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NONLINEAR REFINEMENT OF THE DEFORMATION MODEL OF ORTHOTROPIC MATERIALS, THE RIGIDITY OF WHICH DEPENDS ON THE TYPE OF STRESS STATE

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Annotation. As a result of numerous experimental and theoretical studies, it has been established that when deforming both traditional and new structural materials, which are polymers, composites and synthetic structures used in construction, mechanical engineering, and power engineering devices, complicated mechanical properties manifest themselves.Most of these materials have an orthotropic structure complicated by the dependence of deformation and strength characteristics on the type of stress state, which can be interpreted as deformation anisotropy. These properties contradict the generally accepted theories of deformation. Therefore, a number of models have been specially developed for such materials over the past 56 years, taking into account the complicated properties of materials. However, all of them have disadvantages and certain contradictions with the fundamental rules for constructing equations of state. The previous works of the authors of the presented studies establish general approaches to the construction of energetically nonlinear deformation models of composite materials with recommendations for calculating their constants based on a wide range of experiments.It turned out that the set of necessary experiments should include experiments on complex stress states, most of which are technically unrealizable. In another work of the authors in 2021, using the tensor space of normalized stresses, the deformation potential was formulated in a quasi-linear form, constructed in the main axes of orthotropy of materials, for determining constants, which is sufficient for the data of the simplest experiments. Along with the obvious advantages of the introduced potential, it has one drawback, which is to replace real nonlinear diagrams with direct rays with minimal error. Despite the unconditional adequacy of the quasi-linear potential, the use of this level of approximations leads to quantitative errors.Therefore, simplified nonlinear equations of state for composite materials are proposed here, the simplest experiments are sufficient to determine the material functions of which. These equations are based on the general laws of mechanics, on the basis of which the constants of material polynomials for a carbon-graphite composite are calculated, taking into account the Drucker constraints.

Keywords: material functions, nonlinear deformation anisotropy, dimensional orthotropy, defining equations, strain and stress tensors, least squares method, main axes of orthotropy

НЕЛИНЕЙНОЕ УТОЧНЕНИЕ МОДЕЛИ ДЕФОРМИРОВАНИЯ ОРТОТРОПНЫХ МАТЕРИАЛОВ, ЖЕСТКОСТЬ КОТОРЫХ ЗАВИСИТ ОТ ВИДА НАПРЯЖЕННОГО СОСТОЯНИЯ

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Аннотация. В результате многочисленных экспериментальных и теоретических исследований установлено, что при деформировании как традиционных, так и новых конструкционных материалов, которыми являются полимеры, композиты и синтетические структуры, используемые в строительстве, машиностроении и энергетических устройствах, проявляются сложные механические свойства. Большинство из этих материалов имеют ортотропную структуру, осложненную зависимостью деформационных и прочностных характеристик от типа напряженного состояния, что можно интерпретировать как деформационную анизотропию. Эти свойства противоречат общепринятым теориям деформирования. Поэтому за последние 56 лет для таких материалов был специально разработан ряд моделей, учитывающих усложненные свойства материалов. Однако все они имеют недостатки и определенные противоречия с фундаментальными правилами построения уравнений состояния. Предыдущие работы авторов представленных исследований устанавливают общие подходы к построению энергетически нелинейных моделей деформирования композиционных материалов с рекомендациями по расчету их констант на основе широкого спектра экспериментов. Оказалось, что набор необходимых экспериментов должен включать эксперименты по сложным напряженным состояниям, большинство из которых технически нереализуемы. В другой работе авторов в 2021 году с использованием тензорного пространства нормированных напряжений потенциал деформации был сформулирован в квазилинейной форме, построенный в главных осях ортотропии материалов и установлено, что для определения констант достаточно данных простейших экспериментов. Наряду с очевидными преимуществами введенного потенциала, у него есть один недостаток, который заключается в замене реальных нелинейных диаграмм прямыми лучами с минимальной погрешностью. Несмотря на безусловную адекватность квазилинейного потенциала, использование такого уровня приближений приводит к количественным ошибкам. Поэтому здесь предлагаются упрощенные нелинейные уравнения состояния для композиционных материалов при определенияи материальных функций которых достаточно простейших экспериментов. Эти уравнения основаны на общих законах механики, на основе которых рассчитываются константы материальных полиномов для углеграфитового композита с учетом ограничений Друкера.

Ключевые слова: материальные функции, нелинейная деформационная анизотропия, структурная ортотропия, определяющие уравнения, тензоры деформаций и напряжений, метод наименьших квадратов, главные оси ортотропии

1. INTRODUCTION

The designs of unique structures, especially critical parts of apparatuses and machines are continuously being improved, which has caused the appearance of special materials with increased deformation and strength characteristics with a minimum mass of the objects being examined. Along with this, similar materials have certain properties that contradict classical theories [1 – 7]. Mainly, these materials include chaotically filled with nano-modifiers and fibers, fibrous composites, fiberglass, graphitoplastics and boroplastics.

The discrepancy between the mechanical properties of these new, as well as many materials traditionally used in engineering, such as concrete and reinforced concrete [8, 9], cast iron [10], fluoroplast [11], carbon-graphite [12, 13], is the sensitivity of their physical and mechanical properties to the type of stress state. Some of these compositions are isotropic [8 – 12], and most are anisotropy of the type of orthotropy and transversal isotropy [1 – 8, 13]. At the same time, over the past 50 years, equations of state have been proposed for isotropic materials in many studies based on various hypotheses, which are largely contradictory, and as demonstrated in [8], all these equations have serious drawbacks and are contradictory. In contrast to these theories, in the works [8, 14, 15], sufficiently universal and free from such shortcomings defining relations were formulated and investigated for applicability to the calculation of a wide class of materials. These relations have energy consistency and are formulated in vector and tensor normalized stress spaces. At the same time, classes of structurally anisotropic materials have been covered by theoretical studies only in individual publications, often under-researched to the level of applicability to design calculations and are not fully systematized [1 – 7, 13, 16, 17].

In the light of the above, the purpose of the presented publication is to demonstrate the application of the normalized tensor space of normalized stresses for postulating nonlinear equations of state in relation to orthotropic materials with deformation anisotropy.

2. FORMULATION OF THE PROBLEM: METHODS AND RESULTS OF CONSTRUCTING NONLINEAR STATE EQUATIONS FOR COMPOSITE MATERIALS. DEFINITION OF MATERIAL FUNCTIONS AND THEIR LIMITATIONS

Research [18] was devoted to an attempt to apply the technique of normalized stress tensor spaces to the formulation of a nonlinear version of the theory of deformation of orthotropic materials with deformation anisotropy acquired during loading in compliance with all the rules for postulating the form of specific additional stress work for continuous deformable media [19]. It has been demonstrated [8, 18] that in equations with full consideration of physical nonlinearity, even with quasi-linear approximations of experimental deformation diagrams, it is necessary to involve a wide range of experiments, including complex stress-strain states, in calculating the constants of the equations of state. Often, for this purpose, it requires the use of wide sets of tests in a plane stress state, spatial (triaxial) experiments in the main planes of orthotropy, including both stretching and compression. However, this is not the most difficult. It is much more difficult to perform tests for joint shifts in two or three main material planes of orthotropy. Conducting such tests at the current level of development of the experimental base is practically impossible for technical reasons and therefore there is not enough information in the scientific literature about the results of such experiments. In the work of the authors published in 2021 [20], using the tensor space of normalized stresses, a quasi-linear form of the deformation potential for orthotropic materials is formulated, taking into account the manifestation of deformation anisotropy. In addition to the potential itself and the equations of state arising from it, a method for calculating the constants of these equations using experimentally determined technical characteristics is also developed there, and the limitations arising from the complete proof of the uniqueness theorem are evaluated. This potential and the equations of state resulting from it have a sufficiently high accuracy, do not contradict the laws of classical mechanics and are free from obvious drawbacks characteristic of well-known theories proposed by other authors. However, it is impossible to achieve further improvement and increase the accuracy of computational models of structures made of composite materials without replacing quasi-linear equations of state with nonlinear ones, which, undoubtedly, should have the smallest error of theoretical predictions relative to experimental data. At the same time, a simple statement of the problem will not be enough, since the difficulties that arise in obtaining experimental data in full do not allow us to establish a set of necessary technical constants. Therefore, it is proposed here to formulate direct nonlinear equations of the relationship between strain and stress tensors for composite orthotropic materials sensitive to the type of stress state in a form close to the generalized Hooke's law with modification by the type of the theory of small elastic-plastic deformations, but using normalized stress spaces. The proposed version of the equations of state should be universal for any type of stress state and allows to establish one-to-one relationships between deformations and stresses with the involvement of a system of simple experiments, technically feasible and sufficient to fully determine the nonlinear material functions introduced into these equations and taking into account the complex of complicated mechanical properties.

The proposed nonlinear coupling equations of strain and stress tensors for an orthotropic material with deformation anisotropy are recommended to be formulated as:

$$e_{11} = \left[A_{1111}(\sigma_i) + B_{1111}(\sigma_i)\,\alpha_{11}\right]\sigma_{11} + \left[A_{1122}(\sigma_i) + B_{1122}(\sigma_i)(\alpha_{11} + \alpha_{22})\right]\sigma_{22} +$$
$$+\left[A_{1133}(\sigma_i) + B_{1133}(\sigma_i)(\alpha_{11} + \alpha_{33})\right]\sigma_{33};$$
$$e_{22} = \left[A_{1122}(\sigma_i) + B_{1122}(\sigma_i)(\alpha_{11} + \alpha_{22})\right]\sigma_{11} +$$

$$
\begin{aligned}
&+\left[A_{2222}(\sigma_i)+B_{2222}(\sigma_i)\,\alpha_{22}\right]\sigma_{22}+\\
&+\left[A_{2233}(\sigma_i)+B_{2233}(\sigma_i)\left(\alpha_{22}+\alpha_{33}\right)\right]\sigma_{33};\\
e_{33}&=\left[A_{1133}(\sigma_i)+B_{1133}(\sigma_i)\left(\alpha_{11}+\alpha_{33}\right)\right]\sigma_{11}+\\
&+\left[A_{2233}(\sigma_i)+B_{2233}(\sigma_i)\left(\alpha_{22}+\alpha_{33}\right)\right]\sigma_{22}+\\
&+\left[A_{3333}(\sigma_i)+B_{3333}(\sigma_i)\,\alpha_{33}\right]\sigma_{33};\\
\gamma_{12}&=C_{1212}(\sigma_i)\,\tau_{12};\quad \gamma_{23}=C_{2323}(\sigma_i)\,\tau_{23};\ \gamma_{13}=C_{1313}(\sigma_i)\,\tau_{13},
\end{aligned}
\tag{1}
$$

where $A_{ijkm}(\sigma_i)$, $B_{ijkm}(\sigma_i)$ and $C_{ijkm}(\sigma_i)$ – material nonlinear functions of stress intensity determined by the deformation characteristics of an orthotropic material; $\alpha_{ij}=\sigma_{ij}/S$ – components of normalized stresses; $\sigma_i=\sqrt{3S_{ij}S_{ij}/2}$ – intensity of stresses; $S_{ij}=\sigma_{ij}-\delta_{ij}\sigma$ – stress deviator; $S=\sqrt{\sigma_{ij}\sigma_{ij}}$ – the modulus of the total stress vector (the norm of the tensor stress space); (i, j, k, m=1,2,3).

According to the nature of the experimental data changes, it is found that it is advisable to represent the nonlinear material functions of the equations of state (1) as power polynomials, whose coefficients depend on the experimentally established deformed characteristics of the material under study obtained from experiments on uniaxial stretching and compression along the main axes of orthotropy and on shifts in the three main material planes. Mathematical processing of empirical results was carried out using the least squares method implemented in the Microcal Origin Pro 8.0 software package (Microcal Software Inc.). The method of calculating the constants of the equations of state considered in [18], in the case of physical nonlinearity of orthotropic materials in the presence of deformation anisotropy, material functions can be determined in the form of dependencies associated with the experiments carried out:

$$
\begin{aligned}
A_{kkkk}(\sigma_i)&=0,5\left[1/E_k^{+}(\sigma_i)+1/E_k^{-}(\sigma_i)\right];\\
B_{kkkk}(\sigma_i)&=0,5\left[1/E_k^{+}(\sigma_i)-1/E_k^{-}(\sigma_i)\right];\\
A_{kkmm}(\sigma_i)&=-0,5\left[\nu_{km}^{+}(\sigma_i)/E_m^{+}(\sigma_i)+\nu_{km}^{-}(\sigma_i)/E_m^{-}(\sigma_i)\right];\\
B_{kkmm}(\sigma_i)&=-0,5\left[\nu_{km}^{+}(\sigma_i)/E_m^{+}(\sigma_i)-\nu_{km}^{-}(\sigma_i)/E_m^{-}(\sigma_i)\right];\\
C_{kmkm}(\sigma_i)&=1/G_{km}(\sigma_i);\quad k,m=1,2,3,
\end{aligned}
\tag{2}
$$

where $E_k^{\pm}(\sigma_i)=a_k^{\pm}+m_k^{\pm}\sigma_i+n_k^{\pm}\sigma_i^2$;

$$\nu_{km}^{\pm}(\sigma_i)=\lambda_{km}^{\pm}+\beta_{km}^{\pm}\sigma_i+\mu_{km}^{\pm}\sigma_i^2;$$

$$G_{km}(\sigma_i)=g_{km}+p_{km}\sigma_i+q_{km}\sigma_i^2;$$

$E_k^{\pm}(\sigma_i)$, $\nu_{km}^{\pm}(\sigma_i)$, $G_{km}(\sigma_i)$ – functions that depend on the intensity of stresses and represent nonlinear interpretations of deformation modules, transverse deformation coefficients and shear modules of orthotropic material in the corresponding directions of the main material axes and planes (the «+» sign corresponds to the characteristics obtained from experiments on axial tension, and the «–» sign corresponds to axial compression); $a_k^{\pm}$, $m_k^{\pm}$, $n_k^{\pm}$, $\lambda_{km}^{\pm}$, $\beta_{km}^{\pm}$, $\mu_{km}^{\pm}$, g_{km}, p_{km}, q_{km} – coefficients of approximation polynomials obtained as a result of processing experimental data on the deformation of materials by the least squares method.

It is advisable to carry out the determination of material functions, taking into account approximate limitations traditional for the mechanics of orthotropic materials [19]:

$$
\begin{aligned}
\nu_{km}^{+}(\sigma_i)/E_m^{+}(\sigma_i)&=\nu_{mk}^{+}(\sigma_i)/E_k^{+}(\sigma_i);\\
\nu_{km}^{-}(\sigma_i)/E_m^{-}(\sigma_i)&=\nu_{mk}^{-}(\sigma_i)/E_k^{-}(\sigma_i).
\end{aligned}
\tag{3}
$$

The deformation of the composite «carbon fiber-carbon AVCOMod 3a» was considered as a reference material with a twofold anisotropy, experimental data for which were borrowed from works [6, 13], which presented the results of tests for uniaxial tension and compression in the directions along the main axes of orthotropy and for a net shift in three mutually orthogonal main material planes. Mathematical processing of experimental data on the deformation of the AVCOMod 3 composite allowed us to present the coefficients of the polynomials in Table. 1. Deformation diagrams of the composite under consideration, constructed according to experimental data and their theoretical representations according to model (1) are shown in Fig. 1 – 4.

In the previous work [20], the uniqueness theorem of solutions for the quasilinear deformation potential, which determines approximate relationships of strain and stress tensors for composite materials with complicated properties of the type under consideration, was substantiated and proved. In this [20] and other works [8, 18, 21] it is established that the proof of the uniqueness theorem when postulating any equations of state is finally brought to the verification of Drucker's postulate

$$\delta e_{ij} \delta \sigma_{ij} \geq 0, \qquad (4)$$

within the framework of which restrictions are defined, imposing on material functions. For the accepted nonlinear equations of state (1), the components of the complete Sylvester matrix take the form:

$$D_{11} = \frac{\partial e_{11}}{\partial \sigma_{11}};\ D_{12} = \frac{\partial e_{11}}{\partial \sigma_{22}};\ D_{13} = \frac{\partial e_{11}}{\partial \sigma_{33}};$$
$$D_{14} = \frac{\partial e_{11}}{\partial \tau_{13}};\ D_{15} = \frac{\partial e_{11}}{\partial \tau_{23}};\ D_{16} = \frac{\partial e_{11}}{\partial \tau_{12}};$$
$$D_{22} = \frac{\partial e_{22}}{\partial \sigma_{22}};\ D_{23} = \frac{\partial e_{22}}{\partial \sigma_{33}};\ D_{24} = \frac{\partial e_{22}}{\partial \tau_{13}};$$
$$D_{25} = \frac{\partial e_{22}}{\partial \tau_{23}};\ D_{26} = \frac{\partial e_{22}}{\partial \tau_{12}};\ D_{33} = \frac{\partial e_{33}}{\partial \sigma_{33}};$$
$$D_{34} = \frac{\partial e_{33}}{\partial \tau_{13}};\ D_{35} = \frac{\partial e_{33}}{\partial \tau_{23}};\ D_{36} = \frac{\partial e_{33}}{\partial \tau_{12}};$$
$$D_{44} = \frac{\partial \gamma_{13}}{\partial \tau_{13}};\ D_{45} = \frac{\partial \gamma_{13}}{\partial \tau_{23}};\ D_{46} = \frac{\partial \gamma_{13}}{\partial \tau_{12}};$$
$$D_{55} = \frac{\partial \gamma_{23}}{\partial \tau_{23}};\ D_{56} = \frac{\partial \gamma_{23}}{\partial \tau_{12}};\ D_{66} = \frac{\partial \gamma_{12}}{\partial \tau_{12}};$$
$$D_{ij} = D_{ji}. \qquad (5)$$

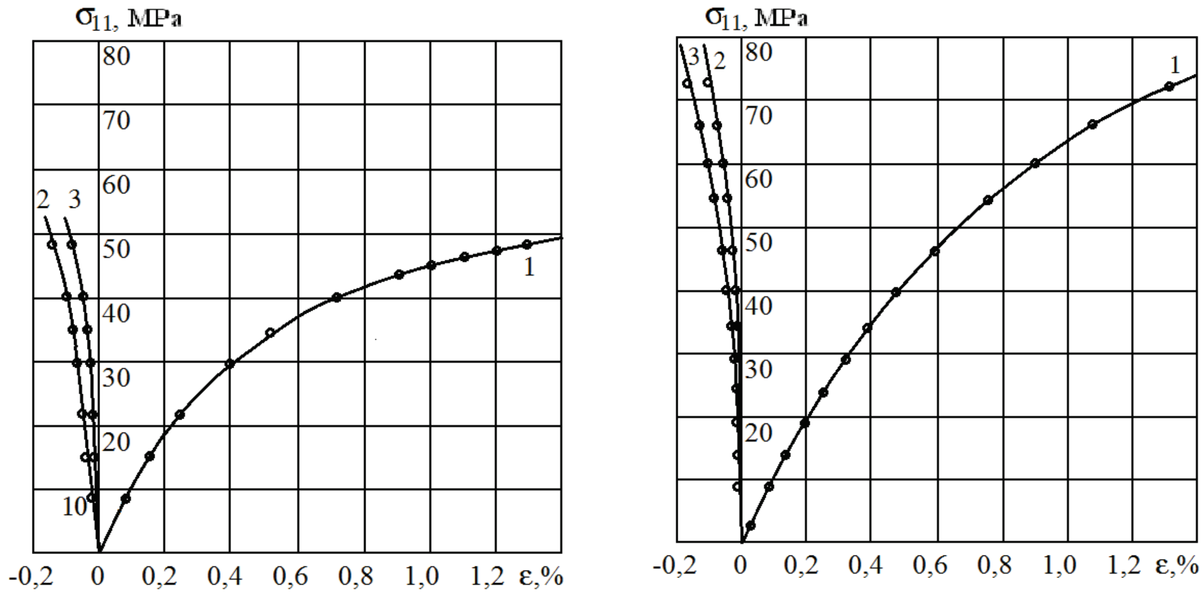
It is established that the polynomial constants representing the material functions of the AVCOMod 3a composite [6, 13] given in Table 1, taking into account the quadratic forms (5), satisfy the Sylvester criterion arising from the requirements of the Drucker postulate [8, 18, 20, 21].

3. DISCUSSION OF THE ADEQUACY OF THE INTRODUCED EQUATIONS OF STATE TO THE EXPERIMENTAL DATA

Figures 1-4 confirm the adequacy of the postulated equations of state (1) to experimentally established diagrams for stretches, compressions in the main axes of orthotropy and shifts in the corresponding planes for reference samples of the AVCOMod 3a composite [6, 13]. It should be noted that the nonlinear model (1) has a number of advantages over the known equations formulated by other authors, such as S.A.Ambartsumyan [16], C.W.Bert–L.N.Reddy [2, 13], R.M.Jones–D.A.R.Nelson [4, 6] and A.A.Zolochevsky [7] in relation to calculations, especially of spatial structures made of orthotropic composite AVCOMod 3a [6, 13]. In particular, the model (1) is free from physically unjustified restrictions imposed on the coefficients of material functions, the absence of piecewise equations of state and the presence of a mechanism for the influence of changes in the type of stress state on the dependence of deformations on stresses. In addition, the error of theoretical approximations (1) of reference experimental diagrams compared with other theories [2, 3, 6, 7, 13], judging by Fig. 1 – 4, it is minimal, and when the coordinate system rotates, it has a higher accuracy.

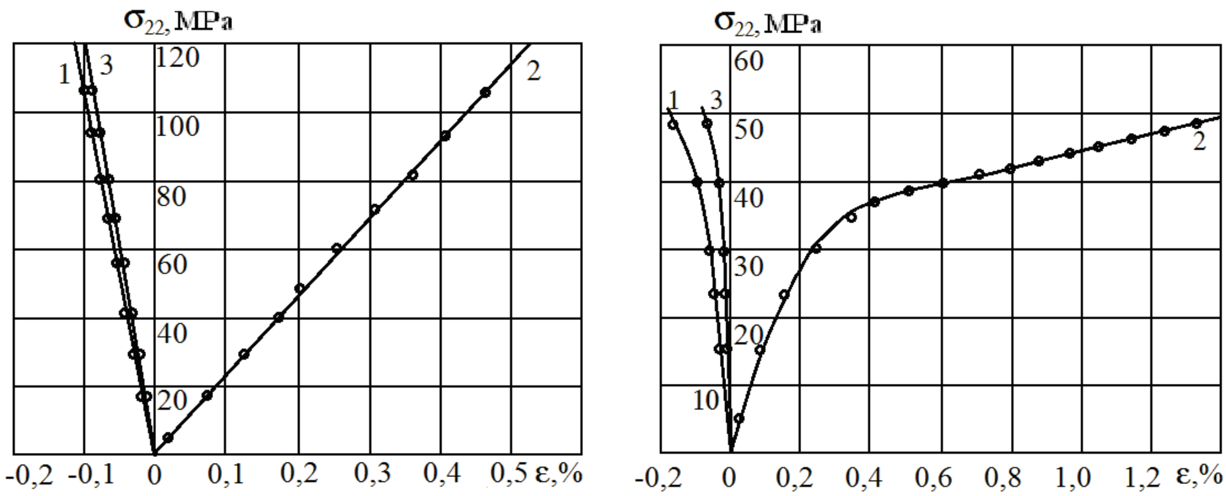
Table 1. Constants of composite material AVCOMod 3a [6, 13]

Type of sample test	Technical parameter	Constants of polynomials of material functions		
Uniaxial stretching along the main axes of orthotropy	$E_k^+(\sigma_i)$	a_1^+=1,058·10^{10}, Pa	m_1^+=62,829	n_1^+=1,535·10^{-6}, Pa^{-1}
		a_2^+=2,864·10^{10}, Pa	m_2^+=–105,476	n_2^+=5,893·10^{-7}, Pa^{-1}
		a_3^+=2,301·10^{10}, Pa	m_3^+=88,349	n_3^+=3,711·10^{-6}, Pa^{-1}
	$\nu_{km}^+(\sigma_i)$	λ_{12}^+=0,158	β_{12}^+=–3,1·10^{-9}, Pa^{-1}	μ_{12}^+=2,19·10^{-17}, Pa^{-2}
		λ_{21}^+=0,103	β_{21}^+=–1,8·10^{-9}, Pa^{-1}	μ_{21}^+=9,1·10^{-18}, Pa^{-2}
		λ_{13}^+=0,203	β_{13}^+=2,15·10^{-9}, Pa^{-1}	μ_{13}^+=6,15·10^{-17}, Pa^{-2}
		λ_{23}^+=0,104	β_{23}^+=0,87·10^{-10}, Pa^{-1}	μ_{23}^+=6,74·10^{-17}, Pa^{-2}
		λ_{31}^+=0,146	β_{31}^+=–0,15·10^{-10}, Pa^{-1}	μ_{31}^+=6,97·10^{-17}, Pa^{-2}
		λ_{32}^+=0,1884	β_{32}^+=–1,15·10^{-2}, Pa^{-1}	μ_{32}^+=1,914·10^{-4}, Pa^{-2}
Uniaxial compression along the main axes of orthotropy	$E_k^-(\sigma_i)$	a_1^-=9,988·10^{9}, Pa	m_1^-=–12,943	n_1^-=6,71·10^{-7}, Pa^{-1}
		a_2^-=2,326·10^{10}, Pa	m_2^-=–436,81	n_2^-=–6,08·10^{-7}, Pa^{-1}
		a_3^-=5,14·10^{9}, Pa	m_3^-=–129,15	n_3^-=–78,3·10^{-6}, Pa^{-1}
	$\nu_{km}^-(\sigma_i)$	λ_{12}^-=0,118	β_{12}^-=–1,46·10^{-9}, Pa^{-1}	μ_{12}^-=2,14·10^{-17}, Pa^{-2}
		λ_{21}^-=0,06	β_{21}^-=1,77·10^{-9}, Pa^{-1}	μ_{21}^-=2,95·10^{-17}, Pa^{-2}
		λ_{13}^-=0,264	β_{13}^-=–1,12·10^{-9}, Pa^{-1}	μ_{13}^-=3,01·10^{-17}, Pa^{-2}
		λ_{23}^-=0,189	β_{23}^-=2,16·10^{-9}, Pa^{-1}	μ_{23}^-=2,1·10^{-17}, Pa^{-2}
		λ_{31}^-=0,134	β_{31}^-=–0,46·10^{-10}, Pa^{-1}	μ_{31}^-=5,82·10^{-17}, Pa^{-2}
		λ_{32}^-=0,07793	β_{32}^-=–0,465·10^{-2}, Pa^{-1}	μ_{32}^-=9,0167·10^{-5}, Pa^{-2}
Shift in the main planes of orthotropy	$G_{km}(\sigma_i)$	g_{12}=4,07·10^{9}, Pa	p_{12}=–1,6	q_{12}=–8,38·10^{-6}, Pa^{-1}
		g_{23}=1,723·10^{9}, Pa	p_{23}=16,899	q_{23}=–1,1·10^{-5}, Pa^{-1}
		g_{31}=2,43·10^{9}, Pa	p_{31}=–54,455	q_{31}=–1,97·10^{-5}, Pa^{-1}



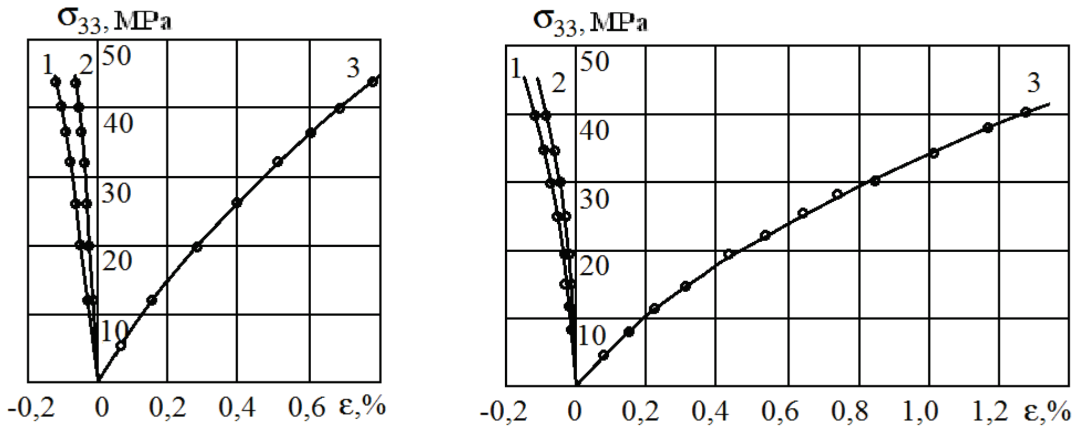
(a) *(b)*

Figure 1. Deformation diagrams along the x_1 axis: (a) stretching; (b) compression; 1 – longitudinal deformation ε_{11}; 2, 3 – transverse deformations ε_{22} and ε_{33}; ○ – experimental data; —— – nonlinear approximations



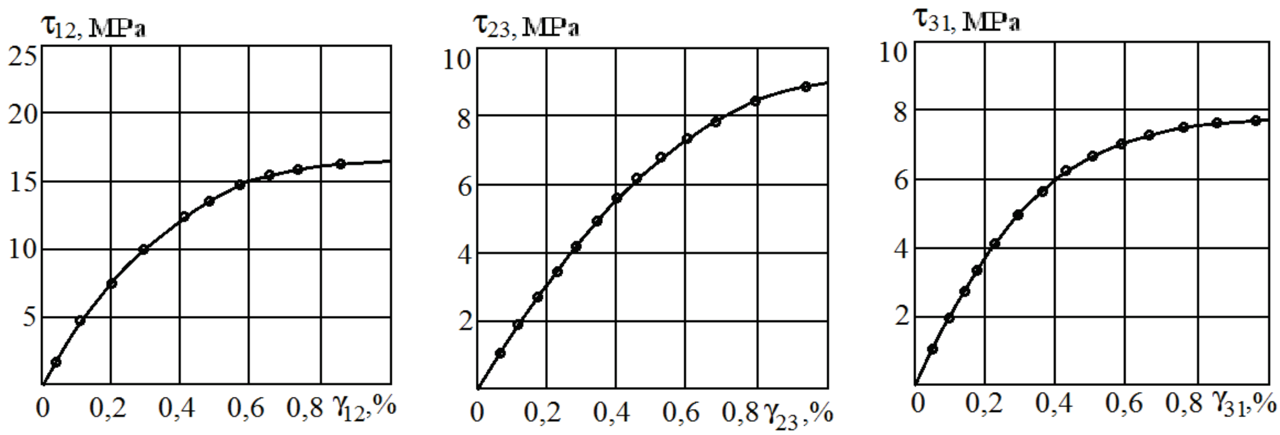
(a) *(b)*

Figure 2. Deformation diagrams along the x_2 axis: (a) stretching; (b) compression; 1, 3 – transverse deformations ε_{11} and ε_{33}; 2 – longitudinal deformation ε_{22}; ○ – experimental data;—— – nonlinear approximation



(a) *(b)*

Figure 3. Deformation diagrams along the x_3 axis: (a) stretching; (b) compression; 1, 2 – transverse deformations ε_{11} and ε_{22}; 3 – ongitudinal deformation ε_{33}; ⚬ – experimental data; —— – nonlinear approximation



(a) *(b)* *(c)*

Figure 4. Deformation diagrams on a net shift in the main planes of orthotropy: (a) in the plane x_1x_2; (b) n the plane x_2x_3; (c) n the plane x_1x_3; ⚬ – experimental data; —— – nonlinear approximation

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