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PROBLEM OF PLATE BENDING IN THE MOMENT ASYMMETRIC THEORY OF ELASTICITY

Nelly N. Rogacheva, Yulia G. Zheglova

National Research Moscow State University of Civil Engineering, Moscow, RUSSIA

Abstract. For a number of materials used in modern practice, calculations according to the classical theory of elasticity give incorrect results. To ensure the reliable operation of structures, there is a need for new theories. At present, of particular interest for practical applications is the asymmetric moment theory of elasticity. In the work, by the method of hypotheses, the three-dimensional equations of the moment asymmetric theory of elasticity are reduced to the equations of the theory of plates. The hypotheses of the theory of plates in the moment theory of elasticity are formulated on the basis of previously obtained our results of the reduction of three-dimensional equations to two-dimensional theories by a mathematical method. Just as in the classical theory of elasticity, the complete problem of the moment theory of plates is divided into two problems - a plane problem and a problem of plate bending. The equations of the plane problem have been obtained in many papers. The situation is different with the construction of the theory of plate bending in the moment theory of elasticity. In this work, for the first time, substantiated hypotheses are formulated and a consistent theory of plate bending is presented. A numerical calculation of the bending of a rectangular hinged plate is carried out according to the obtained applied theory. The calculation results are presented in the form of graphs.

Keywords: plate bending, moment asymmetric theory of elasticity

ЗАДАЧА ИЗГИБА ПЛАСТИНЫ В МОМЕНТНОЙ АСИММЕТРИЧНОЙ ТЕОРИИ УПРУГОСТИ

Н.Н. Рогачева, Ю.Г. Жеглова

Национальный исследовательский Московский государственный строительный университет, г. Москва, РОССИЯ

Аннотация. Для ряда материалов, используемых в современной практике, расчеты по классической теории упругости дают неверные результаты. Для обеспечения надежной работы конструкций необходимы новые теории. В настоящее время особый интерес для практических приложений представляет асимметричная моментная теория упругости. В работе методом гипотез трехмерные уравнения моментной несимметричной теории упругости сведены к уравнениям теории пластин. Гипотезы теории пластин моментной теории упругости сформулированы на основе ранее полученных нами результатов сведения математическим методом трехмерных уравнений к двумерным теориям. Как и в классической теории упругости, полная задача теории моментов пластин делится на две задачи — плоскую задачу и задачу об изгибе пластин. Уравнения плоской задачи получены во многих работах . Иначе обстоит дело с построением теории изгиба пластин в моментной теории упругости. В данной работе впервые сформулированы обоснованные гипотезы и представлена непротиворечивая теория изгиба пластин. По полученной прикладной теории проведен численный расчет на изгиб прямоугольной откидной пластины. Результаты расчета представлены в виде графиков.

Ключевые слова: изгиб пластин, моментная ассиметричная теория упругости

1. INTRODUCTION

In connection with the appearance and application of new materials interest in non-classical theories of asymmetric elasticity has recently grown [1-13]. Experiments confirm that for a number of materials, for example, polymers, porous materials, the moment asymmetric theory of elasticity should be used [6, 8] . Many papers focus on plates and shells

is often constructed by means of hypotheses [1, 2, 3. 10, 12, 13]. As a rule, the well known Kirchhoff's hypotheses are used for the sought-for quantities of symmetric elasticity and new hypotheses are adopted for quantities which do not exist in the symmetric theory of elasticity, such as the components of moment stress tensor and an independent rotation vector. The use of a false hypotheses leads to considerable errors.
The methods to reduce 3D equations to 2D type will be further called reduction methods.
Another reduction method is the asymptotic method. This method allows to qualitatively investigate the behavior stress-strain state, passing from 3D to 2D problem without applying any hypotheses, next to estimate the errors of the resulting theories. The asymptotic reduction method was tested for many problems [14, 15].
We reduced the 3D problem of asymmetric elasticity to the plate theory by the asymptotic method in [9]. Two iterative processes are used in order to pass from a 3D problem to a 2D problem. The first iterative process allows to construct the internal stress-strain state which can be approximately described by the plate theory in asymmetric elasticity. The second iterative process is used to create a quickly decreasing stress-strain state; i.e. the boundary layer localized near the edge. The boundary layer and the internal stress-strain state interact at the edge provided 3D boundary conditions are satisfied. The investigation of this interaction gives the boundary conditions for the plate theory and boundary layer. Note that the linear problem of asymmetric elasticity has the only solution and the unique asymptotics. It is very important to choose a correct asymptotics. Incorrect asymptotics leads to a wrong theory.

2. BASIC EQUATIONS

We write a complete system of equations for asymmetric theory of elasticity in a tensor form [1].
The equilibrium equations:

$$\sigma_{nm,n} = 0, \;\; \mu_{nm,n} + \ni_{mnk}\, \sigma_{nk} = 0; \qquad (1)$$

The constitutive relations:

$$\begin{aligned} \sigma_{nm} &= (\mu+\alpha)e_{nm} + (\mu-\alpha)e_{mn} + \lambda e_{kk}\delta_{nm}, \\ \mu_{nm} &= (\gamma+\varepsilon)\chi_{nm} + (\gamma-\varepsilon)\chi_{mn} + \beta\chi_{kk}\delta_{nm}. \end{aligned} \qquad (2)$$

The geometrical relations:

$$e_{nm} = u_{m,n} - \ni_{knm}\, \omega_k, \quad \chi_{nm} = \omega_{m,n}. \qquad (3)$$

Here σ_{nm}, μ_{nm} are the components of the force and moment stress tensors, e_{nm} are the components of the asymmetric strain tensor, χ_{nm} are the components of the asymmetric bending-torsion tensor, $\lambda, \mu, \alpha, \beta, \gamma, \varepsilon$ are the constants of the material of asymmetric elasticity, u_n are the components of the displacement vector, ω_n are the components of the rotation vector at the point of the body, $\ni_{nmk}$ is the Levi-Civita symbol. The subscripts *n, m* after a comma denote differentiation with respect to the coordinates x_n and x_m respectively. Here and henceforth, the subscripts *n, m, k* take the values 1, 2, 3 and the subscripts *i, j* ($(i \neq j)$ take the values 1, 2.
A thin plate is located in Cartesian coordinates. The plane $x_1\, x_2$ coincides with the middle plane of the plate and the x_3 axis is orthogonal to the middle plane.
The boundary conditions on the plate faces $x_3 = \pm h$ are written as

$$\sigma_{i3} = q_i^{\pm}, \; \sigma_{33} = q_3^{\pm}, \;\; \mu_{3i} = m_i^{\pm}, \; \mu_{33} = m_3^{\pm}, \qquad (4)$$

where $q_n^{\pm}$, $m_n^{\pm}$ are the components of the given external forces and moments.
On the edge of the plate $x_i = const$ six boundary conditions are given. For example, they can be written at the free edge as follows:

$$\begin{aligned} &\sigma_{ii} = 0, \; \sigma_{ji} = 0, \; \sigma_{i3} = 0, \\ &\mu_{ii} = 0, \; \mu_{ji} = 0, \; \mu_{i3} = 0, \end{aligned} \qquad (5)$$

and at the rigidly fixed edge they can be written at the free edge as follows:

$$\begin{aligned} &u_i = 0,\ u_j = 0,\ u_3 = 0, \\ &\omega_i = 0,\ \omega_j = 0,\ \omega_3 = 0. \end{aligned} \tag{6}$$

3. CONSTRUCTING THE EQUATIONS OF THE PLATE THEORY

Here we will use the hypotheses method since the asymptotical method is complex and requires special knowledge. The hypotheses way is very simple and clear.
The plate theory in asymmetric elasticity was constructed by asymptotical method in [9].
Using the results of the mathematical reduction 3D asymmetric elasticity theory to 2D plate theory we can formulate the correct hypotheses.
In the considered asymmetric elasticity, the equations of state include six physical constants $\gamma, \beta, \varepsilon$, μ, λ, α. We assume that the quantities μR^2, λR^2, $\alpha R^2, \gamma, \beta, \varepsilon$ are of the same order, where value R is the characteristic size of the plate.
Just as in the classical theory of elasticity, in the moment asymmetric theory of elasticity, the complete system of plate equations is divided into two independent systems for a plane problem and plate bending.
We divide all the required values of the theory of plates into two groups (7) and (8)

$$u_i,\ \omega_3,\ e_{ij},\ e_{ii},\ \chi_{i3},\ \chi_{3i},\ \sigma_{ij},\ \sigma_{ii},\ \sigma_{33},\ \mu_{i3},\ \mu_{3i} \tag{7}$$

$$u_3, \omega_i, e_{i3}, e_{3i}, \chi_{ij}, \chi_{ii}, \sigma_{i3}, \sigma_{3i}, \mu_{ii}, \mu_{ij}, \mu_{33}. \tag{8}$$

In the plane problem, the required quantities (7) are even functions of the x_3 coordinate, and the quantities (8) are odd functions of the coordinate x_3. In the bending problem, the opposite is true: quantities (7) are odd functions of the x_3 coordinate, and quantities (8) are even functions of the coordinate x_3.

Let us formulate hypotheses for constructing the equations of the plane problem for the asymmetric theory of elasticity
The first hypothesis.
The tangential displacement u_i does not depend on the x_3-coordinate (it is constant along the thickness)

$$u_i\big|_{x_3=0} = v_i$$

The second hypothesis.
The angle of rotation ω_3 does not depend on x_3

$$\omega_3\big|_{x_3=0} = \psi_3$$

The third hypothesis.
The components of moment stresses μ_{i3} do not depend on x_3.
From the equilibrium equation (1) it can be seen that if generalized displacements u_i, ω_3, do not depend on x_3, then from formulas (3) it is obvious that deformations e_{ii}, e_{ij}, χ_{i3}, also do not depend on x_3.
Let us apply these hypotheses to the 3D equation of asymmetric elasticity, just as it is done when constructing the theory of Kirchhoff plate. Then we will move on to the forces and moments accepted in the theory of plate according to the following formulas:

$$\begin{aligned} &T_{ii} = \int_{-h}^{+h} \sigma_{ii} dx_3, \quad T_{ij} = \int_{-h}^{+h} \sigma_{ij} dx_3, \\ &M_{i3} = \int_{-h}^{+h} \mu_{i3} dx_3, \quad i \neq j = 1,2. \end{aligned} \tag{9}$$

As a result of simple transformations, we obtain the following equations of a plane problem in the asymmetric theory of elasticity
The equilibrium equations

$$\frac{\partial T_{ii}}{\partial x_i}+\frac{\partial T_{ji}}{\partial x_j}+X_i=0,$$
$$\frac{\partial M_{13}}{\partial x_1}+\frac{\partial M_{23}}{\partial x_2}+T_{12}-T_{21}+\mathrm{M}=0, \qquad (10)$$
$$X_i=q_i^+ + q_i^-, \quad \mathrm{M}=m_3^+ + m_3^-.$$

The constitutive relations

$$T_{ii}=\frac{4h\mu(\mu+\lambda)}{2\mu+\lambda}\varepsilon_i+\frac{2h\mu\lambda}{2\mu+\lambda}\varepsilon_j+\frac{h\lambda}{2\mu+\lambda}(q_3^+ - q_3^-),$$
$$T_{ij}=2h[(\mu+\alpha)\varepsilon_{ij}+(\mu-\alpha)\varepsilon_{ji}], \qquad (11)$$
$$M_{i3}=\frac{8h\gamma\varepsilon}{\gamma+\varepsilon}\varphi_{i3}+\frac{h(\gamma-\varepsilon)}{(\gamma+\varepsilon)}(m_i^+ - m_i^-).$$

The geometrical relations

$$\varepsilon_i=\frac{\partial v_i}{\partial x_i},\ \varepsilon_{ij}=\frac{\partial v_j}{\partial x_i}+(-1)^i\psi_3,\ \phi_{i3}=\frac{\partial\psi_3}{\partial x_i} \qquad (12)$$

where

$$\phi_{i3}=\chi_{i3}\big|_{x_3=0},\ \psi_3=\omega_3\big|_{x_3=0}.$$

Equations (10)-(12) have been obtained in many papers [1, 2, 3. 10, 12, 13]. This is explained by the fact that in this case the hypotheses very similar to those of Kirchhoff are used for the construction classical plane plate theory. There is a difference only in relations for the forces T_{ii} and the moments M_{i3}. In these relations we have small additional terms taking into account the external load components $(q_3^+ - q_3^-)$ and $(m_i^+ - m_i^-)$.

Let us formulate hypotheses for constructing equations for the problem of plate bending.

The first hypotheses.

The components of generalized displacements u_3, ω_i do not depend on the coordinate x_3, i.e. they are constant across the plate thickness

$$u_3\big|_{x_3=0}=w,\ \omega_i\big|_{x_3=0}=\psi_i \qquad (13)$$

The second hypothesis.

The components of the stress tensor σ_{i3} are much larger than the stress σ_{3i}.

By virtue of hypothesis 2, in the second relation (14), the quantity σ_{3i} can be neglected

$$\sigma_{i3}=(\mu+\alpha)e_{i3}+(\mu-\alpha)e_{3i},$$
$$\sigma_{3i}=(\mu+\alpha)e_{3i}+(\mu-\alpha)e_{i3}. \qquad (14)$$

Resolving after discarding σ_{3i} equation (14) with respect to the deformation components e_{i3}, e_{3i} we obtain the following formulas:

$$e_{3i}=-\frac{(\mu-\alpha)}{(\mu+\alpha)}e_{i3},\quad \sigma_{i3}=\frac{4\mu\alpha}{(\mu+\alpha)}e_{i3} \qquad (15)$$

By virtue of the first hypothesis, the quantities u_3 and ω_i do not depend on the variable x_3, so it follows from relations (15), that e_{3i}, e_{i3} and σ_{i3} do not depend on the variable x_3.

Let us introduce the notation adopted in the theory of plates

$$N_{i3}=\int_{-h}^{+h}\sigma_{i3}dx_3,\quad N_{3i}=\int_{-h}^{+h}\sigma_{3i}dx_3,$$
$$G_{ii}=\int_{-h}^{+h}\sigma_{ii}x_3dx_3,\quad G_{ij}=\int_{-h}^{+h}\sigma_{ij}x_3dx_3, \qquad (16)$$
$$M_{ii}=\int_{-h}^{+h}\mu_{ii}dx_3,\quad M_{ij}=\int_{-h}^{+h}\mu_{ij}dx_3,$$
$$i\neq j=1,2,\ w=v_3\big|_{x_3=0},\ \psi_i=\omega_i\big|_{x_3=0}.$$

Continuing such simplifications in the equations of the 3D asymmetric theory of elasticity, we obtain the following 2D equations of the bending problem in the notation of the theory of plates:

The equilibrium equations

$$\frac{\partial N_{13}}{\partial x_1}+\frac{\partial N_{23}}{\partial x_2}+Z=0,$$
$$\frac{\partial M_{ii}}{\partial x_i}+\frac{\partial M_{ji}}{\partial x_j}+\mathrm{M}_i+(-1)^j N_{j3}=0, \quad (17)$$
$$\frac{\partial G_{ii}}{\partial x_i}+\frac{\partial G_{ji}}{\partial x_j}-N_{3i}+Y_i=0,$$

where Z, M_i, Y_i are the surface load

$$Z=h(q_3^+ + q_3^-), \quad \mathrm{M}_i=(m_i^+ + m_i^-),$$
$$Y_i=h(q_i^+ - q_i^-).$$

The constitutive relations

$$N_{i3}=\frac{8h\mu\alpha}{(\mu+\alpha)}\varepsilon_{i3},$$
$$M_{ii}=2h\left(\frac{4\gamma(\gamma+\beta)}{2\gamma+\beta}\phi_i+\frac{2\gamma\beta}{2\gamma+\beta}\phi_j\right),$$
$$M_{ij}=2h[(\gamma+\varepsilon)\phi_{ij}+(\gamma-\varepsilon)\phi_{ji}],$$
$$G_{ii}=\frac{2h^3}{3}\left(\frac{4\mu(\mu+\lambda)}{2\mu+\lambda}\kappa_{ii}+\frac{2\mu\lambda}{2\mu+\lambda}\kappa_{jj}\right)+$$
$$+\frac{\lambda}{2\mu+\lambda}\frac{h^2}{3}(q_3^+ + q_3^-), \quad (18)$$
$$G_{ij}=\frac{2h^3}{3}\left((\mu+\alpha)\kappa_{ij}+(\mu-\alpha)\kappa_{ji}\right).$$

The geometrical relations

$$\varepsilon_{i3}=\frac{\partial w}{\partial x_i}+(-1)^j\psi_j, \quad \varphi_i=\frac{\partial \chi_i}{\partial x_i}, \quad \varphi_{ij}=\frac{\partial \psi_j}{\partial x_i}$$
$$\kappa_{ii}=-\frac{(\mu-\alpha)}{(\mu+\alpha)}\frac{\partial^2 w}{\partial x_i^2}+(-1)^j\frac{2\alpha}{(\mu+\alpha)}\varphi_{ij},$$
$$\kappa_{ij}=-\frac{(\mu-\alpha)}{(\mu+\alpha)}\frac{\partial^2 w}{\partial x_i \partial x_j}+(-1)^i\frac{2\alpha}{(\mu+\alpha)}\frac{\partial \psi_i}{\partial x_i}- \quad (19)$$
$$-(-1)^i\frac{\beta}{2\gamma+\beta}\left(\frac{\partial \psi_1}{\partial x_1}+\frac{\partial \psi_2}{\partial x_2}\right).$$

For this system of differential equations of the sixth order, it is necessary to specify three boundary conditions at each edge. For example, the conditions at the free edge and the rigidly fixed edge $x_i=const$ have the form respectively components

$$N_{i3}=0, \quad M_{ji}=0, \quad M_{ii}=0,$$
$$w=0, \quad \frac{\partial w}{\partial x_i}+(-1)^j\psi_j=0, \quad \psi_i=0.$$

3. NUMERICAL EXAMPLE

Let us perform the calculation of the stress-strain state of the bend of a rectangular hinge-operated plate along all edges of the plate. Длины сторон пластины равны l_i $(-l_i \le x_i \le l_i)$.

We introduce dimensionless coordinates

$$x_i=l_i\xi_i \quad (20)$$

Let us write down all the required quantities in the dimensionless form

$$w_*=\frac{w}{l_1}, \quad \psi_{i*}=\psi_i,$$
$$N_{i3*}=N_{i3}\frac{\mu+\alpha}{8h\mu\alpha}, \quad N_{3i*}=N_{3i}\frac{\mu+\alpha}{8h\mu\alpha},$$
$$M_{ii*}=\frac{M_{ii}}{l_1}\frac{\mu+\alpha}{8h\mu\alpha}, \quad M_{ij*}=\frac{M_{ij}}{l_1}\frac{\mu+\alpha}{8h\mu\alpha}, \quad (21)$$
$$G_{ii*}=\frac{G_{ii}}{l_1}\frac{\mu+\alpha}{8h\mu\alpha}, \quad G_{ij*}=\frac{G_{ij}}{l_1}\frac{\mu+\alpha}{8h\mu\alpha},$$
$$\varepsilon_{i3*}=\varepsilon_{i3}, \quad \kappa_{ii*}=l_1\kappa_{ii}, \quad \kappa_{ij*}=l_1\kappa_{ij},$$
$$\varphi_{i*}=l_1\varphi_i, \quad \varphi_{ij*}=l_1\varphi_{ij}.$$

We write out a complete system of equations with respect to dimensionless desired quantities. The equilibrium equations

$$\frac{\partial N_{13*}}{\partial \xi_1}+\frac{l_1}{l_2}\frac{\partial N_{23*}}{\partial \xi_2}+Z_*=0,$$
$$\frac{\partial M_{11*}}{\partial \xi_1}+\frac{l_1}{l_2}\frac{\partial M_{21*}}{\partial \xi_2}+\mathrm{M}_{1*}+N_{23*}=0,$$
$$\frac{l_1}{l_2}\frac{\partial M_{22*}}{\partial \xi_2}+\frac{\partial M_{12*}}{\partial \xi_1}+\mathrm{M}_{2*}-N_{13*}=0, \quad (22)$$
$$\frac{\partial G_{11*}}{\partial \xi_1}+\frac{l_1}{l_2}\frac{\partial G_{21*}}{\partial \xi_2}-N_{31*}+Y_{1*}=0,$$
$$\frac{l_1}{l_2}\frac{\partial G_{22*}}{\partial \xi_2}+\frac{\partial G_{12*}}{\partial \xi_1}-N_{32*}+Y_{2*}=0,$$
$$Z_*=\frac{l_1}{h}\frac{\mu+\alpha}{8\mu\alpha}Z,\ \mathrm{M}_{i*}=\frac{\mu+\alpha}{8h\mu\alpha}\mathrm{M}_i,\ Y_{i*}=\frac{\mu+\alpha}{8h\mu\alpha}Y_i.$$

The constitutive relations

$$N_{i3*}=\varepsilon_{i3*},\quad M_{ii*}=A_1\phi_{i*}+A_2\phi_{j*}$$
$$M_{ij*}=A_{12}\phi_{ij*}+A_{21}\phi_{ji*} \quad (23)$$
$$G_{ii*}=c_1\kappa_{ii*}+c_2\kappa_{jj*}+cZ_*,\quad G_{ij*}=c_{12}\kappa_{ij*}+c_{21}\kappa_{ji*}.$$

The geometrical relations

$$\varepsilon_{i3*}=\frac{\partial w_*}{\partial \xi_i}+\psi_{j*},\ \varphi_{i*}=\frac{\partial \psi_{i*}}{\partial \xi_i},\ \varphi_{ij*}=\frac{\partial \psi_{j*}}{\partial \xi_i},$$
$$\kappa_{ii*}=-\frac{(\mu-\alpha)}{(\mu+\alpha)}\frac{\partial^2 w_*}{\partial \xi_i^2}-(-1)^i\frac{2\alpha}{(\mu+\alpha)}\phi_{ij*}, \quad (24)$$
$$\kappa_{ij*}=-\frac{l_1}{l_2}\frac{(\mu-\alpha)}{(\mu+\alpha)}\frac{\partial^2 w_*}{\partial \xi_i \partial \xi_j}+(-1)^i\frac{2\alpha}{(\mu+\alpha)}\varphi_{i*}-$$
$$-(-1)^i\frac{\beta}{2\gamma+\beta}\left(\varphi_{i*}+\varphi_{j*}\right).$$

In formulas (23), the following designations are introduced:

$$A_1=\frac{1}{l_1^{\,2}}\frac{(\mu+\alpha)}{\mu\alpha}\frac{\gamma(\gamma+\beta)}{2\gamma+\beta},$$
$$A_2=\frac{1}{l_1 l_2}\frac{(\mu+\alpha)}{2\mu\alpha}\frac{\gamma\beta}{2\gamma+\beta},$$
$$A_{12}=\frac{1}{l_1^{\,2}}\frac{(\mu+\alpha)}{4\mu\alpha}(\gamma+\varepsilon),$$
$$A_{21}=\frac{1}{l_1 l_2}\frac{(\mu+\alpha)}{4\mu\alpha}(\gamma-\varepsilon),$$
$$c_1=\frac{h^2}{3l_1^2}\frac{(\mu+\alpha)(\mu+\lambda)}{\alpha(2\mu+\lambda)},\quad c_2=\frac{h^2}{6l_1^2}\frac{(\mu+\alpha)\lambda}{\alpha(2\mu+\lambda)},$$
$$c_{12}=\frac{h^2}{12l_1^2}\frac{(\mu+\alpha)^2}{\mu\alpha}\quad c_{21}=\frac{h^2}{12l_1^2}\frac{(\mu^2-\alpha^2)}{\mu\alpha},$$
$$c=\frac{h^2}{3l_1^2}\frac{(\mu+\alpha)\lambda}{\mu\alpha(2\mu+\lambda)}.$$

The solution of the problem that satisfies the boundary conditions will be sought in the following form

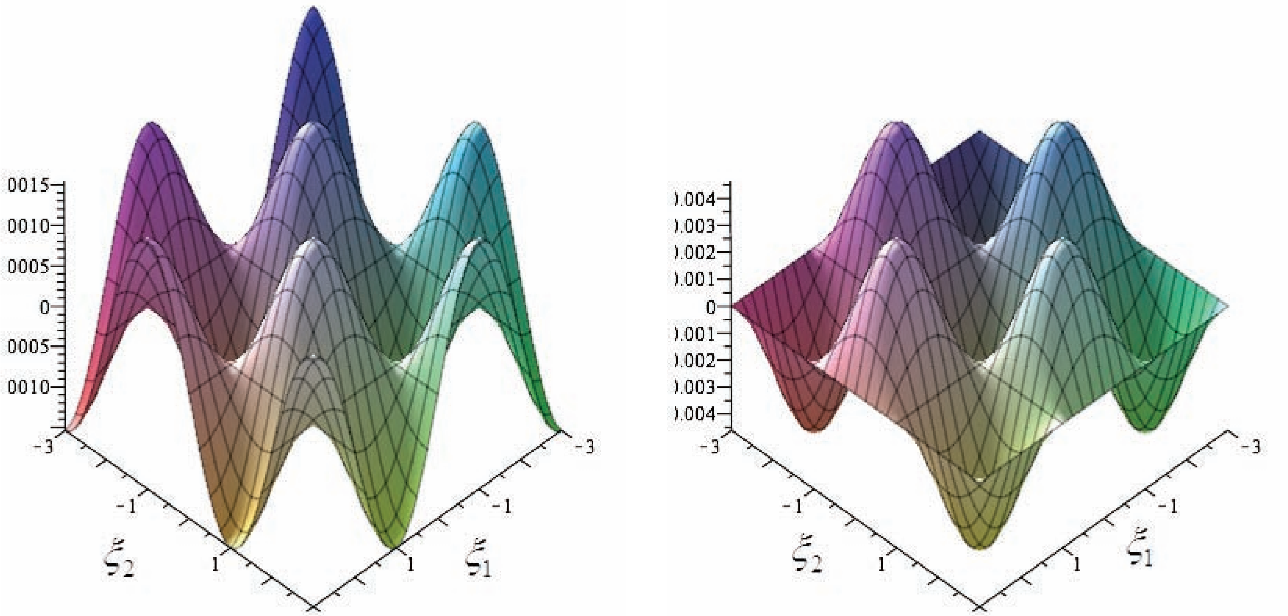
$$f_{1*}=f_{1nm}\cos k_n\xi_1\cos k_m\xi_2,$$
$$f_{2*}=f_{2nm}\cos k_n\xi_1\sin k_m\xi_2,$$
$$f_{3*}=f_{3nm}\sin k_n\xi_1\cos k_m\xi_2, \quad (25)$$
$$f_{4*}=f_{4nm}\sin k_n\xi_1\sin k_m\xi_2,$$

where $k_n=\frac{\pi}{2l_1}(2n-1)$, $k_m=\frac{\pi}{2l_2}(2m-1)$.

In formulas (25), f_{1*} means the quantities w_*, ϕ_{ij*}, κ_{ii*}, G_{ii*}, M_{ij*}, Z_*; f_{2*} means the quantities ψ_{1*}, ε_{23*}, N_{23*}, N_{32*}, Y_{2*}, M_{1*}; f_{3*} means the quantities ψ_{2*}, ε_{13*}, N_{13*}, N_{31*}, Y_{1*}, M_{2*}; f_{4*} means the quantities ϕ_{i*}, M_{ii*}, G_{ij*}.

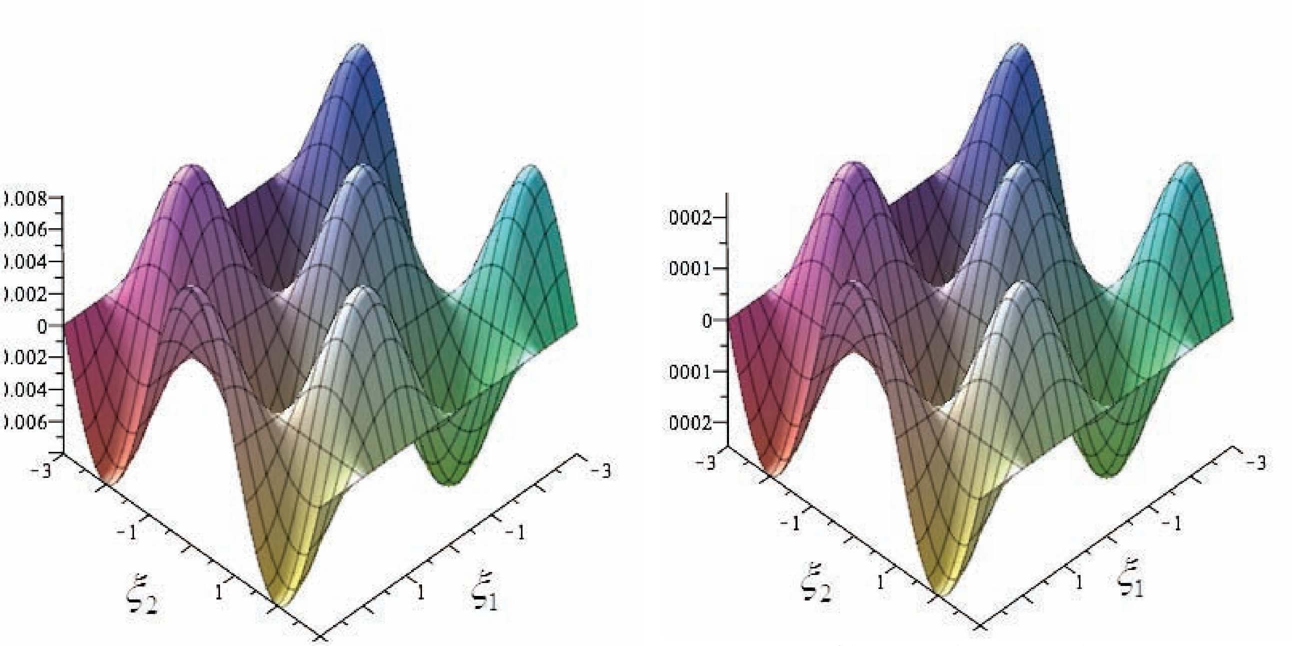
The solution of the problem for the required quantities was reduced to solving the system of linear algebraic equations. Below are the results of the calculation in the form of graphs of the desired values as functions of the dimensionless coordinates ξ_1, ξ_2 the middle plane of the plate.

During the numerical example, we took $\alpha=1{,}6\cdot 10^6$ N/m², $\mu=2\cdot 10^6$ N/m², $\lambda=3\cdot 10^6$ N/m², $\beta=1{,}2$ kN, $\gamma=\varepsilon=3$ kN.



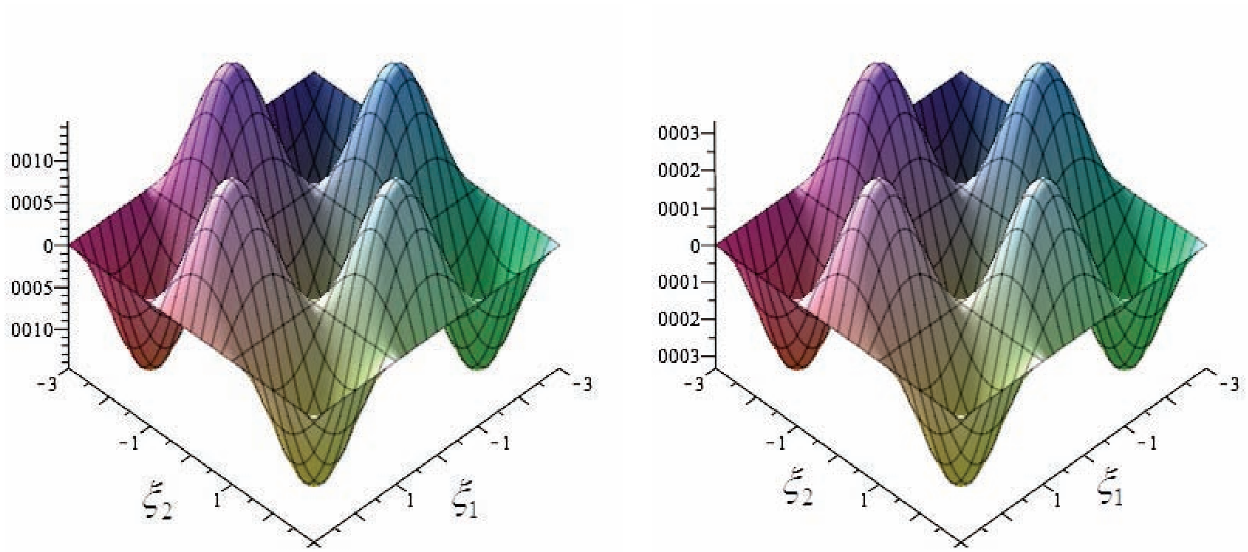
a) M_{11^*}, (n=1, m=2) b) M_{12^*}, (n=1, m=2)

Figure 1. The moments M_{11^*} fig. 1(a) and M_{12^*} fig. 1(b) is a function of the coordinates x_1, x_2.



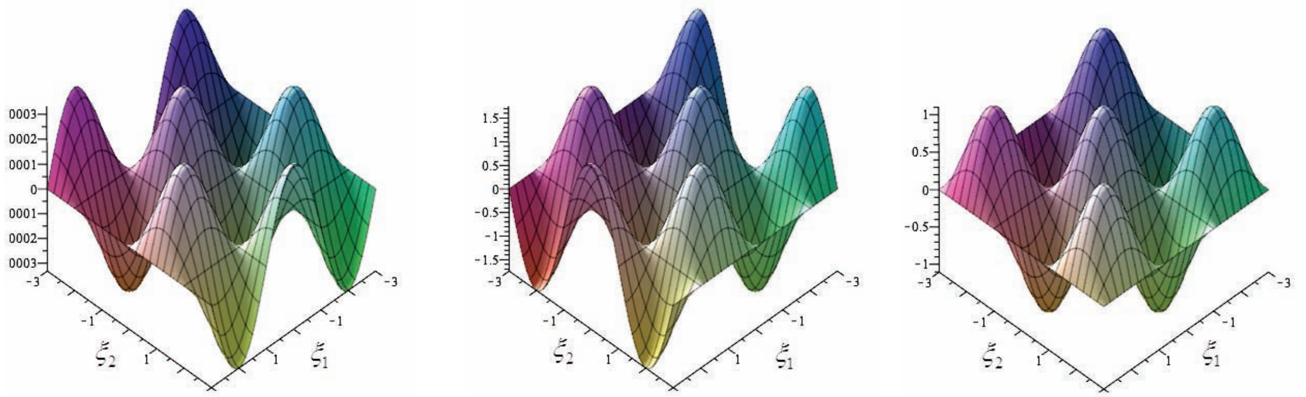
a) N_{13^*}, (n=1, m=2) b) N_{31^*}, (n=1, m=2)

Figure 2. Graphs of N_{13^*} fig. 2(a) and N_{31^*} fig. 2(b) is a function of the coordinates x_1, x_2.



a) G_{11*}, (n=1, m=2) b) G_{12*}, (n=1, m=2)

Figure 3. Graphs of G_{11*} fig. 3(a) and G_{12*} fig. 3(b) is a function of the coordinates x_1, x_2.



a) ψ_{1*}, (n=1, m=2) b) ψ_{2*}, (n=1, m=2) c) w_*, (n=1, m=2)

Figure 4. The dependence generalized displacement ψ_{1*}, ψ_{2*}, w_* on the variables x_1, x_2.

The obtained dependences of the dimensionless desired quantities on the coordinates of the middle plane of the plate make it possible to understand the behavior of the stress-strain state. We especially note that for forces and moments, the symmetry conditions known in the classical theory of elasticity are not satisfied, the transverse forces N_{i3*} are significantly greater than the forces N_{3i*}.

4. CONCLUSIONS

It is known that the classical theory of elasticity is unsuitable for describing the behavior of structures made of a number of materials widely used in practice. As the results of experiments show (for example, [6, 8]), in these cases one should use the moment asymmetric theory of elasticity. Thin-walled structures according to the three-dimensional asymmetric theory of elasticity are almost impossible to calculate in view of the complexity of the problem.

For the construction of applied theories of plates, as in the classical theory of elasticity, a three-dimensional problem is reduced to a two-dimensional problem. As a rule, the reduction is performed by the method of hypotheses, using some simplifying assumptions. Hypotheses are accepted by analogy with Kirchhoff's hypotheses. The choice of hypotheses for a complex problem based on known hypotheses for a simple problem often leads to incorrect hypotheses for the desired quantities that are present in a complex problem and are absent in a simple problem. For example, when constructing the theory of thin-walled electroelastic elements by the hypothesis method for entire classes of problems, incorrect hypotheses were adopted for the desired electrical quantities. The reason is that there are no electrical quantities in the theory of elasticity and there are no analogues for the corresponding hypotheses. Mathematically grounded theories of electroelastic thin-walled elements were first constructed without using hypotheses by the asymptotic method of reducing 3D problems of the theory of electroelasticity to 2D problems [15]. The same situation has developed in the moment asymmetric theory of elasticity: the correct equations of the plane problem of the moment theory of elasticity were obtained by the method of hypotheses by many authors in view of the simplicity of the problem. In the problem of plate bending, the correct theory was obtained only by mathematical method [9]. Here, this theory of plate bending in the asymmetric theory of elasticity is presented by the method of hypotheses for simplicity.

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Nelly N. Rogacheva, Doctor of Physical and Mathematical Sciences, laureate of the State Prize of the Russian Federation in the field of science and technology, Associate Professor, Department of Informatics and Applied Mathematics, Moscow State University of Civil Engineering, 129337, Russia, Moscow, Yaroslavskoe Shosse, 26, tel. +7(499)1835994, e-mail: rogachevann@mgsu.ru;

Yulia G. Zheglova, Candidate of Tehnical Sciences, Associate Professor, Department of Information Systems, Technologies and Automation in Construction, Moscow State University of Civil Engineering, 129337, Russia, Moscow, Yaroslavskoe Shosse, 26, tel. +7(495)2874919, e-mail: uliagermanovna@yandex.ru.

Рогачева Нэлля Николаевна, доцент, доктор физико-математических наук, лауреат Государственной премии Российской Федерации в области науки и техники, кафедра информатики и прикладной математики Национального исследовательского Московского государственного строительного университета; 129337, Россия, г. Москва, Ярославское шоссе, д. 26; тел. +7(499)1835994; e-mail: rogachevann@mgsu.ru;

Жеглова Юлия Германовна, доцент, кандидат технических наук, кафедра информационных систем, технологий и автоматизации в строительстве Национального исследовательского Московского государственного строительного университета; 129337, Россия, г. Москва, Ярославское шоссе, д. 26; тел. +7(495)2874919 доб. 30-44; e-mail: uliagermanovna@yandex.ru.