DOI:10.22337/2587-9618-2023-19-3-173-183

# DEVELOPMENT OF COMPUTATIONAL SCHEMES OF GENERALIZED KINEMATIC DEVICES THAT PRECISELY REGULATE THE NATURAL FREQUENCY SPECTRUM OF ELASTIC SYSTEMS WITH A FINITE NUMBER OF DEGREES OF MASS FREEDOM, IN WHICH THE DIRECTIONS OF MOTION ARE PARALLEL, BUT DO NOT LIE IN THE SAME PLANE
# PART 1: THEORETICAL FOUNDATIONS

***Leonid S. Lyakhovich* [1], *Pavel A. Akimov* [2], *Zaur R. Galyautdinov* [1], *Andrey S. Plyaskin* [1]**

[1] Tomsk State University of Architecture and Civil Engineering, Tomsk, RUSSIA

[2] National Research Moscow State University of Civil Engineering, Moscow, RUSSIA

**Abstract:** To date, for some elastic systems with a finite number of degrees of mass freedom, in which the directions of mass movement are parallel and lie in the same plane, methods have been developed for creating additional generalized targeted constraints and generalized targeted kinematic devices. Each generalized targeted constraint increases, and each generalized targeted kinematic device reduces the value of only one selected natural frequency to a predetermined value, without changing the remaining natural frequencies and natural modes. Earlier, for elastic systems with a finite number of degrees of mass freedom, in which the directions of mass motion are parallel, but do not lie in the same plane (for example, plates), an approach for the computing of a matrix of additional stiffness and a method for the development of computational schemes of additional generalized targeted constraints were developed. Also earlier, for such systems, an approach was proposed for the computing of a special matrix with allowance for additional inertial forces that determine a generalized targeted kinematic device. At the same time, the method of development of computational schemes of kinematic devices was not proposed. The distinctive paper is devoted to approach, that makes it possible to develop computational schemes of generalized targeted kinematic devices for such systems as well. A variant of the computational scheme of constraint for the rod system with one degree of activity, is considered. Some special properties of such targeted kinematic devices are revealed.

**Keywords:** localization, natural frequency, natural mode, additional generalized targeted constraint, stiffness coefficients, inertial forces, generalized targeted kinematic device

# ФОРМИРОВАНИЕ РАСЧЕТНЫХ СХЕМ ОБОБЩЕННЫХ КИНЕМАТИЧЕСКИХ УСТРОЙСТВ, ПРИЦЕЛЬНО РЕГУЛИРУЮЩИХ СПЕКТР ЧАСТОТ СОБСТВЕННЫХ КОЛЕБАНИЙ УПРУГИХ СИСТЕМ С КОНЕЧНЫМ ЧИСЛОМ СТЕПЕНЕЙ СВОБОДЫ МАСС, У КОТОРЫХ НАПРАВЛЕНИЯ ДВИЖЕНИЯ ПАРАЛЛЕЛЬНЫ, НО НЕ ЛЕЖАТ В ОДНОЙ ПЛОСКОСТИ

***Л.С. Ляхович* [1], *П.А. Акимов* [2], *З.Р. Галяутдинов* [1], *А.С. Пляскин* [1]**

[1] Томский государственный архитектурно-строительный университет, г. Томск, РОССИЯ

[2] Национальный исследовательский Московский государственный строительный университет, г. Москва, РОССИЯ

**Аннотация:** К настоящему времени для некоторых упругих систем с конечным числом степеней свободы масс, у которых направления движения масс параллельны и лежат в одной плоскости, разработаны методы создания дополнительных обобщенных прицельных связей и обобщенных прицельных кинематических устройств. Каждая обобщённая прицельная связь увеличивает, а каждое обобщенное прицельное кинематическое устройство уменьшает величину лишь одной выбранной собственной частоты до наперед заданного значения, не изменяя при этом остальные собственные частоты и формы собственных колебаний. Ранее для упругих систем с конечным числом степеней свободы масс, у которых направления движения масс параллельны, но не лежат в одной плоскости (например, пластины), разработан подход образования матрицы дополнительных жесткостей и метод формирования расчетных схем дополнительных обобщенных прицельных связей. Также ранее для таких систем предложен подход образования матрицы учета действия дополнительных инерционных сил, определяющих обобщенное прицельное кинематическое устройство. При этом способ формирования расчетных схем кинематических устройств, предложен не был. В данной статье предлагается подход, позволяющий формировать расчетные схемы обобщенных прицельных кинематических устройств и для таких систем. Рассмотрен вариант расчетной схемы связи, представленный стержневой системой с одной степенью активности. Выявлены некоторые особые свойства таких прицельных кинематических устройств.

**Ключевые слова:** частота собственных колебаний, форма собственных колебаний, дополнительная обобщенная прицельная связь, коэффициенты жесткости, инерционные силы, обобщенное прицельное кинематическое устройство

## INTRODUCTION

As is known [1-9], one of the methods for releasing a certain frequency interval from one or more natural frequencies can be the introduction of generalized targeted constraints and generalized targeted kinematic devices. In the mentioned research works, for elastic systems with a finite number of degrees of freedom of masses, in which the directions of mass motion are parallel and lie in the same plane (for example, elastic rods), methods and algorithms for development of targeted constraints and generalized targeting kinematic devices were proposed. In [3-9] for systems with a finite number of degrees of freedom of masses, in which the directions of mass motion are parallel, but do not lie in the same plane, an approach is proposed to the computing of a matrix of additional stiffness coefficients with allowance for targeted constraints and an algorithm for development of computational scheme for this constraint. For generalized targeted kinematic devices an approach to the computing of a special matrix with allowance for additional inertial forces was also considered. Besides, a method for development of computational schemes of kinematic devices for such systems has not been proposed.

The distinctive paper contains description of an algorithm that makes it possible to develop computational schemes of kinematic devices for such systems as well. A variant of the computational scheme, represented by a statically determinate rod system with one degree of activity, is considered. Some special properties of such kinematic devices have been revealed. The possibility of taking into account the influence of the elastic properties of the rods of a kinematic device on the values of the natural frequencies and the coordinates of natural modes is considered.

Let us present, in accordance with [4, 5], the algorithm of computing of corresponding matrix with allowance for additional inertial forces. For the considering systems (for instance, Figure 1*a*), the main system of the displacement method is selected (Figure 1*b*). The equations of the displacement method are written in the conventional form for systems with a finite number of degrees of freedom

$$
\begin{aligned}
&(r[1,1]+m[1]\omega^2)v_\omega[1,j]+r[1,2]v_\omega[2,j]+\ldots+r[1,q]v_\omega[q,j]+\ldots+r[1,n]v_\omega[n,j]=0\\
&r[2,1]v_\omega[1,j]+(r[2,2]+m[2]\omega^2)v_\omega[2,j]+\ldots+r[2,q]v_\omega[q,j]+\ldots+r[2,n]v_\omega[n,j]=0\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&r[n,1]v_\omega[1,j]+r[n,2]v_\omega[2,j]+\ldots+r[n,q]v_\omega[q,j]+\ldots+(r[n,n]+m[n]\omega^2)v_\omega[n,j]=0
\end{aligned}
\qquad (1)
$$

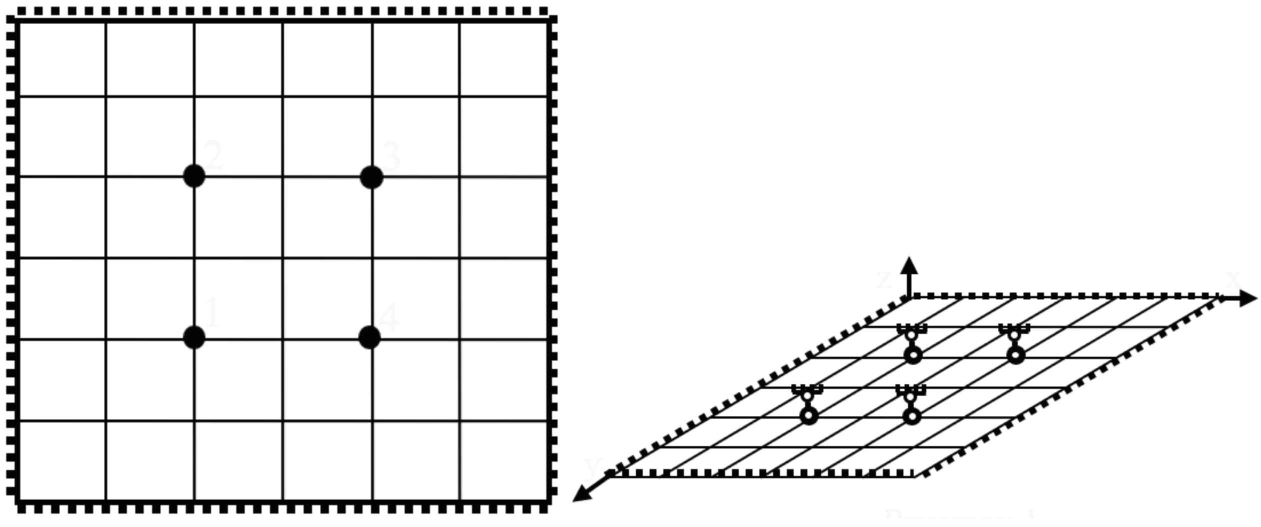

*Figure 1. Considering system: a) initial problem; b) the main system of the displacement method.*

In (1) the quantities $r[i,k]$ form a matrix of stiffness coefficients $A=\|r[i,k]\|$; the mass values $m[i]$ form a diagonal matrix $M=\|m[i]\|$; $\omega$ is the natural frequency of the system; $v_\omega[k,j]$ are displacements in the direction of movement of the masses in the $j$ th ($j=1,2,..,q,,..,n$) form of natural oscillations.

Roots of the equation

$$\left|A-\omega^2 M\right|=0 \tag{2}$$

determine the natural frequency spectrum of the system

$$\omega[1],\omega[2],...,\omega[q-1],\omega[q],\omega[q+1],...\omega[n]. \tag{3}$$

In [4], [5] it is shown that a kinematic device with one degree of activity transmits a generalized impact inertial force to the considering type of structure, which reduces the value of only one natural frequency to a given value $\omega_S$, leaving the remaining frequencies of the spectrum unchanged.

The device is formed on the basis of a special matrix with allowance for additional inertial forces

$$M_0=M_{m0}*M_m, \tag{4}$$

where we have

$$M_m=\|m_0[i,k]\|_{i,k=1}^{n}. \tag{5}$$

The matrix $M_0$ has special properties [4], [5]. If the introduced kinematic device is "targeted" at the $q$ th natural frequency, then the coefficients of the matrix $M_m$ ($\|m_0[i,k]\|_{i,k=1}^{n}$) are orthogonal to the coordinates of the natural modes corresponding to the remaining frequencies of the spectrum. That is

$$\sum_{k=1}^{n} m_0[i,k]*v_\omega[k,j]=0, \quad (i=1,2,..,n,\ j=1,2,..,q-1,q+1,..,n). \tag{6}$$

In relation to the $q$ th natural frequency, at which the kinematic device is "targeted" we have

$$\sum_{k=1}^{n} m_0[i,k]*v_\omega[k,q]\neq 0, \quad (i=1,2,..,n). \tag{7}$$

In [4], [5] it is shown that conditions (5) and (6) will be satisfied by the coefficients

$$m_0[i,k]=m[i]m[k]v_\omega[i,q]v_\omega[k,q]. \tag{8}$$

The value of the multiplier $M_{m0}$ is found as the root of the equation

$$\left|(A-\omega_S^2 * M)-M_{m0} * (\omega_S^2 * M_m)\right|=0. \quad (9)$$

The value $M_{m0}$ is determined by the dependency

$$M_{m0}=\frac{\sum_{i=1}^{n}\sum_{k=1}^{n}(a[i,k]-\omega_S^2 m[i,k])v_\omega[i,(q)]v_\omega[k,(q)]}{\sum_{i=1}^{n}\sum_{k=1}^{n}\omega_S^2 m_0[i,k]v_\omega[i,(q)]v_\omega[k,(q)]} \quad (10)$$

Result of solving the equation

$$\left|A-\omega^2(M+M_{m0}M_m)\right|=0 \quad (11)$$

must confirm that the natural modes have not changed, and the "target" frequency has decreased to the value $\omega_S$.

Kinematic device, which will correspond to a special matrix of coefficients with allowance for additional inertial forces

$$M_0=M_{m0}M_m, \quad M_m=\left\|m_0[i,k]\right\|_{i,k=1}^{n}, \quad (12)$$

should ensure the relationship between nodal displacements is the same as between the coordinates of the $q$ th natural mode of the original system. In [2, 4, 5] it is also shown that such a relationship will be realized if the kinematic device transmits inertial forces $\overline{R}[i](i=1,..,n)$ to the nodes, the ratios between which are proportional to the ratios between the forces

$$R_0[i]=m[i]v_\omega[i,q]. \quad (13)$$

The magnitude of the forces $\overline{R}[i](i=1,..,n)$ [2] is determined from the relation

$$\overline{R}[i]=\sum_{k=1}^{n}M_{mO}\omega_S^2 m_0[i,k]v_\omega[k,(q)]. \quad (14)$$

Computational schemes of generalized targeted kinematic devices are multivariate and depend on the geometry of the base system, the location of the masses and other features of the object.

One of the options for the computational scheme of a generalized targeted kinematic device can be a statically determined hinge-rod system, to which additional mass $M_{add}$ is applied.

The geometry of the computational scheme is determined by the lengths of the main racks installed in the nodes where the masses are located, in the direction of movement of the masses $l_{st}[i]\ (i=1,2,..,n)$, the coordinates of the node $X_0,Y_0,Z_0$ for applying the additional mass, as well as the coordinates $X[i],Y[i],Z[i]$ of the articulated fixed supports located outside the original system and determining the directions of the additional rods connecting the tops of some of the main racks with these supports.

After forming a matrix $M_0$ of coefficients of additional inertial forces (4), (5), (8), (10) and a setting of values $\overline{R}_0[i]$ (12), it is required, by varying the lengths of the racks $l_{st}[i]\ (i=1,2,..,n)$ and coordinates $X_0,Y_0,Z_0$, $X[i],Y[i],Z[i]$ to find their values at which the inertial forces $N_{st}[i](i=1,..,n)$ in racks, will be equal to the forces $\overline{R}_0[i](i=1,..,n)$ with the accepted level of approach.

Thus, it is proposed to solve the problem of forming a computational scheme of a kinematic device based on minimizing the sum of the squares of the differences between the inertial forces arising in the main racks in the process of forming the kinematic device, and the quantities $\overline{R}[i](i=1,..,n)$, the relationships between which will be proportional to the relationships between the forces $\overline{R}_0[i](i=1,..,n)$. To do this, in the space of varied parameters $l_{st}[i]\ (i=1,2,..,n)$, $X_0,Y_0,Z_0$, $X[i],Y[i],Z[i]$ the target function is selected

$$f_0 = \sum_{i=1}^{i=n} (Nst[i] - \bar{R}[i])^2 . \qquad (15)$$

To assess the proximity of the target function (14) to the minimum, a small value $OOO$ is assigned.

In accordance with the above, a version of the computational scheme of the kinematic device is formed in the form as a statically determinable hinged-rod system. In this case, the computational scheme should provide the additional mass with only one degree of freedom, namely the ability to move in parallel with the movements of the remaining masses in the process of natural oscillations.

According to the design conditions, the length of one of the main racks (for example, the $g$ th rack) is specified. Let's call this rack the base one, $l_{st}[g] = l_{st0}[g]$. The initial values of the remaining variable parameters are selected, that is, the lengths of the racks

$$\bar{l}_{st}[i] \; (i = 1, 2, .., (g-1), (g+1), .., n)$$

and coordinates $X_0, Y_0, Z_0$, $X[i], Y[i], Z[i]$.

These steps determine the initial geometry of the computational model of the kinematic device.

To further solve the problem, it is necessary to choose a method for finding the minimum of the target function (15) (steepest descent [10-15], random search [16-18], or others).

The algorithm for implementing further steps to develop the computational model of the kinematic device is similar to the algorithm of development of the computational model of the targeted constraints [6-9].

*The first step*. Increments are set to variable lengths of racks

$$l_{st}[i] = \bar{l}_{st}[i] + \Delta l (i = 1, 2, .., (g-1), (g+1), .., n)$$

and coordinates

$$X_0 = X_0 + \Delta X_0 , \quad Y_0 = Y_0 + \Delta Y_0 ,$$

$$Z_0 = Z_0 + \Delta Z_0 ;$$

$$X[i] = X[i] + \Delta X[i], \quad Y[i] = Y[i] + \Delta Y[i],$$

$$Z[i] = Z[i] + \Delta Z[i], (i = 1, 2, ...n_1) ;$$

Thus, the geometry of the computational scheme of the kinematic device is updated.

*The second step*. A system of equilibrium equations is compiled to determine the forces in the elements of the updated computational scheme of a statically determinate kinematic device.

*The third step*. From the equilibrium equations, the forces in the elements of the computational scheme of the kinematic device are found.

*The fourth step*. The value of the target function (15) is computed.

*The fifth step*. If we have $f_0 > OOO$, then in accordance with the chosen method of searching for the minimum of the target function (15), the varied parameters and thereby the computational scheme of the kinematic device and the system of equilibrium equations are adjusted. Then we go to the third step and the process continues.

*The sixth step*. If the value $f_0$ is less than a preselected small value $OOO$, then the process ends, and the computational scheme of the targeted kinematic device is developed with a given error ($OOO$).

Within development a computational scheme or creation of targeted constraints situations are known in which the values of the lengths of the main racks turn out to be of different signs. Structurally, such a computational scheme is almost impossible to implement. In these cases, within creation of targeted constraints, the constraint shifted in the direction of mass movement in a positive or negative direction by an amount at which the lengths of all main racks became of the same sign [7, 9]. Within development of computational scheme of kinematic devices, a similar the solution is not always applicable in such situations.

Allowance for the features of kinematic devices, when the above situation occurs, represents a separate problem and will be considered separately in subsequent paper.

Let us consider an approach to determining the value of additional mass ( $M_{add}$ ) from the condition that the kinetic energy of additional inertial forces is equal to the work of the inertial force created by the additional mass during natural vibrations.
It is obvious that the kinetic energy of additional inertial forces ( $E_k$ ) is determined by the dependence

$$E_k = M_{m0}\left[\sum_{i=1}^{n}\sum_{k=1}^{n}\omega_S^2 m_0[i,k]v_\omega[i,(q)]v_\omega[k,(q)]\right]. \quad (16)$$

Inertial force created by additional mass is equal to

$$N_{in}(t) = -M_{add}\frac{\partial^2 v(t)}{(\partial t)^2}. \quad (17)$$

Movement of additional mass in (17) is equal to

$$v(t) = v_m \sin(\omega_S t). \quad (18)$$

In (18) the amplitude value of the movement of additional mass during vibrations.
From (17) and (18) it follows that the amplitude value of the inertial force created by the additional mass

$$N_{in} = M_{add} v_m \omega_S^2. \quad (19)$$

Equating the kinetic energy of additional inertial forces (16) to the work of the inertial force $N_{in}$ on displacement $v_m$, we obtain $E_k = N_{in} v_m$ or $E_k = v_m^2 \omega_S^2$. Thus, we get

$$M_{add} = \frac{M_{m0}\left[\sum_{i=1}^{n}\sum_{k=1}^{n}\omega_S^2 m_0[i,k]v_\omega[i,(q)]v_\omega[k,(q)]\right]}{M_{add} v_m^2 \omega_S^2}. \quad (20)$$

In order to use dependence (20) it is necessary to determine firstly the displacement value $v_m$.
Let us consider the option of determining the value $v_m$ under the condition that the kinematic device moves as an absolutely rigid body only due to the displacement of the nodes in which the masses of the original system are located. To do this, in the generated scheme of the kinematic device, a unit force is introduced in the node for applying additional mass, directed parallel to the movement of the masses. The action of this force determines the forces in the main racks

$$N_{st,1}[i] \quad (i = 1,2,..,n).$$

The movements of the nodes, where the main racks are installed, are determined by the coordinates of the shape of the natural mode corresponding to the frequency at which the kinematic device is ‘targeted”

$$\mathbf{v_\omega[i,(q)] \quad (i = 1,2,..,n)}.$$

Thus, we have

$$v_m = \sum_{i=1}^{n} v_\omega[i,(q)]N_{st,1}[i], \quad (21)$$

$M_{add}$ is determined by (20). The determination did not take into account the compliance of the kinematic device rods. Therefore, the result of changes in the spectrum of natural frequencies will depend on the rigidity parameters of its elements.
Let us consider the possibility of taking into account the influence of the compliance of the rods of the kinematic device on the amount of movement of additional mass. Let us denote the amplitude of movement of additional mass during oscillations by $v_{m,0}$. The value $v_{m,0}$ consists of two components

$$v_{m,0} = v_m + v_{m,d}, \quad (22)$$

where $v_m$ is the component corresponding to the movement of the kinematic device as an absolutely rigid body (21); $v_{m,d}$ is the component of the movement of additional mass corresponding to the deformation of the rods of the kinematic device.
In order to determine $v_{m,d}$ the forces in all the rods of the developed diagram of the kinematic device from the action of a unit force introduced at the node of application of additional mass and directed parallel to the movement of the masses

$$N_{st,1}[i], N_{p,1}[j], N_{d,1}[k].$$

are found.
Here $N_{st,1}[i], N_{p,1}[j], N_{d,1}[k]$ are the corresponding forces are in the racks, belt rods and additional rods. The amplitude values of the forces

$$N_{st}[i], N_p[j], N_d[k],$$

that arise during natural vibrations in the rods of the kinematic device are determined at the end of the process of its creation. It should be noted that $N_{st}[i], N_p[j], N_d[k]$ are respectively the forces in the racks, belt rods and additional rods. Since the linear theory of small vibrations is used, the forces in the rods of the kinematic device are determined according to the undeformed scheme.
Thus, the displacement $v_{m,d}$ is determined by the constraint

$$v_{md} = \left[ \sum_{i=1}^{n} \frac{N_{st}[i] N_{st,1}[i] \left| l_{st}[i] \right|}{EF_{st}[i]} + \right. \\ + \sum_{j=1}^{n_2} \frac{N_p[j] N_{p,1}[j] l_p[j]}{EF_p[j]} + \\ \left. + \sum_{k=1}^{n_1} \frac{N_d[k] N_{d,1}[j] l_d[k]}{EF_d[k]} \right], \tag{23}$$

where $E, F_{st}[i], F_p[j], F_d[k]$ are respectively the elastic modulus of the material of the kinematic device and the cross-sectional area of the racks, belt and additional rods.
The total displacement is presented in (22).
The expression for determining the amplitude value of the inertial force created by the additional mass can be written in the following form

$$N_{in,0} = M_{add} v_{m,0} \omega_S^2. \tag{24}$$

The energy balance is now based on the equality of the sum of the kinetic energy of additional inertial forces ($E_k$) and the potential energy of deformation of the rods of the kinematic device ($E_d$) to the work of the inertial force $N_{in,0}$ on displacement $v_{m,0}$. This condition takes the form

$$E_k + E_d = N_{in,0} v_{m,0} = M_{add} v_{m,0}^2 \omega_S^2; \tag{25}$$

$$v_{md} = \left[ \sum_{i=1}^{n} \frac{N_{st}^2[i] l_{st}[i]}{EF_{st}[i]} + \sum_{j=1}^{n_2} \frac{N_p^2[j] l_p[j]}{EF_p[j]} + \right. \\ \left. + \sum_{k=1}^{n_1} \frac{N_d^2[k] l_d[k]}{EF_d[k]} \right]. \tag{26}$$

Finally, we have

$$M_{add} = \frac{E_k + E_d}{v_{m,0}^2 \omega_S^2}. \tag{27}$$

Before the formation of the kinematic device, a system with $n$ degrees of freedom was considered. Let us call the natural frequencies of this system (3) “basic”. If the kinematic device is created in such a way that the additional mass is able to move only in parallel with the movements of the remaining masses, then only one more degree of freedom is added to the considering system. Thus, the considering system has $(n+1)$ degrees of freedom. This circumstance causes the appearance of another “additional” natural frequency.
The targeting of the kinematic device is realized only in relation to the “basic” natural frequencies. The value of the “additional” natural frequency depends on the compliance parameters of the rods of the kinematic device and can be either

greater than the values of all "basic" frequencies or less than any of them.

## REFERENCES


1. **Lyahovich L.S., Maletkin O.Ju.** O pricel'nom regulirovanii sobstvennyh chastot uprugih system [On targeted control of natural frequencies of elastic systems]. // Izvestija vuzov. Stroitel'stvo i arhitektura, 1990, No. 1, pp. 113-117 (In Russian).
2. **Lyahovich L.S.** Osobye svojstva optimal'nyh sistem i osnovnye napravlenija ih realizacii v metodah rascheta sooruzhenij [Special properties of optimal systems and the main directions of their implementation in the methods of structural analysis]. Tomsk, TGASU, 2009. – 372 pages (In Russian).
3. **Lyakhovich L.S., Akimov P.A.** Aimed control of the frequency spectrum of eigenvibrations of elastic plates with a finite number of degrees of freedom of masses by superimposing additional constraints. // International Journal for Computational Civil and Structural Engineering, 2021, Volume 17, Issue 2, pp. 76-82.
4. **Lyakhovich L.S., Akimov P.A.** Aimed control of the frequency spectrum of eigenvibrations of elastic plates with a finite number of degrees of freedom by introducing additional generalized kinematic devices. // International Journal for Computational Civil and Structural Engineering, 2021, Volume 17, Issue 3, pp. 14-20.
5. **Akimov P.A, Lyahovich L.S.** Pricel'noe regulirovanie spektra chastot sobstvennyh kolebanij uprugih plastin s konechnym chislom stepenej svobody mass putem vvedenija dopolnitel'nyh obobshhennyh svjazej i obobshhennyh kinematicheskih ustrojstv [Precision control for eigen-frequency of elastic plates with finite number of mass degrees of freedom by using additional generalized connections and kinematic devices]. // Vestnik Tomskogo gosudarstvennogo arkhitekturno-stroitel'nogo universiteta. Journal of Construction and Architecture, 2021, Vol. 23, No. 4, pp. 57-67 (In Russian).
6. **Lyakhovich L.S., Akimov P.A.** About Development of Computational Schemes of Some Additional Constraints for Elastic Systems. Part 1: Theoretical Foundations [О формировании расчетных схем некоторых дополнительных связей для упругих систем. Часть 1: Теоретические основы подхода]. // Promyshlennoe i Grazhdanskoe Stroitelstvo, 2022, No. 9, pp. 4-10 (in Russian).
7. **Lyakhovich L.S., Akimov P.A.** About Development of Computational Schemes of Some Additional Constraints for Elastic Systems. Part 2: Samples of Analysis [О формировании расчетных схем некоторых дополнительных связей для упругих систем. Часть 2: Примеры расчетов]. // Promyshlennoe i Grazhdanskoe Stroitelstvo, 2022, No. 9, pp. 4-10 (in Russian).
8. **Lyakhovich L.S., Akimov P.A.** Formation of Computational Schemes of Additional Targeted Constraints That Regulate The Frequency Spectrum of Natural Oscillations of Elastic Systems With a Finite Number of Degrees of Mass Freedom, the Directions of Movement of Which are Parallel, But Do Not Lie in the Same Plane. Part 1: Theoretical Foundations. // International Journal for Computational Civil and Structural Engineering, 2022, Volume 18, Issue 2, pp. 183-193.
9. **Lyakhovich L.S., Akimov P.A.** Formation of Computational Schemes of Additional Targeted Constraints That Regulate The Frequency Spectrum of Natural Oscillations of Elastic Systems With a Finite Number of Degrees of Mass Freedom, the Directions of Movement of Which are Parallel, But Do Not Lie in the Same Plane. Part 2: The First Sample of Analysis. // International Journal for Computational Civil and Structural Engineering, 2022, Volume 18, Issue 3, pp. 137-146.
10. **Bertola M.** Nonlinear steepest descent approach to orthogonality on elliptic curves. //

Journal of Approximation Theory, 2022, Vol. 276, 105717.
11. **Chen Z., Fang Y., Kong X., Dehg L.** Identification of multi-axle vehicle loads on beam type bridge based on minimal residual norm steepest descent method. // Journal of Sound and Vibration, 2023, Vol. 563, 117866.
12. **Lapucci M., Mansueto P.** Improved front steepest descent for multi-objective optimization. // Operations Research Letters, 2023, Vol. 51, Issue 3, pp. 242-247.
13. **Mittal G., Gibi A.K.** A modified steepest descent method for solving non-smooth inverse problems. // Journal for Computational and Applied Mathematics, 2023, Vol. 424, 114997.
14. **Mittal G., Gibi A.K.** Convergence analysis of an optimally accurate frozen multi-level projected steepest descent iteration for solving inverse problems. // Journal of Complexity, 2023, Vol. 75, 101711.
15. **Ren Q.** Seismic acoustic full waveform inversion based on the steepest descent method and simple linear regression analysis. // Journal of Applied Geophysics, 2022, Vol. 203, 104686.
16. **Do B., Ohsaki M.** A random search for discrete robust design optimization of linear-elastic steel frames under interval parametric uncertainty. // Computers & Structures, 2021, Vol. 249, 106506.
17. **Oztas G.Z., Erdem S.** Random search with adaptive boundaries algorithm for obtaining better initial solutions. // Advances in Engineering Software, 2022, Vol. 169, 103141.
18. Wosniack M.E., Raposo E.P., Viswanathan G.M., da Luz M.G.E. A parallel algorithm for random searches. // Computer Physics Communications, 2015, Vol. 196, pp. 390-397.

## СПИСОК ЛИТЕРАТУРЫ

1. **Ляхович Л.С., Малеткин О.Ю.** О прицельном регулировании собственных частот упругих систем. // Известия вузов. Строительство и архитектура, 1990, № 1, с. 113-117.
2. **Ляхович Л.С.** Особые свойства оптимальных систем и основные направления их реализации в методах расчета сооружений. Монография. – Томск: Издательство ТГАСУ, 2009. - 372 с.
3. **Lyakhovich L.S., Akimov P.A.** Aimed control of the frequency spectrum of eigenvibrations of elastic plates with a finite number of degrees of freedom of masses by superimposing additional constraints. // International Journal for Computational Civil and Structural Engineering, 2021, Volume 17, Issue 2, pp. 76-82.
4. **Lyakhovich L.S., Akimov P.A.** Aimed control of the frequency spectrum of eigenvibrations of elastic plates with a finite number of degrees of freedom by introducing additional generalized kinematic devices. // International Journal for Computational Civil and Structural Engineering, 2021, Volume 17, Issue 3, pp. 14-20.
5. **Акимов П.А., Ляхович Л.С.** Прицельное регулирование спектра частот собственных колебаний упругих пластин с конечным числом степеней свободы масс путем введения дополнительных обобщенных связей и обобщенных кинематических устройств. // Вестник Томского государственного архитектурно-строительного университета, 2021, том 23, № 4, с. 57-67.
6. **Ляхович Л.С., Акимов П.А.** О формировании расчетных схем некоторых дополнительных связей для упругих систем. Часть 1. Теоретические основы подхода. // Промышленное и гражданское строительство, 202, №9, с. 4-10.
7. **Ляхович Л.С., Акимов П.А., Мешеулов Н.В.** О формировании расчетных схем некоторых дополнительных связей для упругих систем. Часть 2. Примеры расчёта. // Промышленное и гражданское строительство, 2022, №9, с. 11-19.

8. **Lyakhovich L.S., Akimov P.A.** Formation of Computational Schemes of Additional Targeted Constraints That Regulate The Frequency Spectrum of Natural Oscillations of Elastic Systems With a Finite Number of Degrees of Mass Freedom, the Directions of Movement of Which are Parallel, But Do Not Lie in the Same Plane. Part 1: Theoretical Foundations. // International Journal for Computational Civil and Structural Engineering, 2022, Volume 18, Issue 2, pp. 183-193.
9. **Lyakhovich L.S., Akimov P.A.** Formation of Computational Schemes of Additional Targeted Constraints That Regulate The Frequency Spectrum of Natural Oscillations of Elastic Systems With a Finite Number of Degrees of Mass Freedom, the Directions of Movement of Which are Parallel, But Do Not Lie in the Same Plane. Part 2: The First Sample of Analysis. // International Journal for Computational Civil and Structural Engineering, 2022, Volume 18, Issue 3, pp. 137-146.
10. **Bertola M.** Nonlinear steepest descent approach to orthogonality on elliptic curves. // Journal of Approximation Theory, 2022, Vol. 276, 105717.
11. **Chen Z., Fang Y., Kong X., Dehg L.** Identification of multi-axle vehicle loads on beam type bridge based on minimal residual norm steepest descent method. // Journal of Sound and Vibration, 2023, Vol. 563, 117866.
12. **Lapucci M., Mansueto P.** Improved front steepest descent for multi-objective optimization. // Operations Research Letters, 2023, Vol. 51, Issue 3, pp. 242-247.
13. **Mittal G., Gibi A.K.** A modified steepest descent method for solving non-smooth inverse problems. // Journal for Computational and Applied Mathematics, 2023, Vol. 424, 114997.
14. **Mittal G., Gibi A.K.** Convergence analysis of an optimally accurate frozen multi-level projected steepest descent iteration for solving inverse problems. // Journal of Complexity, 2023, Vol. 75, 101711.
15. **Ren Q.** Seismic acoustic full waveform inversion based on the steepest descent method and simple linear regression analysis. // Journal of Applied Geophysics, 2022, Vol. 203, 104686.
16. **Do B., Ohsaki M.** A random search for discrete robust design optimization of linear-elastic steel frames under interval parametric uncertainty. // Computers & Structures, 2021, Vol. 249, 106506.
17. **Oztas G.Z., Erdem S.** Random search with adaptive boundaries algorithm for obtaining better initial solutions. // Advances in Engineering Software, 2022, Vol. 169, 103141.
18. Wosniack M.E., Raposo E.P., Viswanathan G.M., da Luz M.G.E. A parallel algorithm for random searches. // Computer Physics Communications, 2015, Vol. 196, pp. 390-397.

*Leonid S. Lyakhovich*, Full Member of the Russian Academy of Architecture and Construction Sciences, Professor, DSc, Professor of Department of Structural Mechanics, Tomsk State University of Architecture and Building; 2, Solyanaya St., 2, Tomsk, 634003, Russia; E-mail: lls@tsuab.ru

*Pavel A. Akimov*, Full Member of the Russian Academy of Architecture and Construction Sciences, Professor, Dr.Sc.; Rector of National Research Moscow State University of Civil Engineering; 26, Yaroslavskoe Shosse, Moscow, 129337, Russia; phone: +7(495) 651-81-85; Fax: +7(499) 183-44-38; E-mail: AkimovPA@mgsu.ru, rector@mgsu.ru, pavel.akimov@gmail.com.

*Zaur R. Galyautdinov,* Associate Professor, DSc, Professor of Department of Reinforced Concrete and Stone Structures, Tomsk State University of Architecture and Building; 2, Solyanaya St., 2, Tomsk, 634003, Russia;
E-mail: gazr@yandex.ru.

*Andrey S. Plyaskin*, Associate Professor, Ph.D., Head of Department of Metal and Wooden Structures, Tomsk State University of Architecture and Building; 2, Solyanaya St., 2, Tomsk, 634003, Russia;
E-mail: plyaskinandrei@mail.ru.

*Ляхович Леонид Семенович*, академик РААСН, профессор, доктор технических наук, профессор кафедры строительной механики, Томский государственный архитектурно-строительный университет; 634003, Россия, г. Томск, Соляная пл. 2;
E-mail: lls@tsuab.ru

*Акимов Павел Алексеевич*, академик РААСН, профессор, доктор технических наук; ректор Национального исследовательского Московского государственного строительного университета; 129337, Россия, г. Москва, Ярославское шоссе, дом 26;
телефон: +7(495) 651-81-85; факс: +7(499) 183-44-38;
Email: AkimovPA@mgsu.ru, rector@mgsu.ru, pavel.akimov@gmail.com.

*Галяутдинов Заур Рашидович*, доцент, доктор технических наук, доцент кафедры железобетонных и каменных конструкций, Томский государственный архитектурно-строительный университет; 634003, Россия, г. Томск, Соляная пл. 2; E-mail: gazr@yandex.ru.

*Пляскин Андрей Сергеевич*, доцент, кандидат технических наук, заведующий кафедрой металлических и деревянных конструкций, Томский государственный архитектурно-строительный университет; 634003, Россия, г. Томск, Соляная пл. 2;
E-mail: plyaskinandrei@mail.ru.