

# FEATURES OF DEVELOPMENT OF DYNAMIC MODELS OF HYDRO STRUCTURES AND THEIR VERIFICATION ACCORDING TO FIELD MEASUREMENT DATA USING THE EXAMPLE OF SAYANO-SHUSHENSKAYA HPP

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**Abstract:** This article presents the results of the development and verification of a dynamic finite element model of a high-pressure concrete hydraulic structure (Sayano-Shushenskaya HPP). A brief analysis of previously developed models of the hydraulic structure was carried out. An algorithm for sequential refinement of the results of dynamic calculations of hydraulic structures, significant factors and approaches to modeling the dynamic stress-strain state of hydraulic structures is presented.

**Keywords:** Arch-gravity dam, dynamic characteristics, added masses, natural frequencies and mode shapes

# ОСОБЕННОСТИ РАЗРАБОТКИ ДИНАМИЧЕСКИХ МОДЕЛЕЙ ГИДРОТЕХНИЧЕСКИХ СООРУЖЕНИЙ И ИХ ВЕРИФИКАЦИЯ ПО ДАННЫМ НАТУРНЫХ ИЗМЕРЕНИЙ НА ПРИМЕРЕ САЯНО-ШУШЕНСКОЙ ГЭС

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**Аннотация:** В настоящей статье представлены результаты разработки и верификации динамической конечно-элементной модели высоконапорного бетонного гидротехнического сооружения (Саяно-Шушенской ГЭС). Проведен краткий анализ разработанных ранее моделей гидроузла. Представлен алгоритм последовательного уточнения результатов динамических расчетов ГТС, значимые факторы и подходы к моделированию динамического НДС гидротехнических сооружений.

**Ключевые слова:** Арочно-гравитационная плотина, динамические характеристики, присоединенные массы, собственные частоты и формы колебаний

## 1. INTRODUCTION

Today, increased attention is being paid to the safety of high-pressure concrete hydraulic structures all over the world. Reliable and trouble-free operation of high-pressure dams is a key issue for researchers and practitioners in the field of construction and operation of such structures. Since a significant part of concrete dams were designed and built in the middle of

the last century [1] and have been in operation for several decades, monitoring their condition is fundamental to ensuring an adequate level of safety.

Assessing the dynamic characteristics of concrete water-retaining structures is of great importance in monitoring the safety of such structures. The regulatory legal acts in force in Russia [2] oblige regular dynamic testing of hydraulic structures and the preparation of dynamic

passports of such objects. In addition to the results of dynamic testing, the dynamic passport includes calculated natural frequencies and vibration modes obtained from numerical models. Having information about changes in the dynamic characteristics of a structure, one can draw conclusions about its condition and changes in the stress-strain state (SSS) of the object, which increases the overall level of reliability and safety of the object [3, 4].

In world practice, there are examples of the use of dynamic monitoring systems of concrete dams as a basis for diagnostic, mathematical models of hydraulic structures that can adapt according to the readings of instrumentation equipment (CAI) and conduct a predictive analysis of changes in the SSS of a structure [5, 6, 7].

Another important aspect of using the dynamic characteristics of a structure is the possibility of verifying the developed numerical models of the structure using natural frequencies and shapes measured in situ (if they are determined experimentally).

Therefore, the issue of calculation and full-scale determination of the dynamic characteristics of concrete pressure dams is an urgent and sought-after task in the hydraulic engineering industry.

In this article, using the example of the Sayano-Shushenskaya HPP (SSHHP) hydroelectric complex, we consider the important features of creating dynamic computational models of hydraulic structures, the disadvantages of simplified methods for modeling dynamic stress-strain state and the most significant factors that must be taken into account when creating dynamic models of hydraulic structures. The article also presents the results of a comparison of the dynamic characteristics obtained from calculation models developed in 2023 with the results of field studies at the SSHHP dam.

## 2. REVIEW OF PREVIOUSLY DEVELOPED SSHHP MODELS

During the operation of the SSHHP, a number of calculation models of the “dam-base” system

were built, in various software systems, with different formulations and details [8, 9, 10, 11, 12], which were developed in the period from 2024 to 2014. Natural frequencies of dam vibrations obtained from these models differed significantly from field measurements [13, 14]. These significant differences were due to simplified approaches to modeling the dynamic characteristics of a structure and insufficient detail of the calculation models.

To bring the calculated eigenvalues closer to the natural ones, approaches related to adjusting the added mass of water on the pressure face were forced to pressure face [8]. Criticism of the application of the distribution of hydrodynamic pressure on the pressure face of arch dams, obtained from the Westergaard solution [15], is associated with the obvious simplification of this method. This approach considers the motion of an absolutely rigid body in a fluid, but in reality the rigidity of arched dams is not infinite. Thus, the Center for Geodynamic Observations in the Electric Power Industry, when calculating the model in 2014, proposed to significantly change the distribution of hydrodynamic pressure on the pressure face (figure 1) in order to bring the calculated dynamic characteristics closer to those measured in situ [8].

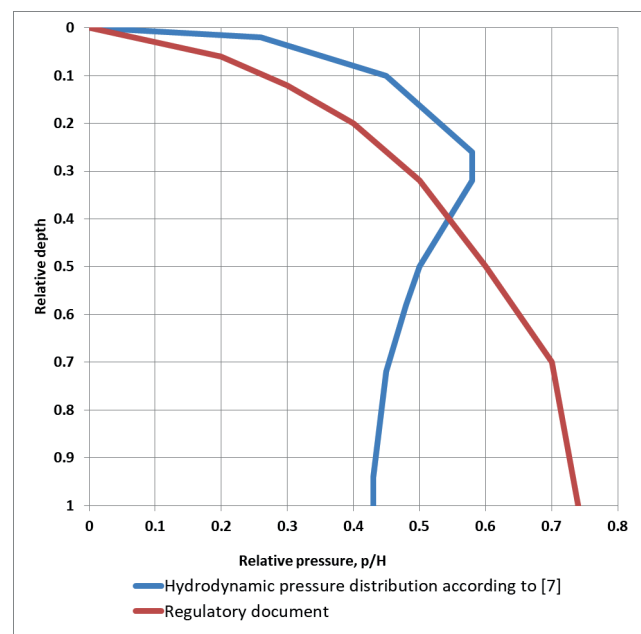


Figure 1. Distribution of hydrodynamic pressure along the height of the dam

The disadvantages of the outdated approach to taking into account fluid and added masses when performing dynamic calculations were demonstrated in a number of works devoted to determining the dynamic characteristics of high-pressure hydraulic structures [16, 17]. In most cases, taking into account the hydrodynamic pressure on the pressure face of the hydraulic structure, calculated according to Westergard, gives overestimated values at higher natural frequencies.

In another case, on a site built by a group of specialists. VNIIG B.E. Vedenev finite element models of the system “dam-base of the SSHHPP” in the ANSYS software package [9], when performing calculations in the period from 2004 to 2013. «Attempts were constantly made to calibrate the model in such a way as to bring the calculated data closer to the field measurement data» [18].

In many cases, to determine the elastic modulus of a structure or foundation, “calibration” of the calculation model is used, during which, by changing the properties of the materials of the foundation and structure, a number of design parameters are achieved to match the field measurement data [19]. Such “calibration” is in some cases unfounded and is a necessary measure in the absence of initial data or deficiencies in the calculation model. Below (Table 1) are the lowest natural frequencies of the SSHHPP dam when varying (significantly increasing) the value of the elastic modulus of concrete, obtained in [9].

*Table 1. Values of the first five natural frequencies of the SSHHPP dam, Hz [9]*

Excluding water		Headwater level 500 m		Headwater level 540 m	
E <sub>concr</sub> 50 GPa	E <sub>concr</sub> 45 GPa	E <sub>concr</sub> 50 GPa	E <sub>concr</sub> 45 GPa	E <sub>concr</sub> 50 GPa	E <sub>concr</sub> 45 GPa
1.568	1.481	1.384	1.312	1.159	1.099
1.625	1.532	1.481	1.398	1.265	1.196
2.019	1.895	1.862	1.751	1.580	1.490
2.531	2.372	2.351	2.208	1.999	1.882
3.164	2.972	2.999	2.768	2.520	2.374

It must be taken into account that a model “calibrated” in such a free manner, in the absence of

justification for this, cannot be considered verified, even if the calculated parameters correspond to field measurements.

### 3. NEW DYNAMIC MODEL OF SSHHPP

As part of the contract with the Branch of JSC Institute Hydroproject - employees of R&D Centre StaDyO in the period 2020 - 2023, a volumetric finite element model of the system “base - reservoir - dam - station building of the Sayano-Shushenskaya HPP” was developed and verified in ANSYS.

The main goal of the research work was to assess the occurrence of resonance phenomena in the structures of the station building during equipment operation or hydrodynamic phenomena in the flow part. Since the SSS of the station is directly influenced by the SSHHPP dam, which is confirmed by the readings of control and measuring equipment, it was decided to create a general (basic) model of the hydroelectric complex, including all the main structures. And after verification of the basic model, proceed to the analysis of the station building within the framework of a local model, refined in terms of building structures and the foundation mass under the structure.

Two families of basic models were created: dynamic and static models, which have differences related to the features of calculations (for example, properties of materials, loads, the need to take into account the sequence of construction, etc.).

When creating the basic dynamic model, a series of computational studies were carried out with sequential consideration of various factors and assessment of their influence on the natural frequencies and vibration modes of the SSHHPP arch-gravity dam. As a result, more than ten groups of dynamic models were prepared, which reflect the evolution of the basic model in terms of clarifying the criterion dynamic characteristics.

To consistently refine the dynamic model, the following factors were taken into account:

1. Use of volumetric finite elements of the 2nd order. Grid Convergence Analysis
  3. Taking into account a number of design features of the dam body.
  4. Various methods of accounting for fluid on models with and without a rock base. Assessing the impact of boundary conditions settings for FLUID30 acoustic elements on the results.
  5. Taking into account a homogeneous rock base.
  6. Taking into account the heterogeneous rock base, the structure and characteristics of the geological elements of which correspond to the current results of geophysical research.
  7. Taking into account additional factors: masses of water in water pipelines, gantry cranes on the crest and other loads.
- An analysis of the influence of most of the above factors on the dynamic characteristics of the dam has not been previously carried out. It is the consistent “complication” of the calculation model with an assessment of the contribution of each of the factors under consideration that is a mandatory requirement when preparing dynamic models.

*Estimation of grid convergence of the basic dynamic model.*

When conducting these studies, the reservoir was not taken into account, and the foundation was assumed to be absolutely rigid and massless. After a series of calculations, a comparison was made of the natural vibration frequencies of the dam for variants of the model with a free breakdown (1,578 thousand nodes) and the model with an optimized mesh in part of the dam body (243 thousand nodes). When comparing the models, agreement was noted in the shapes of natural vibrations, and the differences in the first 10 frequencies do not exceed 0.25% (Table 2). For further research, a variant with an optimized mesh and second-order volumetric finite elements was adopted.

*Table 2. Estimation of grid convergence*

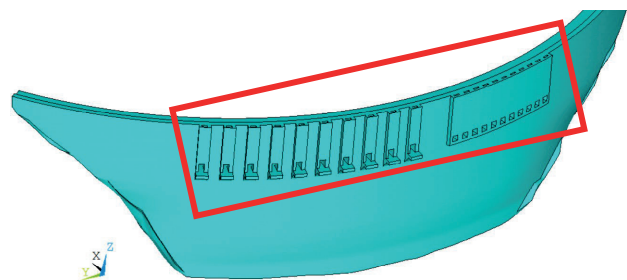
№	Free mesh 1 578 000 nodes	Optimized model 243 000 nodes	$\Delta, \%$
1	1.0037	1.0012	0.25
2	1.1241	1.1218	0.21
3	1.4106	1.4108	0.01
4	1.7986	1.8006	0.11
5	2.2581	2.2588	0.03
6	2.4948	2.4888	0.24
7	2.7343	2.7349	0.02
8	2.8931	2.8916	0.05
9	3.2307	3.2327	0.06
10	3.4157	3.4135	0.06

*Assessment of the influence of the design features of the body of the SSHHPP dam.*

In the previously developed models of the SSHHPP, the arch-gravity dam was assumed to be simplified (with a smooth pressure face), because taking into account the protruding elements of a structural element in the area of water intake holes (figure 2) increases the dimension of the problem, and the influence of these elements on the dynamic characteristics was considered insignificant (although such assessments were not carried out).

When assessing the influence of this factor, three models were considered:

- 1) Without taking into account these structural elements. The surface of the dam on the upstream side is “smooth”.
- 2) Taking into account the protruding structural element in the area of the water intake openings of the spillway.
- 3) Taking into account the protruding structural element in the area of the water intake openings of the spillway and pressure water conduits (figure 2).



*Figure 2. Structural element on the pressure face of the dam*

The base was assumed to be absolutely rigid, the reservoir was not taken into account.

With the contribution of these design features to the total mass of the dam at the level of 1.4%, the influence on the lower natural frequencies of the dam turned out to be significantly higher - up to 3.65% (Table 3).

*Table 3. Comparison of natural frequencies*

№	Model 1	Model 2	Model 3	Model 1 / 3 $\Delta, \%$
	Natural frequency, Hz			
1	1.5301	1.5073	1.4791	3.33%
2	1.6122	1.5825	1.5533	3.65%
3	1.9492	1.9402	1.9120	1.91%
4	2.4125	2.4129	2.3942	0.76%
5	2.9750	2.9921	2.9683	0.23%

The studies carried out showed a significant influence of this factor on the lower frequencies of natural vibrations of the arch-gravity dam of the SSHHPP. For further research, a model was adopted that took into account the structural elements in the area of the spillway inlet openings and pressure conduits.

*Assessment of the impact of different liquid accounting methods*

In assessing this factor, a series of calculations were carried out in which the dimensions of the acoustic reservoir, boundary conditions for the reservoir elements and the properties of the foundation were varied.

Due to the inability to present the results of all calculations performed in the article, the results for the model with a homogeneous elastic foundation,  $E = 50 \text{ GPa}$ , are presented below.

Two methods of accounting for the reservoir were considered:

1. Added masses (with distribution according to Westerhard [15]). This method was used on all previously developed models of the SSHHPP hydroelectric power station.
2. Using acoustic finite elements (FLUID30). Previously, such a method was not used for SSHPP models.

For the acoustic elements of the reservoir that are in contact with the structure and foundation,

additional degrees of freedom in the form of displacements are specified [20].

Analysis of natural frequencies showed that when the reservoir is taken into account with added masses, the natural values of the dam are significantly lower than when modeling the fluid with acoustic elements. At lower frequencies the difference reaches 10% (Table 4). The first 3 modes of the dam's natural vibrations coincide; at higher frequencies in model No. 2, forms appear that are not detected in model No. 1. Some natural vibration modes are similar.

*Assessment of the influence of additional factors.*

A separate series of studies was carried out to assess the influence on the dynamic characteristics of the SSHHPP dam of such factors as the position of gantry cranes on the dam crest and the presence of water in the water conduits.

Based on the results of the studies, it was revealed that the position of the gantry cranes on the crest affects the change in the lower natural frequencies of the dam by no more than 0.38%, and the added mass of water in the water conduits significantly affects the dynamic characteristics of the water conduits themselves, but not on the lower natural frequencies and vibration patterns of the SSHHPP dam.

*The final dynamic model of SSHHPP.*

The final dynamic model of the system "base – reservoir – dam – station building of the Sayano-Shushenskaya HPP" was made on an elastic heterogeneous foundation. Dynamic properties are based on reasonable grounds based on geophysical research data. The dam material was adopted based on the results of laboratory studies of cores from the dam body with the following characteristics  $E=41.5 \text{ GPa}$ ,  $\nu=0.15$ ,  $\rho=2.45 \text{ t/m}^3$ . The model takes into account 4 reservoir filling levels: 500 m, 520 m and 540 m and 539 m. The volume of the reservoir is taken into account by FLUID30 acoustic finite elements. All significant design features of the dam are taken into account.

*Table 4. Comparison of natural frequencies/ Matching shapes are shown in color*

№	Model 1	Model 2	Model 1/2 $\Delta, \delta$ %
	Added mass	Acoustic elem.	
Natural frequency, Hz			
1	1.0011	1.1071	9,57
2	1.1217	1.2069	7,06
3	1.4108	1.5297	7,77
4	1.8006	1.6786	8,14
5	2.2588	1.9601	8,39
6	2.4874	2.1063	
7	2.7348	2.2677	8,99
8	2.8911	2.4657	10,03
9	3.2327	2.5349	8,97
10	3.4134	2.8993	8,95
11	3.5415	2.9788	
12	3.8096	3.0051	
13	4.0371	3.2133	
14	4.2018	3.3561	
15	4.4122	3.5331	
16	4.5372	3.5513	
17	4.7068	3.6151	
18	4.9933	3.6864	
19	5.0325	3.7491	
20	5.1021	3.9496	

**4. RESULTS**

For the final basic dynamic model of the system, a comparison was made between the calculated dynamic characteristics and the available set of results of instrumental studies of the actual dynamic properties of the dam, obtained in different years (table 5, 6) (figure 3, 4). The natural mode shapes were compared with field data obtained in 1999 using the standing wave method and in 2022 using the frequency domain decomposition method (table 7).

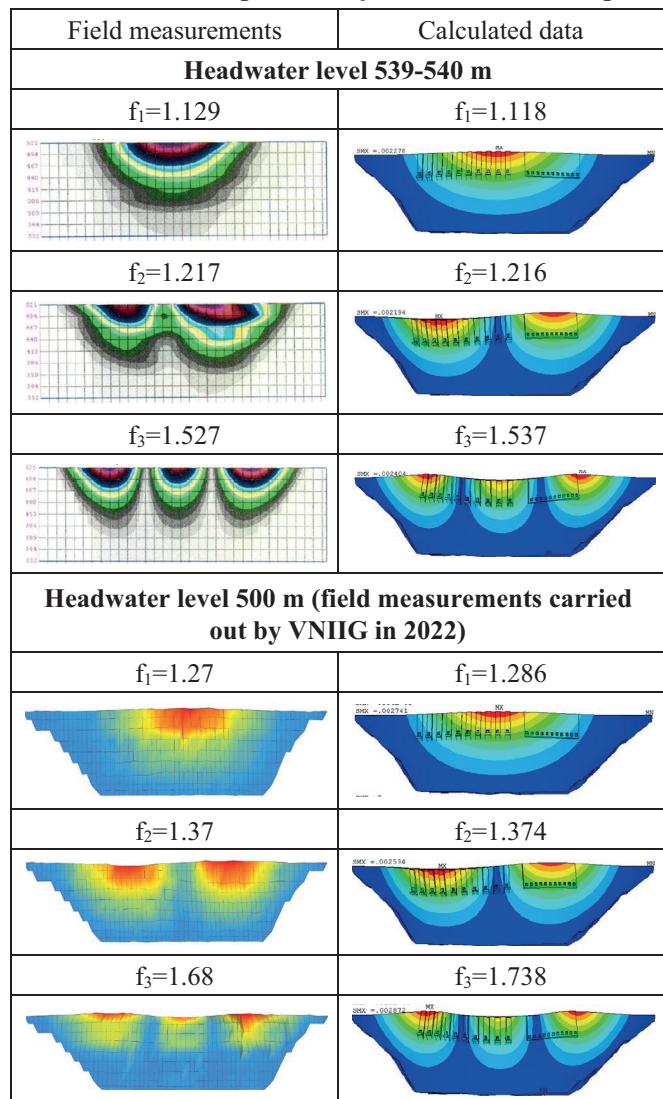
*Table 5. Comparison of natural frequencies. Headwater level 539-540*

№	Field measurements, 1999 г, Hz	Calculated data, Hz	$\Delta$ , Hz	$\delta$ , %
1	1.129	1.1181	0.0109	0.97
2	1.217	1.2162	0.0008	0.07
3	1.527	1.5365	0.0095	0.62
4	1.921	1.9651	0.0441	2.24
5	2.448	2.4709	0.0229	0.93
6	2.972	3.0077	0.0357	1.19

*Table 6. Comparison of natural frequencies. Headwater level 500*

№	Field measurements, 1999 г, Hz	Calculated data, Hz	$\Delta$ , Hz	$\delta$ , %
1	1.236	1.2861	0.0501	3.90
2	1.325	1.3738	0.0488	3.55
3	1.633	1.7373	0.1043	6.00
4	2.035	2.2044	0.1694	7.68
5	2.531	2.7457	0.2147	7.82
6	3.064	3.3118	0.2478	7.48

*Table 7. Comparison of natural mode shapes*



Features of Development of Dynamic Models of Hydro Structures and Their Verification According to Field Measurement Data, Using the Example of Sayano-Shushenskaya HPP.

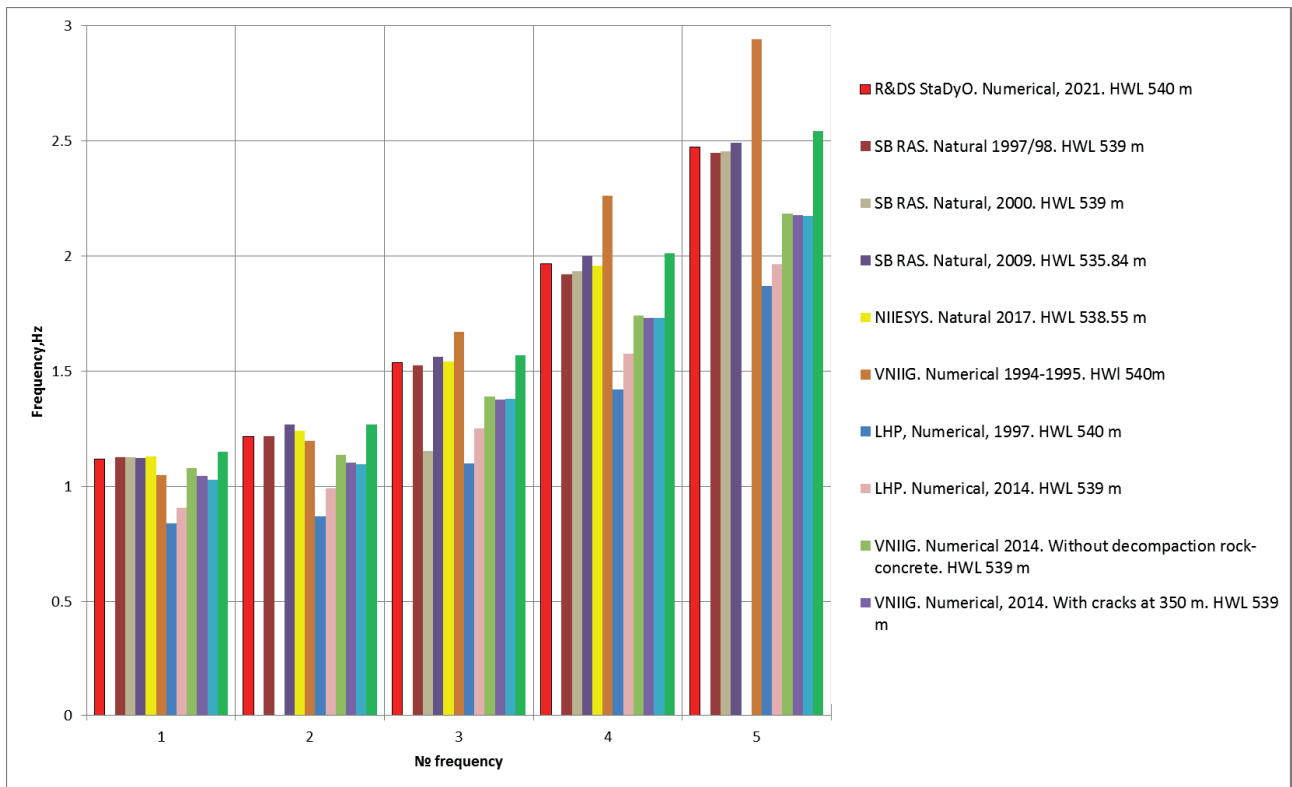


Figure 3. Results of previous studies. Headwater level 539-540 m

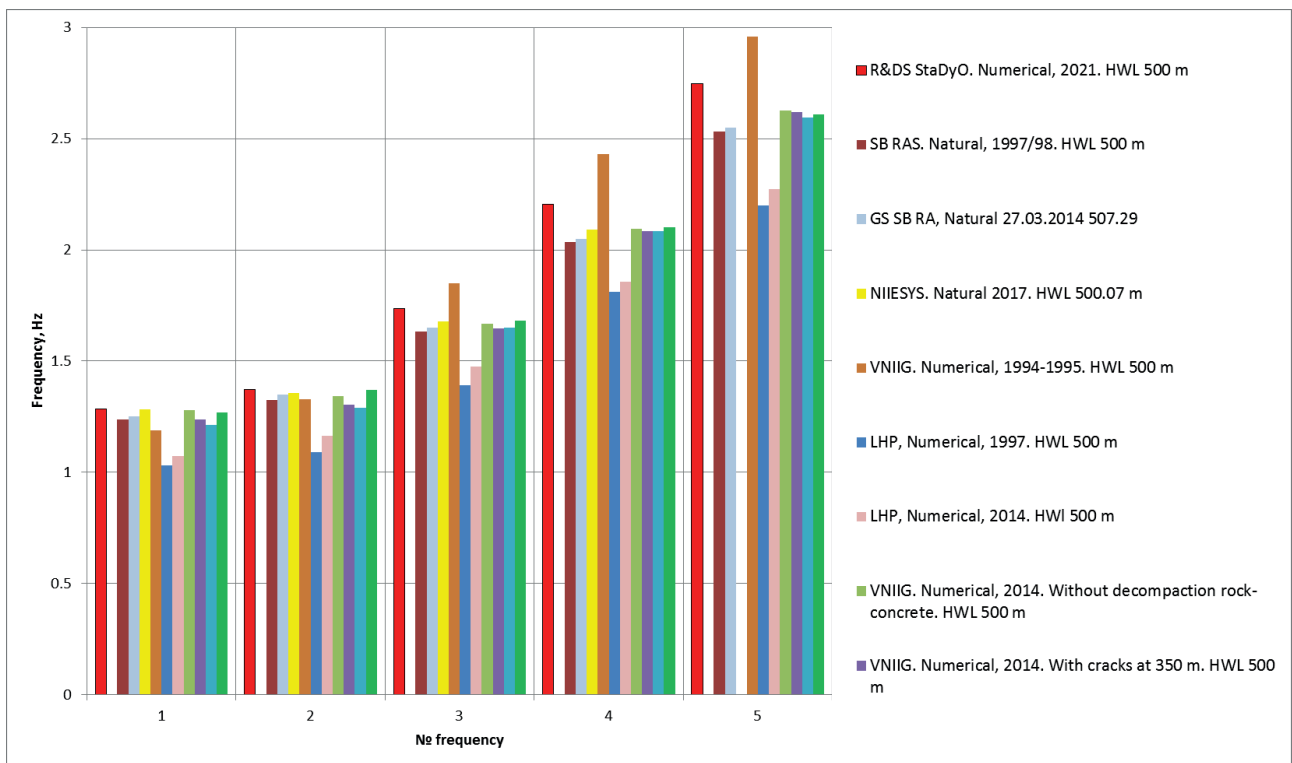


Figure 4. Results of previous studies. Headwater level 500 m

## CONCLUSION

A multiparameter and multifactor verification of the spatial finite element base model of the system “foundation – reservoir – dam – station building of the Sayano-Shushenskaya HPP” developed and implemented in the ANSYS Mechanical software package was carried out using field measurement data: significant dynamic characteristics (spectrum of natural frequencies and vibration modes) of the dam.

The accepted and tested verification methodology excludes any “calibration” of model parameters, but represents a systematic movement from the initial basic model to an increasingly more detailed and/or complex one – until an acceptable correspondence of the criterion calculation results to the field data is achieved, or until the source of such discrepancies is identified, associated with the unreliability of the original information.

As a result of the research, the most significant factors were identified that must be taken into account when creating dynamic models of hydraulic structures.

The results obtained for the SSHHPP dam (natural frequencies and vibration modes) exceed in accuracy and correspondence to nature all previously obtained results.

## REFERENCES

1. **Perera D. et al.** Ageing water storage infrastructure: An emerging global risk //UNU-INWEH Report Series. – 2021. – T. 11. – C. 25.
2. Code of Practice 358.1325800.2017 “Hydraulic structures. Rules for design and construction in seismic areas.”.
3. **Prakash G. et al.** Recent advancement of concrete dam health monitoring technology: A systematic literature review //Structures. – Elsevier, 2022. – T. 44. – C. 766-784.
4. **Dmitriev D. S., Uchevatkin A. A.** Numerical modeling as a basis for monitoring systems for the safety of hydraulic structures and automated stress-strain state control systems // Vestnik MGSU. – 2021. – T. 16. – No. 12. – pp. 1582-1591.
5. **Oliveira, S., & Alegre, A.** Seismic and structural health monitoring of dams in Portugal / S. Oliveira, A. Alegre. / Seismic Structural Health Monitoring. – Springer, 2019. – C. 87-113.
6. **Oliveira S., Alegre A.** Seismic and structural health monitoring of Cabril dam. Software development for informed management // Journal of Civil Structural Health Monitoring. 2020. T. 10. №. 5. C. 913-925.
7. **Alegre, A., Carvalho, E., Matsinhe, B., Mendes, P., Oliveira, S., & Proença, J.** Monitoring vibrations in large dams.
8. **Bronshtein V.I. et al.** Study of the influence of various factors on the stress-strain state and seismic resistance of the Sayano-Shushenskaya HPP dam using the numerical modeling method //Hydrotechnical Construction. – 2014. – No. 12. – pp. 35-43.
9. **Khrapkov A. A. et al.** On the construction of a mathematical model of the arch-gravity dam of the Sayano-Shushenskaya HPP // News of the All-Russian Scientific Research Institute of Hydraulic Engineering named after. BE Vede-neeva. – 2011. – T. 264. – P. 56-69.
10. **Toloshinov A.V. et al.** Construction of a finite element calculation model for assessing the stress-strain state of the Sayano-Shushenskaya HPP dam // News of Higher Educational Institutions. Construction. – 2006. – No. 7. – pp. 38-47.
11. **Yakubson V.M.** Analysis of the stress-strain state of the Sayano-Shushenskaya HPP dam // Engineering and Construction Journal. – 2012. – No. 1. – P. 2-2.7.
12. **Kozinets G.L.** Determination of the dynamic characteristics of structures in con-

- tact with water, using the example of the arched concrete dam of the Sayano-Shushenskaya HPP // Magazine of Civil Engineering. – 2011. – No. 5. – pp. 43-48.
13. **Zavalishin S.I., Shablinsky G.E.** Field studies and calculation of the dynamic characteristics of the Sayano-Shushenskaya HPP dam //Hydrotechnical Construction. – 2015. – No. 2. – pp. 52-59.
  14. **Liseykin A.V. et al.** Determination of natural vibration frequencies of structures using low-amplitude seismic signals (using the example of the Sayano-Shushenskaya hydroelectric power station dam according to monitoring data from 2001-2021) // Russian Seismological Journal. – 2023. – Т. 5. – No. 2. – P. 32-50.9.
  15. **Westergaard H.M.** Water pressures on dams during earthquakes / H.M. Westergaard // Proc. Am. Soc. Civ. Engrs. – 1931. – Т. 57. – № 9.
  16. **Belostotsky A. M. et al.** Comparative analysis of methods for numerical modeling of the dynamics of three-dimensional systems “base-arch dam-reservoir” under seismic impacts // International Journal for Computational Civil and Structural Engineering. – 2016. – Т. 12. – No. 2. – pp. 65-74.
  17. ICOLD, International Benchmark Workshop on Numerical Analysis of Dams. Theme A – Fluid Structure Interaction Arch Dam – Reservoir at Seismic loading. Graz University of Technology, 2013, pp.15-188.
  18. **Kostylev V.S.** Forecast of the stress-strain state of concrete dams using mathematical modeling and natural data: dis. – All-Russian Scientific Research Institute of Hydraulic Engineering named after. BE Vedeneva, 2013.
  19. **Ivashintsov D.A. et al.** Parametric identification of calculation models of hydraulic structures // St. Petersburg: Publishing House of OJSC “VNIIG im. B.E. Vedeneva. – 2001. – P. 432.
  20. **Belostotsky A. M., et al.** Verification of the methodology for numerical modeling of the dynamic stress-strain state of three-dimensional “structure-liquid” systems // International Journal for Computational Civil and Structural Engineering. – 2015. – Т. 11. – No. 4. – pp. 124-132.

#### СПИСОК ЛИТЕРАТУРЫ

1. **Perera D. et al.** Ageing water storage infrastructure: An emerging global risk //UNU-INWEH Report Series. – 2021. – Т. 11. – С. 25.
2. СП 358.1325800.2017 «Сооружения гидротехнические. Правила проектирования и строительства в сейсмических районах».
3. **Prakash G. et al.** Recent advancement of concrete dam health monitoring technology: A systematic literature review //Structures. – Elsevier, 2022. – Т. 44. – С. 766-784.
4. **Дмитриев Д. С., Учеваткин А. А.** Численное моделирование в основе систем мониторинга безопасности гидротехнических сооружений и автоматизированные системы управления напряженно-деформированным состоянием //Вестник МГСУ. – 2021. – Т. 16. – №. 12. – С. 1582-1591.
5. **Oliveira, S., & Alegre, A.** Seismic and structural health monitoring of dams in Portugal / S. Oliveira, A. Alegre. / Seismic Structural Health Monitoring. – Springer, 2019. – С. 87-113.
6. **Oliveira S., Alegre A.** Seismic and structural health monitoring of Cabril dam. Software development for informed management // Journal of Civil Structural Health Monitoring. 2020. Т. 10. №. 5. С. 913-925.

7. **Alegre, A., Carvalho, E., Matsinhe, B., Mendes, P., Oliveira, S., & Proença, J.** Monitoring vibrations in large dams.
8. **Бронштейн В.И. и др.** Исследование влияния различных факторов на напряженно-деформированное состояние и сейсмостойкость плотины Саяно-Шушенской ГЭС методом численного моделирования //Гидротехническое строительство. – 2014. – №. 12. – С. 35-43.
9. **Храпков А. А. и др.** О построении математической модели арочно-гравитационной плотины Саяно-Шушенской ГЭС //Известия Всероссийского научно-исследовательского института гидротехники им. Б.Е. Веденеева. – 2011. – Т. 264. – С. 56-69.
10. **Толошинов А. В. и др.** Построение конечно-элементной расчетной модели для оценки напряженно-деформированного состояния плотины Саяно-Шушенской ГЭС //Известия высших учебных заведений. Строительство. – 2006. – №. 7. – С. 38-47.
11. **Якубсон В.М.** Анализ напряженно-деформированного состояния плотины Саяно-Шушенской ГЭС //Инженерно-строительный журнал. – 2012. – №. 1. – С. 2-2.
12. **Козинец Г.Л.** Определение динамических характеристик сооружений, контактирующих с водой, на примере арочной бетонной плотины Саяно-Шушенской ГЭС //Magazine of Civil Engineering. – 2011. – №. 5. – С. 43-48.
13. **Завалишин С.И., Шаблинский Г. Э.** Натурные исследования и расчет динамических характеристик плотины Саяно-Шушенской ГЭС //Гидротехническое строительство. – 2015. – №. 2. – С. 52-59.
14. **Лисейкин А. В. и др.** Определение частот собственных колебаний сооружений по малоамплитудным сейсмическим сигналам (на примере плотины Саяно-Шушенской ГЭС по данным мониторинга 2001-2021 гг.) //Российский сейсмологический журнал. – 2023. – Т. 5. – №. 2. – С. 32-50.
15. **Westergaard H.M.** Water pressures on dams during earthquakes / H.M. Westergaard // Proc. Am. Soc. Civ. Engrs. – 1931. – Т. 57. – № 9.
16. **Белостоцкий А. М. и др.** Сравнительный анализ методик численного моделирования динамики трехмерных систем" основание-арочная плотина-водохранилище" при сейсмических воздействиях //International Journal for Computational Civil and Structural Engineering. – 2016. – Т. 12. – №. 2. – С. 65-74.
17. ICOLD, International Benchmark Workshop on Numerical Analysis of Dams. Theme A – Fluid Structure Interaction Arch Dam – Reservoir at Seismic loading. Graz University of Technology, 2013, pp.15-188.
18. **Костылев В.С.** Прогноз напряжённо-деформированного состояния бетонных плотин с использованием математического моделирования и натурных данных: дис. – Всероссийский научно-исследовательский институт гидротехники им. Б.Е. Веденеева, 2013.
19. **Ивашинцов Д.А. и др.** Параметрическая идентификация расчетных моделей гидротехнических сооружений //СПб.: Изд-во ОАО "ВНИИГ им. Б.Е. Веденеева. – 2001. – С. 432.
20. **Белостоцкий А. М., и др.** Верификация методики численного моделирования динамического напряженно-деформированного состояния трехмерных систем «сооружение-жидкость» //International Journal for Computational Civil and Structural Engineering. – 2015. – Т. 11. – №. 4. – С. 124-132.

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