

METHOD OF EQUAL RATE SURFACES IN STRUCTURAL AERODYNAMICS

*Vadim K. Akhmetov*¹, *Viktor Y. Shkadov*²

¹ Moscow State University of Civil Engineering, Moscow, RUSSIA

² Lomonosov Moscow State University, RUSSIA

Abstract. The problem of combustion of natural fuels is now an extremely important environmental issue. Flue gas emitted into the atmosphere has a negative impact on the environment. The consequences can range from precipitation containing acidic compounds to increased levels of lung disease in the population. A solution to this problem of fuel combustion in thermal power plants can be a design called a combined high-rise structure. It combines two components: the cooling tower and the chimney itself. The study of the flow gases nature in such a structure is an extremely important task, since the process of mixing the smoke with the additionally supplied air in the pipe determines the concentration of the emission of harmful substances into the atmosphere. This article is devoted to the numerical study of gas dynamics in the structures under consideration. A generalization of equal rate surfaces method to describe the internal aerodynamics of the structure is presented. The problem statement is based on the viscous compressible gas model. The flow is described by the Navier-Stokes system of equations, which is written in a parabolic form and supplemented by an algebraic turbulence model. The numerical solution is carried out by the finite difference method. Various variants of the structure geometry and initial conditions are considered. The favorable effect of the initial swirling of the flow on the intensive mixing of flue gases and the decrease in the concentration of harmful substances at the outlet of the chimney was noted. Based on this research, recommendations for the design and profiling of structures under construction have been developed.

Keywords: high-rise structures, numerical calculation methods, swirling flows, gas mixing

МЕТОД ПОВЕРХНОСТЕЙ РАВНЫХ РАСХОДОВ В СТРОИТЕЛЬНОЙ АЭРОДИНАМИКЕ

*В.К. Ахметов*¹, *В.Я. Шкадов*²

¹ Национальный исследовательский Московский государственный строительный университет, г. Москва, РОССИЯ

² Московский государственный университет им. М.В. Ломоносова, г. Москва, РОССИЯ

Аннотация. Проблема сжигания природного топлива в настоящее время является крайне важной экологической задачей. Дымовой газ, выбрасываемый в атмосферу, негативно влияет на окружающую среду. Последствия этого могут быть самыми разнообразными, от выпадения осадков, содержащих кислотные соединения, до повышения уровня легочных заболеваний населения. Решением проблемы сжигания топлива при выработке энергии в тепловых электростанциях может служить конструкция, называемая комбинированным высотным сооружением. Она объединяет в себе две составляющие: градирню и, непосредственно, дымовую трубу. Исследование характера течения газов в таком сооружении является чрезвычайно важной задачей, так как от процесса смешения дыма с дополнительно подаваемым воздухом в трубу зависит концентрация выброса вредных веществ в атмосферу. Данная статья посвящена численному изучению газодинамики в рассматриваемых сооружениях. Представлено обобщение метода поверхностей равных расходов для описания внутренней аэродинамики сооружения. В основе математической модели используется вязкий сжимаемый газ. Течение описывается системой уравнений Навье-Стокса, записанной в параболизированной форме, дополненной алгебраической моделью турбулентности. Численное решение проводится методом конечных разностей. Рассмотрены различные варианты геометрии конструкции и начальных условий. Отмечено благоприятное воздействие начальной закрутки потока на интенсивность смешения дымовых газов и уменьшение концентрации вредных веществ на выходе из трубы. На основе проведенных исследований разработаны рекомендации для проектирования и профилирования возводимых сооружений.

Ключевые слова: высотные сооружения, численные методы расчета, закрученные течения, смешение газов

1. INTRODUCTION

The use of natural fuel in the combustion process has a significant negative impact on the environment. The standards established for the maximum permissible content of harmful substances in the flue gases discharged. In particular, for sulfur compounds the limit is not more than 200-400 mg/m³ SO₂. Devices for flue gas discharge should be designed to ensure the fulfillment of these requirements.

In the mid-seventies, the first studies were carried out to improve the technology for reducing harmful flue gas emissions by designing combined high-rise structures. This design combines a chimney and a wet cooling tower. The flue gas is pre-cleaned in a special desulfurization device. Then it enters the lower part of the extraction tower, where the heat exchanger is located. The jet of warm air forms the flue gas plume, mixes with it and is removed into the atmosphere due to natural draught. The ratio of flue gas volume to warm air amounts from 1:5 to 1:25. This provides the following advantages compared to a conventional chimney: the system of reheating the smoke and related energy costs are eliminated, the concentration of harmful substances and the temperature of the exhaust gases are significantly reduced.

The efficiency of combined high-rise structures is largely determined by gas dynamic processes of turbulent mixing of flue gases and humid air. To intensify the mixing process, it is preferable to supply flue gas to the exhaust tower with some initial swirling of the flow.

A large number of studies have been devoted to the investigation of swirling flows. Practical applications for various technical devices are considered in [1, 2]. Numerical studies of swirling flows in pipes are presented in [3-7], and experimental results are reflected in [8-10]. The issues of mixing and gas dynamics of flows in swirling combustion chambers are discussed in [11,12], and for flows in gas turbines in [13-15]. One of the most important features of swirling flow is the effect of swirl decay, which

occurs when a certain flow swirl is reached. This phenomenon is associated with the loss of flow stability. Studies of hydrodynamic stability of swirling flows [16-21] support this conclusion.

The objective of this study is to investigate the processes of turbulent mixing of heated flue gases with warm air flow in the field of gravity. For this purpose, it uses a mathematical model based on parabolized Navier-Stokes equations. It presents an efficient numerical method of solution, which allows to pass from a general system of partial differential equations to a system of ordinary differential equations written on current lines.

2. PROBLEM STATEMENT AND RESEARCH METHOD

We consider the gas-dynamic process of mixing of heated flue gases entering the pipe by jet flow. The principle flow diagram is shown in Fig. 1. The flue gas is fed into the central part of the chimney $0 \leq r \leq R_1$ at the base of the $z = 0$.

It may have an initial swirl to improve mixing. Warm air is introduced coaxially into the lower part of the pipe $R_1 \leq r \leq R_0 = R(0)$. This flow is usually supplied without a swirl. The design of the structure has the following typical dimensions: bottom diameter of about 90 m, total height of the structure of about 100 m, flue gas to warm air flow ratio of 300:5000 m³/s, flue gas and moist air temperatures of about 120 °C and 70 °C respectively.

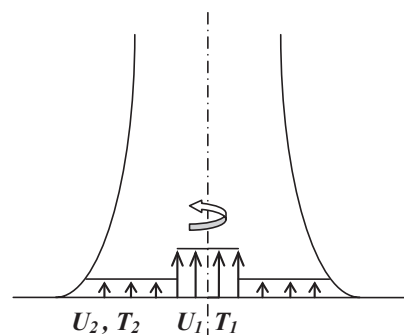


Figure 1. Flow diagram

The system of parabolized Navier-Stokes equations (as an analogue of the Prandtl boundary layer equations) is a set of laws of conservation of mass, momentum and energy. This mathematical model includes the following equations:

$$\begin{aligned}
 & \frac{\partial(r\rho U)}{\partial z} + \frac{\partial(r\rho V)}{\partial r} = 0, \\
 & \frac{\partial[(p+\rho U^2)r]}{\partial z} + \frac{\partial(\rho rUV)}{\partial r} = \frac{\partial}{\partial r}(r\tau) - \rho gr, \\
 & \frac{\partial(r\rho UH)}{\partial z} + \frac{\partial(r\rho VH)}{\partial r} = \\
 & = \frac{\partial(rq)}{\partial r} + \rho VW^2 + \mu r \left(\frac{\partial W}{\partial r} - \frac{W}{r} \right)^2, \quad (1) \\
 & \frac{\partial(r\rho UE)}{\partial z} + \frac{\partial(r\rho VE)}{\partial r} = \frac{\partial(\mu r\gamma_\alpha)}{\partial r}, \\
 & \frac{\partial(r\rho UW)}{\partial z} + \frac{\partial(r\rho VW)}{\partial r} = \\
 & = \frac{\partial}{\partial r} \left(\mu r \frac{\partial W}{\partial r} \right) - \frac{\rho VW}{r} - \mu \frac{W}{r^2}, \\
 & h = c_p T, \quad H = h + \frac{U^2}{2} + gz, \\
 & q = \frac{1}{\sigma} \mu \frac{\partial}{\partial r} (h + 0.5\sigma U^2), \\
 & \gamma_\alpha = \frac{1}{\sigma_\alpha} \frac{\partial E}{\partial r}, \quad \sigma = \frac{1}{\lambda} \mu c_p, \quad \tau = \mu \frac{\partial U}{\partial r}.
 \end{aligned}$$

In this system of equations U , V , W are the velocity components (axial, radial and azimuthal) in the cylindrical coordinate system r, φ, z ; T is the temperature, μ is the dynamic viscosity; h is the enthalpy; g is the gravity acceleration; E is the impurity concentration; τ is the friction force; c_p is the specific heat capacity; q is the heat flux; γ_α is the impurity mass flux; σ , σ_α are the Prandtl numbers.

Let us consider the method of surfaces of equal flow rates. For the first time this method for the description of jet flows was developed in [22]. Later it was widely developed for solving

various problems of hydrodynamics and heat and mass transfer with an interface [23]. For the solution of (1) in the coordinate system r, φ, z , let us introduce smooth lines $r = \delta_n(z)$, $n = 0, 1, 2, \dots, N$, which will be current lines and satisfy Eq:

$$U \frac{\partial \delta_n}{\partial z} = V \quad \text{for } r = \delta_n(z).$$

These lines should be found. It follows from the problem statement that $\delta_0 = 0$ since this is the axis of symmetry, and $\delta_N = R(z)$ defines the pipe wall. We will search for the solution of the system (1) on intermediate lines

$$\begin{aligned}
 r = \delta_{n+1/2}(z) &= \frac{1}{2}(\delta_n + \delta_{n+1}), \\
 n &= 0, 1, 2, \dots, N-1.
 \end{aligned}$$

The system of equations (1) can be represented in compact form:

$$\begin{aligned}
 & \frac{\partial(r\rho UA)}{\partial z} + \frac{\partial(r\rho VA)}{\partial r} = \frac{\partial Q}{\partial r} - \varepsilon_A \omega r, \quad (2) \\
 & A = \{1, U, H, E, W\}, \\
 & Q = \{0, r\tau, rq, r\mu\gamma_\alpha, \mu r \partial W / \partial r\}, \\
 & \varepsilon_A = 1, \quad \omega = \frac{\partial p}{\partial z} + \rho gz \quad \text{for } A = U, \\
 & \varepsilon_A = 1, \quad \omega = -\frac{\rho VW^2}{r} - \mu \left(\frac{\partial W}{\partial r} - \frac{W}{r} \right)^2 \quad \text{for} \\
 & \quad A = H, \\
 & \varepsilon_A = 1, \quad \omega = \frac{\rho VW}{r} + \mu \frac{W}{r} \quad \text{for } A = W, \\
 & \varepsilon_A = 0 \quad \text{for } A = 1, E.
 \end{aligned}$$

Let us introduce the functions

$$f_{n+1/2} = 0.5(\delta_{n+1}^2 - \delta_n^2), \quad n = 0, 1, 2, \dots, N-1.$$

Then the following relationships can be obtained for $\delta_n(z)$:

$$\delta_1^2 = 2f_{1/2}, \quad \delta_2^2 = 2(f_{1/2} + f_{3/2}), \quad \dots,$$

$$\delta_N^2 = 2 \sum_{n=1}^N f_{n-1/2}.$$

Having integrated (2) from r to $r = \delta_n$ taking into account Leibniz's rule, the original system can be represented by a set of ordinary differential equations to be solved on lines $r = \delta_{n+1/2}(z)$:

$$\begin{aligned} U\dot{U} &= \frac{1}{\rho f} R_u - \left(1 - \frac{1}{\gamma}\right) \pi_T \frac{1}{\rho} \dot{p} - \pi_g, \\ U\dot{T} &= \frac{1}{\rho f} R_T - \left(1 - \frac{1}{\gamma}\right) U \frac{1}{\rho} \dot{p} + \frac{1}{\rho} \frac{\pi_w^2}{\pi_T} G_T, \\ U\dot{E} &= \frac{1}{\rho f} R_E, \\ U\dot{W} &= \frac{1}{\rho f} R_w + \frac{1}{\rho} G_w, \\ \frac{\dot{f}}{f} &= -\frac{\dot{p}}{p} + \frac{\dot{T}}{T} - \frac{\dot{U}}{U}. \end{aligned} \quad (3)$$

In system (3), the point denotes the derivative on the coordinate z . Dimensionless variables of velocity, temperature, density, concentration and pressure are referred to the maximum values of U_1 , T_1 , ρ_1 , E_1 , p_1 , W_1 of the internal flue gas flow at $z = 0$, and f – to R_0^2 . Dimensionless parameters of the system (3) are Froude number $\pi_g = R_0 g / U_1^2$, swirl parameter $\pi_w = W_1 / U_1$ and $\pi_T = c_p T_1 / U_1^2$ as an analog of Mach number: $\pi_T = 1 / (M^2(\gamma - 1))$.

For the case of flow without pre-twisting of the flue gas flow, the pressure distribution is determined from the equation $\partial p / \partial r = 0$, and for flow with twisting from the equation of the form:

$$\frac{\partial p}{\partial r} = \frac{\gamma}{\gamma - 1} \frac{\pi_w^2}{\pi_T} \rho \frac{W^2}{r}.$$

It can be integrated as follows:

$$p(z, r) = p^w(z, r) + p_0(z),$$

$$p^w(z, r) = \frac{\gamma}{\gamma - 1} \frac{\pi_w^2}{\pi_T} \int_0^r \rho \frac{W^2}{r} dr.$$

The integral for a function $p^w(z, r)$ is calculated using the trapezoidal formula and then calculated $\dot{p}^w(z, r)$ using recurrence formulas:

$$\begin{aligned} \dot{p}_1^w &= \alpha_{1/2} f_{1/2}, \quad \dot{p}_{n+1}^w = \dot{p}_n^w + \alpha_{n+1/2} f_{n+1/2}, \\ n &= 1, 2, \dots, N-1, \\ \dot{p}_{n+1/2}^w &= 0.5(\dot{p}_n^w + \dot{p}_{n+1}^w), \quad n = 0, 1, 2, \dots, N-1, \\ \alpha_{n+1/2} &= \frac{\gamma}{\gamma - 1} \frac{\pi_w^2}{\pi_T} \frac{\rho f}{r^2} \left(2W\dot{W} - W^2 \frac{\dot{U}}{U} \right) \bigg|_{\delta_{n+1/2}}. \end{aligned}$$

The function $p_0(z)$ is obtained by integrating the equation

$$\begin{aligned} \dot{p}_0 \sum_{n=0}^{N-1} g_{n+1/2} + \sum_{n=0}^{N-1} \dot{p}^w g_{n+1/2} &= \\ = R\dot{R} - \sum_{n=0}^{N-1} \left(\pi_g \frac{f_{n+1/2}}{U^2} + \frac{R_T}{\rho U T} + \right. & \quad (4) \\ \left. + \frac{f_{n+1/2}}{\rho U T} \pi_w^2 \frac{G_T}{\pi_T} - \frac{R_u}{\rho U^2} \right) \end{aligned}$$

where

$$g_{n+1/2} = -\frac{f_{n+1/2}}{\gamma(p^w + p_0)} + \left(1 - \frac{1}{\gamma}\right) \pi_T f_{n+1/2} \frac{1}{\rho U^2}.$$

The following expressions are used to calculate the density ρ at each line:

$$\rho(z, \delta_{n+1/2}) = \frac{p(z, \delta_{n+1/2})}{T(z, \delta_{n+1/2})}. \quad (5)$$

The dissipation terms of equations (3), (4) are calculated by the formulas:

$$\begin{aligned}
R_u &= [r\mu \frac{\partial U}{\partial r}], \quad R_w = [r\mu \frac{\partial W}{\partial r}], \\
R_E &= \frac{1}{\sigma_\alpha} [r\mu \frac{\partial E}{\partial r}], \\
R_T &= \frac{1}{\sigma} [r\mu \frac{\partial T}{\partial r}] + \frac{1}{\pi_T} \left([r\mu U \frac{\partial U}{\partial r}] - U [r\mu \frac{\partial U}{\partial r}] \right), \\
G_T &= \mu \left(\frac{\partial W}{\partial r} - \frac{W}{r} \right)^2 + \frac{\rho V W^2}{r}, \\
G_w &= -\frac{\rho V W}{r} - \mu \frac{W}{r^2}.
\end{aligned} \quad (6)$$

The square brackets in (6) imply the difference between two consecutive values: $[Q] = Q_{n+1} - Q_n$.

To solve the system (3) it is necessary to set boundary conditions on the boundary of the solution region. For the variables of velocity, temperature, concentration it is obvious from the requirement of flow symmetry at $r=0$. On the pipe wall, the boundary layer is assumed to be thin enough. Therefore, the following conditions were used:

$$\begin{aligned}
\frac{\partial A}{\partial r} &= 0 \text{ for } \delta = 0 \text{ and } \delta = R(z), \\
A &= \{U, W, T, E\}.
\end{aligned} \quad (7)$$

An algebraic turbulence model was used to complete the system of equations (3)-(6):

$$l_i^2 = \nu_{ti} \left\{ \left(\frac{\partial V_z}{\partial r} \right)^2 + \left[r \frac{\partial}{\partial r} \left(\frac{V_\phi}{r} \right) \right]^2 \right\}^{-1/2}, \quad (8)$$

where l_i is the mixing path length, $i = z, \phi$.

Finally, the system (3)-(6) of $5N+1$ equations was integrated numerically by Runge-Kutta method.

3. RESEARCH RESULTS

Numerical results of the solution of the boundary value problem (3)-(6) were produced

considering the following distributions of the sought variables for $z=0$:

$$\begin{aligned}
U(r) &= U_1 = 1, \quad W(r) = W_1(r), \\
T(r) &= T_1 = 1, \quad E(r) = E_1 = 1, \quad 0 \leq r \leq r_1, \\
U(r) &= U_2, \quad W(r) = W_2, \\
T(r) &= T_2, \quad E(r) = E_2, \quad r_1 < r \leq 1.
\end{aligned} \quad (9)$$

The dimensionless value of the length of the region was taken as $z_0 = 2.2R_0$. The shape of the lateral surface was specified in two ways: either $R_0 = 1$, or $R(z) = 1 - 0.15z$. The flows were calculated in the range of parameters: $U_2 = 0.05 - 0.4$; $T_2 = 0.5 - 0.9$; $E_2 = 0.1$; $\sigma = 0.72$; $\pi_g = 6.45$; $\pi_T = 5754$; $\pi_w = 0 - 1.35$; $r_1 = 0.15; 0.33$; $N = 50$.

In most of the calculations the values used were $U_2 = 0.1$, $T_2 = 0.8$, $E_2 = 1$, $R_0 = 1$, $r_1 = 0.33$. The velocity profiles at these parameter values are shown in Fig. 2. In all cases, the main effect of the swirled flow is observed as the presence of a local minimum of the axial velocity for $r=0$. This effect increases as the value of initial swirl increases from $\pi_w = 1$ to $\pi_w = 1.3$. For the values $\pi_w > 1.35$, calculation using parabolized equations is not possible. In this case, the full system of Navier-Stokes equations should be used.

In the considered problem, the lifting force has a significant effect on the process of flue gas mixing. Due to this, it is possible to increase the axial velocity of the gas flow through the pipe. For example, for the flow without swirl at $\pi_w = 0$ a small section $z \leq 0.5$, the axial velocity of the flow increases by 1.5 times (Fig. 2, a). The action of lift force due to temperature differences can also contribute to the increase in flow velocity (Fig. 2, b). The addition of an initial swirl reduces this effect.

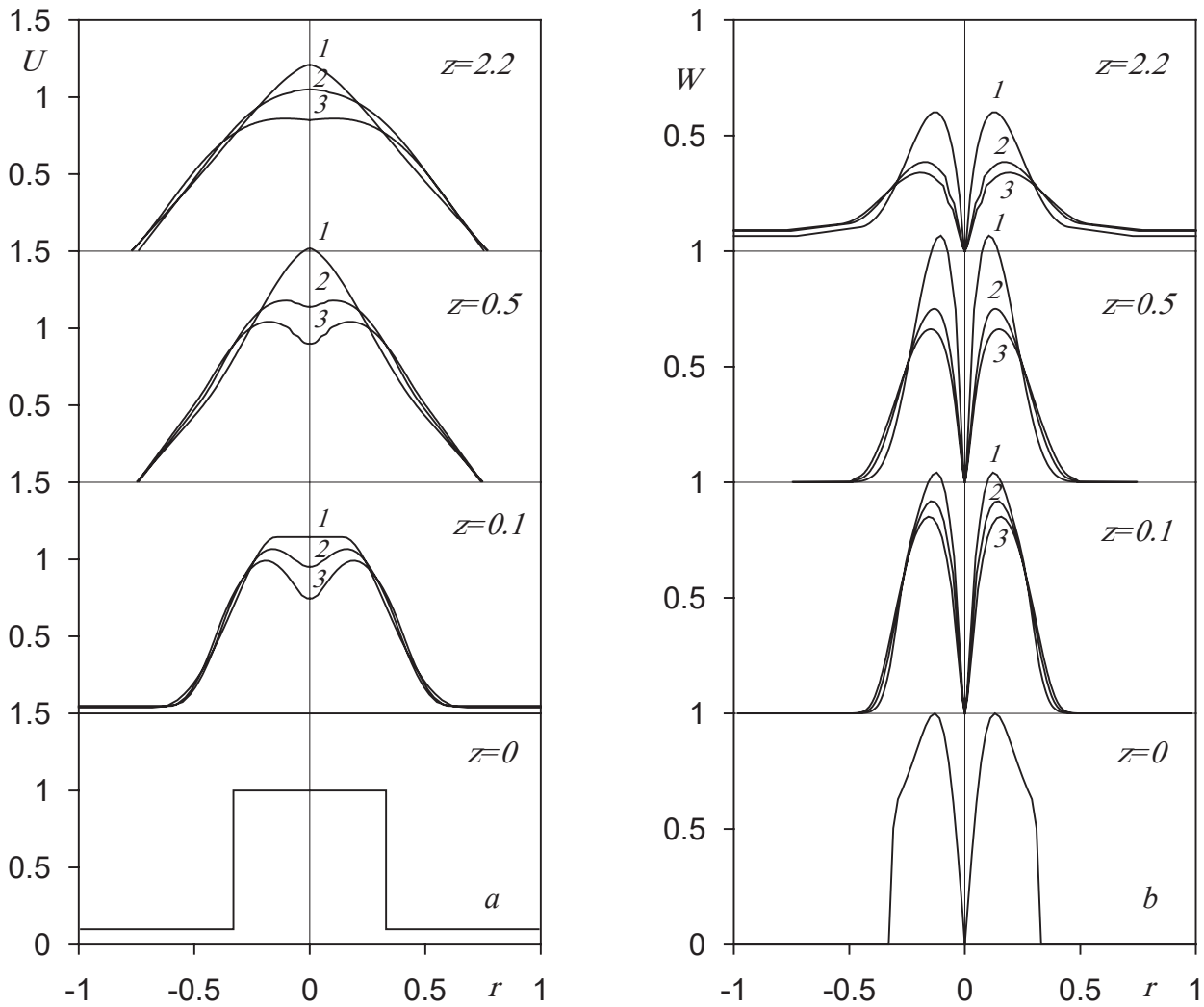


Figure 2. Axial U and azimuthal W velocity profiles at swirling $\pi_w = 0; 1; 1.3$ (a) $\pi_w = 0.2; 1; 1.3$ (b) (curves 1-3) in different cross-sections $z = \text{const}$

With increasing temperature difference T_1/T_2 for the internal jet of flue gas and external flow of warm air, the effect of the lifting force increases. Fig. 3, a shows the relation of axial velocity at $r=0$ for different ratios of temperature difference, where $T_1 = 1$. The axial velocity maximum can increase more than 1.8 times the initial value (Fig. 3, a, curve 4). The lift force can also contribute to the increase in flow swirl. For example, the maximum azimuthal velocity $W_{\max}$ can increase by a

factor of 1.3 compared to the initial value (Fig. 3, b).

Temperature distribution on the flow axis along the pipe for $r=0$ is shown in Fig. 4, a. Due to mixing with warm air the flue gas temperature drops rather quickly. The initial swirl of the flow intensifies this process. A similar result is observed for the concentration distribution of harmful impurities. In the outlet section of the pipe the maximum concentration decreases more than 2 times (Fig. 4, b). This effect is extremely important with regard to the

environmental performance of the combined high-rise structure.

Fig. 5 shows characteristic pictures of the current lines. It can be seen that the current lines converge to the center rather quickly as the

distance z increases. This data can be used for profiling the walls of the exhaust pipe in order to reduce the size of the erected structure, reduce material costs and increase the stability of the structure.

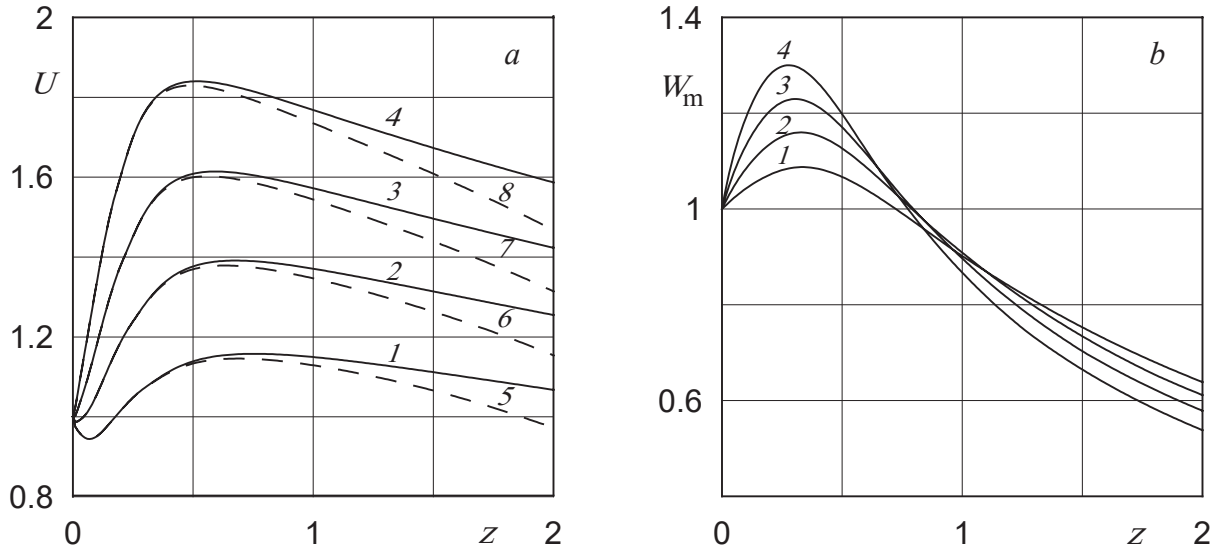


Figure 3. Distribution of axial velocity on axis $r = 0$ for $\pi_w = 1$ (a) and maximum azimuthal velocity (b) for $\pi_w = 0.2$; $T_2 = 0.8; 0.7; 0.6; 0.5$; $R(z) = 1$ (curves 1–4), $R(z) = 1 - 0.15z$ (curves 5–8)

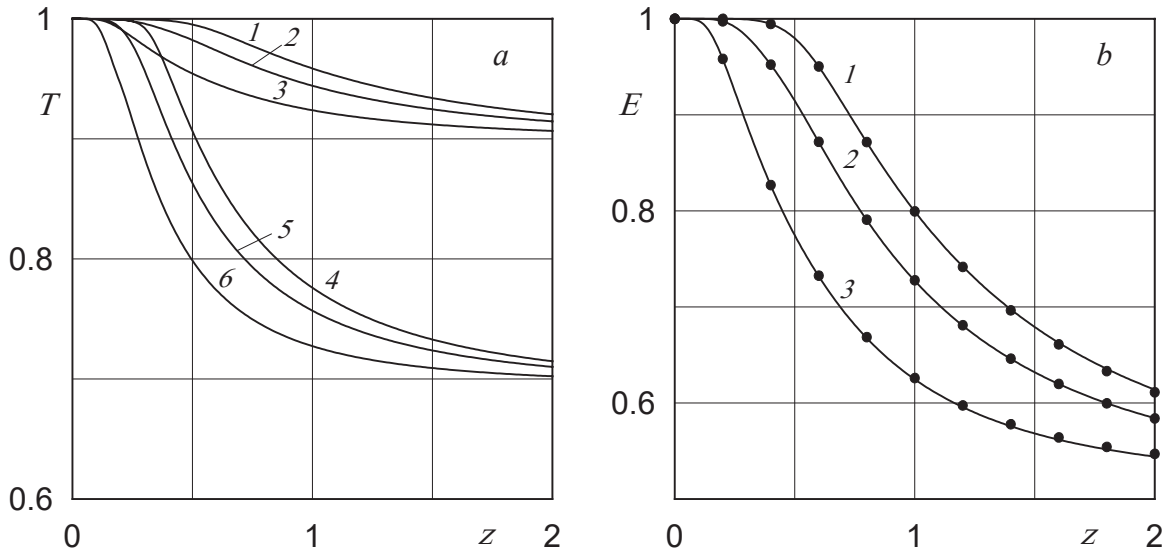


Figure 4. Temperature (a) and concentration (b) distribution on the axis $r = 0$ for $\pi_w = 0.2; 0.5; 1.3$; $T_2 = 0.8$ (curves 1–3) and $T_2 = 0.5$ (curves 4–6)

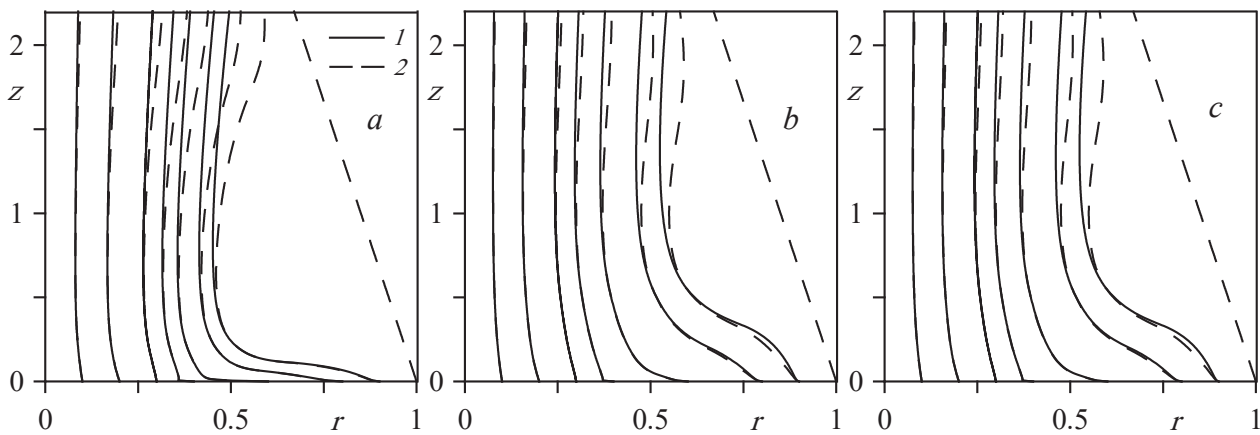


Figure 5. Current lines for $T_2=0.8$; $r_1=0.33$; $U_2=0.1$; 0.2 ; 0.3 (a, b, c), $1 - R(z)=1$; $2 - R(z)=1 - 0.15z$

4. CONCLUSIONS

The presented mathematical model and numerical calculation method allow obtaining necessary and detailed information on the character of flow in the exhaust stack of a combined high-rise building, temperature distribution and concentration of harmful impurities at the stack outlet. The initial swirling of the flue gas flow contributes to the improvement of gas mixing. However, excessively strong twist leads to undesirable flow inhibition and possible disintegration of swirl flow. The developed methodology allows to determine the maximum permissible value of the initial twist to eliminate such phenomenon. The obtained data of gas-dynamic flow pattern can serve as a basis for choosing the optimal shape of the pipe side surface in order to reduce the cost of the constructed structure.

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Ахметов Вадим Каюмович, доктор технических наук, профессор кафедры информатики и прикладной математики Национального исследовательского Московского государственного строительного университета, 129337, г. Москва, Ярославское шоссе, д. 26, тел. +7(499) 183-59-94, e-mail: vadim.akhmetov@gmail.com

Vadim K. Akhmetov, Doctor of Science, Professor, Department of Computer Science and Applied Mathematics, Moscow State University of Civil Engineering, 26, Yaroslavl'skoe Shosse, Moscow, 129337, Russia, tel. +7(499) 183-59-94, e-mail: vadim.akhmetov@gmail.com

Шкадов Виктор Яковлевич, доктор физико-математических наук, профессор кафедры аэромеханики и газовой динамики механико-математического факультета Московского государственного университета им. М.В. Ломоносова, 119991, г. Москва, Ленинские горы, д. 1, tel. +7(495) 939-39-49, e-mail: shkadv@mech.math.msu.su

Viktor Y. Shkadov, Doctor of Sciences, Professor, Department of Aeromechanics and Gas Dynamics, Faculty of Mechanics and Mathematics, Lomonosov Moscow State University, 119991, Moscow, Leninskie Gory, 1, tel. +7(495) 939-39-49, e-mail: shkadv@mech.math.msu.su