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OPTIMAL SEISMIC PROTECTION OF STRUCTURES ON BEARINGS WITH BILINEAR HYSTERETIC CHARACTERISTIC

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Abstract: The design of optimal seismic protection systems on bearings with hysteresis behavior leads to a conditional multidimensional global optimization problem with a nonlinear and non-differentiable objective function. In this paper, bearings with bilinear characteristic with hysteresis are considered, i.e. for each nonlinear bearing there are four parameters to optimize: ultimate elastic displacement and stiffness for the elastic mode and ultimate displacement and stiff-ness for the inelastic mode.

We study the seismic response of an elastic beam on four nonlinear bearings under vertical seismic ground motion. The perturbation functional as an integral of the quadratic form of the state vector with a weight matrix formed from the stiffness and inertia matrices of the system on equivalent elastic supports is considered as the objective function. Minimization of the functional means the minimum of the total energy of the system on equivalent elastic bearings. Genetic algorithm is used to solve the optimization problem.

Keywords: seismic protection, bilinear characteristic, global multidimensional optimization, optimality criterion, seismic action, genetic algorithm

ОПТИМАЛЬНАЯ СЕЙСМОЗАЩИТА КОНСТРУКЦИЙ НА ОПОРАХ С БИЛИНЕЙНОЙ ГИСТЕРЕЗИСНОЙ ХАРАКТЕРИСТИКОЙ

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Аннотация: Проектирование оптимальных систем сейсмозащиты на опорах с гистерезисным поведением приводит к условной многомерной глобальной задаче оптимизации с нелинейной и недифференцируемой целевой функцией. В этой статье рассматриваются опоры с билинейной характеристикой с гистерезисом, т.е. для каждой опоры необходимо оптимизировать четыре параметра: предельное упругое перемещение и жесткость для упругого режима и предельное смещение и жесткость для неупругого режима.

Исследуется сейсмический отклик упругой балки на четырех нелинейных опорах при вертикальном сейсмическом движении грунта. В качестве целевой функции рассматривается функционал возмущения как интеграл квадратичной формы вектора состояния с весовой матрицей, сформированной из матриц жесткости и инерции системы на эквивалентных упругих опорах. Минимизация функционала означает минимум общей энергии системы на эквивалентных упругих опорах. Для решения задачи оптимизации используется генетический алгоритм.

Ключевые слова: сейсмозащита, билинейная характеристика, глобальная многомерная оптимизация, критерий оптимальности, сейсмическое воздействие, генетический алгоритм

1. INTRODUCTION

In this work, the objective function corresponds to the total energy of some elastic structure (called an equivalent system below in the text) whose natural frequencies are very close to the resonant frequencies of the original structure on nonlinear bearings. Scientific publications describe extensive experience in optimal design of structures based on elements with bilinear hysteretic characteristic, such as lead rubber bearings, elastomeric isolators and other devices [1-16]; many researchers apply stochastic optimization methods [11-16], solving the problem of compromise between displacements and stresses in structures. In many works take the sum of dimensionless relative dynamic response parameters (relative displacements, velocities, accelerations, forces, etc.) with weight coefficients as the objective function [4-7]. By adjusting the weighting coefficients, researchers emphasize the most significant parameters. For example, in [4], the ratio of peak roof acceleration to peak ground acceleration is minimized. The optimality criterion given in [7] minimizes both top floor acceleration and bearing displacement.

For optimal seismic protection, we take an objective function as quadratic perturbation functional of the state vector with a weight matrix, which is formed from the stiffness and/or inertia matrices of the equivalent system. Then the minimum of the objective function will be close to the minimum of the potential and/or kinetic energy of the structure on the nonlinear bearings. The complication of the problem is multidimensionality with possible multi-extremality, nonlinearity and non-differentiability of the objective function, parametrical limitations of the structure and vibration protection. Not all optimization methods are suitable for solving this problem. The most effective methods are based on the direct calculation of the objective function, for example, random search methods including population methods (genetic and other evolutionary algorithms). Intelligence algorithms of optimization of nonlinear vibration isolators are applied in many works, for example, in [13-16].

Bilinear force-displacement hysteretic curve of the bearing is shown in Fig. 1. Such system has special properties, for example, as shown for a one-dimensional oscillator in [3], resonant frequencies depend on the amplitude of cyclic load. At small cyclic load amplitudes, the oscillator works in elastic regime $A'OA$ with stiffness c_1 (Fig.1). At large amplitudes, the support goes to the AB line with stiffness c_2 and works in inelastic regime with hysteresis on the loop $ABCDE$. Frequencies $\omega_1 = \sqrt{c_1/M}$ and $\omega_2 = \sqrt{c_2/M}$ correspond to these two regimes accordingly, here M is the mass of the oscillator. Depending on the regime, the resonant frequency of the oscillator will be close to one of the frequencies.

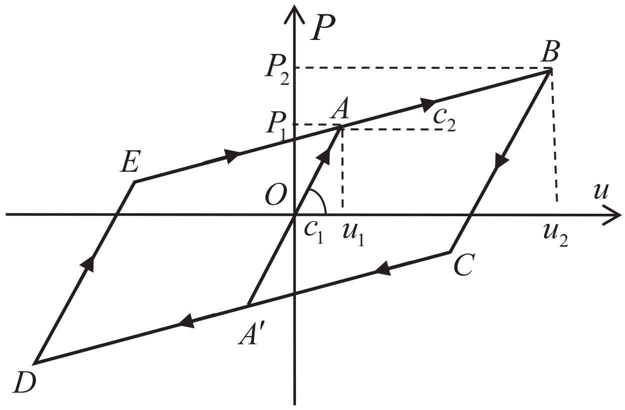


Figure 1. Bilinear curve of cyclic deformation

To design of elastomeric isolators it is common to consider elements with bilinear characteristic as linear–elastic bearings with effective stiffness $c_{eff} = P_2/u_2$, where P_2 is the ultimate force (Fig. 1) and $c_1 > c_{eff} > c_2$ [12]. However, it is very problematic to accurately calculate the resonant frequencies of multidimensional systems because they depend on the amplitude of cyclic loads.

Note that the work of the bearing in plastic regime with large deformations is undesirable because it leads to off-center compression of bearings, residual displacements and the need to restore it after an earthquake. In addition, as follows from [12], the damping of the nonlinear bearing is significantly reduced at large plastic strains. Consequently, the ultimate displacements should ensure that nonlinear elements operate predominantly in the elastic regime with greater damping and more predictable resonant frequencies.

In the study, we assume that each bearing works mainly in elastic regime, therefore the resonant frequencies are close to the natural frequencies of the system on elastic bearings with stiffness c_1. This system is almost equivalent to the system on nonlinear histeretic bearings in terms of resonant frequencies. Despite on the nonlinear "damping - shear deformations" function and new damping models [8], we take a simple model of damping from [12] as $\xi_e = \frac{2}{\pi}\left(\frac{F_1}{F_2} - \frac{d_1}{d_2}\right)$.

Before designing of earthquake–resistant structures, it is necessary to find out whether the nonlinear bearings can reduce the seismic response of the structure? Bearings with bilinear characteristics reduce resonant frequencies, therefore it is necessary to determine natural frequencies of the original structure (on absolutely rigid bearings) and the prevailing frequencies of seismic action. If natural frequencies of the original structure are already far from the prevailing frequencies of seismic action, the bearings are not only unnecessary, but can also seriously harm, enhancing structural dynamic response. The nonlinear bearings are effective when the lowest resonant frequencies of a system with them are lower than the prevailing frequencies of seismic action.

2. OPTIMIZATION PROBLEM

We consider a structure with n degrees of freedom on N bearings. Each bilinear hysteretic bearing has four parameters, which should be optimized: $\mathbf{h}_i = \left[u_1^{(i)} \quad u_2^{(i)} \quad c_1^{(i)} \quad c_2^{(i)}\right]$, where c_1 and u_1 – stiffness of elastic mode and maximal elastic displacement, c_2 and u_2 – stiffness of inelastic mode and maximal displacement of the bearings, $i = 1,..., N$. A simple example of the structure shown in Fig. 2. Vertical seismic ground motion is given by accelerogram $a(t)$ with duration T. At each the moment of time the state of the structure is described by a $2n$–dimensional vector $\mathbf{X}(t) = [\mathbf{q} \quad \dot{\mathbf{q}}]^{\mathrm{T}}$, $\mathbf{q}$ and $\dot{\mathbf{q}}$ are nodal displacements and velocities.

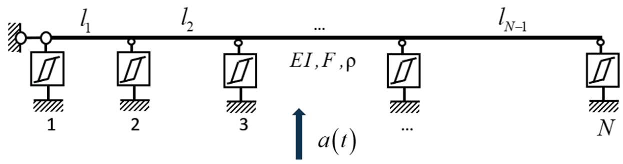


Figure 2. Beam on bilinear hysteretic bearings

We take the objective function as a quadratic functional:

$$f(\mathbf{H}) = \frac{1}{2T}\int_0^T \mathbf{X}^{\mathrm{T}}\mathbf{Q}\mathbf{X}dt,$$

where $\mathbf{H} = [\mathbf{h}_1 \quad \mathbf{h}_2 \quad ... \quad \mathbf{h}_N]$ – vector of optimized parameters, $\mathbf{Q}^{[2n\times 2n]}$ – weight matrix, $\mathbf{X}^{\mathrm{T}}\mathbf{Q}\mathbf{X} \geq 0$. We choose the matrix $\mathbf{Q}$ to give to the objective function an energy meaning. Let the matrix $\mathbf{Q}$ consists four zero blocks of the same size (two rows of two blocks). If we put stiffness matrix of the equivalent system in the first (upper left) block, the objective function will be equal to the average value of the potential energy of elastic deformation U on the interval $[0, T]$. This weight matrix is denoted by $\mathbf{Q}_U$. Minimization of the objective function $f(\mathbf{H})$ using the weight matrix $\mathbf{Q}_U$ leads to a decrease of internal forces and stresses. Similarly, if weight matrix $\mathbf{Q}_{\mathrm{T}}$ has only inertia matrix in the fourth (lower right) block, then objective function $f(\mathbf{H})$ is equal to average value of the kinetic energy T on the interval $[0, T]$. In this case, the kinematic parameters of the structure will be reduced.

The objective functions corresponding to potential, kinetic and total energy are denoted as follows:

$$J_U = f(\mathbf{H}|\mathbf{Q}_U), \quad J_{\mathrm{T}} = f(\mathbf{H}|\mathbf{Q}_{\mathrm{T}}),$$
$$J_E = f(\mathbf{H}|\mathbf{Q}_E), \quad \mathbf{Q}_E = \mathbf{Q}_U + \mathbf{Q}_{\mathrm{T}}.$$

Consider the optimization problem:

$$\min_{\mathbf{H}\in D} f\left(\mathbf{H}\right)=f\left(\mathbf{H}_*\right)=f_*, \qquad (1)$$

where $\mathbf{H}_*-$ optimal solution, f_*- value of the optimized objective function, $D-$ acceptable values of the vector $\mathbf{H}$:

$$D=\left\{\mathbf{H}\middle|\ \mathbf{H}_{low}\le\mathbf{H}\le\mathbf{H}_{up},\ \mathbf{A}_{cond}\mathbf{H}\le\mathbf{b}_{cond}\right\}. \qquad (2)$$

Depending on the weight matrix $\mathbf{Q}_U$, $\mathbf{Q}_{\mathrm{T}}$ or $\mathbf{Q}_E$, problem (1) is the optimality criterion in terms of potential, kinetic or total energy.

Conditions $\mathbf{H}_{low}\le\mathbf{H}\le\mathbf{H}_{up}$ are acceptable parameters of a bilinear diagram (stiffnesses and ultimate displacements):

$$\mathbf{H}_{low}=\left[\mathbf{h}_{1\_low}\quad \mathbf{h}_{2\_low}\quad \dots\quad \mathbf{h}_{N\_low}\right],$$
$$\mathbf{h}_{i\_low}=\left[u_{1\_low}^{(i)}\quad u_{2\_low}^{(i)}\quad c_{1\_low}^{(i)}\quad c_{2\_low}^{(i)}\right],$$
$$\mathbf{H}_{up}=\left[\mathbf{h}_{1\_up}\quad \mathbf{h}_{2\_up}\quad \dots\quad \mathbf{h}_{N\_up}\right],$$
$$\mathbf{h}_{i\_up}=\left[u_{1\_up}^{(i)}\quad u_{2\_up}^{(i)}\quad c_{1\_up}^{(i)}\quad c_{2\_up}^{(i)}\right],$$

where

$$c_{1\_low}^{(i)}<c_1^{(i)}<c_{1\_up}^{(i)},\ c_{2\_low}^{(i)}<c_2^{(i)}<c_{2\_up}^{(i)},$$
$$u_{1\_low}^{(i)}<u_1^{(i)}<u_{1\_up}^{(i)},\ u_{2\_low}^{(i)}<u_2^{(i)}<u_{2\_up}^{(i)},\quad i=1,\dots,N.$$

The conditions $\mathbf{A}_{cond}\mathbf{H}\le\mathbf{b}_{cond}$ correspond to the bilinear hysteresis algorithm and are a matrix form of the following inequalities:

$$u_2^{(i)}-u_1^{(i)}\ge 0,\quad c_1^{(i)}-c_2^{(i)}\ge 0.$$

Both conditions (2) are set before the optimization process. Displacement of $i-$th bearing must not exceed predetermined value $u_*^{(i)}$:

$$\left|u^{(i)}\right|\le u_*^{(i)}. \qquad (3)$$

Constraint (3) cannot be satisfied before the optimization procedure starts. Consequently, the optimization algorithm must include a penalty: if at some time the condition (3) is not satisfied, the calculation stops and some large value (penalty) is assigned to the objective function.

We will not introduce additional constraints related to reliability of the structure, such as strength and stiffness conditions, assuming that the minimization of dynamic response of the structure is provided by the criterion of smallness of perturbations (1). Additional constraints will complicate the optimization procedure. If it found out that the global minimum of the objective function does not satisfy the strength or stiffness conditions, this means that hysteretic bearings is ineffective for this design.

Problem (1) with conditions (2-3) is a multidimensional global conditional optimization problem with a nonlinear non-differentiable objective function. Optimization methods based on the computation of the objective function without computing derivatives, such as random search algorithms, e.g., annealing simulation, hypersphere algorithm, "greedy" adaptive algorithms, genetic and other evolutionary algorithms, are suitable for finding the optimal solution [11]. The following example illustrates the results of solving the optimization problem (1) with conditions (2-3) obtained using the genetic algorithm (GA).

3. EXAMPLE OF OPTIMAL SOLUTION

We consider a steel hinged beam on four bearings, the cross-section profile is I–beam No. 20 (height 200 mm), the lengths of the three spans are 6, 8 and 12 m. The beam is under seismic action determined by the accelerogram of the vertical ground motion of the foundation; we use accelerogram synthesized at the Institute of Physics of the Earth, Moscow, Russia. The dynamic model of the beam was developed by using the finite element method; structural damping was neglected. The seismic response was determined by direct integration of equations of motion in Simulink.

Figure 3 shows the estimates of the seismic ground motion power spectral density: the raw periodogram and its smoothed estimate found using the Welch window transformation with 50% overlap. The first three natural frequencies of the hinged supported beam are marked by red vertical lines (5.76, 14.33 and 22.85 Hz). It can be seen that the first frequency is dangerously close to one of the prevailing frequencies of the seismic action.

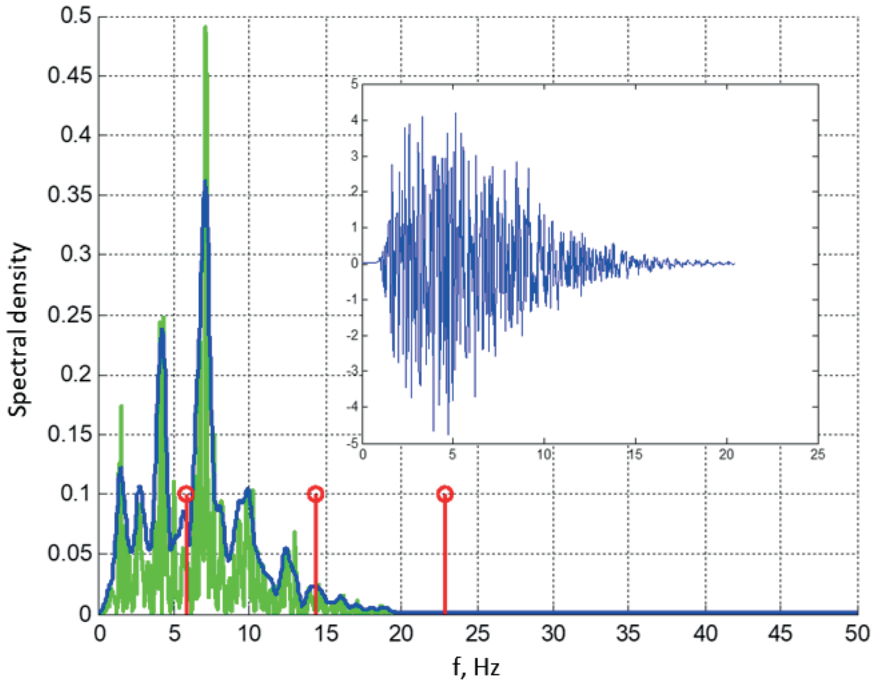


Figure 3. Anapa earthquake Z3G2–A. Spectral density: raw periodogram (green), smoothed periodogram (blue); natural frequencies of the hinged supported beam (red); in the box – accelerogram Z3G2–A (acceleration - m/s^2, time - s)

By analyzing Fig. 3, we conclude that the best way to reduce the dynamic response of the beam is to make the structure more rigid (e.g., add another supports) to shift its natural frequencies to the right. But for the purpose of this study, we place the beam on four bilinear hysteretic bearings (Fig. 2, $N = 4$) and try to find an optimal solution of problem (1-3) by applying a genetic algorithm. To solve this problem, the beam model on the bilinear hysteretic bearings was developed in Simulink. The model is a system with incomplete dissipation, since damping is present only in the bearings.

GA implemented in MATLAB Optimtool was used to find the optimal solution. The optimality criterion is the minimum of the total mechanical energy: $J_E = f\left(\mathbf{H}\middle|\mathbf{Q}_E\right) \to \min$. The dimensionality of the optimization problem is 16 (4 bearings, each with 4 optimized parameters). For conditions (2-3) we assign:

$$\mathbf{h}_{i\_low} = \left(0.02\text{ m} \quad 0.03\text{ m} \quad 10^3\text{ N/m} \quad 10^2\text{ N/m}\right),$$
$$\mathbf{h}_{i\_up} = \left(0.05\text{ m} \quad 0.05\text{ m} \quad 10^7\text{ N/m} \quad 10^5\text{ N/m}\right),$$
$$u_*^{(i)} = 0.10\text{ m},\ i = 1,\ 2,\ 3,\ 4\,.$$

GA parameters: initial population contains 20 agents found manually, selection and mutation operators have uniform distribution, elite is 5 agents, two-point crossover, maximum number of iterations is 100. As a result, the following solution was obtained:

$$\mathbf{H}_* = \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 & \mathbf{h}_4 \end{bmatrix},$$

$$\mathbf{h}_1 = \begin{bmatrix} 0.03\ \text{m} & 0.05\ \text{m} & 10^5\ \text{N/m} & 10^3\ \text{N/m} \end{bmatrix},$$

$$\mathbf{h}_2 = \begin{bmatrix} 0.03\ \text{m} & 0.05\ \text{m} & 1.5\cdot 10^4\ \text{N/m} & 2.5\cdot 10^2\ \text{N/m} \end{bmatrix},$$

$$\mathbf{h}_3 = \begin{bmatrix} 0.03\ \text{m} & 0.05\ \text{m} & 2\cdot 10^4\ \text{N/m} & 10^2\ \text{N/m} \end{bmatrix},$$

$$\mathbf{h}_4 = \begin{bmatrix} 0.03\ \text{m} & 0.05\ \text{m} & 10^5\ \text{N/m} & 5\cdot 10^2\ \text{N/m} \end{bmatrix}.$$

Seismic responses of the initial and optimized system are shown in Fig. 3, 4. Optimization resulted in a reduction of the maximum bending moment by more than a factor of two with a small allowable increase in kinematic parameters. The bearings work mainly in the elastic regime; there are no plastic displacements in bearings 1, 2, 4, and they are minimal in the third bearing. Table 1 summarizes the natural frequencies of the beam on bilinear hysteretic bearings and beams on equivalent elastic bearings.

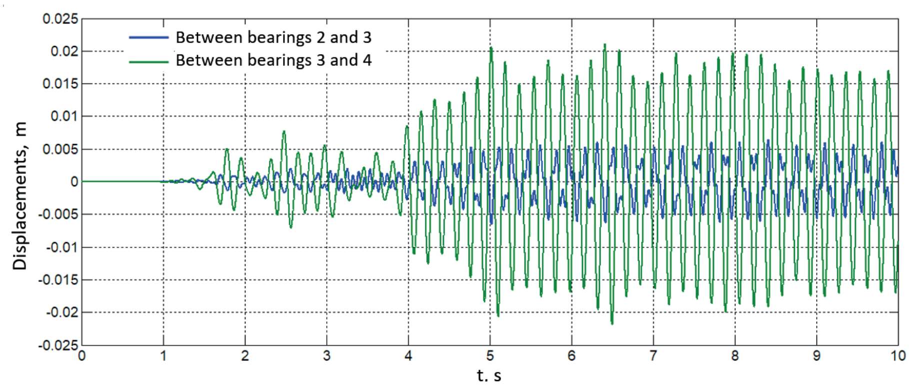
Table 1

N	Natural frequencies, Hz	
	Hinged supported beam	Beam on equivalent elastic bearings
1	5.76	1.80
2	14.33	3.50
3	22.85	6.20
4	26.64	9.17
5	52.04	13.80
6	70.13	21.30

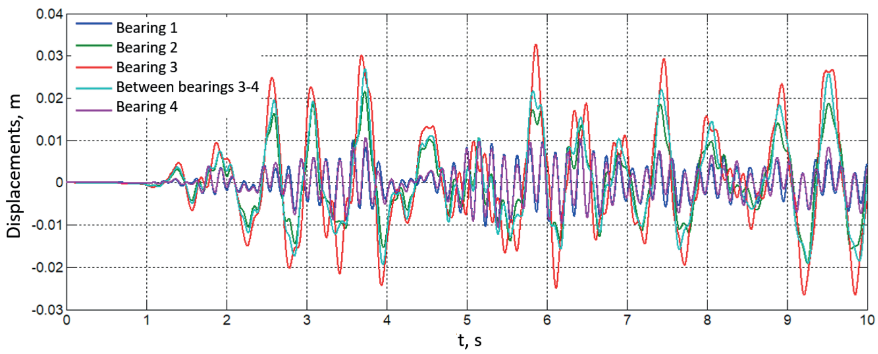
Table 2 shows the values of the objective function J_E before and after optimization, and values of the objective function with weight matrices $\mathbf{Q}_U$ and $\mathbf{Q}_T$.

Table 2. Objective function values before and after optimization

	Hinged supported beam	Beam on equivalent elastic bearings
J_E	3.133e05	1.287e05
J_U	1.570e05	3.029e04
J_T	1.563e05	9.845e04

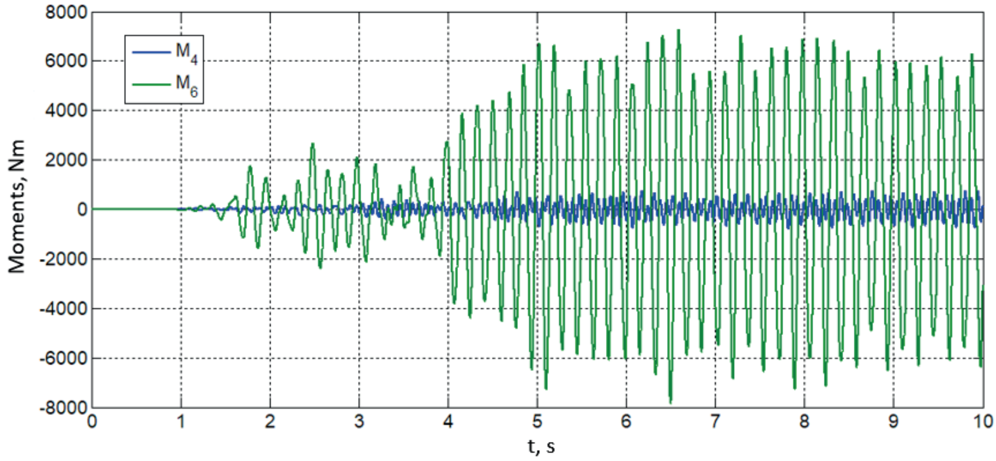


a)

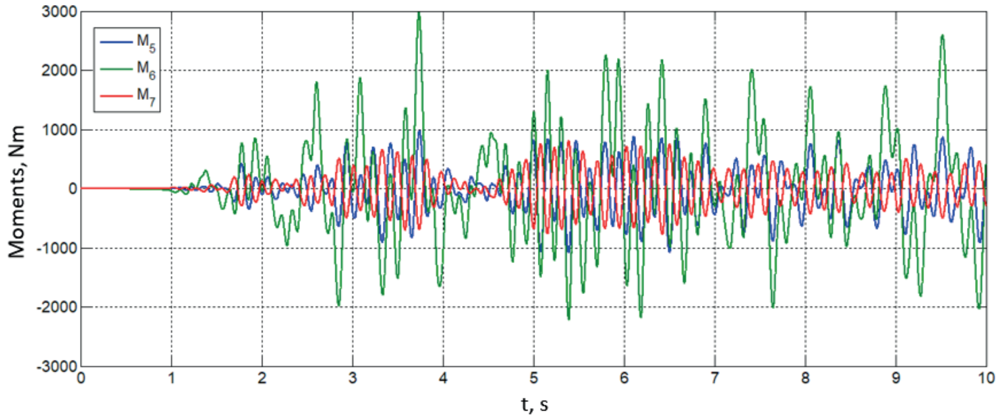


b)

Figure 4. Displacements of cross-sections of: a) the hinged supported beam: $u_{\max} = 0.022$ m, $\sigma_{\max} = 0.010$ m; b) the optimized beam: $u_{\max} = 0.028$ m, $\sigma_{\max} = 0.009$ m.



a)



b)

Figure 5. Bending moments: a) the hinged supported beam: M_4 *– moment at the center of the second span,* M_6 *– moment at the center of the third span,* $M_{\max} = M_6 = 7.84$ *kNm,* $\sigma_{\max} = 3.30$ *kNm; b) the optimized beam:* M_5 *– moment at the center of the second span,* M_6 *– moment over the third bearing,* M_7 *– moment at the center of the third span,* $M_{\max} = M_6 = 2.98$ *kNm,* $\sigma_{\max} = 0.860$ *kNm.*

4. EXPERIENCE OF USING A GENETIC ALGORITHM

Using GA to optimize a multidimensional dynamic system, you should not expect a fast and unambiguous solution. For multidimensional optimization, GA requires multiple calculations of the objective function, for example, for default parameters of the GA in the MATLAB Optimtool, if the number of parameters to be optimized is greater than 5, the initial population is $n = 200$ agents-solutions, maximal number of iterations is equal to $10n$ generations. For the above problem, we need to compute the objective function for 200 variants of the vector $\mathbf{H}$ at one iteration, and there can be 1600 such iterations. To speed up the process, it is necessary take more suitable solutions as the initial population and using elitism. Since the genetic algorithm is a stochastic method, by solving the same problem several times, you can get different solutions. The vector of optimal parameters $\mathbf{H}_*$ isn't a single best solution. Some solutions were rejected for various reasons, because they led, for example, to large plastic deformations. It was necessary to correct the initial population several times and adjust the parameters of genetic operators.

Fig. 6 shows the natural frequencies of the beam on equivalent elastic bearings and the estimates of the spectral density of the Anapa earthquake Z3G2 – A. Comparing the results in Fig. 3 and 6, we can draw a simple conclusion: the optimization resulted in a shift of the natural frequencies of the equivalent system to the minima of the seismic periodogram. To avoid such "adaptability" of the system on the bilinear hysteretic bearings to a single accelerogram, it is necessary to solve the optimization problem on a series of accelerograms. The random nature of earthquakes and the uncertainty with the resonant frequencies of the systems on nonlinear bearings make accurate tuning of resonant frequencies very unreliable. Hence, the nonlinear bearings should be used for the design of earthquake-resistant structures only if the lowest natural frequencies of the system on equivalent elastic bearings are lower than the prevailing seismic frequencies. In this case, a reasonable solution will be obtained without high computational costs.

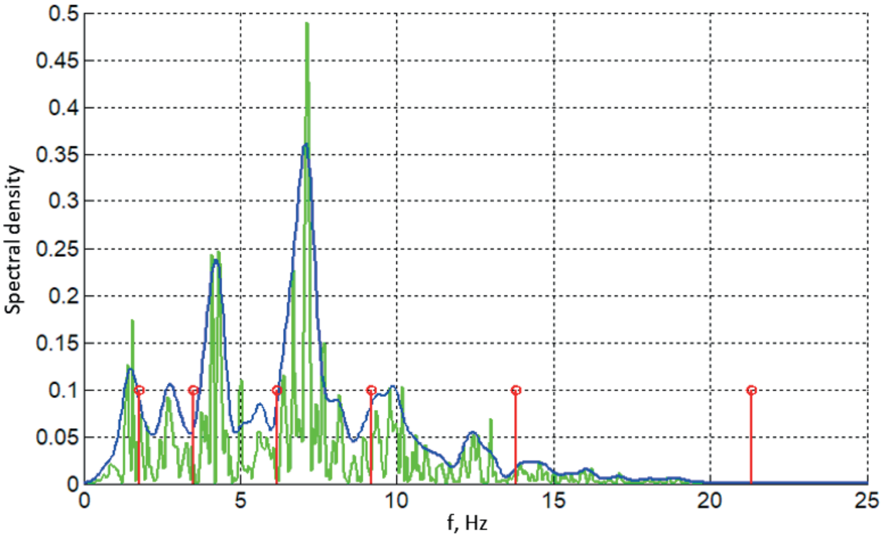


Figure 6. Anapa earthquake Z3G2–A. Spectral density: raw periodogram (green), smoothed periodogram (blue); resonant frequencies of the optimized beam (red)

5. CONCLUSIONS

In the study, an energy approach was formulated to solve the optimization problem of the structure on the bilinear hysteretic bearings. The approach is illustrated on the example of optimization of an earthquake-resistant structure. Multidimensional conditional nonlinear optimization has a complicated mathematical formulation and computationally "hard" solution. However, the optimal solution, as well as the answer to the question about the feasibility of installing a vibration protection system based on the bilinear hysteretic bearings, can be obtained by preliminary frequency analysis of the structure and seismic effects.

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