

DEVELOPMENT OF COMPUTATIONAL SCHEMES OF GENERALIZED KINEMATIC DEVICES THAT PRECISELY REGULATE THE NATURAL FREQUENCY SPECTRUM OF ELASTIC SYSTEMS WITH A FINITE NUMBER OF DEGREES OF MASS FREEDOM, IN WHICH THE DIRECTIONS OF MOTION ARE PARALLEL, BUT DO NOT LIE IN THE SAME PLANE PART 2: SAMPLES OF ANALYSIS

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Abstract: To date, for some elastic systems with a finite number of degrees of mass freedom, in which the directions of mass movement are parallel and lie in the same plane, methods have been developed for creating additional generalized targeted constraints and generalized targeted kinematic devices. Each generalized targeted constraint increases, and each generalized targeted kinematic device reduces the value of only one selected natural frequency to a predetermined value, without changing the remaining natural frequencies and natural modes. Earlier, for elastic systems with a finite number of degrees of mass freedom, in which the directions of mass motion are parallel, but do not lie in the same plane (for example, plates), an approach for the computing of a matrix of additional stiffness and a method for the development of computational schemes of additional generalized targeted constraints were developed. Also earlier, for such systems, an approach was proposed for the computing of a special matrix with allowance for additional inertial forces that determine a generalized targeted kinematic device. At the same time, the method of development of computational schemes of kinematic devices was not proposed. The first part of the paper was devoted to approach, that made it possible to develop computational schemes of generalized targeted kinematic devices for such systems as well. A variant of the computational scheme of constraint for the rod system with one degree of activity was considered. Some special properties of such targeted kinematic devices were revealed. The distinctive second part of the paper is devoted to corresponding samples of analysis.

Keywords: localization, natural frequency, natural mode, additional generalized targeted constraint, stiffness coefficients, inertial forces, generalized targeted kinematic device, samples of analysis

ФОРМИРОВАНИЕ РАСЧЕТНЫХ СХЕМ ОБОБЩЕННЫХ КИНЕМАТИЧЕСКИХ УСТРОЙСТВ, ПРИЦЕЛЬНО РЕГУЛИРУЮЩИХ СПЕКТР ЧАСТОТ СОБСТВЕННЫХ КОЛЕБАНИЙ УПРУГИХ СИСТЕМ С КОНЕЧНЫМ ЧИСЛОМ СТЕПЕНЕЙ СВОБОДЫ МАСС, У КОТОРЫХ НАПРАВЛЕНИЯ ДВИЖЕНИЯ ПАРАЛЛЕЛЬНЫ, НО НЕ ЛЕЖАТ В ОДНОЙ ПЛОСКОСТИ ЧАСТЬ 2: ПРИМЕРЫ РАСЧЕТА

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Аннотация: К настоящему времени для некоторых упругих систем с конечным числом степеней свободы масс, у которых направления движения масс параллельны и лежат в одной плоскости, разработаны методы создания дополнительных обобщенных прицельных связей и обобщенных прицельных кинематических устройств. Каждая обобщенная прицельная связь увеличивает, а каждое обобщенное прицельное кинематическое устройство уменьшает величину лишь одной выбранной собственной частоты до наперед заданного значения, не изменяя при этом остальные собственные частоты и формы собственных колебаний. Ранее для упругих систем с конечным числом степеней свободы масс, у которых направления движения масс параллельны, но не лежат в одной плоскости (например, пластины), разработан подход образования матрицы дополнительных жесткостей и метод формирования расчетных схем дополнительных обобщенных прицельных связей. Также ранее для таких систем предложен подход образования матрицы учета действия дополнительных инерционных сил, определяющих обобщенное прицельное кинематическое устройство. При этом способ формирования расчетных схем кинематических устройств, предложен не был. В первой части статьи был предложен подход, позволяющий формировать расчетные схемы обобщенных прицельных кинематических устройств и для таких систем. Был рассмотрен вариант расчетной схемы связи, представленный стержневой системой с одной степенью активности, выявлены некоторые особые свойства таких прицельных кинематических устройств. В настоящей второй части статьи рассматриваются соответствующие примеры расчетов.

Ключевые слова: частота собственных колебаний, форма собственных колебаний, дополнительная обобщенная прицельная связь, коэффициенты жесткости, инерционные силы, обобщенное прицельное кинематическое устройство, примеры расчетов

Let's consider a hingedly supported rectangular plate measuring 6 m by 6 m , carrying concentrated masses (Figure 1a). The thickness of the plate is equal to 0.12 m . The modulus of elasticity of the plate material is equal to $E = 2.4 \cdot 10^{10}$. Poisson's ratio is equal to $\nu_0 = 0.2$.

Let's select the main system of the displacement method (Figure 1b) and construct the corresponding system of equations (1) from [1] (matrices $A = \|r[i, k]\|$, $M = \|m[i]\|$ from [1]). From equation (2) from [1] we determine the natural frequencies and natural modes of the plate. The values of the frequencies of natural vibrations of the plate and the coordinates of the corresponding natural modes are presented in Table 1 (columns contain natural frequencies and coordinates of natural modes). Such a plate was also considered in [8, 11]. The results, presented here in Table 1 do not differ significantly from the corresponding results in [8, 11]. This difference is due to the different accuracy of computing the coefficients of matrix $A = \|r[i, k]\|$ (matrix (1) from [1]). In [8, 11] the coefficients were determined based on the plate analysis with the use of Ritz method with retention of about thirty terms of the

approximating series. In this work, SCAD software was used to compute the coefficients.

Let's assume that it is required to reduce the value of the first frequency of natural vibrations to 40 sec^{-1} .

To do this, in accordance with (4), (5), (8), (10) from [1], we can form a matrix of coefficients with allowance for the action of additional inertial forces (12). After forming this matrix, we can determine from equation (11) from [1] the modified spectrum of natural frequencies and the corresponding natural modes with allowance for the action of additional inertial forces (12) from [1]. The modified spectrum of natural frequencies and their corresponding natural modes are shown in Table 2.

The results, presented in Table 2, confirm that taking into account the matrix of additional inertial forces did not change any of the natural modes, but only reduced the value of only the first frequency of natural vibrations to a given value of 40 sec^{-1} .

The targeted kinematic device must correspond to the matrix of additional inertial forces (12). One of the variants of computational scheme of targeted kinematic device is shown in Figure 2a and Figure 2b. The accepted variant is represented by a statically determinate rod system.

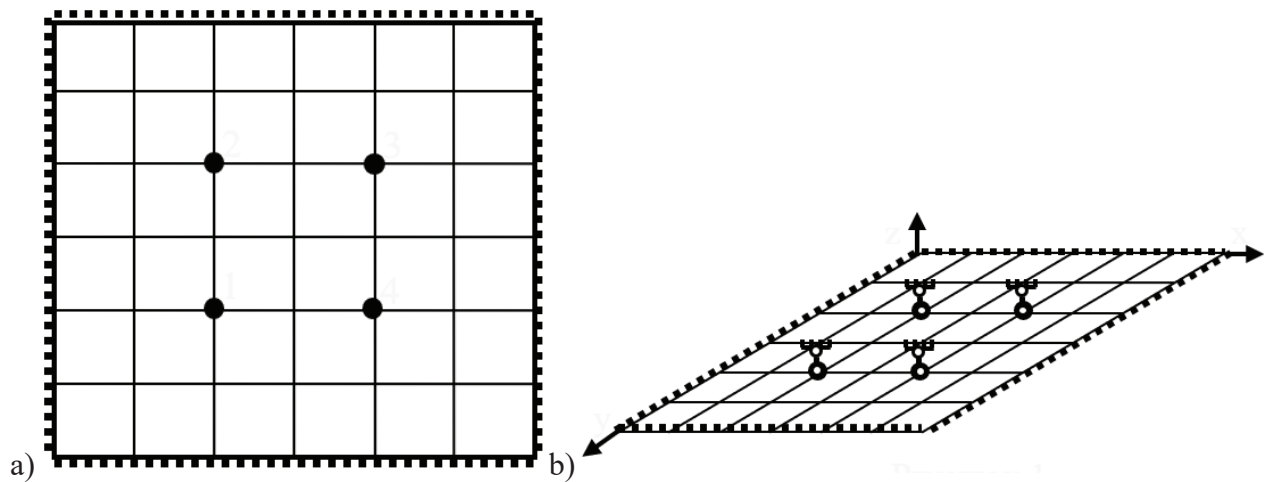


Figure 1. Considering system: a) initial problem; b) the main system of the displacement method.

Table 1. Results of analysis (natural frequencies and natural modes).

ω	60,932	138,865	143,624	196,414
1	0.4905	0.0001	0.7047	-0.5934
2	0.4966	-0.7075	0.0945	0.5167
3	0.5058	-0.0711	-0.7029	-0.4387
4	0.5069	0.7032	0.0190	0.4341

Table 2. Results of analysis (natural frequencies and natural modes).

ω	40.000	138.865	143.624	196.414
1	0.4905	0.0001	0.7047	-0.5934
2	0.4965	-0.7075	0.0945	0.5167
3	0.5058	-0.0711	-0.7029	-0.4387
4	0.5070	0.7032	0.0190	0.4341

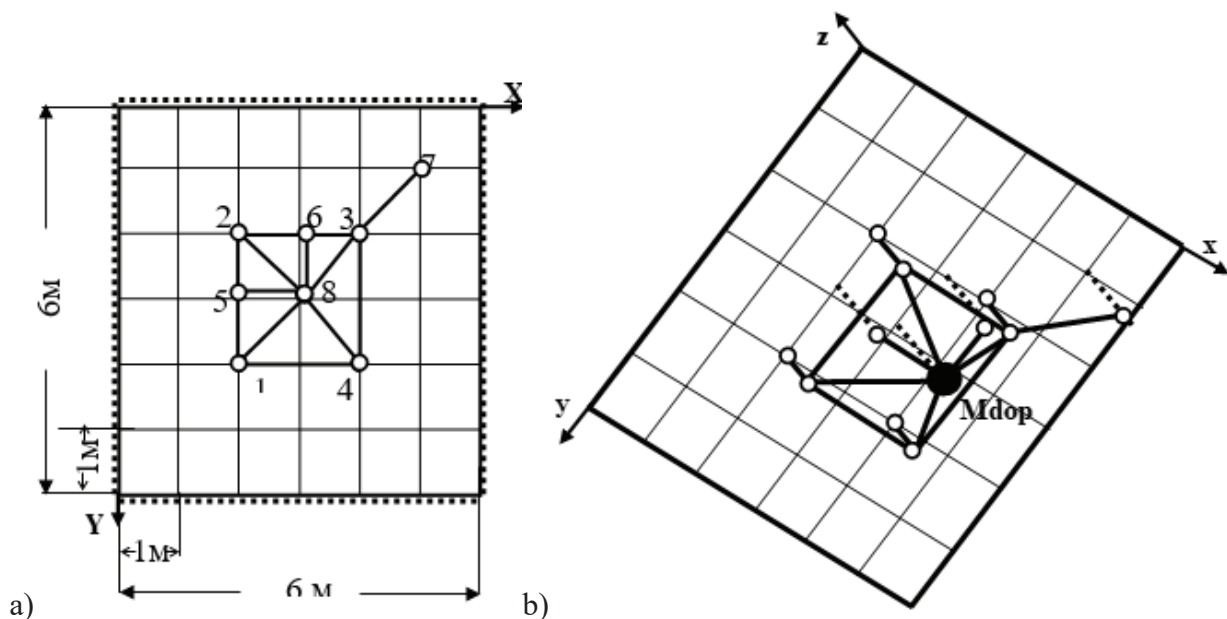


Figure 2. The first sample: a) device plan; b) general view of device.

Additional mass is applied at node 8. Rods 8-5 and 8-6 provide the additional mass with only one degree of freedom, namely the ability to move in parallel with the movements of the remaining masses during their natural vibrations. The geometric parameters of the kinematic device are the lengths of the racks

$$l_{st}[i] \ (i=1,2,...,n)$$

and the coordinates of nodes 5, 6, 7, 8

$$X[i], Y[i], Z[i], \ X_0, Y_0, Z_0.$$

In the considering sample the length of the “base” rack $l_{st}[1] = -0.5 \text{ m}$ and the coordinates of some nodes are selected. Table 3 shows the accepted values of these coordinates in numbers.

Table 3. Coordinates of nodes.

Nodes	Coordinates		
	$X[i]$	$Y[i]$	$Z[i]$
1	2	4	-0.5
2	2	2	$l_{st}[2]$
3	4	2	$l_{st}[3]$
4	4	4	$l_{st}[4]$
5	2	Y_0	Z_0
6	X_0	2	Z_0
7	5	1	Z_0
8	X_0	Y_0	Z_0

The remaining cells show variable geometric parameters that form a space in which their values are found, giving the target function (15) from [1] a minimum. In order to define the target function, it is necessary to determine the inertial forces

$$\bar{R}_0[i] \ (i=1, 2, 3, 4).$$

After the formation of the matrix of additional inertial forces (12) from [1], the values $\bar{R}_0[i] \ (i=1, 2, 3, 4)$ are determined from dependence (14) from [1]. In the considering sample we have

$$\bar{R}_0[1] = -1036326.342; \ \bar{R}_0[2] = -1153948.433; \\ \bar{R}_0[3] = -1228846.563; \ \bar{R}_0[4] = -128231.533.$$

To assess proximity to the minimum, a small value OOO is assigned. In the considering sample we define $OOO = 0.00001$. We have the following initial values of varied parameters:

$$l_{st}[2] = -0.52 \text{ m}; \ l_{st}[3] = -0.4 \text{ m}; \\ l_{st}[4] = -0.42 \text{ m}; \ X_0 = 3.15 \text{ m}; \\ Y_0 = 2.95 \text{ m}; \ Z_0 = -1 \text{ m}.$$

In order to search for the minimum of the target function (15) from [1] using the above algorithm in the space of variables, the random search method was used. Equilibrium equations were constructed for the nodes, located at the tops of the racks and node 8, to which additional mass will be applied. The found lengths of the rods of the kinematic device and forces in them are given in Table 4.

Table 4. Lengths of the rods of kinematic device and forces in them.

$l_{st}[1] = 0.5000$	$N_{st}[1] = -1036326.34$
$l_{st}[2] = 0.5137$	$N_{st}[2] = -1153948.44$
$l_{st}[3] = 0.4026$	$N_{st}[3] = -1228846.56$
$l_{st}[4] = 0.4172$	$N_{st}[4] = -1285231.53$
$l_p[8,5] = 1.1549$	$N_p[8,5] = 0.00$
$l_p[8,6] = 0.9846$	$N_p[8,6] = 0.00$
$l_p[7,3] = 1.5360$	$N_p[7,3] = 0.00$
$l_p[8,1] = 1.6177$	$N_p[8,1] = -2871872.91$
$l_p[8,2] = 1.5942$	$N_p[8,2] = -2918886.49$
$l_p[8,3] = 1.4294$	$N_p[8,3] = -3576078.34$
$l_p[8,4] = 1.4448$	$N_p[8,4] = -3504917.59$
$l_p[1,2] = 1.0001$	$N_p[1,2] = 1802838.58$
$l_p[2,3] = 1.0062$	$N_p[2,3] = 2127441.20$
$l_p[3,4] = 1.0001$	$N_p[3,4] = 2463557.45$
$l_p[1,4] = 1.0034$	$N_p[1,4] = 2057237.61$

From Table 4 it is clear that the forces in the racks in absolute values coincide quite accurately with the forces $\bar{R}_0[i]$. This circumstance confirms the minimum of the target function (12) and the fulfillment of the requirement that the relationships between forces $N_{st}[i]$ be proportional to the relationships between values $\bar{R}_0[i]$.

The values $\bar{R}_0[i]$ and forces in the rods of the kinematic device (14) from [1] depend on the coordinates of the natural modes $v_\omega[k, j]$. The coordinates $v_\omega[k, j]$ are the coordinates of the natural mode normalized to unity. In the process of real small vibrations, the coordinate values will be significantly less than in the normalized form, and therefore the forces in the rods of the kinematic device will also decrease. In the process of creation of kinematic device we defined following values of the remaining variable parameters:

$$X_0 = 3.15486; \quad Y_0 = 2.98459; \\ Z_0 = -1.002116.$$

After creation of computational scheme of the kinematic device, the value of additional mass M_{add} is determined. Before the creation of the kinematic device the considering system had four degrees of freedom. The kinematic device is formed in such a way that M_{add} is able to move only in parallel with the movements of other masses. Therefore, only one more degree of freedom is added in the considering system. Thus, now this system has five degrees of freedom. Now the system has an “additional” frequency. As noted above, the value of the “additional” natural frequency depends on the compliance parameters of the rods of the kinematic device. It can be either greater than the values of all “basic” frequencies or less than any of them.

Two variants of determining the value M_{add} are considered in the sample. Within the first variant, the amount of additional mass is determined without taking into account the influence of the compliance of the rods of the kinematic device. Within the second variant compliance is taken into account according to the method outlined above.

In the first variant the movement of additional mass v_m was determined under the condition that the kinematic device moves as an absolutely rigid body only due to the displacement of the nodes in which the masses of the original system are located. Within this variant the value

$$v_m = \sum_{i=1}^4 v_\omega[i, (q)] N_{st,1}[i] = -0.500478$$

was determined in the sample by (21) from [1]. The forces in the racks from the action of a unit force applied to the node where the additional mass is located in the direction of its movement are determined from the equilibrium conditions of the nodes of the kinematic device. We have

$$N_{st}[1] = 0.220291; \quad N_{st}[2] = 0.245294; \\ N_{st}[3] = 0.261215; \quad N_{st}[4] = 0.273200.$$

The additional mass within this variant is determined by (20) from [1]:

$$M_{add} = 5874.825 \text{ kg}.$$

With this mass, three test analysis were performed with the use of SCAD software for different values of the parameters of rods of the kinematic device. All rods are assumed to have a round solid cross-section.

Modulus of elasticity of the rod material. Different diameters of rod sections were considered. The results are presented in tables 5, 6 and 7. As noted above, the targeting of the kinematic device is realized only in relation to the “basic” frequencies of natural vibrations. Therefore, after placing additional mass and increasing the number of degrees of freedom of the system by one, the normalization of the natural modes is carried out according to the “basic” four coordinates. This normalization allows researcher to compare the proper modes that arose before the creation of the kinematic device with the modes that arose after its introduction. The “additional” frequency and the corresponding natural mode, normalized by the “basic” coordinates, are shown in the rightmost column of the tables.

Table 5. Results of analysis for $D = 100$ mm (natural frequencies and natural modes).

ω	38,79	137,62	142,33	194,68	357,12
1	0.4906	-0.00011	0.7047	-0.5945	0.4908
2	0.4966	0.7074	0.0944	0.5178	0.4965
3	0.5058	0.0711	-0.7029	-0.4350	0.5058
4	0.5069	-0.7032	0.01898	0.4350	0.5067

Table 6. Results of analysis for $D = 60$ mm (natural frequencies and natural modes).

ω	38,21	137,62	142,33	194,68	217,54
1	0.4906	-0.00012	0.7047	-0.5945	0.4913
2	0.4966	0.7074	0.0944	0.5170	0.4961
3	0.5058	0.0711	-0.7029	-0.4384	0.5061
4	0.5069	-0.7032	0.01889	0.4344	0.5063

Table 7. Results of analysis for $D = 20$ mm (natural frequencies and natural modes).

ω	31,67	137,62	142,33	194,68	87,49
1	0.4906	-0.00058	0.7047	-0.5935	0.4905
2	0.4966	0.7075	0.0945	0.5166	0.4965
3	0.5058	0.0711	-0.7029	-0.4388	0.5058
4	0.5069	-0.7031	0.01897	0.4341	0.5070

Table 8. Summary of results.

D	100 mm	60 mm	20 mm
ω	38.79	38.21	31.67
Δ , %	3.025	4.475	20.875

Analysis of the test results confirms the targeting of the kinematic device. In all three test calculations, the shapes of natural oscillations of the “basic” frequencies coincide with the accepted accuracy with those given in Tables 1 and 2. The values of the “basic” natural frequencies, except for the first, changed by fractions of a percent. The first frequency differed from the intended value (40 sec^{-1}) depending on the compliance of rods of the kinematic device. With increasing compliance, the value of the first frequency found decreases (Table 8). In Table 8 we have

$$\Delta = ((40 - \omega_1) / 40) \cdot 100\% .$$

In the SCAD software, the compliance of rods of the kinematic device was taken into account. The additional mass at which the tests were performed in the considered variant was determined without taking into account the compliance of

rods of the kinematic device. The test results confirm that the value of the “additional” natural frequency (ω_D) depends on the compliance parameters of rods of the kinematic device. At $D = 100 \text{ mm}$ and $D = 60 \text{ mm}$ the value does not fall into the “basic” part of the spectrum. At $D = 20 \text{ mm}$ for $\omega_D = 87.49 \text{ sec}^{-1}$ it turns out to be the second quantity in the full spectrum of natural frequencies.

In the sample additional test analysis was performed with the use of SCAD software for different values of the parameters of the rods of the kinematic device, but taking into account the influence of compliance according to the method outlined above. The parameters of rods of the kinematic device were taken to be the same as in previous tests. The results are presented in tables 9, 10 and 11.

Analysis of the results of these tests confirms the targeting of the kinematic device.

Table 9. Results of analysis for $D = 100 \text{ mm}$, $M_{add} = 57.62044 \text{ kN}$ (natural frequencies and natural modes (Z)).

Nodes	39,5	137,52	142,14	194,64	375,93
1	0.4860	0.0379	0.7246	-0.5864	0.4523
2	0.4912	0.7270	0.0969	0.5247	0.4335
3	0.5117	0.0685	-0.6793	-0.4279	0.5720
4	0.5105	-0.6822	0.0641	0.4447	0.5294

Table 10. Results of analysis for $D = 60 \text{ mm}$, $M_{add} = 55.717807 \text{ kN}$ (natural frequencies and natural modes (Z)).

Nodes	39,37	137,49	142,08	194,59	230,37
1	0.4860	0.0494	0.7305	-0.5759	0.4859
2	0.4912	0.7326	0.0973	0.5359	0.3939
3	0.5117	0.0678	-0.6713	-0.4119	0.6060
4	0.5105	-0.6754	0.0788	0.4598	0.5170

Table 11. Results of analysis for $D = 20 \text{ mm}$, $M_{add} = 39.441538 \text{ kN}$ (natural frequencies and natural modes (Z)).

Nodes	38,08	137,66	142,42	194,69	93,97
1	0.4849	-0.0176	0.6955	-0.5944	0.5092
2	0.4912	0.6980	0.09282	0.5155	0.5179
3	0.5123	0.0725	-0.7126	-0.4403	0.4816
4	0.5110	-0.7121	-0.0001	0.4325	0.4905

Table 12. Summary of results.

D	100 mm	60 mm	20 mm
ω	39.5	39.37	38.08
Δ , %	1,225	1.575	4.80

The values of the “basic” natural frequencies, except for the first one, changed by fractions of a percent. The first frequency differed from the intended value (40 sec^{-1}) depending on the compliance of rods of the kinematic device. With increasing compliance, the value of the first frequency found decreases (Table 12). A comparison of the data in Tables 8 and 12 shows that allowance for the compliance of rods of the kinematic device according to the method outlined above allows researcher to reduce the difference between the intended value (40 sec^{-1}) and the results obtained in the tests. Tests also showed that the method of taking into account the compliance of the rods of a kinematic device requires further analysis and improvement.

The results of tests carried out with allowance for the influence of the compliance of the rods of the kinematic device according to the method outlined above also confirm that the value of the “additional” natural frequency (ω_D) depends on the compliance parameters of the rods of the kinematic device. In cases of $D = 100 \text{ mm}$ and $D = 60 \text{ mm}$ the value of the “additional” frequency ω_D does not fall into the “basic” part of the spectrum of natural frequencies. In case of $D = 20 \text{ mm}$ and $\omega_D = 93.97 \text{ sec}^{-1}$ it turns out to be the second value in the full spectrum of natural oscillation frequencies.

The test results with allowance for compliance of the rods of the kinematic device show that the

shapes of natural vibrations are close to those given in Tables 1 and 2.

Thus, [1] and the distinctive paper propose an approach that allows researcher to create computational schemes of targeted kinematic devices for elastic systems with a finite number of degrees of mass freedom, in which the directions of mass motion are parallel, but do not lie in the same plane. The necessity of taking into account the influence of the compliance of the rods of the kinematic device when determining the value of additional mass is shown. The need for further analysis and improvement of the corresponding methodology was also noted.

Some patterns of changes in the spectrum of frequencies of natural vibrations of the system have been identified after the formation of the targeted kinematic device.

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