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EXPERIMENTAL STUDIES OF THE STABILITY OF THIN-WALLED CYLINDRICAL SHELLS OF MEDIUM LENGTH FILLED WITH BULK MATERIAL UNDER TORSION

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Abstract. Experimental studies were carried out to study the effect of bulk filler on the stability of shells during torsion. In the domestic and foreign literature, we have not found publications where the stability of thin-walled shells filled with bulk material under torsion would be considered. To carry out experiments, a special installation has been developed and manufactured that allows testing samples for torsion, bending and complex loading, loading the samples accordingly. Before the tests, samples were taken for the absence of defects in the form of dents, ovality, diversity, constancy of cross-sectional dimensions. It is known that defects in samples significantly affect stability. The samples were tested with a length of 140 mm, 90 mm, 25 mm. The samples were made by deep drawing from a high-plastic aluminum alloy 3004 in the H19 state. In this case, one end of the samples was rigidly fixed to the installation, a torque was applied to the other end, which increased until the samples lost stability. At the beginning and at the end of the test, the samples were loaded in small portions to catch the beginning of movement and the beginning of loss of stability. The installation made it possible to create various boundary conditions of the samples. In experiments, the torque and the twist angle were measured. Linear dependences of the twisting angle on the torque are constructed. So, stability was lost in elasticity. For samples filled with bulk filler, at the beginning of the twisting process, there was a nonlinear relationship between the torque and the twisting angle. This is due to the occurrence of friction forces between the filler and the inner wall of the samples, which created a counteracting torque. Then the graphs for the empty and filled samples were enlarged parallel to each other.
For empty samples [1], critical stresses, critical twisting angles, the number of waves formed, and their direction were calculated. The convergence of the calculated and experimental parameters within 36% is obtained. The bulk filler, in the form of iron powder, increased the value of the critical torque to 17%.

Keywords: torsion, stability, experiment, sample, installation, filler, torque, voltage

ЭКСПЕРИМЕНТАЛЬНЫЕ ИССЛЕДОВАНИЯ УСТОЙЧИВОСТИ ТОНКОСТЕННЫХ ЦИЛИНДРИЧЕСКИХ ОБОЛОЧЕК СРЕДНЕЙ ДЛИНЫ, ЗАПОЛНЕННЫХ СЫПУЧИМ МАТЕРИАЛОМ, ПРИ КРУЧЕНИИ

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Аннотация. Экспериментальные исследования выполнены с целью изучения влияния сыпучего заполнителя на устойчивость оболочек при кручении. В отечественной и зарубежной литературе не обнаружены нами публикации, где рассматривалось бы устойчивость тонкостенных оболочек, заполненных сыпучим материалом, при кручении. Для выполнения экспериментов разработана и изготовлена специальная установка, позволяющая испытывать образцы на кручение, на изгиб и на сложное нагружение, нагружая образцы соответствующим образом. До испытаний производился отбор образцов на отсутствие дефектов в виде вмятин, овальности, разностенности, постоянства размеров поперечного сечения. Известно, что дефекты образцов существенно влияют на устойчивость. Образцы

испытывались длиной 140 мм, 90 мм, 25 мм. Образцы изготавливались глубокой вытяжкой из высокопластичного алюминиевого сплава 3004 в состоянии H19. В данном случае один конец образцов жестко закреплялся к установке, к другому концу прикладывался крутящий момент, который возрастал до потери устойчивости образцов. В начале и в конце испытания нагружали образцы малыми порциями, чтобы уловить начало движения и начало потери устойчивости. Установка позволяла создавать различные граничные условия образцов. В экспериментах измеряли крутящий момент и угол закручивания. Построены линейные зависимости угла закручивания от крутящего момента. Значит устойчивость терялась в упругости. Для образцов, заполненных сыпучим заполнителем, в начале процесса закручивания имело место нелинейная зависимость между крутящим моментом и углом закручивания. Это связано с возникновением сил трения между заполнителем и внутренней стенкой образцов, которые создавали противодействующий крутящий момент. Затем графики для пустых и заполненных образцов увеличивались параллельно друг другу.
Для пустых образцов [1] рассчитывались критические напряжения, критические углы закручивания, число образовавшихся волн, их направление. Получена сходимость расчетных и экспериментальных параметров в пределах 36%. Сыпучий заполнитель, в виде железного порошка увеличивал значение критического крутящего момента до 17%.

Ключевые слова: кручение, устойчивость, эксперимент, образец, установка, заполнитель, крутящий момент, напряжение

INTRODUCTION

Problems of stability of cylindrical shells were considered in numerous works [1-18,22-31, etc.]. In the works of L.H. Donnel, H.M. Mushtari, V.M. Dorevsky and others obtained simple formulas for calculating critical stresses and the number of waves. Studies of the buckling of shells filled with liquid, gas, and other loads were carried out in [12,24,26]. In [9,18,20-23], the stability of shells in bending filled with bulk material was studied. In these studies, the influence of various bulk materials, the geometric dimensions of the shells, the percentage of volume filling on stability was established. The critical bending moment, for shells filled with bulk material, increased by 10-20%. The stability of thin-walled cylindrical shells filled with bulk material during torsion has been little studied [19] and this problem is very relevant. For example, tank semi-trailers are used to transport bulk cargo (flour, grain, sand, etc.). It is much cheaper to transport such goods on tank semi-trailers, they can be transported along rural roads to remote areas. The tank is a thin-walled cylindrical shell, which is attached to the trailer of the car and when the car is moving, it can bend, twist, which leads to loss of stability. **The purpose of this work** is an experimental study of the torsion loss of thin-walled shells filled with bulk material.

EXPERIMENTAL SETUP, SAMPLES, TESTING

A special setup (Figure 1) was made and assembled for the experiments. Sample 1 was rigidly fixed at one end to frame 2. The other end was placed in wall 3, where it could turn, and to stiffen this end, a metal disk was inserted inside, to which rod 4 was attached to create a torque.
Rod 5 was rigidly attached perpendicular to the axis of sample 1, to the end of which a leg of the dial gauge ICH-10 6 was attached to measure displacements and calculate the angle of rotation.
Dynamometers 7 were used to measure the forces. Forces were created by weights 8, which were placed on hangers 9. Hangers 9 were connected by cables 10 by dynamometers 7 and rod 4 at the same distance from the longitudinal axis of the sample. This created torque.
The specimens were made from 3004 aluminum alloy in H19 temper from deep drawn sheets in a die. The samples were of high quality, but before testing they were selected, checked for

the absence of defects in the form of dents, ovality, variation in wall thickness, and constancy of cross-sectional dimensions. The dimensions of the samples are presented in Table 1. The dimensions of the samples are assumed to be similar to tanks manufactured at the enterprises. A photograph of the samples is shown in Figure 2.

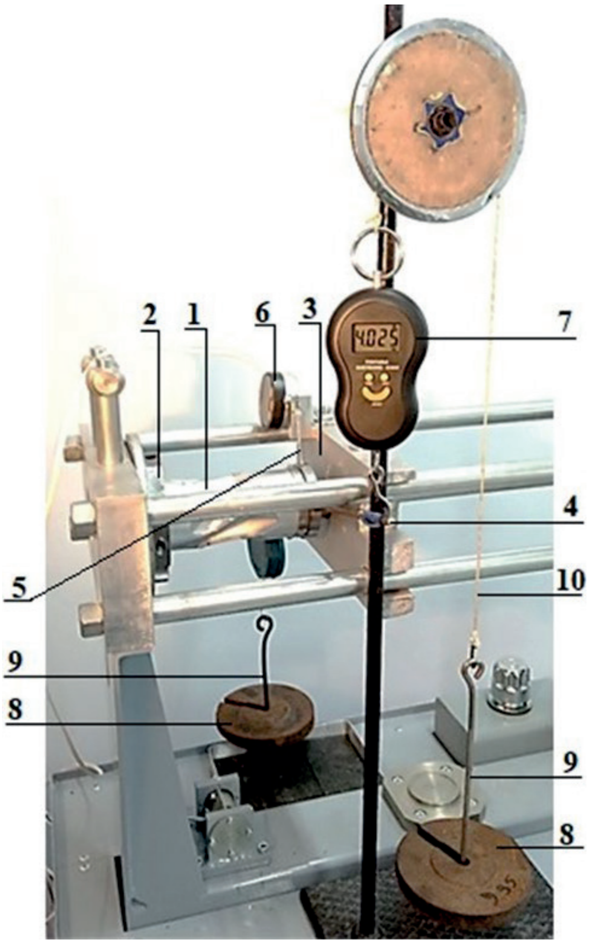

Figure. 1. Test facility.

The samples were tested empty and filled to 90% of the volume with PZh-5 iron powder. Weights 8 were loaded simultaneously from both sides, at the beginning of twisting and when approaching the critical state, they were loaded in small portions.

In the experiments, at each load, the readings of the dynamometers and the indicator of the clock type ICh-10 were recorded.

Table 1. Sample sizes

N series	Radius R, mm	Wall thickness h, mm	Length L, mm	L/R
1	31,5	0,1	140	4,45
2	31,5	0,1	90	2,86
3	31,5	0,1	25	0,79

One end of the sample was pinched, and the other end, when a torque was applied, could only rotate freely, excluding other movements, which is the advantage of the installation. If the wall 3 is removed, bending and bending with torsion tests can be performed

Figure 2. Sample photography: 1-sample before testing, 2-sample 140mm long after testing without loading, 3-sample 90mm long tested with filler, 4-sample 25mm long tested without loading

TESTING

Tests began with a thorough inspection and sampling. The absence of ovality, thickness variation, the presence of dents, scratches, and variation in geometric dimensions was checked. Samples with a dent were immediately rejected. The wall thickness of the samples was 0.1 mm and samples with a wall thickness of 0.11 mm were not allowed for testing. Ovality was not observed. Then the correct fixing of the sample on the installation began. One end of the sample was mounted on the mandrel of frame 2 and crimped from the outside with a clamp to obtain a rigid fastening. To avoid distortion of the sample during the test, the second end was inserted into the bearing hole installed in wall 3, where it could rotate freely. Then, metal rods 4 were installed on both sides of the sample. The coaxiality of the centers of the circles of the ends of the sample was corrected with a laser beam. Hangers 9 were hung on cables 10 to the ends of the rods 4.

At the beginning and at the end of the tests, weights were loaded with a small mass in order to catch the beginning of twisting and the moment of loss of stability. During the test, the coaxiality of the centers of the circles of the ends of the sample was constantly checked by a laser beam so that the conditions of the torsion test were not violated (there was no bending). In the experiments, the magnitude of the load and the corresponding movement of the indicator leg were recorded.

TEST RESULTS

Each series of samples was tested at least five pieces. Average statistical test results are shown in Table 2.

Table 2. Results of test.

sample series	empty		filled		$M_{cr}\cdot10^{-8}$, MN m [1]	n	%
	$M_{cr}\cdot10^{-8}$, MN m	$\varphi_{cr}\cdot10^{-4}$, radian	$M_{cr}\cdot10^{-8}$, MN m	$\varphi_{cr}\cdot10^{-4}$, radian			
1	1094	22,2	1154	37,4	1218	6	11,3
2	1103	29,5	1290	40	1540	8	28,4
3	1745	39,6	1986	46	2294	14	31,5

In table. 2 denotes: M_{cr}-critical torque, φ_{cr}-angle of twist under the action of M_{cr}, n-number of waves after buckling of the sample, %-percentage of divergence of critical torques obtained in experiments and calculated from the literature [1] for empty samples. The percentage of discrepancy increases with decreasing sample length. This is due to the fact that the calculation formulas were obtained for shells of medium length.

As an example, the test results for specimens 90 mm long are shown in Figure.3.

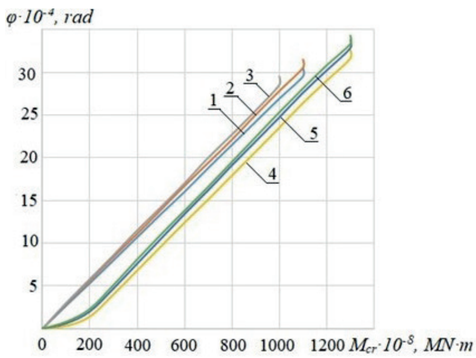

Figure 3. Test results of samples with a length of 90 mm. Graphs 1,2,3 for empty 4,5,6 for filled samples

The angle of twist was calculated as the ratio of the movement of the indicator leg to the distance from the sample axis to the indicator leg.

The graphs are linear, the loss of stability occurred in elasticity. The graphs for empty and filled with iron powder samples are parallel, except for the initial twisting process, where the influence of the friction force between the powder and the inner wall of the sample occurs.

The experimental value of the critical shear stress is:

$$\tau_{cr}^{o} = \frac{M_{cr}}{2\pi R^2 h} = \frac{1103\cdot10^{-8}}{2\cdot3{,}14\cdot31{,}5^2\cdot10^{-6}\cdot0{,}1\cdot10^{-3}} = 17{,}7\ МПа. \qquad (1)$$

Estimated upper critical shear stress [1]:

$$\tau_{cr} = 0{,}78E\frac{h}{R}\sqrt[4]{\frac{\mathrm{Rh}}{L^2}} = 0{,}78\cdot0{,}7\cdot10^5\,\frac{0{,}1}{31{,}5}\sqrt[4]{\frac{31{,}5\cdot0{,}1}{90^2}} = 24{,}27\ MPa, \qquad (2)$$

where $E=0.7\ 10^5$ MPa modulus of elasticity.
The divergence of critical shear stresses is 26.6%.
For samples filled to 90% of the volume with iron powder, the critical torque increased compared to the critical torque for empty samples of the first series by 5.5%, the second series by 17%, and the third series by 14%. The value of M_{cr} increased with decreasing sample length.
The number of generated waves according to [1] for samples of the first series:

$$n = 4{,}2 \cdot \sqrt[8]{1\text{-}\mu^2} \cdot \sqrt{\frac{R}{L}} \cdot \sqrt[4]{\frac{R}{h}} = 4{,}2 \cdot \sqrt[8]{1\text{-}0{,}3^2} \cdot \sqrt{\frac{31{,}5}{140}} \cdot \sqrt[4]{\frac{31{,}5}{0{,}1}} = 8. \quad (3)$$

In experiments n=6.
Second series: n=10, and in experiments n=6.
Third series: n=22, and in experiments n=14.
In experiments, the number of waves is less than in calculations by 25%, 20%, 36%, respectively. For the filled samples, waves first formed in the upper part, where there was no iron powder. Waves then formed on the rest of the wall.
The angle of inclination of the dents to the axis of the specimens of the first series [1]:

$$\gamma = 1{,}73 \cdot \sqrt[4]{\frac{Rh}{L^2}} = 1{,}73 \cdot \sqrt[4]{\frac{31{,}5 \cdot 0{,}1}{140^2}} = 12{,}6. \quad (4)$$

The second and third series $\gamma=14^o$ and $\gamma=26^o$ respectively. In experiments $\gamma=12^o \ldots 24^o$.
The difference between the results is due to the fact that the calculated dependences in [1] were obtained for shells without defects, and the tested samples were not ideal.

CONCLUSION

1. Loose aggregate increases the critical torque value.
2. The stability of the samples is lost in elasticity.
3. When designing and manufacturing tank trucks for the transport of bulk materials, it is necessary to take into account the effect of aggregate on stability.
4. The formulas used in the calculations from [1], which were obtained for shells of medium length, give calculated results for short samples of the third series, which differ significantly from the test results.

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