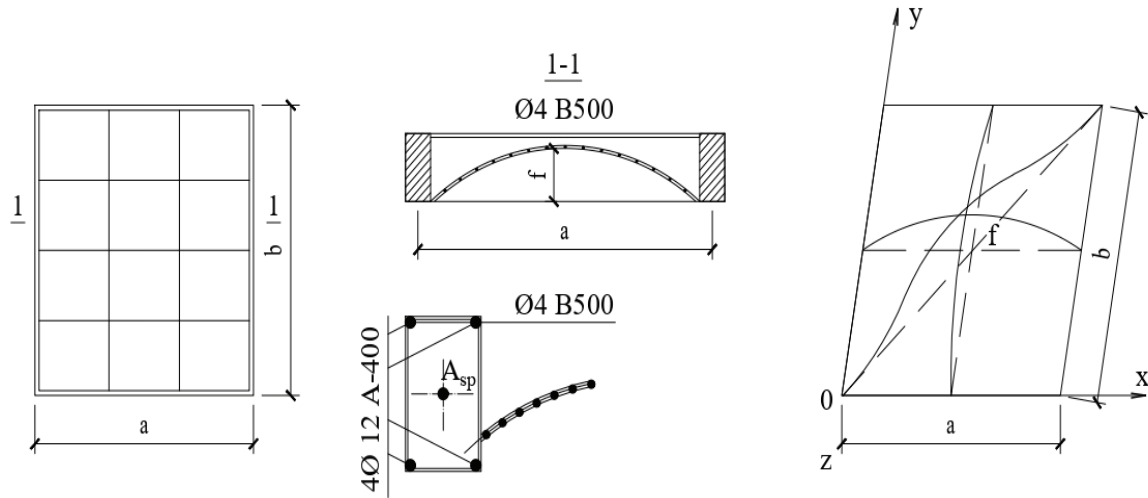
CONTENT OF THE WORK

Shallow shells are applied widely as covers of residential and public buildings due to small rise in a center and flat rectangular contour.

Rational usage of space constructions is reached by providing of their work in non-moment intense state.

As you know, loading capacity of the reinforced shallow shells is approximately defined by the method of limiting balance on the basis of experimental researches.

$$D\Delta^2w - K_x \frac{\partial^2 \varphi}{\partial y^2} - K_y \frac{\partial^2 \varphi}{\partial x^2} + 2K_{xy} \frac{\partial^2 \varphi}{\partial x \partial y} - L(w, \varphi) = q$$



Here the following designations are accepted:
The initial intense state of the shallow shells with variable curvature is described by the following nonlinear differential equations:

$$\frac{1}{Eh} \Delta^2 \varphi + K_x \frac{\partial^2 w}{\partial y^2} + K_y \frac{\partial^2 w}{\partial x^2} + 2K_{xy} \frac{\partial^2 w}{\partial x \partial y} + \frac{1}{2} L(w, w) = 0 \quad (1)$$

D - cylindrical rigidity, ν - is Poisson's ratio, Δ and L differential operators [5,6]:

$$\begin{aligned} D &= K_0 E_b (K_b I_{x,b} + n K_s I_{x,s}) \\ K_b &= 1,5 \left(1 - \frac{1}{3} K_0^2 \right) - \gamma_{bl} \left(1 + \frac{K_0}{2} \right) (1 - K_0); \\ I_{x,b} &= \frac{by^3}{3} \\ I_{x,s} &= A_s (h_0 - y)^2 + \frac{\varepsilon_s}{\varepsilon_{sl}} A_s (y - \hat{a})^2; \\ K_s &= \frac{\varepsilon_{sl}}{\varepsilon_s K_0} \\ K_0 &= 1 \quad \text{and} \quad \gamma_{bl} = 1 : \\ D_0 &= E_b \left[\frac{by^3}{3} + n A_s (h_0 - y)^2 \right] \end{aligned}$$

The height of the compressed zone of concrete is determined by solving the following quadratic equation:

$$\xi^2 + 2n\mu_s \xi - 2n\mu_s = 0$$

here

$$\mu_s = \frac{A_s}{bh_0} ; \quad n = \frac{E_s}{E_b}$$

$$D_p = K_0 E_b (K_b I_{x,b} + n K_s I_{x,s})$$

The height of the compressed zone of concrete during bending is determined by the formula [5,6]:

$$\xi = \frac{n_R \mu_s}{1 - K_0 - 0,5(1 - \gamma_{bl})(1 - K_0)}$$

where

$$\begin{aligned} n_R &= \frac{R_s}{R_b} ; \quad K_s = \frac{\varepsilon_{sl}}{\varepsilon_s K_0} ; \quad \varepsilon_s = \varepsilon_{b2} \frac{h_0 - y}{y} \\ \Delta(\dots) &= \frac{\partial^2(\dots)}{\partial x^2} + \frac{\partial^2(\dots)}{\partial y^2} \\ L(w, \varphi) &= \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 w}{\partial y^2} \frac{\partial^2 \varphi}{\partial x^2} \\ &\quad - 2 \frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 \varphi}{\partial x \partial y} \end{aligned}$$

K_x , K_y , and K_{xy} - variable curvatures.

Equation of the middle surface of the shallow shells with a flat rectangular contour is accepted in the following form:

$$Z(x, y) = -\frac{16f}{a^2 b^2} (ax - x^2)(by - y^2) \quad (2)$$

The curvatures of the middle surface of the shells can be defined by the following formulas because of shallowness:

$$\begin{aligned} K_x &= \frac{32f}{a^2b^2}(by - y^2); & K_y &= \frac{32f}{a^2b^2}(ax - x^2); \\ K_{xy} &= -\frac{16f}{a^2b^2}(a - 2x)(b - 2y) \end{aligned} \quad (3)$$

The end conditions on a contour can be written in the form:

On line $x=0$ and $x=a$:

$$\begin{aligned} w &= 0; & \frac{\partial^3 \varphi}{\partial x^3} + (2 + \nu) \frac{\partial^3 \varphi}{\partial x \partial y^2} &= \frac{Eh}{B_h} \varphi; \\ \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} &= 0; \\ \frac{\partial^2 \varphi}{\partial x^2} - \nu \frac{\partial^2 \varphi}{\partial y^2} &= \frac{PEh}{EbA_0} \left(1 - \frac{6e_0}{H}\right) \end{aligned} \quad (4)$$

The end conditions have the similar form on a contour along the line $y=0$ and $y=b$.

The bending forces of the shallow shells considerably are decreased in these end conditions.

The solution of the nonlinear differential equations with variable coefficients (1) presents large mathematical difficulties in these end conditions (4). Therefore the approximate method is used for the solution of the equations (1), which was developed in the work [2, 3], and the aim of which consists of the following:

The functions of stress and deflection are presented in the following form:

$$\begin{aligned} \varphi &= \Phi(x, y) + \bar{\Phi}(x, y); \\ w &= W(x, y) + \bar{W}(x, y) \end{aligned}$$

Here W and Φ functions of stress and deflection are appropriate to initial intense state of the shell. $\bar{\Phi}$ and $\bar{W}$ is the increase of these functions at the moment of losing the loading capacity of the shell, which are considered small.

Substituting ferreted functions (5) in this initial system of differential equations (1), neglecting the small members, we shall receive linear differential equations, from the solution of which the loading capacity of reinforced

concrete shallow shells will be determined by the conditions of the loss of stability:

$$\begin{aligned} D\Delta^2 \bar{W} - K_x \frac{\partial^2 \bar{\Phi}}{\partial y^2} - K_y \frac{\partial^2 \bar{\Phi}}{\partial x^2} + 2K_{xy} \frac{\partial^2 \bar{\Phi}}{\partial x \partial y} + \\ + N_x^0 \frac{\partial^2 \bar{W}}{\partial x^2} + N_y^0 \frac{\partial^2 \bar{W}}{\partial y^2} + 2N_{xy}^0 \frac{\partial^2 \bar{W}}{\partial x \partial y} = 0, \\ \frac{1}{Eh} \Delta^2 \bar{\Phi} + K_x \frac{\partial^2 \bar{W}}{\partial y^2} + K_y \frac{\partial^2 \bar{W}}{\partial x^2} + 2K_{xy} \frac{\partial^2 \bar{W}}{\partial x \partial y} - \\ - \frac{1}{2} L(W, \bar{W}) = 0 \end{aligned}$$

Here N_x^0 , N_y^0 and N_{xy}^0 are the initial membrane forces before losing the loading capacity of the reinforced concrete shallow shells, which are defined by the solution of the nonlinear differential equations and are considered known.

So, the problem of investigation of loading capacity of the shallow shells is led to integration of the nonlinear differential equations (1) and (6).

The nonlinear differential equations, describing the initial moment state, are solved by us using small parameter method in the works [2, 3]

The geometrical sizes and reinforcing of the shallow shells are produced on the basis of calculation, which are received from solution of the equations (1), and then the loading capacity of considered design of the shell is checked up on the basis of the solution of the differential equations (6).

In practical calculations the initial intense state of the shallow shells, depending on the accuracy of calculation, the loading capacity of the reinforced shallow shells are accepted as following states:

a) Initial condition is non-moment state (5).

The initial differential equation is simplified in this case and accepts the form [1, 2]:

$$K_x \frac{\partial^2 \Phi}{\partial y^2} - K_y \frac{\partial^2 \Phi}{\partial x^2} + 2K_{xy} \frac{\partial^2 \Phi}{\partial x \partial y} = -q \quad (7)$$

The initial membrane forces N_x^0 , N_y^0 and N_{xy}^0 are defined by solving of this equation through stress function by the known formulas Erie.

b) Initial condition is moment and to linear state.

The nonlinear differential operators are neglected in the initial differential equations (1)

and known differential equations of the theory of the shallow shells are received.

$$\begin{aligned} D\Delta^2 W - K_x \frac{\partial^2 \Phi}{\partial y^2} - K_y \frac{\partial^2 \Phi}{\partial x^2} + 2K_{xy} \frac{\partial^2 \Phi}{\partial x \partial y} &= q \\ \frac{1}{Eh} \Delta^2 \Phi + K_x \frac{\partial^2 W}{\partial y^2} + K_y \frac{\partial^2 W}{\partial x^2} + 2K_{xy} \frac{\partial^2 W}{\partial x \partial y} &= 0 \end{aligned} \quad (8)$$

Solution of the differential equations (8) was received by many authors and is resulted in the literature [4].

c) Initial condition is nonlinear and moment state.

The initial equations up to the before critical state are described by the nonlinear equations (1). Solution of these nonlinear equations were indicated in the work [2,3] on the basis of a small parameter method. The solution of the

considered problem is difficult in such setting a task. Therefore the gained results present the theoretical and practical interest in designing of the reinforced concrete shallow shells.

When the solution of the equations describing initial state of the shallow shells is found or known, then it is possible to find and investigate the loading capacity of the shell.

Differential equations (6) is large in appearance, form the point of view of initial nonlinear and moment state, and therefore it isn't given in this article.

Increasing of functions $\bar{\Phi}$ and $\bar{W}$ describing the state in losing of loading capacity we shall accept in the following form:

$$\bar{W}(x, y) = \bar{B}_{mn} \sin(\lambda_m x) \sin(\mu_n y);$$

$$\begin{aligned} \bar{\Phi}(x, y) = \bar{A}_{mn} \sin(\lambda_m x) \sin(\mu_n y) - \sum_m \left\{ \left[\frac{1}{\lambda_m^2} - \frac{vb^2}{2} \left(\frac{y^2}{b^2} - \frac{y}{b} \right) \right] X_m - \right. \\ \left. - \frac{Ehb^2}{2} \left(\frac{y^2}{b^2} - \frac{y}{b} \right) P_m \right\} \sin(\lambda_m x) - \\ - \sum_n \left\{ \left[\frac{1}{\mu_n^2} - \frac{va^2}{2} \left(\frac{x^2}{a^2} - \frac{x}{a} \right) \right] \xi_n - \frac{Eha^2}{2} \left(\frac{x^2}{a^2} - \frac{x}{a} \right) \theta_n \right\} \sin(\mu_n y) \end{aligned} \quad (9)$$

$$\lambda_m = \frac{m\pi}{a}, \quad \mu_n = \frac{n\pi}{b}$$

The unary series in the function of stress (9) takes into account the influences of prestressing of the contour elements and horizontal reinforcing excluding horizontal displacements of the contour elements.

Substituting (9), received from (6), in the system of differential equations and then using end conditions (4), from the condition of a non-trivial solution of the received systems of algebraic equations, we'll receive the following in the general form:

$$q = \frac{1}{K(m, n)} \left(D\Delta_{mn}^2 + \frac{Eh\Delta_{kmn}^2}{\Delta_{mn}^2} \right) + P(m, n) + N(m, n)$$

As it is visible, the loading capacity of the shell depends on the form deformation at losing of

stability. In varying m and n we find the least significance q and form of losing of loading capacity of the shell.

The offered method is general and it is possible in particular case to receive the known solutions for various classes of shells from it.

On the basis of the manual [4] the reinforced thin-walled shells are calculated on the action of an external load q on linear moment theory and then their designing and reinforcing are performed. In this case the significance of external load q should not exceed the significance of loading capacity q_u .

For example, we'll consider a cylindrical shell, fixed above by horizontal reinforcing, providing immobility of the contour elements. In the considered example the prestressing of the contour elements are absent.

Initial state of the shell is non-moment. The initial up to critical state of the shell is described by the following equation:

$$\frac{1}{R} \frac{\partial^2 \Phi}{\partial y^2} = -q. \quad (11)$$

Decision of the equation (11) we accept in the form:

$$\Phi(x, y) = \sum_m \sum_n A_{mn} \sin(\lambda_m x) \sin(\mu_n y) - \sum_n \left[\frac{1}{\mu_n^2} - \frac{va^2}{2} \left(\frac{x^2}{a^2} - \frac{x}{a} \right) \right] \xi_n \sin(\mu_n y) \quad (12)$$

The coefficients of series of the solution (12) have the following significances satisfying the end conditions (4) and (11):

$$\begin{aligned} A_{mn} &= \frac{16qR}{mn\pi^2\lambda_m^2} + \frac{4}{m\pi\lambda_m^2} \left(1 + v \frac{\lambda_m^2}{\mu_n^2} \right) \xi_n \\ &= \sum_m \frac{16\Delta_{mn}^{(2+v)}}{mn\pi^2\lambda_m^2} qR = \left[\frac{v(2+v)}{2} a\mu_n^2 - \sum_m \frac{4\Delta_{mn}^{(2+v)}}{n\pi\lambda_m^2} \right] \xi_n \\ &\quad \left(1 + v \frac{\lambda_m^2}{\mu_n^2} \right) \\ \Delta_{mn}^{(2+v)} &= \lambda_m^3 + (2+v)\lambda_m\mu_n^2 \end{aligned}$$

Differential equations for the determination of loading capacity (6) for considered case accept the following form:

$$\begin{aligned} D\Delta^2 \bar{W} - \frac{1}{R} \frac{\partial^2 \bar{\Phi}}{\partial y^2} - q \left\{ \frac{\partial^2 \bar{W}}{\partial x^2} \sum_m \sum_n a_{mn} \sin(\lambda_m x) \sin(\mu_n y) - \sum_n \left[1 - \frac{v\mu_n^2}{2} (x^2 - ax) \right] \xi_n \sin(\mu_n y) + \right. \\ \left. \frac{\partial^2 \bar{W}}{\partial y^2} \left[v \sum_n \xi_n \sin(\mu_n y) - \sum_m \sum_n a_{mn} \lambda_m^2 \sin(\lambda_m x) \sin(\mu_n y) \right] - 2 \frac{\partial^2 \bar{W}}{\partial x \partial y} \left[\frac{v\mu_n}{2} (2x - a) \xi_n \cos(\mu_n y) \right] + \right. \\ \left. + a_{mn} \lambda_m \mu_n \cos(\lambda_m x) \cos(\mu_n y) \right\} = 0, \quad (13) \end{aligned}$$

$$\frac{1}{Eh} \Delta^2 \bar{\Phi} + \frac{1}{R} \frac{\partial^2 \bar{W}}{\partial x^2} = 0$$

We accept the function of deflection $\bar{W}$ and stress $\bar{\Phi}$ in the form:

$$\begin{aligned} \bar{W} &= \bar{B}_{mn} \sin(\lambda_m x) \sin(\mu_n y), \\ \bar{\Phi} &= \bar{A}_{mn} \sin(\lambda_m x) \sin(\mu_n y). \end{aligned} \quad (14)$$

Substituting (14) in the (13) we find significances of the critical force or loading capacity of the shell.

It is very interesting that in the initial non-moment state the following state is realized:

$$N_x^0 = -qR; \quad N_y^0 = 0; \quad N_{xy}^0 = 0$$

Then the equation of stability accepts known form, indicated in the reference [3]. The significance of critical force:

$$q = \frac{1}{R\mu_n^2} \left(D\Delta_{mn}^2 + \frac{Eh\mu_n^2}{R^2\Delta_{mn}^2} \right),$$

$$\Delta_{mn}^2 = (\lambda_m^2 + \mu_n^2)^2$$

It is supposed that the cylindrical shell simply supported on the all contour, and the initial state is non-moment. Then in the initial state, the solution of the differential equation accepts the following form:

$$\Phi = \sum_m \sum_n qR a_{mn} \sin(\lambda_m x) \sin(\mu_n y)$$

and the critical force accepts following significances:

$$q = \frac{1}{K_0(m,n)} \left(D\Delta_{mn}^2 + \frac{Eh\mu_n^2}{R^2\Delta_{mn}^2} \right) \quad (16)$$

m, n odd numbers.

The loading capacity of the cylindrical shallow shell determined by the formula (16) is a new solution and differs from the known formula

(15) by attendance $K(m,n)$, which expresses influence of the initial non-moment state on significance of loading capacity of the shell. Certainly, in real terms of the work of the shell the above-mentioned intense state even with top horizontal reinforcing will not be realized. We have the following in the considered variant:

$$q = \frac{1}{K_0(m,n)} \left(D\Delta_{mn}^2 + \frac{Eh\mu_n^2}{R^2\Delta_{mn}^2} \right) + P(m,n) + N(m,n)$$

Here, the second item takes into account the increasing of loading capacity of the cylindrical shallow shell because of producing of the condition with horizontal reinforcing.

CONCLUSION

So, we developed the new method of determination of loading capacity of the shallow shells on the basis of the geometrical nonlinear theory of the shallow shells. Methods of increasing of the loading capacity of the reinforced concrete shallow shells offered by us are very simple and will be easily realized in practice. They give an opportunity of their wide usage in housing construction as covers.

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Seyfullayev Khanlar Kurban-Doktor of technical sciences, prof., Azerbaijan Scientific-Research Institute of Construction and Architecture, ASRICA, Fizuli street 65, Baku, 1014, Azerbaijan, xanlar.seyfullayev@mail.ru.

Сейфуллаев Ханлар Курбан оглы- доктор технических наук, профессор, зав.отделом Азербайджанского научно-исследовательского института строительства и архитектуры, АзНИИСА.Улица Физули 65, Баку,1014,Азербайджан. xanlar.seyfullayev@mail.ru.

Jabrayilova Gulnara Khanlar- PhD, assos. Prof, dosent Azerbaijan University of Architecture and Construction, AzUAC, A. Sultanova street 5, 1073, Baku, Azerbaijan, gulnara.djebrailova@gmail.com.

Джебраилова Гюльнара Ханлар кызы-кандидат технических наук, доцент кафедры механики, Азербайджанский Архитектурный и Строительный Университет, АзАиСУ. Улица А.Султанова 5, Баку, 1073, Азербайджан. gulnara.djebrailova@gmail.com.