

THE IMPACT OF COUPLING THERMOELASTICITY EQUATIONS ON SETTLEMENT OF STRUCTURES ON FROZEN SOIL

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Abstract: This paper seeks to propose a numeric model for the determination of structure's settlement on frozen soil. The settlement is driven by the dead weight of the soil, weight of the erected building and thawing of the soil underneath the building due to its heating. A benchmark task was solved both in coupled and uncoupled problem settings. A special analysis is dedicated to the impact of thermoelasticity equations being coupled or independent.

Key words: multiyear frozen soils, thermoelasticity, structure settlement, thawing

ВЛИЯНИЕ СВЯЗАННОСТИ УРАВНЕНИЙ ТЕРМОУПРУГОСТИ НА ОСАДКУ СООРУЖЕНИЯ НА МЕРЗЛОМ ГРУНТЕ

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Аннотация: В данной работе предложена численная модель для определения осадки сооружения на мерзлом грунте. Осадка происходит за счет действия собственного веса грунта, веса возведенного сооружения и за счет оттаивания грунта под фундаментом сооружения из-за его разогрева. Была решена модельная задача в связанной и несвязанной постановке. Проанализировано влияние эффекта связанности уравнений термоупругости.

Ключевые слова: многолетнемерзлые грунты, термоупругость, осадка сооружения, процесс оттаивания

1. PROBLEM SETTING

Expression [1] presents thermoelasticity equations linearized around the initial temperature T_0 of the undeformed state. These equations describe soil deformation driven by the temperature gradient T and bulk forces. By generalizing the equations for the case of partially frozen soil, we obtain the following system:

$$\begin{cases} \frac{\partial}{\partial x_j} \left(C_{ijkl}^{th(f)} \frac{\partial u_k}{\partial x_l} \right) = 3\alpha^{th(f)} K^{th(f)} \frac{\partial (T - T_0)}{\partial x_i} - \rho F_i \\ \frac{\partial}{\partial x_i} \left(\lambda_{ij}^{th(f)} \frac{\partial (T - T_0)}{\partial x_j} \right) + q_V = 3\alpha^{th(f)} K^{th(f)} T_0 \frac{\partial}{\partial t} \left(\frac{\partial u_i}{\partial x_i} \right) + \\ + \rho c^{th(f)} \frac{\partial}{\partial t} (T - T_0) \end{cases} \quad (1)$$

where

$$C_{ijkl}^{th(f)} = \frac{\nu^{th(f)} E^{th(f)}}{(1 + \nu^{th(f)})(1 - 2\nu^{th(f)})} \delta_{ij} \delta_{kl} + \frac{E^{th(f)}}{2(1 + \nu^{th(f)})} (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk})$$

– elastic modulus tensor of frozen or thawed soil; $k^{th(f)}$ – tensor permeability coefficient of thawed or frozen soil; $\alpha^{th(f)}$ – linear thermal expansion coefficient of thawed or frozen soil; $c^{th(f)}$ – specific heat capacity of thawed or frozen soil;

$$K^{th(f)} = \frac{E^{th(f)}}{3(1-2\nu^{th(f)})}$$

– cubical expansion (compression) coefficient of thawed or frozen soil; q_V – power of inner heat sources in a volumetric unit; ρ – density of dry soil. The target values of these equations are soil displacements \vec{u} and temperature T . Based on the papers [2-4], the analytical dependency (2) for the heat capacity coefficient takes account of ice-water phase transitions when crossing the ground thawing temperature:

$$c^{th(f)} = c^{(f)} + L_0 \frac{\partial W_w}{\partial T}, \quad (2)$$

where $L_0 = 335 \cdot 10^6 \text{ J} / \text{m}^3$, while the total content of the unfrozen water depends on the temperature and is given by the formula:

$$W_w = k_w W_p, \quad (3)$$

where W_p – humidity at the lower plastic limit of soil when rolled into threads, W_w – soil humidity due to unfrozen water, k_w – unfrozen water content coefficient identified for various temperatures according to Table 3.1. in [5]. The analytical dependency established between the Young’s modulus $E^{th(f)}$ and temperature and interpolating experimental data is presented in [6]. Analogous curves were constructed for Poisson’s ratio $\nu^{th(f)}$ and thermal conductivity $\lambda^{th(f)}$ in [7]. The problem setting is finalized by standard initial and boundary conditions. The equations (1) of the task are coupled. They do not allow for the possibility to ascertain the thermal field independently without finding the soil deformations.

While accepting a simplified hypothesis that thermal field is independent from soil deformations, we can relax it and solve the equations

independently one from the other: first, we determine the thermal field based on the heat propagation equation, and then identify structure’s settlement via the Duhamel Neumann-type equation of equilibrium. In this case, the system of equations in question takes the following form:

$$\begin{cases} \frac{\partial}{\partial x_j} \left(C_{ijkl}^{th(f)} \frac{\partial u_k}{\partial x_i} \right) = 3\alpha^{th(f)} K^{th(f)} \frac{\partial (T - T_0)}{\partial x_i} - \rho F_i \\ \frac{\partial}{\partial x_i} \left(\lambda_{ij}^{th(f)} \frac{\partial (T - T_0)}{\partial x_j} \right) + q_V = \rho c^{th(f)} \frac{\partial (T - T_0)}{\partial t}. \end{cases} \quad (4)$$

2. NUMERICAL SOLUTION

For solving the system (1), Euler’s implicit method was applied for time discretization of the task $t^{n+1} = t^n + \Delta t$:

$$\begin{cases} \frac{\partial}{\partial x_j} \left(C_{ijkl}^{th(f)} \frac{\partial u_k^{n+1}}{\partial x_i} \right) - \\ - 3\alpha^{th(f)} K^{th(f)} \frac{\partial (T^{n+1} - T_0)}{\partial x_i} = -\rho F_i \\ \frac{\partial}{\partial x_i} \left(-3\alpha^{th(f)} K^{th(f)} T_0 \frac{u_i^{n+1}}{\Delta t} + \lambda_{ij}^{th(f)} \frac{\partial (T^{n+1} - T_0)}{\partial x_j} \right) - \\ - \frac{\rho c^{th(f)}}{\Delta t} (T^{n+1} - T_0) = \\ = -3\alpha^{th(f)} K^{th(f)} T_0 \frac{u_i^n}{\Delta t} - \frac{\rho c^{th(f)}}{\Delta t} (T^n - T_0) - q_V, \end{cases}$$

Where the superscript $n+1$ of the target k^{th} displacement vector component u_k^{n+1} as well as of the temperature T^{n+1} means that the values of these functions were utilized till the $n+1^{\text{th}}$ time increment. This equation can be presented as a block matrix:

$$\begin{pmatrix} \frac{\partial}{\partial x_j} \left(C_{ijkl}^{th(f)} \frac{\partial(\cdot)}{\partial x_l} \right) & -\frac{\partial}{\partial x_i} (3\alpha^{th(f)} K^{th(f)}(\cdot)) \\ -\frac{\partial}{\partial x_k} \left(3\alpha^{th(f)} K^{th(f)} T_0 \frac{(\cdot)}{\Delta t} \right) & \frac{\partial}{\partial x_i} \left(\lambda_{ij}^{th(f)} \frac{\partial(\cdot)}{\partial x_j} \right) - \frac{\rho c^{th(f)} n}{\Delta t} (\cdot) \end{pmatrix} \begin{pmatrix} u_k^{n+1} \\ T^{n+1} - T_0 \end{pmatrix} = \begin{pmatrix} \bar{f}_1 \\ f_2 \end{pmatrix},$$

Where the vector of the right hand side is equal to

$$\begin{pmatrix} \bar{f}_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} -\rho F_i \\ -3\alpha^{th(f)} K^{th(f)} T_0 \frac{u_i^n}{\Delta t} - \frac{\rho c^{th(f)}}{\Delta t} (T^n - T_0) - q_v \end{pmatrix}.$$

The discretization of the spatial variables is accomplished by the method of finite elements and use of standard isoparametric brick-type finite elements. To find a solution, which is a column vector of variables

$$\begin{pmatrix} u_k^{n+1} \\ T^{n+1} - T_0 \end{pmatrix},$$

using Gaussian method for solving systems of linear algebraic equations proves sufficient. The problem's coefficients are temperature functions. Their value for the current time increment is calculated based on the solution to the problem found for the previous time increment. For this reason, an internal iteration process was introduced, so that the numeric and exact solutions would match at each time increment.

3. BENCHMARK PROBLEM

The problem of finding the building settlement on frozen soil makes use of the numeric model constructed earlier. The settlement is driven by the dead load of the soil, weight of the erected facility and soil thawing under the building's foundation due to its heating. The foundation without heat insulation is warmed up because the temperature inside the facility exceeds that of outside. The calculation performed for the model employs both coupled and independent equations. Figure 1 displays one fourth of the

computational domain. The soil is modeled as a three-layer environment with thermomechanical constants, the values of which can be found in Table 1. Initial temperature distribution T_0 within the domain (see Figure 2) is considered at the first stage of the numeric algorithm employed when solving the thermal conductivity equation for a specific temperature and heat flow at the boundary of the domain. The upper horizontal boundary of the domain (soil surface) maintains the temperature $+2\text{ }^{\circ}\text{C}$, while the lower horizontal boundary of the domain (foot of the multiyear frozen soil) keeps the temperature $-2\text{ }^{\circ}\text{C}$. This temperature range suggests that the multiyear frozen soil commences 10 m below the earth surface. The second stage of the numeric algorithm solves the problem of heating the surface layer of soil under the foundation bed by the temperature $+20\text{ }^{\circ}\text{C}$ applied continuously. The second stage starts after the construction has been completed and the structure has been commissioned. The calculation is performed till the temporal solution is found, which is assumed to be 30 years in this model. Figures 3-6 display the vertical displacement pattern and temperature pattern 30 years after the construction completion. The data in Figures 3 and 4 were obtained when solving the task in the coupled problem setting, while those of Figures 5 and 6 were derived from the solution in the uncoupled problem setting. The analysis of the obtained data brings about the conclusion that the benchmark problem features the soil thawing depth being equal to 15 m for the coupled problem setting and 14 m for the uncoupled problem setting. Relative differential settlement of the facility for the solution in the coupled problem setting is 17,5 cm, while the same for the uncoupled task setting is 13 cm.

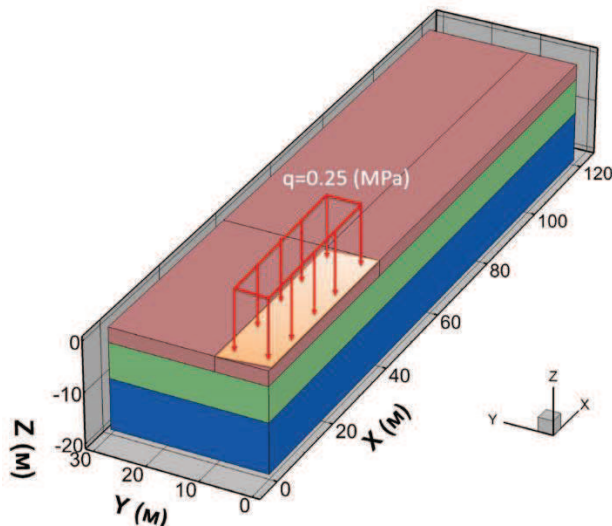


Figure 1. 1/4 of the computational domain, with distributed surface pressure applied. The surface taken up by the structure is depicted in yellow.

	1	2	3	4
Linear thermal expansion coefficient $\alpha^{th(f)}$, $1/^\circ\text{C}$		0	0	0.0009

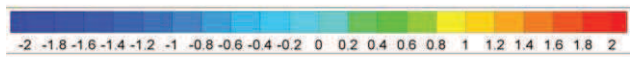
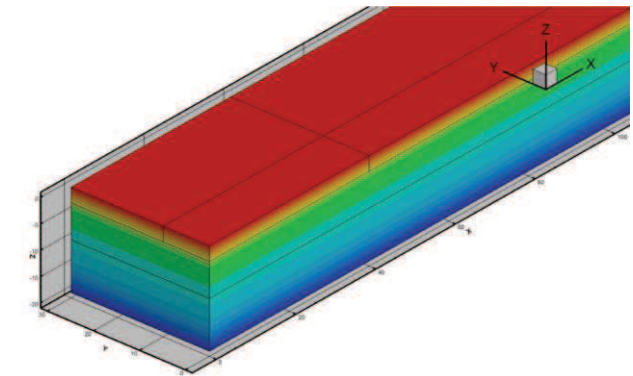
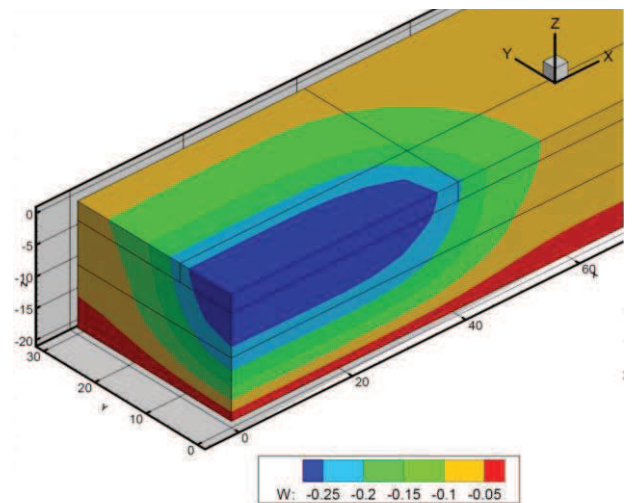


Figure 2. Initial temperature distribution across the computational domain, $^\circ\text{C}$.

Constants of soil layer	Crushed stone or grus fill material	Clayey sand	Grus with clayey sand fill (multiyear frozen)
1	2	3	4
Soil depth, m	3	7	10
Density ρ , kg/m^3	2220	2120	2360
Deformation modulus $E^{th(f)}$, mPa	40	30.6	30
Poisson coefficient $\nu^{th(f)}$	0.4	0.4	0.4
Specific heat capacity $c^{th(f)}$, $\text{J}/(\text{kg}\cdot^\circ\text{C})$	918	1076.6	960
Heat transmission $\lambda^{th(f)}$, $\text{W}/(\text{m}\cdot^\circ\text{C})$	1.8	1.74	1.48

Figure 3. Vertical displacements (m) in the soil 30 years after construction completion. Solution in the coupled problem setting. Relative settlement equal to 17.5 cm.



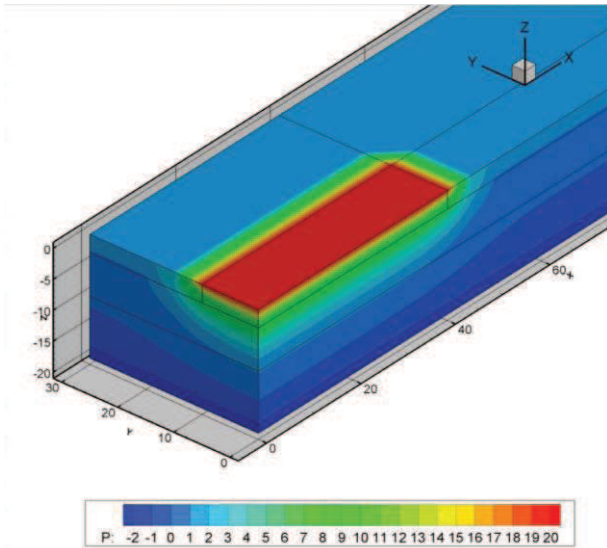


Figure 4. Temperature distribution ($^{\circ}\text{C}$) in the soil 30 years after construction completion. Solution in the coupled problem setting. Thawing depth found to be 15 m.

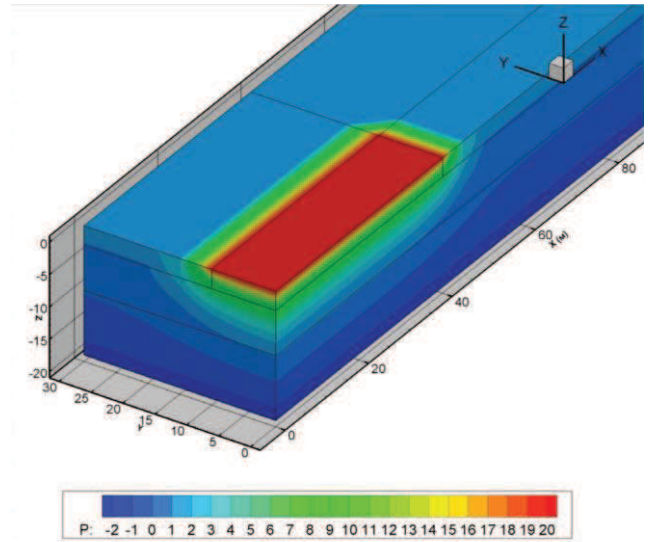


Figure 6. Temperature distribution ($^{\circ}\text{C}$) in the soil 30 years after construction completion. Solution in the uncoupled problem setting. Thawing depth found to be 14 m.

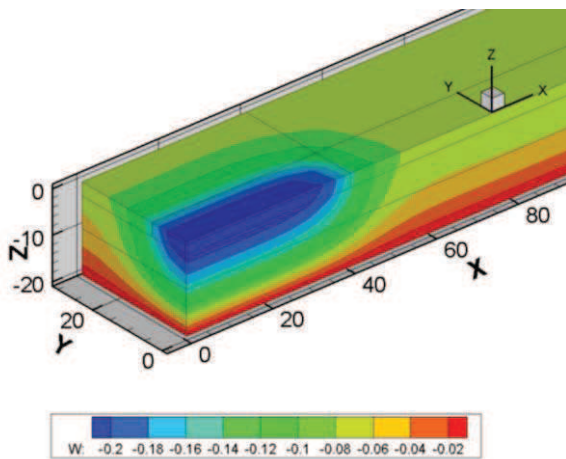


Figure 5. Vertical displacements (m) in the soil 30 years after construction completion. Solution in the uncoupled problem setting. Relative settlement of the structure found to be 13 cm.

4. CONCLUSION

The analysis of the results obtained gives rise to the following conclusions. If taken account of, the thermomechanical term

$$3\alpha^{th(f)}K^{th(f)}T_0\frac{\partial}{\partial t}\left(\frac{\partial u_i}{\partial x_i}\right)$$

of the heat propagation equation changes the value of thawed soil settlement up to 25%. The effect of coupling the thermoelasticity equations intensifies with the increase in deformation exerted on the soil by the bulk and surface forces in a unit of time. Equilibrium equations of elasticity theory for the determination of soil settlement make it possible to estimate 3-dimensional soil thawing-driven deformation. In contrast, the same cannot be achieved via models employing the method of layer-by-layer addition.

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