DETERMINATION OF CONCRETE RHEOLOGICAL PARAMETERS USING NONLINEAR OPTIMIZATION METHODS

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Abstract. The article proposes a method for processing concrete creep curves based on the nonlinear equation of V.M. Bondarenko. The experimental data of A.V. Yashin is used. The problem of finding rheological parameters and the nonlinearity function is posed as a nonlinear optimization problem. The objective function represents the sum of the squared deviations of the experimental values of the creep strain from the theoretical values for all creep curves for one concrete at different stress levels. The minimum of the objective function is found using the interior point method, the surrogate optimization method, the pattern search method, the genetic algorithm, and the particle swarm method. It has been established that the first of these methods has the greatest efficiency. The proposed approach provides high quality approximation of experimental curves at all stress levels. It is shown that for concrete the nonlinearity of creep deformations is more pronounced than the nonlinearity of instantaneous deformations, and the same function cannot be used to describe these two types of nonlinearity.

Keywords: concrete, creep, viscoelasticity, viscoplasticity, nonlinearity

ОПРЕДЕЛЕНИЕ РЕОЛОГИЧЕСКИХ ПАРАМЕТРОВ БЕТОНА ПРИ ПОМОЩИ МЕТОДОВ НЕЛИНЕЙНОЙ ОПТИМИЗАЦИИ

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Аннотация: В статье предлагается методика обработки кривых ползучести бетона на основе нелинейного уравнения В.М. Бондаренко. Используются экспериментальные данные А.В. Яшин. Задача поиска револогических параметров и функции нелинейности ставится как задача нелинейной оптимизации. Целевая функция представляет сумму квадратов отклонений экспериментальных значений деформации ползучести от теоретических по всем кривым ползучести для одного бетона при различных уровнях напряжений. Минимум целевой функции отыскивается при помощи метода внутренней точки, метода суррогатной оптимизации, метода шаблонного поиска, генетического алгоритма и метода роя частиц. Установлено, что наиболее эффективно обладает первый из указанных методов. Предлагаемый подход обеспечивает высокое качество аппроксимации экспериментальных кривых при всех уровнях напряжений. Показано, что для бетона нелинейность деформаций ползучести более выражена, чем нелинейность мгновенных деформаций, и для описания этих двух видов нелинейности нельзя использовать одну и ту же функцию.

Ключевые слова: бетон, ползучесть, вязкоупругость, вязкопластичность, нелинейность

INTRODUCTION

There are a large number of different theories of nonlinear concrete creep in the literature, which differ in the set of hypotheses used and approaches to their construction [1-10]. One of the first variants of the nonlinear theory of concrete creep, which generalized the linear Harutyunyan-Maslov equation, was the theory of N.Kh. Harutyunyan and P.I. Vasiliev [11-13]:

\[ \varepsilon(t) = \frac{\sigma(t)}{E(t)} - \int_{\tau_0}^{t} \frac{\partial}{\partial \tau} \left[ \frac{1}{E(\tau)} \right] \sigma(\tau) d\tau - \int_{\tau_0}^{t} \frac{\partial C(t, \tau)}{\partial \tau} F[\sigma(\tau)] d\tau, \]

(1)

where \( C(t, \tau) \) is the measure of creep, \( F[\sigma(\tau)] \) is the nonlinearity function, \( \tau_0 \) the
initial moment of time at which the load is applied. Equation (1) takes into account the nonlinear component of creep deformations, but does not take into account the instantaneous nonlinearity of deformation.

Another version of the Harutyunyan-Maslov equation generalization, which allows taking into account both nonlinear creep and instantaneous deformation nonlinearity, was proposed by Yu. N. Rabotnov [14]:

$$f\left[\varepsilon(t)\right] = \frac{\sigma(t)}{E(t)} - \int_{\tau_0}^{t} \frac{\partial}{\partial \tau} \left[ \frac{1}{E(\tau)} \right] \sigma(\tau) d\tau -$$

$$- \int_{\tau_0}^{t} \frac{\partial C(t, \tau)}{\partial \tau} \sigma(\tau) d\tau,$$  

(2)

where $f(\varepsilon)$ is the non-linear deformation function. Close in essence to equation (2) is the viscoelastic-plastic model of hereditary aging of concrete, proposed by A.G. Tamrazyan [15]:

$$\varepsilon(t) = \frac{f(\sigma(t))}{E(t)} - \int_{\tau_0}^{t} \frac{\partial C(t, \tau)}{\partial \tau} f(\sigma(\tau)) d\tau.$$  

(3)

Equations (2) and (3) postulate the same nature of nonlinearity for instantaneous and long-term deformations, which does not correspond to the real rheological behavior of concrete. More general is the equation of V.M. Bondarenko [16], having the form:

$$\varepsilon(t) = \frac{\int f[s(\tau)]}{E(t)} - \int_{\tau_0}^{t} \frac{\partial}{\partial \tau} \left[ \frac{1}{E(\tau)} \right] f_1[s(\tau)] d\tau -$$

$$- \int_{\tau_0}^{t} \frac{\partial C(t, \tau)}{\partial \tau} f_2[s(\tau)] d\tau,$$  

(4)

where $s(t) = \sigma(t) / R(t)$, $R(t)$ is the instant concrete strength, $f_1$ and $f_2$ are nonlinear functions corresponding to instantaneous and long-term deformation.

The instantaneous nonlinearity of concrete deformation is well studied, and the definition of the function $f_1$ does not cause great difficulties, which cannot be said about the function $f_2$.

The purpose of this work is to develop a methodology for finding the function $f_2$, as well as other rheological parameters that determine the measure of creep, from concrete creep curves at various stress levels.

**METHODS**

Let us consider the procedure for processing creep curves using the example of the curves presented in the work of A.V. Yashin [17]. In this paper, concrete with the compressive strength $R_0 = 30$ MPa was tested at the age of $\tau_0 = 28$ days, with an initial modulus of elasticity $E_0 = 4 \cdot 10^4$ MPa at stress levels $\sigma / R_0$ from 0.4 to 0.8 with a step of 0.1.

Thus, the total number of experimental curves for one class of concrete was 5.

As a measure of creep, the expression proposed by A.G. Tamrazyan was used [15]:

$$C(t, \tau) = C \left( \frac{e^{\alpha t} - e^{\alpha \tau}}{e^{\alpha \tau} - 1} + B \left( e^{-\gamma t} - e^{-\gamma \tau} \right) \right).$$  

(5)

This expression contains 4 rheological parameters $C, B, \alpha, \gamma$ to be determined. The advantage of the creep measure (5) is that its exponential form allows to pass from the integral creep law to the differential one and to calculate creep strains using the Euler or Runge-Kutta method. This transition is shown in [18]. Since the experimental data of A.V. Yashin do not contain information about the aging of concrete (change in time of its modulus of elasticity), then we will neglect this effect, i.e. we omit in equation (4) the term containing

$$\frac{\partial}{\partial \tau} \left[ \frac{1}{E(\tau)} \right].$$
Then the dependence of the creep strain on time at a constant stress level $s = \sigma / R$ takes the form:

$$
\varepsilon_{cr}(t) = f_2(s) \left( C \frac{e^{\alpha_1 t} - e^{\alpha_2 t}}{e^{\alpha_1} - 1} + B \left( e^{-\gamma_1 t} - e^{-\gamma_2 t} \right) \right). (5)
$$

We pose the problem of determining the parameters $C, B, \alpha, \gamma$, as well as the function $f_2(s)$, as a problem of nonlinear optimization. The objective function $Z$ is built, the input parameters of which are $C, B, \alpha, \gamma$, as well as 5 coefficients $k_i$ according to the number of experimental curves, taking into account the nonlinear creep component depending on the stress level.

For the time points $t_j$ at which the strains were measured, the objective function calculates the theoretical values of the creep strains using the formula:

$$
\varepsilon_{cr,j}(t_j) = k_i \sigma_j \left( C \frac{e^{\alpha_1 t_j} - e^{\alpha_2 t_j}}{e^{\alpha_1} - 1} + B \left( e^{-\gamma_1 t_j} - e^{-\gamma_2 t_j} \right) \right),
$$

(6)

where $\sigma_j$ is the stress in $i$-th experiment.

Next, the objective function $Z$ calculates the sum of the squared deviations of the theoretical values of creep strains from the experimental values, multiplied by 1000, over all experimental curves:

$$
Z = \sum_{j=1}^{n} \sum_{i=1}^{m} \left( [\varepsilon_{cr,j,\text{theor}}(t_j) - \varepsilon_{cr,j,\text{test}}(t_j)] \cdot 1000 \right)^2. (7)
$$

The objective function $Z$ must reach a minimum.

The coefficients $k_i$ must be greater than or equal to 1. We accept the range from 1 to 20 for them. The ranges for the parameters $C, B, \alpha, \gamma$ are presented in Table 1.

### Table 1. Rheological parameters ranges

<table>
<thead>
<tr>
<th>parameter</th>
<th>$\alpha$, 1/day</th>
<th>$\gamma$, 1/day</th>
<th>$C$, MPa$^{-1}$</th>
<th>$B$, MPa$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>min</td>
<td>$10^{-4}$</td>
<td>$10^{-4}$</td>
<td>$10^{-5}$</td>
<td>$10^{-5}$</td>
</tr>
<tr>
<td>max</td>
<td>0.1</td>
<td>0.1</td>
<td>$10^{-4}$</td>
<td>$10^{-4}$</td>
</tr>
</tbody>
</table>

The solution to the problem of creep curves processing was implemented in the MATLAB environment using the Optimization Toolbox and Global Optimization Toolbox packages. The following optimization methods were used to find the minimum of the objective function:

1. Interior point method [19]
2. Surrogate optimization method [20]
3. Pattern search method [21]
4. Genetic algorithm [22]
5. Particle swarm method [23-24].

The first of these methods allows to find a local minimum in the vicinity of the starting point of the search, the remaining methods are searching for a global minimum.

At the starting point of the search, the coefficients $k_i$ were taken equal to 1. The values recommended in [15] were taken as the initial values for $C, B, \alpha, \gamma$. They are given in Table 2.

### Table 2. Values of rheological parameters at the starting point of the search

<table>
<thead>
<tr>
<th>$\alpha$, 1/day</th>
<th>$\gamma$, 1/day</th>
<th>$C$, MPa$^{-1}$</th>
<th>$B$, MPa$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.032</td>
<td>0.062</td>
<td>$3.77 \cdot 10^{-5}$</td>
<td>$5.68 \cdot 10^{-5}$</td>
</tr>
</tbody>
</table>

Note that the quality of the A.V. Yashin’s experimental curves approximation with the indicated in the Table 2 values is unsatisfactory. They are probably defined for concrete of a different class or composition.

### RESULTS AND DISCUSSION

Table 3 shows the values of the objective function obtained as a result of solving the problem of nonlinear optimization by the five methods indicated above.
It can be seen from Table 3 that the most effective of the methods used was the interior point method. The values of the constants $C, B, \alpha, \gamma$, as well as the correction factors $k_1...k_5$, obtained by the internal point method, are given in Table 4.

Table 4. Rheological parameters of concrete obtained by the internal point method

<table>
<thead>
<tr>
<th>$\alpha$, 1/day</th>
<th>$\gamma$, 1/day</th>
<th>$C$, MPa$^{-1}$</th>
<th>$B$, MPa$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0014</td>
<td>0.0061</td>
<td>1.02$\times$10$^{-5}$</td>
<td>1.78$\times$10$^{-5}$</td>
</tr>
<tr>
<td>$k_1$</td>
<td>$k_2$</td>
<td>$k_3$</td>
<td>$k_4$</td>
</tr>
<tr>
<td>1.03</td>
<td>1.51</td>
<td>2.57</td>
<td>3.61</td>
</tr>
</tbody>
</table>

Fig. 1 shows the theoretical creep curves constructed using formula (6) and the parameters presented in Table 4. Experimental points are marked with round markers.

![Figure 1. Comparison of theoretical creep curves with experimental results](image1)

The reliability of the approximation $R^2$ for each stress level is given in Table 5.

Table 5. Reliability of experimental curves approximation at different stress levels

<table>
<thead>
<tr>
<th>$\sigma/R_b$</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$</td>
<td>0.918</td>
<td>0.990</td>
<td>0.994</td>
<td>0.982</td>
<td>0.991</td>
</tr>
</tbody>
</table>

Fig. 2 shows the graph of the stress function $f_2(s)$, built on the basis of the calculated values of the coefficients $k_1...k_5$. For comparison, the graph of the function $f_1(s)$ is also shown, which corresponds to the Sargin formula [25] used in the Eurocodes to describe the instantaneous nonlinearity of concrete deformation:

$$f_1(s) = \frac{E_0\varepsilon_R}{2} (k - (k - 2) \cdot s - \sqrt{[(k - 2) \cdot s - k]^2 - 4 \cdot s}),$$ (8)

where $E_0$ is the concrete initial modulus of elasticity (at $\sigma = 0$), $\varepsilon_R$ is the deformation at the top of the diagram $\sigma - \varepsilon$, $k = E_0\varepsilon_R / R_b$ is the coefficient characterizing the curvature of the diagram $\sigma - \varepsilon$.

The deformation $\varepsilon_R$ during plotting the $f_1(s)$ graph was determined by the empirical formula [26]:

$$\varepsilon_R = 0.058 \left( \frac{R_b}{E_0} \right)^{0.5}.$$ (9)

![Figure 2. Comparison of the functions $f_1(s)$ and $f_2(s)$](image2)
It can be seen from Fig. 2 that for concrete, the nonlinearity of creep deformations is much more pronounced than the instantaneous nonlinearity of deformation, and these two types of nonlinearity cannot be described by one function, as is done in equations (2) and (3). The dependence for $f_2(s)$ obtained by us is well approximated by the polynomial

$$f_2(s) = s \cdot (a \cdot s^2 + b \cdot s + c)$$

at $a = 203.6$ MPa, $b = -51.25$ MPa, $c = 30.19$ MPa. This approximation is shown in Fig. 2 by dashed line. Reliability of approximation is $R^2 = 0.986$.

CONCLUSIONS

A technique for processing concrete creep curves based on the nonlinear theory of V.M. Bondarenko using nonlinear optimization methods is proposed. To solve the problem of nonlinear optimization, the interior point method, the surrogate optimization method, the pattern search method, the genetic algorithm, and the particle swarm method were applied. It has been established that the most effective method for the considered problem is the interior point method. On the basis of experimental concrete creep curves obtained by A.V. Yashin, its rheological parameters are determined. The use of the interior point method made it possible to achieve a high quality of approximation of the experimental curves. It is shown that the nonlinearity of creep strains as a function of stress is more pronounced than the instantaneous nonlinearity of deformation, and these two types of nonlinearity cannot be described by a single function. On the basis of experimental data, the nonlinearity function for creep deformations was selected.

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