

# MATHEMATICAL MODELING OF RELAXATION PROCESS IN CONCRETE

*Sergey B. Krylov<sup>1</sup>, Petr D. Arleninov<sup>1,2</sup>*

<sup>1</sup> JSC Research Center of Construction NIIZH named after A.A. Gvozdev, Moscow, RUSSIA

<sup>2</sup> National Research Moscow State University of Civil Engineering, Moscow, RUSSIA

**Abstract:** The study provides experimental and theoretical investigations of stress relaxation in concrete under preset constant deformation of specimens and proposes an approach to the mathematical determination of stress relaxation in concrete. In addition to mathematical modeling, parallel long-term tests were performed on concrete specimens of the same class under four different regimes: determination of concrete creep in compression (concrete prism specimens in spring installations) according to GOST 24544; determination of concrete relaxation in compression (concrete prism specimens in special installations, determination of concrete creep in bending (concrete specimens-beams in rack-type installations working in bending and loaded with gravity load) according to GOST 24544; determination of concrete relaxation in bending (concrete specimen beams, with applied initial deformation in the middle of the span

Determination of stress drop was performed using electronic dynamometers built between the specimen and the point of load application. A new concept, "relaxation measure  $R_m$ ", was proposed. This value is similar in meaning to the creep measure and characterizes the degree of stress reduction in time due to relaxation when loaded with a unit relative strain.

The introduction of relaxation measure allows to simplify the relaxation equation and significantly simplifies its solution. According to the results of calculation by the proposed computational algorithms for determining the relaxation measure in comparison with the experimental data, a satisfactory convergence of the results is obtained. Qualitative conclusions about the applicability of the methodology of SP 63.13330 for reducing the modulus of elasticity of concrete in creep and relaxation.

**Keywords:** concrete, reinforced concrete, creep of concrete, relaxation of concrete

## МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ПРОЦЕССА РЕЛАКСАЦИИ В БЕТОНЕ

*С.Б. Крылов<sup>1</sup>, П.Д. Арленинов<sup>1,2</sup>*

<sup>1</sup> Научно-исследовательский, проектно-конструкторский и технологический институт бетона и железобетона (НИИЖБ) им. А.А. Гвоздева АО «НИЦ «Строительство», г. Москва, РОССИЯ

<sup>2</sup> ФГБОУ ВО «Национальный исследовательский Московский государственный строительный университет» Минобрнауки России (НИУ МГСУ), г. Москва, РОССИЯ

**Аннотация:** Рассматривается подход по математическому определению падения напряжения в бетонных образцах при заданной постоянной деформации образца – релаксации напряжений в бетоне. Работа, в рамках которой были проведены данные исследования, является экспериментально-теоретической. В дополнение к математическому моделированию были проведены параллельные длительные испытания на образцах из бетона одного класса в четырех различных режимах: определение ползучести бетона при сжатии (бетонные образцы-призмы в пружинных установках) по ГОСТ 24544; определение релаксации бетона при сжатии (бетонные образцы призмы в специальных установках), определение ползучести бетона при изгибе (бетонные образцы-балки в установках стеллажного типа, работающие на изгиб и нагруженные гравитационной нагрузкой) по ГОСТ 24544; определение релаксации бетона при изгибе (бетонные образцы-балки, с приложенной начальной деформации в середине пролета

Определение падения напряжений выполнялось с помощью электронных динамометров, встроенных между образцом и точкой приложения нагрузки. Предложено новое понятие – «мера релаксации  $R_m$ ». Эта величина по смыслу подобна мере ползучести и характеризует степень снижения напряжений во времени из-за релаксации при загрузении единичной относительной деформацией. Введение меры релаксации позволяет упростить уравнение релаксации и существенно упрощает его решение. По результатам расчета по предложенным вычислительным алгоритмам, для определения меры релаксации

при сравнении с опытными данными, получена удовлетворительная сходимость результатов. Сделаны качественные выводы о применимости методики СП 63.13330 по снижению модуля упругости бетона при ползучести и при релаксации.

**Ключевые слова:** бетон, железобетон, ползучесть бетона, релаксация напряжений в бетоне

## INTRODUCTION

Usually, during the construction of unique buildings and structures using new high-strength concretes, experimental studies of creep and shrinkage characteristics of concrete are carried out. This direction is quite well studied [1-12]. At the same time, creep of concrete is just one of the deformation properties of a concrete specimen when working under the action of a long-term load. Another important property is stress relaxation.

The use of normative deformation characteristics of concretes even of the same strength class in compression in critical structures is not quite correct. Modern concretes having the same strength class may differ greatly in composition, having different modifiers and additives. Therefore, creep, relaxation and elasticity characteristics for them may be slightly different. This can be easily seen by analyzing the tabulated values of the initial modulus of elasticity, tensile strength of concrete (standard and design) - these figures depend directly on the compressive strength class of concrete and are in fact their minimum values with the required security. Thus, for example, in works [13-14] shows a significant increase in modulus of elasticity, relative to that specified in SP 63.13330 due to the use of stronger coarse aggregate. Thus, for concrete B100, values of initial modulus of elasticity of more than 50GPa are achieved. In works [15-16] the true tensile behavior of concrete is investigated; the obtained values also differ from the tabulated values given in the norms.

The creep testing of concrete specimens itself, on the basis of which all subsequent processing is carried out, also has two serious disadvantages. The first is the need to constantly check that a constant compressive force is maintained in the test rig. And since this condition is violated, the

compressive force must be adjusted at all times (this is especially true for spring systems, which are the most widely used for such tests). The second disadvantage is that the test setup is quite complex. As a result, only a few organizations in our country can perform such tests. Relaxation tests require simpler installations, in which specified deformations of specimens are maintained and forces in specimens are measured. Constant control and maintenance of such installations during testing is not required. This improves the accuracy of the results and makes the tests much simpler and more affordable.

In temperature and force redistribution problems, relaxation plays a more important role than creep. When performing such calculations, first the creep measure of concrete is determined, then the creep kernel is determined through the creep measure, and the relaxation kernel is determined through the creep kernel, then the predicted stresses in concrete at an arbitrary point in time are determined.

The introduction of the new test methodology will improve the accuracy of the results obtained, which in turn will make it possible to upgrade the accuracy of calculations, while at the same time with sufficient reliability will lead to a cheaper design. Simplification of the test method will allow the tests to be performed more widely. This in turn will enable objective data to be obtained on the properties of concrete over a wide range of compositions and strengths. In this regard, the topic under consideration is relevant and necessary for the development of modern construction industry.

Such studies have not been carried out either in our country or abroad. Therefore, the methodology of the proposed tests, their mathematical processing and results are new and essentially unique.

**METHODS**

The basic equations of modern creep theory have the form of three integral expressions (in addition to the equations of creep theory we can include expression (4) for the creep measure  $C(t, \tau)$  through which the relaxation kernel function is already determined) and consist of the creep equation, the relaxation equation and the equation connecting the creep and relaxation kernels  $L(t, \tau), R(t, \tau)$ , they are based on a large number of experimental and theoretical works [17-22]. According to the first equation, the relative strains through elastic modulus, stresses and the function responsible for the creep strain buildup, the creep kernel, are determined. Using the second equation it is possible to determine the stresses, respectively through the relative strains, elastic modulus and the function responsible for the stress drop - the relaxation kernel. Using the third equation, it is possible to define separately, for example, the relaxation kernel through the creep kernel, the data for which are more readily available.

$$\varepsilon(t) = \frac{\sigma(t)}{E(t)} - \int_{t_0}^t \sigma(\tau) L(t, \tau) \partial\tau \quad (1)$$

$$\sigma(t) = \varepsilon(t)E(t) + \int_{t_0}^t \varepsilon(\tau) R(t, \tau) \partial\tau \quad (2)$$

$$L(t, \tau) - R(t, \tau) = \int_{t_0}^t R(\tau) L(t, \tau) \partial\tau \quad (3)$$

$$C(t, \tau) = C_{\infty}^{28} \cdot \Omega(t_0) \cdot F(t - t_0) \quad (4)$$

In the above equations, the following notations are used:

$t$  - test time,  $\tau$  - loading time,  $C_{\infty}^{28}$  is the limiting value of the creep measure  $\Omega(\tau)$  is the ageing function,  $F(t - t_0)$  is the function determining the shape of the creep curve.

Based on the above equations, let us introduce a new concept - relaxation measure (5) and give it the following definition "Relaxation measure is a function that determines the stress drop-in

time when the specimen is loaded with a constant relative strain  $\varepsilon = 1$ . This equation is similar in structure to the creep measure. In this expression:

$\Omega(\tau)$  - the ageing function is assumed to be the same as in the calculation of the creep measure,  $Rm_{\infty}^{28}$  - defines the limiting value of the relaxation measure at the observation time  $\Phi(t - t_0)$  - defines the shape of the relaxation function and is to be determined. The following notations are used in the equations:

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$C_{\infty}^{28}$  is the limiting value of the creep measure  $\Omega(\tau)$  is the ageing function,  $\Phi(t - t_0)$  is the function determining the shape of the creep curve.

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$$Rm(t, \tau) = Rm_{\infty}^{28} \cdot 1 \frac{1}{\Omega(t_0)} \cdot \Phi(t - t_0) \quad (5)$$

In general, the functions  $Rm_{\infty}^{28}$  and  $\Phi(t - t_0)$  should be determined from experimental data, just as the functions  $C_{\infty}^{28}$  and  $F(t - t_0)$  were determined at one time. Since it is not possible to set up a large-scale experiment on concrete compositions with different strengths involving different laboratories, it is advisable to determine the above functions computationally from the creep equation (1) and the equation derived from it (8). This is due to the fact that detailed experimental data are available for the determination of all the quantities included in this equation. Therefore, it is reasonable to use

the same data to determine the relaxation parameters.

Next, let us derive an expression for the determination of the relaxation measure through the creep measure, for this purpose we write down the expressions for the total relative strain (6) at unit stresses and stresses at unit relative strain loading (7).

$$\lambda(t, \tau) = \frac{1}{E(t)} + Cm(t, \tau) \quad (6)$$

$$s(t, \tau) = E(t) - Rm(t, \tau) \quad (7)$$

Then the creep and relaxation equations will take the form (8), (9). According to the introduced definition, the relaxation measure is a function that determines the stress drop when the specimen is loaded with a constant relative strain  $\varepsilon = 1$ , then the creep equation will be written in the form 10.

$$\varepsilon(t, t_0) = \sigma(t_0) \cdot \lambda(t, t_0) + \int_{t_0}^t \frac{\partial}{\partial \tau} \sigma(\tau) \lambda(t, \tau) d\tau \quad (8)$$

$$\sigma(t, \tau) = \varepsilon(t_0) \cdot s(t, t_0) - \int_{t_0}^t \frac{\partial}{\partial \tau} \varepsilon(\tau) s(t, \tau) d\tau \quad (9)$$

$$s(t_0, t_0) \lambda(t, t_0) + \int_{t_0}^t \frac{\partial}{\partial \tau} s(t, \tau) \lambda(t, \tau) d\tau = 1 \quad (10)$$

Substituting (6) and (7) into (10), and considering that  $\frac{\partial}{\partial \tau} E(t) = 0$  (partial derivative on a variable that is not included in the function), and the value of  $Rm(t_0, t_0) = 0$  (relaxation does not have time to manifest itself at the initial time at loading), performing elementary transformations finally we obtain:

$$(E(t)) \left( \frac{1}{E(t)} + Cm(t, t_0) \right) - \int_{t_0}^t \frac{\partial}{\partial \tau} Rm(t, \tau) \left( \frac{1}{E(t)} + Cm(t, \tau) \right) d\tau = 1 \quad (11)$$

In this expression, the calculation of  $Rm(t, \tau)$  values are performed step by step at individual time points.

Within the framework of this study, a set of long-term tests was carried out. The research was carried out at the experimental site of JSC "VNIIG named after B.E. Vedeneev". The experimental part of the study is described in detail in [23].

Concrete specimens were made of two classes of compressive strength (B30 and B60), the tests were carried out from the age of 28 days. Compressive and flexural creep tests were carried out in accordance with GOST 24544. In parallel, tests were carried out to determine stress relaxation in concrete and creep in compression and bending for subsequent mathematical processing of the results.

Plants for determining the relaxation of concrete prisms in compression were prepared by dismantling spring creep test plants with the removal of springs in them to fix the required initial strains

Combined relaxation or flexural creep rigs were prepared by upgrading existing rack-type rigs for flexural creep testing. The upgraded plant allows flexural creep testing of beam specimens on the lower tier with constant gravity loading (using shot pails as loading devices) and concrete relaxation testing of beam specimens on the upper tier with constant strain loading.

Additionally, units were made to determine the relaxation in flexural mode by means of reverse loading. The essence of the method is that the specimen in the setup is in an inverted state to minimize creep strain, and the initial strain loading is from bottom to top.

Figure 1 below shows the compression and flexural relaxation test scheme, Figure 2 shows the test process.

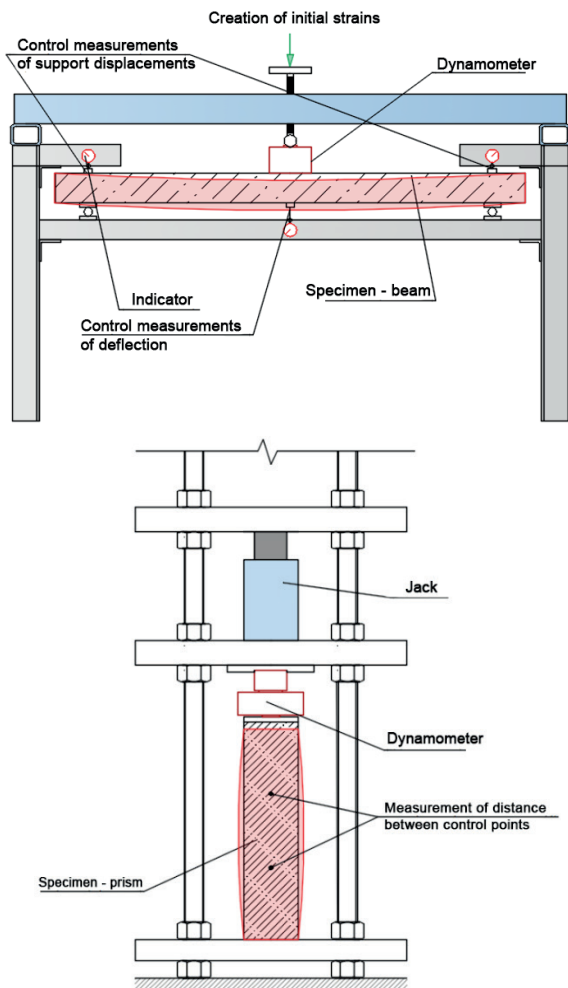


Figure 1. Schematic diagram of the relaxation test facilities

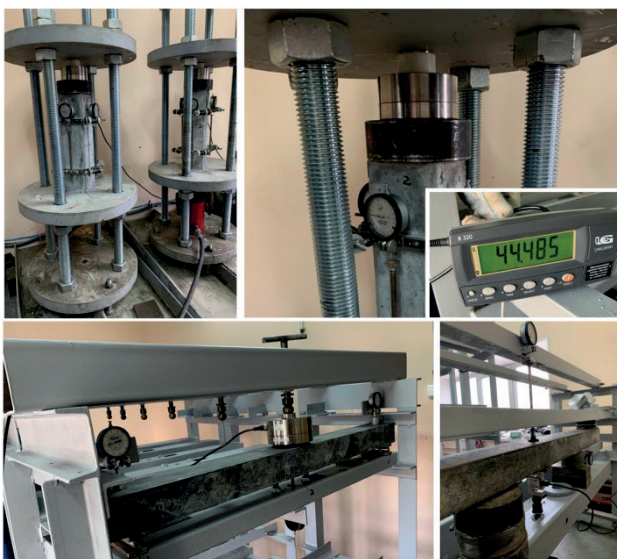


Figure 2. Compression (top) and bending (bottom) tests

## RESULTS AND DISCUSSION

Using formula (11), the relaxation measure was determined by calculation. Figure 3 below shows graphically the exact and approximate plot (based on the proposed methodology) of stress drop over time when  $R_m$  is calculated based on the calculation for concrete of class B30. It can be seen that the limiting value of the relaxation measure can be expressed through the limiting value of the creep measure. The ratio of the limiting degree of increase in the initial relative strains (at infinitely long time compared to the moment of loading) to the degree of decrease in the limiting stresses (also at infinitely long time compared to the moment of loading) is 1.98, i.e. practically 2. Taking this into account leads to the relations:

$$R_m(\infty, 28) \cdot C_m(\infty, 28) = 2 \quad (12)$$

$$R_m(\infty, 28) = \frac{2}{C_m(\infty, 28)} \quad (13)$$

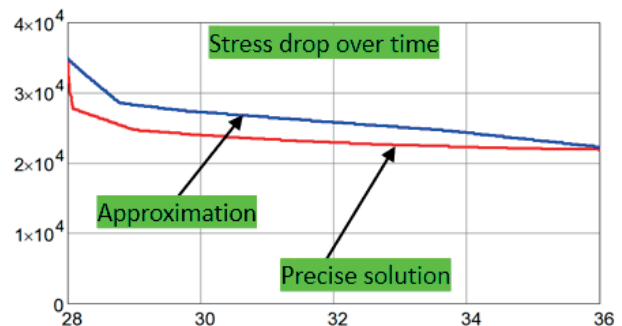


Figure 3. Calculated stresses (MPa) versus time (day) according to exact and approximate formula

Figure 4 shows a comparison of the calculation results using the proposed method with the simplified formula for the relaxation measure with the experimental curve. The graphs show that the error in the first day reaches 20%. After 7 days the error practically disappears. Taking into account that in technical and regulatory literature there are various expressions for the measure of creep, including those giving both very gentle and very steep graphs, calculations even at the initial stage after loading according to the above dependencies are acceptable.

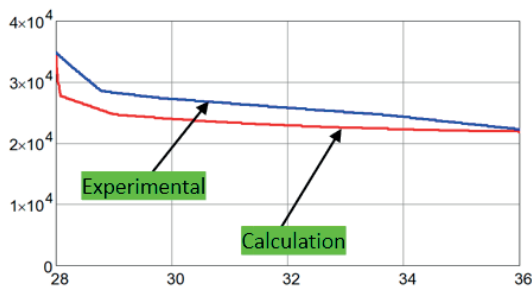


Figure 4. Comparison of stress relaxation curves (kN) over time (days)

## CONCLUSION

1. According to calculations and experimental data, the rate of development of creep deformations is faster than the rate of stress drop during relaxation (approximately by a factor of 2). Therefore, strictly speaking, it is impossible to take both into account by reducing the modulus of elasticity of concrete according to the methodology of SP 63.13330 [21]. The reduction of modulus of elasticity at creep and at relaxation should be different.
2. To simplify the calculations of deflections of structures, it is proposed to introduce the concept of relaxation measure  $R_m(t, \tau)$ . This value is similar in meaning to the creep measure and characterises the degree of stress reduction in time due to relaxation when loaded with a unit relative strain.
3. Calculation of  $R_m(t, \tau)$  values by the above equation (11) is performed step by step at separate time points. For calculations of stresses in real structures, it is necessary to convert the set of calculation results for the mentioned points into a function by selecting a suitable approximation.
4. Taking into account the relations between the limit values of the relaxation and creep measures, as well as the fact that the creep and relaxation phenomena are mutually related, it is proposed to use a function, also constructed on the basis of the creep measure, to approximate the relaxation measure at other points of time. This makes it possible to obtain an expression for the relaxation measure with acceptable accuracy without complicated calculations. Calculations show that good results can be obtained for the expression of the relaxation measure according to formula (5)

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*Krylov S.B*, Doctor of Engineering Sciences. JSC Research Center of Construction NIIZHВ named after A.A. Gvozdev, Head of Laboratory of Reinforced Concrete Mechanics, 109428, 6, building 5., 2nd Institutskaya str., Moscow, Russia, e-mail: Krylov\_s\_b@mail.ru, telephone +7-499-174-74-07

*Крылов Сергей Борисович*, доктор технических наук, заведующий лабораторией Механики железобетона, Научно-исследовательский, проектно-конструкторский и технологический институт бетона и железобетона (НИИЖБ) им. А. А. Гвоздева АО «НИЦ «Строительство», 109428, Москва, 2-я Институтская ул., д.6, корп. 5, тел.: +7-499-174-74-07, e-mail: Krylov\_s\_b@mail.ru

*Arleninoff P.D.*, Department of Reinforced Concrete and Masonry Structures, National Research Moscow State University of Civil Engineering, 129337, Yaroslavskoye shosse, 26, Moscow, Russia; JSC Research Center of Construction NIIZHВ named after A.A. Gvozdev, laboratory of reinforced concrete mechanics, 109428, 6, building 5., 2nd Institutskaya str., Moscow, Russia, e-mail: arleninoff@gmail.com, telephone +7-499-174-74-07

*Арленинов Петр Дмитриевич*, кандидат технических наук, ФГБОУ ВО «Национальный исследовательский Московский государственный строительный университет» Минобрнауки России (НИУ МГСУ), 129337, Москва, Ярославское шоссе, д. 26; заместитель заведующего лаборатории Механики железобетона, Научно-исследовательский, проектно-конструкторский и технологический институт бетона и железобетона (НИИЖБ) им. А.А. Гвоздева АО «НИЦ «Строительство», 109428, Москва, 2-я Институтская ул., д.6, корп. 5. тел.: +7-499-174-74-07, e-mail: arleninoff@gmail.com