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COMPUTER MODELING THE STRESS STATE OF OPERATING TANKS USING GEOMETRIC INTERPOLATORS

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Abstract. On the example of modeling the stress state of cylindrical tanks for the storage of petroleum products with geometrical imperfections, a method for the numerical study of thin-walled shells of engineering structures is implemented. It includes the compilation and numerical solution of differential equations array, followed by interpolation of the results and determination of the displacements extreme values (or stresses) that occur in the reservoir from the action of a hydrostatic load, taking into account geometric and structural non-linearity. To implement the proposed method, the geometric theory of multidimensional interpolation and approximation is used, including a new way of processing the initial conditions of the differential equation. It consists in the parallel transfer of the numerical solution to the desired point, the coordinates of which correspond to the initial conditions. As a result, it was possible to achieve a significant increase in the speed of the numerical solution while maintaining an accuracy sufficient for engineering calculations.

Keywords: computer modeling, differential equation, geometric interpolant, parallel transfer, response surface, stress state, operating tank

КОМПЬЮТЕРНОЕ МОДЕЛИРОВАНИЕ НАПРЯЖЕННОГО СОСТОЯНИЯ ЭКСПЛУАТИРУЕМЫХ РЕЗЕРВУАРОВ С ПОМОЩЬЮ ГЕОМЕТРИЧЕСКИХ ИНТЕРПОЛЯНТОВ

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Аннотация. На примере моделирования напряжённого состояния эксплуатируемых цилиндрических резервуаров для хранения нефтепродуктов с несовершенствами геометрической формы реализован метод численного исследования тонкостенных оболочек инженерных сооружений, который включает составление и численное решение массива дифференциальных уравнений с последующей интерполяцией результатов и определением экстремальных значений перемещений (или напряжений), возникающих в резервуаре от действия гидростатической нагрузки с учётом геометрической и конструктивной нелинейности. Для реализации предложенного метода используется геометрическая теория многомерной интерполяции и аппроксимации, включая новый способ учёта начальных условий дифференциального уравнения. Последний заключается в параллельном переносе численного решения в нужную точку, координаты которой соответствуют начальным условиям. В результате удалось достичь значительного повышения быстродействия численного решения при сохранении достаточной для инженерных расчётов точности.

Ключевые слова: компьютерное моделирование, напряжённое состояние, эксплуатируемый резервуар, дифференциальное уравнение, геометрический интерполянт, поверхность отклика

INTRODUCTION

In engineering practice, steel thin-walled shells of revolution are widely used. Such structures also include steel vertical cylindrical tanks for storing petroleum products. The existing cases of destruction of the above engineering structures have led to the need for periodic monitoring of their technical condition throughout the entire period of operation.

Among the methods for diagnosing the technical condition of cylindrical tanks, taking into account the imperfections of the geometric shape, the most widespread methods are based on modeling both general and local imperfections of the geometric shape [1-4]. In [5-7], a technique is proposed according to which the interpolation of the tank surface is performed using closed and open contours of the first order of smoothness, followed by the calculation of the stressed state (SS) of the shell in the system of finite element analysis. This approach has a number of advantages and opens up new opportunities for analyzing the technical condition of thin-walled shells of engineering structures, taking into account geometric and structural nonlinearity, and for predicting the impact of the geometric shape imperfections development on the strength and stability of the shell under study. On the other hand, the method proposed in [5-7] contains a number of features. Firstly, the calculation of the SS of a storage tank for petroleum products, containing 65854 finite elements in the form of rectangular plates, in a nonlinear formulation (taking into account geometric and structural nonlinearity) took more than 25 hours on a computer running an Intel Core i5-2400 processor (4-core processor with a maximum clock speed of 3.40 GHz), which is long enough for engineering tasks. Secondly, the complexity of considering constructive nonlinearity, leading to the need to implement a special staged loading scheme proposed in the work [6].

All this leads to the need to improve numerical modeling methods to assess the technical condition of the exploited steel thin-walled shells of engineering structures and develop effective measures to increase their safe operation life.

1. METHODS

In the domestic literature, the model for determining the SS of an elastic cylindrical shell under axisymmetric loading has become widespread [8, 9]. In this case, a differential equation is obtained, which has an exact mathematical solution, taking into account the initial and boundary conditions [9]:

$$D\frac{d^4w}{dx^4}+\frac{Ehw}{r^2\left(1-\alpha\mu\right)}=p,$$

where w is the desired displacements due to hydrostatic load;

x is the height coordinate of the wall, counting from the rim weld of the tank;

r is the radius of a cylindrical tank;

h is the wall thickness of a cylindrical tank;

p is the evenly distributed load;

$E=2.1\cdot10^{11}$ is the Young's modulus for steel;

$\mu=0.3$ is the Poisson's ratio;

α is the parameter, which, in accordance with [8], in a uniaxial stress state is taken equal to 0 (hydrostatic pressure in an open cylindrical vessel), and at internal gas pressure in a closed cylindrical vessel is taken to be 0.5;

$D=\frac{Eh^3}{12\left(1-\mu^2\right)}$ is the cylindrical stiffness.

At the same time, the presence of even minor imperfections in the geometric shape, which are expressed in the form of tank wall deviations from the vertical, lead to the fact that the loading of the shell becomes non-axisymmetric. In addition, the presence of geometric nonlinearity in this case leads to the need to take into account also the constructive nonlinearity that occurs in the process of filling the reservoir with liquid [6]. Therefore, it becomes necessary to refine the original differential equation, taking into account the initial deviations of the surface of the cylindrical shell from the vertical.

We compose the initial equilibrium equation in accordance with Figure 1, which shows an infinitesimal element of a cylindrical tank with all the acting loads and forces in the sections:

$$2N_2\sin\frac{\alpha}{2}+\frac{dQ_1}{dx}-pdxdy=0.$$

In accordance with [8], we take the size of a square element $dx = dy = \alpha r = 1$. For small values of the angle, the value $\sin\frac{\alpha}{2} = \frac{\alpha}{2} = \frac{1}{2r}$. Then the original differential equation takes the following form:

$$\frac{N_2}{r} + \frac{dQ_1}{dx} - p = 0.$$

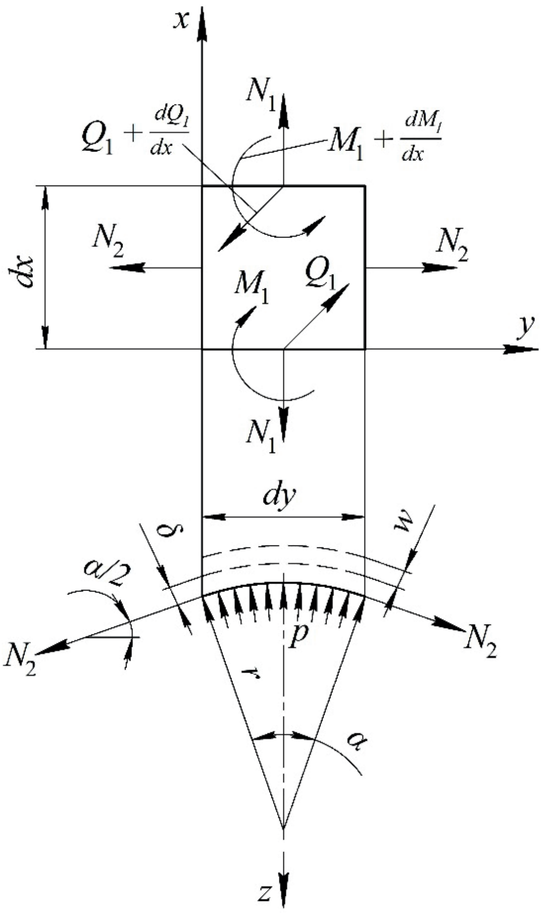

Figure 1. Scheme for compiling a differential equation for the equilibrium of a cylindrical tank with geometric imperfections element

Let us express the force N_2 in terms of the corresponding deformation ε_2. Taking into account the initial deviations from the vertical δ, we obtain:

$$\varepsilon_2 = \frac{\alpha\left(r + \delta + w\right) - \alpha r}{\alpha r} = \\ = \frac{w + \delta}{r} = \frac{1}{kE}\left(\frac{N_2}{h} - \mu\frac{N_1}{h}\right),$$

where $\delta = \delta(x)$ is the function of initial deviations of a cylindrical tank from the vertical;
k is the correction factor that takes into account the geometric and structural non-linearity [5-7] and the stresses that arise in the upper chord of the shell due to its interaction with the tank roof.
Based on the similarity of right triangles (Fig. 1), we have $N_1 = N_2 \sin\frac{\alpha}{2} = \frac{\alpha}{2} N_2$. Then we get:

$$\varepsilon_2 = \frac{w + \delta}{r} = \frac{N_2}{kEh}\left(1 - \alpha\mu/2\right) \Rightarrow N_2 = \frac{kEh\left(w + \delta\right)}{r\left(1 - \alpha\mu/2\right)}.$$

Taking into account that $Q_1 = \frac{dM_1}{dx}$ and, therefore, $\frac{dQ_1}{dx} = \frac{d^2 M_1}{dx^2}$, we get:

$$\frac{d^2 M_1}{dx^2} + \frac{kEh\left(w + \delta\right)}{r^2\left(1 - \alpha\mu/2\right)} = p.$$

The last equation contains two unknowns: M_1 and w. Eliminating one of them, we express the effort in terms of displacements. According to Bernoulli's law, we have:

$$\frac{1}{\rho} = \frac{M_1}{D},$$

where ρ is the radius of curvature when the shell is bent in the meridional direction.
On the other hand, geometrically, the curvature is given by the following expression:

$$K = \frac{1}{\rho} = \frac{d^2 w / dx^2}{\left[1 + \left(dw / dx \right)^2 \right]^{3/2}}.$$

Small displacements cause small curvatures K. Then the quantity $\left(dw / dx \right)^2$ is so small compared to unity that it can be neglected. As a result, we get: $1/\rho \approx d^2 w / dx^2$.

Differentiating the twice approximate value of the curvature, we have:

$$\frac{d^2 M_1}{dx^2} = D \frac{d^4 w}{dx^4}.$$

As a result, we obtain a differential equation with an unknown displacement function $w = w(x)$:

$$D \frac{d^4 w}{dx^4} + \frac{kEh(w+\delta)}{r^2 \left(1 - \alpha\mu / 2\right)} = p.$$

Let us define the uniformly distributed hydrostatic load p by the linear relation: $p = -\gamma g (d - x)$, where γ is the density of stored liquid; g is the free fall acceleration (we take 9.81 m/s^2); d is the height of the liquid level in the tank.

Then the final differential equation takes the following form:

$$D \frac{d^4 w}{dx^4} + \frac{kEh(w+\delta)}{r^2 \left(1 - \frac{\alpha\mu}{2}\right)} = \gamma g (x - d). \quad (1)$$

Given that the initial deviations δ and the desired displacements w are functions of the variable x, the mathematically exact solution proposed in [9] gives significant errors (Fig. 2), which leads to the need to solve it numerically.

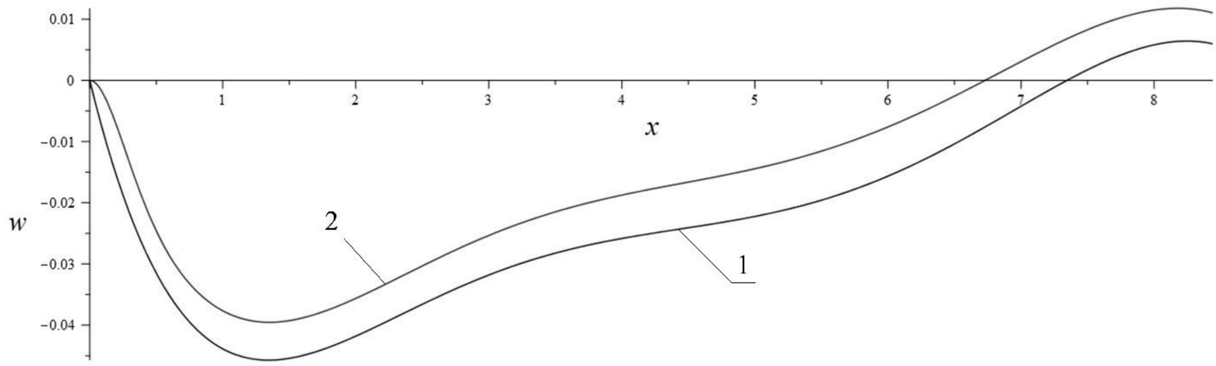

Figure 2. Comparison of the results of solving the differential equation (1): 1 – reference solution; 2 – mathematically exact solution proposed in [9]

The reference solution was obtained by approximating the values of displacements from the action of a hydrostatic load, taking into account the geometric and structural nonlinearity [5-7] when modeling the SS in the SCAD finite element analysis software package. The calculations were carried out in accordance with the strength theory of octahedral shear stresses or the specific energy of deformation (the energy theory of Huber-Henki-Mize).

A method for the numerical solution of differential equations using geometric interpolants was proposed in the works [1010,11]. It can be attributed to the category of superelement methods [12-14] used to solve a wide range of engineering and applied problems. A feature of the method proposed in

[10, 11] is that a geometric interpolant is used as an approximating function - a geometric object passing through predetermined points (interpolation nodes). Thus, a multidimensional geometric interpolant is a superelement that includes information about both geometric and physical parameters of its state. Only in this case, the interpolation nodes are not known in advance. They are calculated from the condition of compliance with the original differential equation. Correspondence of intermediate points to the original differential equation is provided automatically due to interpolation. Thus, the more interpolation nodes, the closer the geometric interpolant is to the desired numerical solution of the differential equation. This approach, by analogy with the isogeometric method [15-18] proposed by Tom Hughes, eliminates the need to coordinate geometric information in the process of interaction between CAD and FEA systems.

In this case, the solution to the differential equation (1) is the displacement function $w = w(x)$, which is a one-parameter set of points – a curved line. We define the displacement function using an algebraic curve passing through predetermined points as a one-parameter geometric interpolant [19, 20]. Based on the conditions for differentiating the displacement function in equation (1), the curve must be at least 5th order.

As a test of the simulation results, each of the reference curves should pass through 7 points. Therefore, a 6th order curve is needed to describe them. Based on these considerations, we take as an approximating function a polynomial of the 6th degree, which describes an algebraic curve of the 6th order. In accordance with the geometric theory of multidimensional interpolation [19], such a curve is determined by the following point equation:

$$\begin{aligned} M = M_1p_1 + M_2p_2 + M_3p_3 + \\ +M_4p_4 + M_5p_5 + M_6p_6 + M_7p_7, \end{aligned} \qquad (2)$$

where M is the current point of the arc of the 6th order algebraic curve;

M_i are the interpolation nodes;

$$\begin{aligned}
&p_1 = \bar{t}^6 - 8.7\bar{t}^5 t + 22.7\bar{t}^4 t^2 - 22.7\bar{t}^3 t^3 + \\
&+8.7\bar{t}^2 t^4 - \bar{t}t^5; \\
&p_2 = 36\bar{t}^5 t - 133.2\bar{t}^4 t^2 + 151.2\bar{t}^3 t^3 - \\
&-61.2\bar{t}^2 t^4 + 7.2\bar{t}t^5; \\
&p_3 = -45\bar{t}^5 t + 301.5\bar{t}^4 t^2 - 418.5\bar{t}^3 t^3 + \\
&+184.5\bar{t}^2 t^4 - 22.5\bar{t}t^5; \\
&p_4 = 40\bar{t}^5 t - 308\bar{t}^4 t^2 + 600\bar{t}^3 t^3 - \\
&-308\bar{t}^2 t^4 + 40\bar{t}t^5; \\
&p_5 = -22.5\bar{t}^5 t + 184.5\bar{t}^4 t^2 - 418.5\bar{t}^3 t^3 + \\
&+301.5\bar{t}^2 t^4 - 45\bar{t}t^5; \\
&p_6 = 7.2\bar{t}^5 t - 61.2\bar{t}^4 t^2 + 151.2\bar{t}^3 t^3 - \\
&-133.2\bar{t}^2 t^4 + 36\bar{t}t^5; \\
&p_7 = -\bar{t}^5 t + 8.7\bar{t}^4 t^2 - 22.7\bar{t}^3 t^3 + \\
&+22.7\bar{t}^2 t^4 - 8.7\bar{t}t^5 + t^6;
\end{aligned}$$

t is the current parameter, which changes from 0 to 1;

$\bar{t} = 1 - t$ is the addition of parameter t to 1.

Next, we need to move from a point equation, which is a symbolic notation, to a system of parametric equations through a coordinate-wise calculation:

$$\begin{cases} x = x_1p_1 + x_2p_2 + x_3p_3 + x_4p_4 + \\ +x_5p_5 + x_6p_6 + x_7p_7 \\ w = w_1p_1 + w_2p_2 + w_3p_3 + w_4p_4 + \\ +w_5p_5 + w_6p_6 + w_7p_7 \end{cases}. \qquad (3)$$

Taking into account the special properties of algebraic curves passing through predetermined points, obtained on the basis of Bernstein polynomials [19], the first equation of the system takes a linear dependence in the case of a uniform distribution of the interpolation nodes coordinates along the axis Ox: $x = td$. Then, to determine the geometric interpolant explicitly, it suffices to perform a change of variables in the second equation of the system: $t = \frac{x}{d}$, where d is the height of the liquid level in the tank.

Next, it is necessary to divide the curve into 7 equal parts along the Ox axis and alternately substitute the obtained values x_j into the original differential equation (1), where j varies from 0 to 7. Thus, a system of linear algebraic equations (SLAE) with 7 unknowns is formed w_j. After solving the, the obtained values w_j must be substituted into the second equation of the system (3), which is the numerical solution of the differential equation (1) in the range of the variable from 0 to d.

A separate issue is the use of initial and boundary conditions to refine the numerical solution of differential equation (1). Traditionally, these conditions are used to determine the constants of integration [9]. But in this case, this is precisely the source of errors in the solution (Fig. 2). In the proposed method, a numerical solution is obtained, which corresponds to the initial differential equation (1) at the nodal points of interpolation. It should be noted that this solution does not take into account the initial conditions and therefore the resulting curve is arbitrarily in the plane xOy. To satisfy the initial condition $w(0)=0$, it is necessary to transfer the resulting curved line to the origin of coordinates. For this, it is necessary to subtract the free term of the approximating polynomial function from the obtained solution $w = w(x)$.

2. RESULTS AND DISCUSSION

As an example, consider a vertical cylindrical steel tank No. 1 for storing gasoline with a volume of 1000 m^3 of the integrated collection point «Beshensky».

The initial data for modeling are the geometric dimensions of the tank (tank radius $r = 6.165$ m; shell thickness $h = 0.005$ m), the height of the liquid level in the tank ($d = 8.44$ m), the physical properties of the stored liquid (usually tanks are tested with water, so we take the water density $\gamma = 1000$ kg/m^3) and the initial deviations of the tank wall from the vertical δ.

To determine the functions of the initial deviations of a cylindrical tank from the vertical at reference points along the circumference of the tank $\delta_i = \delta_i(x)$, where $i = 0,\ 30,\ 60,\ ...,\ 330$, the approximation tool Fit from the Statistics package of the Maple computer algebra system was used.

The initial data entry was organized using the MS Excel software package. Thus, 12 equations were obtained that characterize the functions of the initial deviations of the cylindrical tank from the vertical at 12 reference points along the circumference of the tank. As an example, we give one of the equations (at the point 0°), taking into account the rounding of polynomial coefficients:

$$\delta_0 = 4.188 \cdot 10^{-6} x^6 - 0.00017 x^5 +$$
$$+0.0025 x^4 - 0.018 x^3 + 0.061 x^2 - 0.089 x.$$

Considering that in the process of approximation the amount of initial data corresponded to the degree of the polynomial, the coefficients of determination for all 12 lines were obtained $R^2 = 1$. That is, in fact, using the Fit function, the initial deviations were interpolated δ.

Similarly, 12 equations were obtained that characterize the functions of displacements of a cylindrical tank from the vertical at 12 reference points along the circumference of the tank, which were then used as reference curves to verify the simulation results. As an example, we give the reference equation at the point 0°, taking into account the rounding of polynomial coefficients:

$$\omega_0 = 3.71 \cdot 10^{-6} x^6 - 0.00016 x^5 +$$
$$+0.0024 x^4 - 0.017 x^3 + 0.06 x^2 - 0.089 x.$$

As a result of computational experiments, taking into account the initial data, 12 equations were obtained that characterize the functions of displacements of a cylindrical reservoir from the vertical at 12 reference points along the circumference of the reservoir. For comparison, we present the numerical solution of the differential equation (1) at the point 0°, taking into account the rounding of polynomial coefficients:

$$w_0 = 4.19 \cdot 10^{-6} x^6 - 0.00017x^5 + 0.0025x^4 - \\ -0.018x^3 + 0.061x^2 - 0.09x - 0.007.$$

Comparison of graphical interpretations of the obtained numerical solution with the reference one (Fig. 3) shows that the curves are very similar, but are at some distance from each other. Based on this, we can conclude that the solution was obtained correctly, but is in the wrong place. To agree on them, it is necessary to fulfill the initial conditions. In our case, this condition is – $w(0) = 0$, which says that it is necessary to perform a parallel transfer of the resulting curve to the origin. To do this, it suffices to subtract the free term from the obtained numerical solution.

As an example, we present the numerical solution of the differential equation (1) at the point 0° after applying the initial condition $w(0) = 0$:

$$w_0 = 4.19 \cdot 10^{-6} x^6 - 0.00017x^5 + \\ +0.0025x^4 - 0.018x^3 + 0.061x^2 - 0.09x. \quad (4)$$

As can be seen when comparing the equations w_0 and ω_0, almost all polynomial coefficients in pairs have very close values. Graphical visualization of the comparison results also shows a high level of similarity of algebraic curves (Fig. 4).

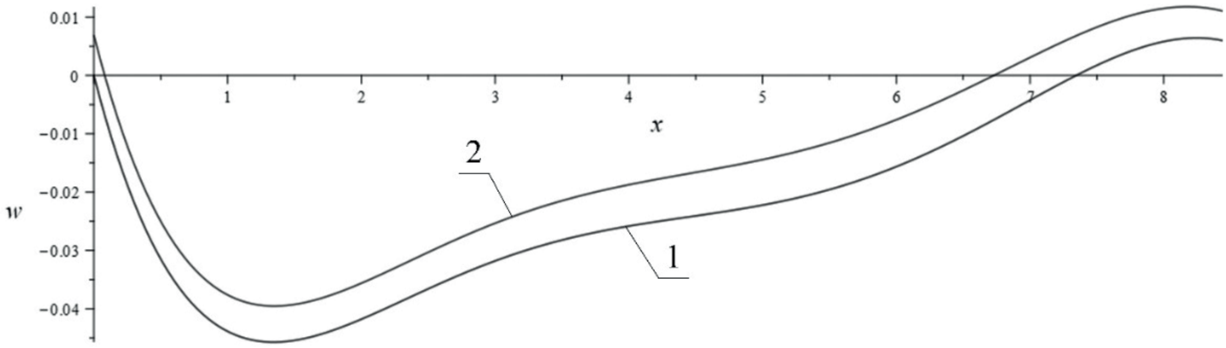

Figure 3. Comparison of the results of solving the differential equation (1): 1 – reference solution; 2 – numerical solution

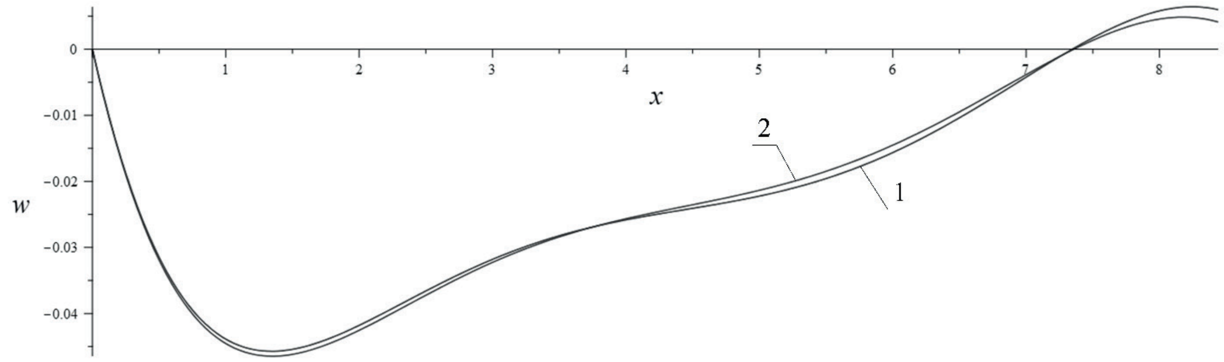

Figure 4. Comparison of the results of solving the differential equation (1) after applying the initial condition $w(0) = 0$: reference solution; 2 – numerical solution

To numerically determine the degree of similarity of the reference and numerical solutions, we use the method of comparing multidimensional geometric objects using the determination coefficient R^2, proposed in the work [5]. We discretize the curves with subsequent comparison of two-point sets using the coefficient of determination. As a result, when comparing algebraic curves, the coefficient of determination was achieved $R^2 = 0.998$. A similar comparison was made for all 12 lines around the circumference of the tank and high values of the coefficient of determination were obtained, confirming the reliability of the results.

In the process of examining the technical condition of steel cylindrical tanks, an important task is to determine the places of maximum displacements and, accordingly, maximum stresses in the tank wall. To solve this problem, one can use the methods of two-dimensional interpolation with the subsequent determination of the extreme points of the response surface. Given the distortion of the geometric shape of the actual surface of a cylindrical tank, it is convenient to build a response surface by presenting the tank in an unfolded form. Only along the abscissa axis we will plot not the length of the arc, which will constantly change due to distortions of the cylindrical surface, but the angle φ, which also determines the position of each point on the tank surface. On the y-axis we will plot the height of the tank d, on the applicate axis - the required displacements of the tank wall w.

The guide lines of the required response surface are defined as functions of the displacements $w_i = w_i(x)$, where $i = 0°, 30°, 60°, ..., 360°$, which are obtained as a result of the numerical solution of the differential equation (1). An example of such a function is equation (4) at the point 0°. For convenience, let's move from the variable x to the parameter v, which changes from 0 to 1. This transition is provided by the change of variables: $x = v \cdot d$. For example, for displacements at the point 0°, taking into account the rounding of polynomial coefficients, we get:

$$w_0 = 1.514v^6 - 7.314v^5 + 12.847v^4 - \\ -10.628v^3 + 4.347v^2 - 0.762v.$$

In this case, 12 such equations were found in a similar way along the circumference of the tank. Since the reservoir with imperfections is a closed shell, the last displacement function corresponds to the first one. $w_0 = w_{360}$. At the same time, the required number of guide lines can be increased to achieve the required calculation accuracy.

Taking into account a large number of guide lines, in order to avoid unplanned oscillations, we will interpolate them using one-dimensional open contours of the 1st order of smoothness. Their geometric construction algorithms are presented in [5]. Based on this, we obtain the following computational algorithm for constructing the generatrix of the response surface:

1. Determine the coordinates of the initial points A_j along the x-axis:

$$\varphi_{A_{j+1}} = 360 \frac{j}{n}, \quad j = 0,1,...,n,$$

where n is the number of response surface guide lines.

2. Determine the length of the segment A_jA_{j+1}:

$$\left|A_jA_{j+1}\right| = \sqrt{\left(\varphi_{A_j} - \varphi_{A_{j+1}}\right)^2 + \left(w_{A_j} - w_{A_{j+1}}\right)^2},$$
$$j = 1,2,...,n-1,$$

where w_{A_j} is the response surface guide line equations;

$\left|A_{j+1}A_{j+2}\right|$ determined from the same point by a shift by one.

3. Determine the length of the segment A_jA_{j+2} :

$$\left|A_jA_{j+2}\right| = \sqrt{\left(\varphi_{A_j} - \varphi_{A_{j+2}}\right)^2 + \left(w_{A_j} - w_{A_{j+2}}\right)^2},$$

where $j = 1, 2, ..., n-2$

4. Determine the coordinates of the points B_{j+1}, C_{j+1}, forming tangents at the points of the curve arc tracing:

$$\begin{cases} \varphi_{B_j} = \left(\varphi_{A_{j+2}} - \varphi_{A_j}\right)\dfrac{\left|A_{j+1}A_{j+2}\right|}{\pi\left|A_jA_{j+2}\right|} + \varphi_{A_{j+1}} \\ w_{B_j} = \left(w_{A_{j+2}} - w_{A_j}\right)\dfrac{\left|A_{j+1}A_{j+2}\right|}{\pi\left|A_jA_{j+2}\right|} + w_{A_{j+1}} \end{cases},$$

$$\begin{cases} \varphi_{C_j} = \left(\varphi_{A_j} - \varphi_{A_{j+2}}\right)\dfrac{\left|A_jA_{j+1}\right|}{\pi\left|A_jA_{j+2}\right|} + \varphi_{A_{j+1}} \\ w_{C_j} = \left(w_{A_j} - w_{A_{j+2}}\right)\dfrac{\left|A_jA_{j+1}\right|}{\pi\left|A_jA_{j+2}\right|} + w_{A_{j+1}} \end{cases},$$

where $j = 1, 2, ..., n-1$.

5. Define the curve contour arcs for the first and last sections:

$$\begin{cases} \varphi_1 = \varphi_{A_1}\bar{u}^2 + 2\varphi_{C_1}u\bar{u} + \varphi_{A_2}u^2 \\ d_1 = d \cdot v \\ w_1 = w_{A_1}\bar{u}^2 + 2w_{C_1}u\bar{u} + w_{A_2}u^2 \end{cases},$$

$$\begin{cases} \varphi_n = \varphi_{A_n}\bar{u}^2 + 2\varphi_{B_{n-1}}u\bar{u} + \varphi_{A_{n+1}}u^2 \\ d_n = d \cdot v \\ w_n = w_{A_n}\bar{u}^2 + 2w_{B_{n-1}}u\bar{u} + w_{A_{n+1}}u^2 \end{cases},$$

where $\bar{u} = 1 - u$ is the addition of parameter u to 1.

6. Define the curve contour arcs for intermediate sections:

$$\begin{cases} \varphi_j = \varphi_{A_j}\bar{u}^3 + 3\varphi_{B_{j-1}}\bar{u}^2u + 3\varphi_{C_j}\bar{u}u^2 + \varphi_{A_{j+1}}u^3 \\ d_j = d \cdot v \\ w_j = w_{A_j}\bar{u}^3 + 3w_{B_{j-1}}\bar{u}^2u + 3w_{C_j}\bar{u}u^2 + w_{A_{j+1}}u^3 \end{cases},$$

where $j = 2, 3, ..., n-1$.

As a result of the execution of the above computational algorithm, we obtain a response surface that characterizes displacements in the wall of a steel cylindrical tank with imperfections (Fig. 5).

As can be seen from Figure 5, the maximum displacements occur in the lower part of the tank in the range from 150° to 300° around the circumference of the tank. Let us determine the values of the maximum displacements and their exact position by the methods of mathematical analysis. To do this, it is necessary to solve a system of two equations with partial derivatives with respect to the parameters u and v, on the interval of their change from 0 to 1. Thus, the maximum displacements of 92.2 mm occur at a height of 2.298 m and 258.6° around the circumference of the tank.

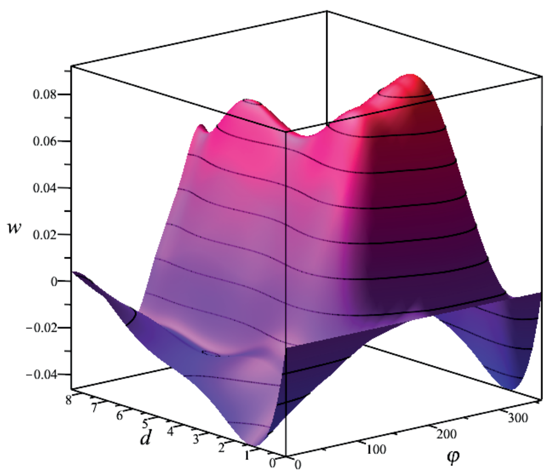

Figure 5. Visualization of the response surface of the tank wall movements with imperfections from the hydraulic load

It is also possible to determine the maximum displacements of the tank wall from the action of hydrostatic load for each belt along its height. To do this, it suffices to cut the resulting response surface with vertical planes parallel to the axis φ. Analytically, this is determined by step-by-step fixing the height values of each tank belt with the determination of the maximum displacements at each stage.

The paper proposes a method for determining the SS of an oil storage tank with imperfections based on displacements. If necessary, it is quite easy to switch to stresses. In accordance with [22], for this it is sufficient to multiply the resulting displacements by the quotient from dividing the Young's modulus by the tank radius. But since the radius varies along the height of the tank due to imperfections in the geometric shape, when moving from displacements to stresses, it is necessary to take into account the functions of the initial deviations $\delta_j = \delta_j(x)$. Thus, Young's modulus must be divided by $r + \delta_j$.

CONCLUSIONS

Improvement of the differential equation and numerical solution, including a new way of taking into account the initial conditions, which are described in the paper, are not the only possible ones. In a similar way, other designs of thin-walled shells of engineering structures can be studied, taking into account the imperfections of the geometric shape or without them, except for vertical cylindrical tanks. It is also possible to model other types of loads (for example, wind load, snow load, structural self-weight load, etc.) using interpolation and approximation methods. This makes it possible to avoid the need for expensive full-scale experiments, which in some cases are not only not profitable, but even impossible.

A feature of the proposed method is that the model is quite simply described by a 4th order differential equation using interpolation and approximation methods. This equation has a simple numerical solution in the form of a 6th degree polynomial. At the same time, there is no need to perform long and complex calculations for modeling thin-walled shells, taking into account both geometric and structural non-linearity.

Due to the use of interpolation and approximation methods for solving the problem, it was possible to achieve sufficient numerical simulation accuracy for engineering calculations, increasing their speed. The calculation of an array consisting of 12 numerical solutions of differential equations that define 12 guide lines of the response surface (as well as the construction of its generatrix using one-dimensional contours of the first order of smoothness, taking into account visualization) takes about 20 seconds even without parallelization of computational threads. If necessary, the number of guide lines of the response surface can be significantly increased to achieve greater accuracy of engineering calculations. At the same time, the proposed method is easily amenable to parallelization of computational operations. This became possible due to the fact that it combines the potential of constructive methods of geometric modeling, which are able to provide parallelization of geometric constructions by tasks (message passing), and the mathematical apparatus "Point calculus", capable of implementing parallelization according to data due to coordinate-wise calculation (data parallel). In the future, this makes it possible to realize the entire available computing potential of modern multi-core processors.

Based on the results of the research, a new engineering technique for examining the technical condition of an oil storage tank with geometric imperfections is proposed based on the numerical solution of differential equations using geometric interpolants [23].

REFERENCE

1. **Aleshina O.O., Ivanov V.N., Cajamarca-Zuniga D.** Stress state analysis of an equal slope shell under uniformly distributed tangential load by different methods // Structural Mechanics of Engineering Constructions and Buildings. 2021. Vol. 17. No. 1. pp. 51-62. DOI: 10.22363/1815-5235-2021-17-1-51-62.
2. **Maraveas C., Balokas G.A., Tsavdaridis K.D.** Numerical evaluation on shell buckling of empty thin-walled steel tanks

under wind load according to current american and european design codes // Thin-Walled Structures. 2015. Vol. 95. pp.152-160. DOI: 10.1016/j.tws.2015.07.007.

3. Behaviour of vertical cylindrical tank with local wall imperfections / A. Šapalas, G. Šaučiuvėnas, K. Rasiulis [et al.] // Journal of Civil Engineering and Management. 2019. 25(3). pp. 287-296. DOI: 10.3846/jcem.2019.9629.
4. **Gorban N.N., Vasiliev G.G., Salnikov A.P.** Accounting actual geometric shape of the tank shell when evaluating its fatigue life // Oil Industry. 2018. No. 8. pp. 75-79. DOI: 10.24887/0028-2448-2018-8-75-79.
5. **Krysko A.A.** Geometricheskoe i komp'yuternoe modelirovanie ekspluatiruemyh konstrukcij tonkostennyh obolochek inzhenernyh sooruzhenij s uchyotom nesovershenstv geometricheskoj formy [Geometric and computer modeling of operating structures of thin-walled shells of engineering structures with regard to geometric form imperfections] // Makeyevka: DONNACEA, 2016. 191 p. (In Russian).
6. **Krysko A.A.** Analiz napryazhenno-deformirovannogo sostoyaniya stenki rezervuara s geometricheskimi nesovershenstvami pri dejstvii gidrostaticheskoj nagruzki [Calculation of the intense deformed state of tank’swall under the action of the hydrostatic load in anonlinear setting with geometric imperfections] // Metallicheskie konstrukcii. 2017. Vol. 23. No. 3. pp. 97-106. (In Russian).
7. **Krysko A.A.** Chislennye issledovaniya mestnyh nesovershenstv geometricheskoj formy vertikal'nogo cilindricheskogo [Numerical studies of local defects in the geometric shape of a vertical cylindrical tank] // Stroitel' Donbassa. 2020. No. 1(10). pp. 13-17. (In Russian).
8. **Lessig E.N., Lileev A.F., Sokolov A.G.** Listovye metallicheskie konstrukcii [Sheet metal structures] // Moscow: Strojizdat, 1970. 488 p. (In Russian).
9. **Timoshenko S.P., Vojnovskij-Kriger S.** Plastinki i obolochki [Plates and shells] // Moscow: Nauka, 1966. 636 p. (In Russian).
10. **Konopatskiy E.V., Voronova O.S., Shevchuk O.A., Bezditnyi A.A.** About one method of numeral decision of differential equalizations in partials using geometric interpolants // CEUR Workshop Proceedings. 2020. Vol. 2763. pp. 213-219. DOI: 10.30987/conferencearticle_5fce27708eb353.92843700.
11. **Konopatskiy E.V., Bezditnyi A.A., Shevchuk O.A.** Modeling geometric varieties with given differential characteristics and its application // CEUR Workshop Proceedings. 2020. Vol. 2744. DOI: 10.51130/graphicon-2020-2-4-31.
12. **Shamloofard M., Hosseinzadeh A., Movahhedy M.R.** Development of a shell superelement for large deformation and free vibration analysis of composite spherical shells // Engineering with Computers. 2021. Vol. 37. No. 4. pp. 3551-3567. DOI: 10.1007/s00366-020-01015-w.
13. **Hughes P.J., Kuether R.J.** Nonlinear interface reduction for time-domain analysis of Hurty/Craig-Bampton superelements with frictional contact // Journal of Sound and Vibration. 2021. Vol. 507. DOI: 10.1016/j.jsv.2021.116154.
14. **Nielsen M.B., Sahin E.** A simple procedure for embedding seismic loads in foundation superelements for combined wind, wave and seismic analysis of offshore wind turbine structures // Paper presented at the COMPDYN Proceedings. 2019. Vol. 3. pp. 4628-4640. DOI: 10.7712/120119.7255.19324.
15. Isogeometric analysis of large-deformation thin shells using RHT-splines for multiple-patch coupling / N. Nguyen-Thanh, K. Zhou, X. Zhuang [et al.] // Computer Methods in Applied Mechanics and

Engineering. 2017. Vol. 316. pp. 1157-1178. DOI: 10.1016/j.cma.2016.12.002.
16. A NURBS-based inverse analysis for reconstruction of nonlinear deformations of thin shell structures / N. Vu-Bac, T.X. Duong, T. Lahmer [et al.] // Computer Methods in Applied Mechanics and Engineering. 2018. Vol. 331. pp. 427-455. DOI: 10.1016/j.cma.2017.09.034.
17. **Leonetti L., Liguori F., Magisano D., Garcea G.** An efficient isogeometric solid-shell formulation for geometrically nonlinear analysis of elastic shells // Computer Methods in Applied Mechanics and Engineering. 2018. Vol. 331. pp. 159-183. DOI: 10.1016/j.cma.2017.11.025.
18. **Li W., Nguyen-Thanh N., Zhou K.** Geometrically nonlinear analysis of thin-shell structures based on an isogeometric-meshfree coupling approach // Computer Methods in Applied Mechanics and Engineering. 2018. Vol. 336. pp. 111-134. DOI: 10.1016/j.cma.2018.02.018.
19. **Konopatskiy E.V., Bezditnyi A.A.** Geometric modeling of multifactor processes and phenomena by the multidimensional parabolic interpolation method // Journal of Physics: Conf. Series 1441 (2020) 012063. DOI: 10.1088/1742-6596/1441/1/012063.
20. **Konopatskiy E.V., Bezditnyi A.A.** Application of mixed geometric interpolants for modeling the strength characteristics of steel fiber concrete // Journal of Physics: Conf. Series 1546 (2020) 012037. DOI: 10.1088/1742-6596/1546/1/012037.
21. An approach to comparing multidimensional geometric objects / I.V. Seleznev, E.V. Konopatskiy, O.S. Voronova [et al.] // Proc. of the 31st Int. Conf. on Computer Graphics and Vision (GraphiCon 2021), Nizhny Novgorod, September 27-30, 2021. Vol. 3027. pp. 682-688. DOI: 10.20948/graphicon-2021-3027-682-688.
22. **Chepur P.V.** Napryazhenno-deformirovannoe sostoyanie rezervuara pri razvitii neravnomernyh osadok ego osnovaniya [Stress-strain state of the tank during the development of non-uniform settlements of its base] // Moscow: RGU nefti i gaza (NIU) imeni I.M. Gubkina, 2015. 181 p. (In Russian).
23. **Konopatskiy E.V., Shevchuk O.A., Krysko A.A.** Modeling of the Stress-Strain State of Steel Tank with Geometric Imperfections // Construction of Unique Buildings and Structures. 2022. No. 2(100). P. 10001. DOI: 10.4123/CUBS.100.1. (In Russian).

СПИСОК ЛИТЕРАТУРЫ

1. **Aleshina O.O., Ivanov V.N., Cajamarca-Zuniga D.** Stress state analysis of an equal slope shell under uniformly distributed tangential load by different methods // Structural Mechanics of Engineering Constructions and Buildings. 2021. Vol. 17. No. 1. pp. 51-62. DOI: 10.22363/1815-5235-2021-17-1-51-62.
2. **Maraveas C., Balokas G.A., Tsavdaridis K.D.** Numerical evaluation on shell buckling of empty thin-walled steel tanks under wind load according to current american and european design codes // Thin-Walled Structures. 2015. Vol. 95. pp.152-160. DOI: 10.1016/j.tws.2015.07.007.
3. Behaviour of vertical cylindrical tank with local wall imperfections / A. Šapalas, G. Šaučiuvėnas, K. Rasiulis [et al.] // Journal of Civil Engineering and Management. 2019. 25(3). pp. 287-296. DOI: 10.3846/jcem.2019.9629.
4. **Gorban N.N., Vasiliev G.G., Salnikov A.P.** Accounting actual geometric shape of the tank shell when evaluating its fatigue life // Oil Industry. 2018. No. 8. pp. 75-79. DOI: 10.24887/0028-2448-2018-8-75-79.

5. **Крысько А.А.** Геометрическое и компьютерное моделирование эксплуатируемых конструкций тонкостенных оболочек инженерных сооружений с учётом несовершенств геометрической формы. – Макеевка: ДОННАСА, 2016. – 191 с.
6. **Крысько А.А.** Анализ напряженно-деформированного состояния стенки резервуара с геометрическими несовершенствами при действии гидростатической нагрузки // Металлические конструкции, 2017. – Т. 23. – № 3. – С. 97-106.
7. **Крысько А.А.** Численные исследования местных несовершенств геометрической формы вертикального цилиндрического // Строитель Донбасса, 2020. – № 1(10). – С. 13-17.
8. **Лессиг Е.Н., Лилеев А.Ф., Соколов А.Г.** Листовые металлические конструкции. – М.: Стройиздат, 1970. – 488 с.
9. **Тимошенко С.П., Войновский-Кригер С.** Пластинки и оболочки. – М.: Наука, 1966. – 636 с.
10. **Konopatskiy E.V., Voronova O.S., Shevchuk O.A., Bezditnyi A.A.** About one method of numeral decision of differential equalizations in partials using geometric interpolants // CEUR Workshop Proceedings. 2020. Vol. 2763. pp. 213-219. DOI: 10.30987/conferencearticle_5fce27708eb353.92843700.
11. **Konopatskiy E.V., Bezditnyi A.A., Shevchuk O.A.** Modeling geometric varieties with given differential characteristics and its application // CEUR Workshop Proceedings. 2020. Vol. 2744. DOI: 10.51130/graphicon-2020-2-4-31.
12. **Shamloofard M., Hosseinzadeh A., Movahhedy M.R.** Development of a shell superelement for large deformation and free vibration analysis of composite spherical shells // Engineering with Computers. 2021. Vol. 37. No. 4. pp. 3551-3567. DOI: 10.1007/s00366-020-01015-w.
13. **Hughes P.J., Kuether R.J.** Nonlinear interface reduction for time-domain analysis of Hurty/Craig-Bampton superelements with frictional contact // Journal of Sound and Vibration. 2021. Vol. 507. DOI: 10.1016/j.jsv.2021.116154.
14. **Nielsen M.B., Sahin E.** A simple procedure for embedding seismic loads in foundation superelements for combined wind, wave and seismic analysis of offshore wind turbine structures // Paper presented at the COMPDYN Proceedings. 2019. Vol. 3. pp. 4628-4640. DOI: 10.7712/120119.7255.19324.
15. Isogeometric analysis of large-deformation thin shells using RHT-splines for multiple-patch coupling / N. Nguyen-Thanh, K. Zhou, X. Zhuang [et al.] // Computer Methods in Applied Mechanics and Engineering. 2017. Vol. 316. pp. 1157-1178. DOI: 10.1016/j.cma.2016.12.002.
16. A NURBS-based inverse analysis for reconstruction of nonlinear deformations of thin shell structures / N. Vu-Bac, T.X. Duong, T. Lahmer [et al.] // Computer Methods in Applied Mechanics and Engineering. 2018. Vol. 331. pp. 427-455. DOI: 10.1016/j.cma.2017.09.034.
17. **Leonetti L., Liguori F., Magisano D., Garcea G.** An efficient isogeometric solid-shell formulation for geometrically nonlinear analysis of elastic shells // Computer Methods in Applied Mechanics and Engineering. 2018. Vol. 331. pp. 159-183. DOI: 10.1016/j.cma.2017.11.025.
18. **Li W., Nguyen-Thanh N., Zhou K.** Geometrically nonlinear analysis of thin-shell structures based on an isogeometric-meshfree coupling approach // Computer Methods in Applied Mechanics and Engineering. 2018. Vol. 336. pp. 111-134. DOI: 10.1016/j.cma.2018.02.018.
19. **Konopatskiy E.V., Bezditnyi A.A.** Geometric modeling of multifactor processes and phenomena by the

multidimensional parabolic interpolation method // Journal of Physics: Conf. Series 1441 (2020) 012063. DOI: 10.1088/1742-6596/1441/1/012063.

20. **Konopatskiy E.V., Bezditnyi A.A.** Application of mixed geometric interpolants for modeling the strength characteristics of steel fiber concrete // Journal of Physics: Conf. Series 1546 (2020) 012037. DOI: 10.1088/1742-6596/1546/1/012037.
21. An approach to comparing multidimensional geometric objects / I.V. Seleznev, E.V. Konopatskiy, O.S. Voronova [et al.] // Proc. of the 31st Int. Conf. on Computer Graphics and Vision (GraphiCon 2021), Nizhny Novgorod, September 27-30, 2021. Vol. 3027. pp. 682-688. DOI: 10.20948/graphicon-2021-3027-682-688.
22. **Чепур П.В.** Напряженно-деформированное состояние резервуара при развитии неравномерных осадок его основания. – М.: РГУ нефти и газа (НИУ) имени И.М. Губкина, 2015. – 181 с.
23. **Konopatskiy E.V., Shevchuk O.A., Krysko A.A.** Modeling of the Stress-Strain State of Steel Tank with Geometric Imperfections // Construction of Unique Buildings and Structures, 2022. – No. 2(100). – P. 10001. – DOI: 10.4123/CUBS.100.1.

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