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MATHEMATICAL MODELING OF UNDRAINED BEHAVIOR OF SOILS

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Abstract: Mathematical modeling of the undrained behavior of soils is carried out on the basis of theoretical paths of effective stresses under undrained deviatory loading in a triaxial compression chamber. The recommendations of normative sources and scientific papers on the application of undrained calculations in practice are analyzed. The basic laws of soil mechanics are considered in calculations taking into account the formation of excess pore pressures in the base. Theoretical calculations obtained by A. Skempton for the law of effective stresses of C. Terzaghi are applied for mathematical modeling of paths. Based on the results of mathematical modeling of an ideal elastic-plastic body it is shown that an accurate description of the paths of effective soil stresses using the elastic theory does not correspond to real soil tests. The influence of the law of volumetric plastic deformation on the paths of effective stresses and on the undrained shear strength is analyzed. The formula for determining the undrained strength parameter for the Modified Cam Clay model is presented. Attention is drawn to the fact that in addition to volumetric plastic deformation which affects the undrained calculation it is necessary to take into account the shear component of plastic deformation which is decisive for the calculations of excavations. Simulation of laboratory tests of soils in the Soil Test for the Mohr-Coulomb, Modified Cam Clay and Hardening Soil Models was carried out. A comparison of the obtained results with the data of laboratory tests is presented. The influence of the choice of soil model on the value of resistance to undrained shear is shown. Recommendations are given for choosing a soil model for numerical simulation based on the results of laboratory triaxial consolidated undrained tests.

Keywords: undrained behavior, Skempton's parameters, effective stress path, soft soil, soil model, cup loading surface

МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ НЕДРЕНИРОВАННОГО ПОВЕДЕНИЯ ГРУНТОВ

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Аннотация: Выполнено математическое моделирование недренированного поведения грунтов на основе теоретических траекторий эффективных напряжений при недренированном девиаторном загружении в камере трехосного сжатия. Проанализированы рекомендации нормативных источников и научных трудов о применении недренированных расчетов в практике. Рассмотрены основные законы механики грунтов при расчетах с учетом образования избыточных поровых давлений в основании. Для математического моделирования траекторий применены теоретические выкладки, полученные А. Скемптоном для закона эффективных напряжений К. Терцаги. На основе результатов математического моделирования идеально-упруго-пластического тела показано, что точное описание траектории эффективных напряжений грунта с помощью теории упругости не соответствует реальным испытаниям грунтов. Проанализировано влияние закона объемного пластического деформирования на траекторию эффективных напряжений и на сопротивление недренированному сдвигу. Представлена формула определения параметра недренированной прочности для модели Modified Cam Clay. Обращается внимание на то, что кроме объемного пластического деформирования, влияющего на недренированный расчет, необходимо учитывать сдвиговую составляющую пластических деформаций, являющуюся определяющей для расчетов котлованов. Выполнено моделирование лабораторных испытаний грунтов в ПК «Soil Test» для моделей Мора-Кулона, Modified Cam Clay и Hardening Soil Model. Представлено сравнение полученных результатов с данными лабораторных испытаний. Показано влияние выбора модели грунта на значение сопротивления недренированному сдвигу. Даны рекомендации для выбора модели грунта для численного моделирования на основании результатов лабораторных трехосных консолидированно-недренированных испытаний.

Ключевые слова: недренированное поведение, параметры Скемптона, траектория эффективных напряжений, слабый грунт, модель грунта, шатровая поверхность нагружения

1. INTRODUCTION

The construction of deep pits in cities has become widespread over the past twenty years. To avoid significant margins occurring in assessment of the bearing capacity of the foundation, it is necessary to increase the accuracy of the geotechnical forecast.

First of all, this problem arises at the construction of underground structures on a large thickness of soft soils, which are typical for such a metropolis as St. Petersburg [1].

The geological section of St. Petersburg is represented by a large stratum of Quaternary deposits in the form of water-saturated, ribbon, silty-clay soils and water-saturated sands overlying a variable stratum of moraine deposits underlain by Upper Kotlin and Lower Cambrian clays as Fig. 1 presents [2].

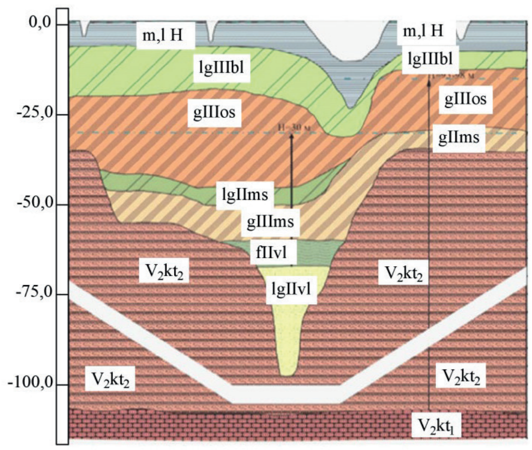

Figure 1. Geological section of the central part of St. Petersburg [2]

Therefore, “soft” glaciolacustrine deposits (lg III) widespread in this area requires a special approach to analysis of foundation pits in St. Petersburg. These deposits have low strength and deformation characteristics. At the same time, they are the main part of the active zone of the base of foundations and structures under construction. This circumstance entails the need to use expensive solutions in the construction of pits and their enclosures.

Figure 2 presents a typical design scheme for the construction of a pit in St. Petersburg. Since lacustrine-glacial deposits, as a rule, are in the active zone of the foundation, the choice of a soil model significantly affects to determination of the forces and displacements of the enclosing structures and additional deformations of the foundation foundations of the surrounding buildings.

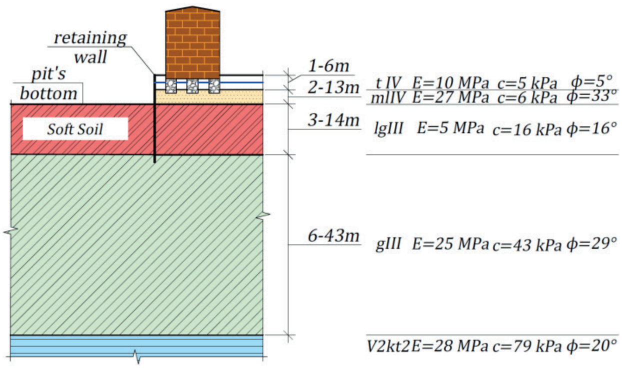

Figure 2. Design scheme for the construction of a pit in St. Petersburg

In modern practice of designing excavation enclosures, the Hardening Soil model is usually used for simulation. However, the mathematical description of this model contains many empirical relations, the validity of which for soft glaciolacustrine deposits of St. Petersburg is practically not confirmed.

On another hand, Models of the Cam-Clay family (Cam-Clay, Modified Cam Clay, Soft Soil, etc.) are widely used in world practice. However, the laws embedded in these models were originally obtained for volumetric compression, which can greatly distort the results for shear deformation. Thus, it is important to pay attention to the limits of its applicability [3], when using any model.

As initial data for models, mechanical characteristics obtained from test schemes that do not correspond to the actual work of the soil or the requirements of the model are often assigned. So, the Hardening Soil model requires triaxial compression tests, and if it is necessary to take into account excess pore pressures. Besides, test results in an undrained mode are required.

Since the process of excavation and installation of underground structures is limited in time, and soft clay soils have rather low filtration coefficients, it makes sense to consider the work of clay soils, considering the possibility of excessive pore pressure.

This study was aimed to develop a computational method for determining the parameters of undrained behavior of soil that is required for numerical simulation of pits in conditions of soft water-saturated soils.

Studies of the undrained behavior of clay soils began with consideration of the behavior of the soil as an ideal elastic-plastic body [4]. This description of the behavior of the soil was based only on the theory of elasticity and did not correlate with the test results. Then models were introduced to take into account the nonlinear deformation and "hardening" of soils, which directly affects the behavior of the pore fluid [5], [6], [7], [8]. Mathematical relations embedded in these models were built on the basis of a series of soil tests.

Proving of the applicability of models for specific soil conditions requires an assessment of the influence of the input soil parameters on the mathematical relations embedded in different models. In the case under consideration, the applicability criterion can be the path of effective stresses under undrained loading or the intersection of the path with the line of the limiting state (the value of undrained shear strength).

In foreign practice, analysis of pits in clay soils is allowed to be carried out both in drained and undrained settings [9]. At the analysis of excavations in clay soils, the decision to calculate excess pore pressures can be made based on the time factor, which depends on the degree of soil consolidation and construction time [10]:

$$T = \frac{c_v}{D^2} t = \frac{k_f \cdot E_{oed}}{\gamma_w \cdot D^2} t \quad (1)$$

where c_v is the factor of primary (filtration) consolidation;

D is the filtration path,

t is the construction time;

k_f is the filtration coefficient;

E_{oed} is the odometric modulus of deformation;

γ_w is the specific gravity of water.

For the values of the time factor $T \geq 0.4$, it is recommended not consider excess pore pressures and to carry out a drained analysis in effective stresses. If $T \leq 0.01$, then it is recommended to take into account excess pore pressures and conduct an undrained analysis. For intermediate values of the time factor, it is necessary to analyze both the drained and undrained behavior of the soil mass during the excavation.

A number of literary sources recommend do not dwell on only the analysis of the undrained behavior, since in this case results can be obtained that do not consider the filtration and consolidation processes occurring in the base over time [11], [12].

Based on the previously stated in this paper, the following tasks were considered during the analysis of mathematical modeling of the undrained behavior of soils:

- study of the path of effective stresses under undrained deviatoric loading for an ideal-elastic-plastic body;
- determination of the influence of the shape of the loading surface on the path of effective stresses;
- study of the path of effective stresses for the most common non-linear soil models;
- determination of criteria for applicability of models in geotechnical calculations.

2. METHODS

2.1. Basic laws of soil mechanics in undrained analysis

In classical soil mechanics, it is customary to consider soil as a multiphase dispersed system.
In the simplest setting, the soil consists of solid particles and a liquid that fills the pores. Such a formulation can be represented as the Terzaghi law of effective stresses:

$$\sigma = \sigma' + p_{active} \quad (2)$$

where σ is the total stresses in the soil mass;
σ' is the stress in the soil skeleton (effective);
p_{active} is the stress in the pore fluid (pore pressure).
In turn, the pore pressure can be separated into hydrostatic one, which is determined by the position of the groundwater level, and excess one, which is formed under loading in the absence of drainage:

$$p_{active} = p_{steady} + p_{excess} \quad (3)$$

where p_{steady} is the steady pore pressure;
p_{excess} – избыточное поровое давление.
Since the main strength criterion for soils as materials subject to plastic flow is shear strength, it is important to consider the effect of excess pore pressure on the stress distribution in the soil.
In accordance with (2), the effect of pore pressure on soil shear strength can be shown using Coulomb's law for the failure site, denoted by *n*:

$$\tau_n = \sigma_n' \cdot \tan(\phi') + c' = (\sigma_n - p_{active})\tan(\phi') + c' \quad (4)$$

where τ_n is the tangent stress acting on fracture area;
σ_n is the normal stress acting on the fracture area.
It is convenient to analyze the soil behavior for different stress-strain states using principal stresses.

In principal stresses, Coulomb's law can be represented as follows:

$$\sin(\phi') = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3' + 2 \cdot c' \cdot \cot(\phi')} \quad (5)$$

where $\sigma_1{}'$ is the largest principal effective stress;
$\sigma_3{}'$ is the least principal effective stress.
An analysis is carried out in total stresses, when it is not possible to assess the effect of pore pressure on soil stresses.
When analyzed in total stresses, the law of shear resistance looks like this:

$$\tau_{\max} = C_u \quad (6)$$

where $\tau_{\max} = \dfrac{\sigma_1 - \sigma_3}{2}$ is the maximum tangent stress;
C_u is the undrained shear resistance (strength).
Figure 3 presents graphs illustrating the law of soil shear resistance. Stress components $\sigma_n^{'}, \tau_n$, presented in formula (5), correspond to the combination of effective principal stresses at the failure site (on the Mohr-Coulomb line). For total stresses, based on formula (6), only the value C_u is known, which is independent of the level of stresses in the soil.

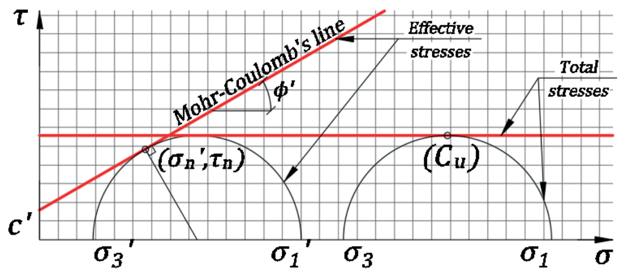

Figure 3. Coulomb's law for soils in effective and total stresses in $\sigma - \tau$ axes

In foreign practice, analysis in "p-q" coordinates is used to assess the stress state of the soil. This allows visually evaluate the "path" of effective and total stresses in the soil sample.

The mean effective stress p` and the stress deviator q are expressed in terms of the principal stresses in triaxial compression as follows:

$$p' = p - p_{active} = \frac{\sigma_1' + 2\sigma_3'}{3} \tag{7}$$

$$q = q' = \sigma_1' - \sigma_3' \tag{8}$$

Representation the Mohr-Coulomb law (5) in "p-q" coordinates:

$$q = \frac{6\sin(\phi')}{3-\sin(\phi')}p' + \frac{6 \cdot c'\cos(\phi')}{3-\sin(\phi')} = Mp' + a \tag{9}$$

where $M = \frac{6\sin(\phi')}{3-\sin(\phi')}$ and $a = \frac{6c'\cos(\phi')}{3-\sin(\phi')}$ are the parameters of the critical state line, depending on the angle of internal friction and specific cohesion.

Figure 4 presents a graph illustrating the Mohr-Coulomb law in "p-q" coordinates.

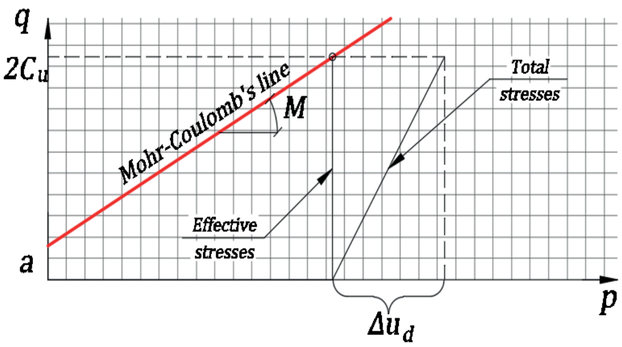

Figure 4. Coulomb's law for soils in effective and total stresses in coordinates $p-q$

2.2. Theoretical path of undrained deviatoric loading for an ideal elastic-plastic body

The path of undrained loading for an ideal elastic-plastic body was described by Skempton [4].

To take into account the distribution of the load between the pore fluid and the soil skeleton in the space of principal stresses during tests of triaxial isotropic compression, Terzaghi's law can be represented as:

$$\Delta p_{active} = B(\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)) \tag{10}$$

where A and B are the Skempton coefficients, the derivation of which is presented below.

Figure 5 provides the scheme of an idealized triaxial test.

The change in pore pressure at the stage of all-round compression is denoted as Δu_a, at the stage of deviatoric loading as Δu_d.

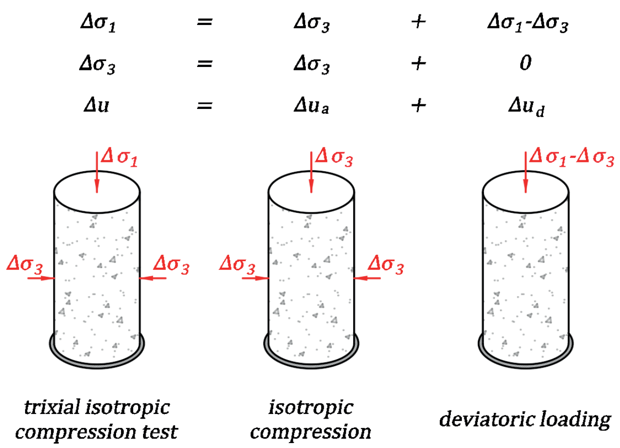

Figure 5. Schemes of idealized triaxial tests

The stage of comprehensive compression is characterized by hydrostatic uniform compression of the soil sample by pressure to restore natural stresses in the soil sample. Let us express the effective stresses in terms of Terzaghi's law:

$$\Delta\sigma_1' = \Delta\sigma_3' = \Delta\sigma_3 - \Delta u_a \tag{11}$$

In accordance with Hooke's law, the relative volumetric deformations of the soil skeleton can be obtained from the following relationship:

$$\varepsilon_V{}^c = \frac{\Delta V}{V} = C_c \cdot \Delta p' \tag{12}$$

where ΔV is the change in sample volume;
V is the initial sample volume;
C_c is the compressibility of the soil skeleton;

Let us represent Hooke's law for the volumetric compression of the pore fluid:

$$\varepsilon_V{}^w = \frac{\Delta V}{V} = C_w \cdot n \cdot \Delta u_a \qquad (13)$$

where C_w is the compressibility of the pore fluid; n is the porosity of soil.
For the stage of comprehensive compression of the soil, taking into account expressions (11) and (12), it was obtained:

$$\Delta V = C_c \cdot V(\Delta\sigma_3 - \Delta u_a) \qquad (14)$$

Assuming that the change in the volume of the soil sample occurs only due to the change in the volume of its pores, expressions (13) and (14) can be equated:

$$\frac{\Delta u_a}{\Delta\sigma_3} = \frac{1}{1+\dfrac{C_w \cdot n}{C_c}} = B \qquad (15)$$

For a completely water-saturated soil, in which there are no gas inclusions in the pore space, the Skempton parameter is $B = 1$.
The Skempton coefficient B depends on the degree of soil consolidation. The coefficient is close to 1, when evaluating the instantaneous strength, i.e., for time 0. This means that the entire mean stress was accepted up by the pore fluid. For a fully drained analysis, the B factor is 0.
At the stage of deviatoric loading, the sample is destroyed due to the development of shear deformations. Using Terzaghi's law, the effective principal stresses are expressed as follows:

$$\Delta\sigma_1' = \Delta q - \Delta u_d \qquad (16)$$

$$\Delta\sigma_3' = -\Delta u_d \qquad (17)$$

Taking into account expressions (16) and (17), the Hooke's law for the stage of deviatoric loading has the form:

$$\Delta V = C_c \cdot V \cdot \frac{1}{3} \cdot (\Delta q - 3\Delta u_d) \qquad (18)$$

Let us represent Hooke's law for volumetric deformations of the pore fluid at the stage of deviatoric loading:

$$\Delta V = C_w \cdot n \cdot V \cdot \Delta u_d \qquad (19)$$

Assuming that the change in the volume of the soil sample occurs only due to the change in the volume of its pores, expressions (18) and (19) can be equated:

$$\frac{\Delta u_d}{\Delta q} = B \cdot \frac{1}{3} = B \cdot A \qquad (20)$$

Then the change in pore pressure at the stage of deviatoric loading can be represented as follows:

$$\Delta u_d = B \cdot A \cdot \Delta q \qquad (21)$$

The Skempton coefficient A determines the excess pore pressure under undrained deviatoric loading.
Since the change in pore pressure during testing is the sum of the change at the stage of all-round compression and at the stage of deviatoric loading, it is possible to obtain the effective stress law of Terzagi, substituting expressions (15) and (21), and considering the Skempton coefficients for principal stresses (10).
The Skempton coefficient *A* shows what part of the load during deviatoric loading (shear) is transferred to the pore fluid. Since soil destruction occurs precisely at the stage of deviatoric loading, for an undrained state it is possible to determine the value of the Skempton coefficient *A* at which destruction will occur. This value of the coefficient is denoted as A_f. This coefficient is fundamental in assessing the momentary stability of prefabricated structures (for example, road embankments) [13].
From expression (20), it follows that for the ideal elastic-plastic behavior of the soil, the value of the coefficient $A_f = \frac{1}{3}$.
Figure 6 presents the stress paths for a triaxial consolidated undrained test. For loading

presented in fig. 5, the path of effective stresses should first represent segment OA (comprehensive compression). Then segment AC (deviator loading). The path of total stresses under deviatoric loading is segment AD.

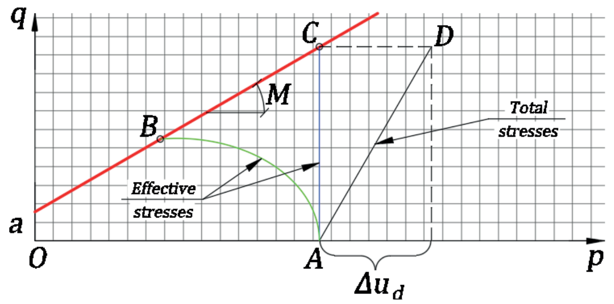

Figure 6. Stress paths: OA - path at the stage of all-round compression; AD - path in full stresses at the stage of deviatoric loading; AC is the theoretical path of effective stresses for an ideal-elastic-plastic body; AB - path obtained from experimental data for normally consolidated soils

2.3. Simulation of Volumetric Plastic Deformation Functions

As noted above, the volumetric deformation of a completely water-saturated soil is due solely to the compressibility of the liquid when it is impossible to squeeze out water from the pores. Thus, it is acceptable to assume that in an undrained triaxial test, the volume of the soil sample remains constant.

In this case, the curve q(p') of the path of undrained deviatoric loading should be an isoline of zero volumetric strains.

Volumetric deformations of the soil mass can be decomposed into two components:

$$\varepsilon_V = \varepsilon_V^e + \varepsilon_V^p \tag{22}$$

where ε_v^e is the elastic component of volumetric deformations;

ε_v^p is the plastic component of volumetric deformations.

With a consolidated undrained test, it is true:

$$\varepsilon_V = \varepsilon_V^e + \varepsilon_V^p \cong 0 \tag{23}$$

The isoline of equal plastic volumetric deformations in accordance with the concept of critical state (CSSM - critical state soil mechanics) can be represented as a loading surface or a “cup” (a surface that limits the zone of a quasi-elastic state under loading) [14].

To analyze the undrained behavior in different soil models, consider the load surfaces in the Cam Clay, Modified Cam Clay, Hardening Soil models. All these models look like cup. The description of the functions of the "cup" is presented in Table. 1.

Table 1. Functions of the loading surface under volume compression

Model	Yield surface for triaxial compression
Cam Clay	$f(p',q) = q + M \cdot p' \cdot \ln(p'/p_p)$
Modified Cam Clay	$f(p',q) = q^2/M^2 + p' \cdot (p' - p_p)$
Hardening Soil	$f(p',q) = q^2/\alpha^2 + p'^2 - p_p^2$

where p_p is the pre-consolidation pressure (represented on the graphs as the point of intersection of the cup and the abscissa axis);

α is the internal parameter of the Hardening Soil model, which is responsible for the shape of the "cup".

Figure 7 presents graphical view of the cup loading surfaces according to the dependencies from Table 1 obtained by the authors in the PC "MathCAD 15".

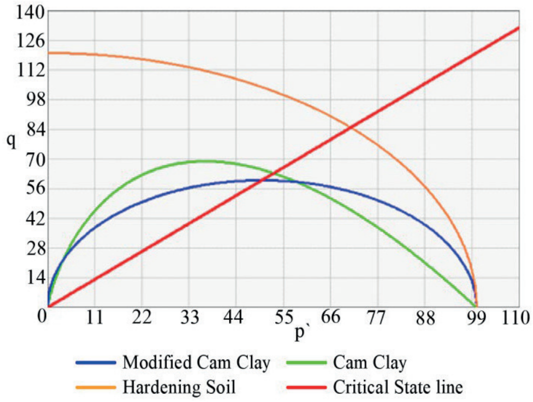

Figure 7. Cup loading surfaces.

2.4. Simulation of the path of undrained deviatoric loading for the Modified Cam Clay model

The Modified Cam Clay model was developed based on the Cam Clay model by Burland [7], [8]. Both models are a consequence of critical state soil mechanics, which is based on the assumption that soils continuously deform under load until a critical state occurs. Then the soil begins to behave like a fluid with a constant internal friction angle and a constant porosity coefficient. Cam Clay served as the basis for further modifications of cup models.

The basic concept of the model is the logarithmic relationship between the mean effective stress and the porosity coefficient, as well as the introduced yield surface, which limits the elastic deformation of the soil.

Figure 8 shows a graphical display of the change in the loading surface with a change in the porosity coefficient in coordinates obtained by the authors in the MathCAD 15 PC.

To determine the path of the effective stresses of the "cup" model, it is necessary to obtain the point of intersection of the path with the line of the critical state. The following mathematical transformations are required for this purpose.

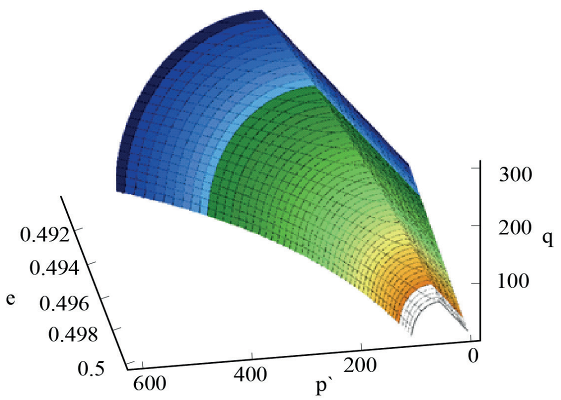

Figure 8. Load Surface for Modified Cam Clay Model

The equation of the cup loading surface is as follows:

$$\left(\frac{q}{M \cdot p'}\right)^2 - \frac{p_p}{p'} + 1 = 0 \qquad (24)$$

The Coulomb-Mohr fracture surface is represented by the equation:

$$q = M \cdot p' \qquad (25)$$

Substituting expression (25) into (24), the intersection point of the "cup" and the destruction surface is found as follows:

$$\left(\frac{Mp'}{Mp'}\right)^2 - \frac{p_p}{p'} + 1 = 0 \rightarrow p_p = 2p_f^{`} \qquad (26)$$

where $p_f^{`}$ is the mean effective stress at failure.

To find the path of undrained deviatoric loading, the terms in equation (23) were written. Based on the logarithmic law of model deformation, the elastic component was written as:

$$\varepsilon_v^e = \kappa^* \cdot \ln(p'/p_0) \qquad (27)$$

where p_0 is the mean effective stress in natural state.

κ^* is the modified recompression ratio:

$$\kappa^* = \frac{\kappa}{1 + e_0} \qquad (28)$$

where κ is the recompression ratio;

e_0 is the soil porosity coefficient in natural state.

The plastic component is represented as the difference between the total deformation and the elastic deformation:

$$\varepsilon_v^p = (\lambda^* - \kappa^*) \cdot \ln(p_p/p_0) \qquad (29)$$

where λ^* is the modified compression ratio:

$$\lambda^* = \lambda/(1 + e_0) \qquad (30)$$

where λ is the compression ratio.

Then expression (23) can be represented as:

$$\kappa \cdot \ln(\frac{p'}{p_0}) = -(\lambda - \kappa) \cdot \ln(\frac{p_p}{p_0}) \quad (31)$$

Substituting expression (26) into expression (31) and expressing p_p, the following relation was obtained:

$$p_f^{`} = 2^{\frac{\kappa-\lambda}{\lambda}} \cdot p_0 = 2^{\Lambda} \cdot p_0 \quad (32)$$

where $\Lambda = (\kappa - \lambda)/\lambda$ is the soil deformability parameter.

The value of the limiting stress deviator can be represented using expression (32) as:

$$q_f = C_u/2 = M \cdot p_f^{`} = M \cdot 2^{\Lambda} \cdot p_0 \quad (33)$$

Then the path can be represented as an ellipse passing through the origin, p_p and $q(p_f^{`})$.

Figure 9 shows the path of the undrained loading of the Modified Cam Clay model, obtained by the authors in the MathCAD 15 PC.

The value of the Skempton coefficient A_f at fracture can be determined from the following relationship:

$$A_f = \frac{\Delta u_d}{\Delta q} = \frac{p_p}{M \cdot 2^{\Lambda} \cdot p_0} + \frac{1}{3} - \frac{1}{M} \quad (34)$$

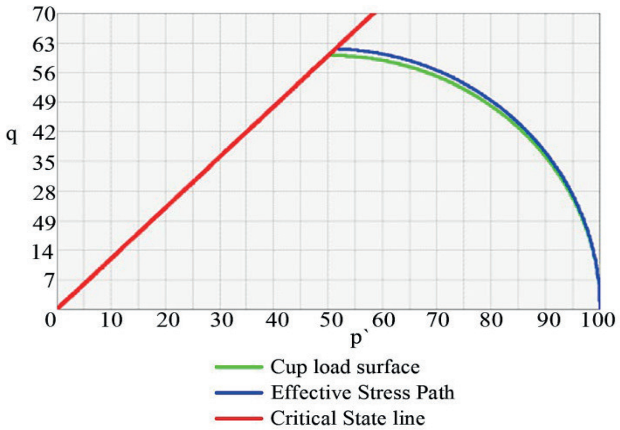

Figure 9. Path of undrained deviatoric loading in the Modified Cam Clay model

2.5. Modeling the path of undrained deviatoric loading during virtual soil testing

For a visual comparison of the work of specific soil models with laboratory experiments, the Soil Test module for the Plaxis PC was used.

Let's perform virtual consolidated-undrained triaxial soil tests in the Modified Cam Clay model. The pressure of triaxial compression has been accepted equal to $p_0 = 100kPa$.

The tests for soil with similar mechanical parameters presented in Table 2 were carried out. Triaxial compression pressure during the test was $100kPa$. The results of the comparison are shown in fig. 10.

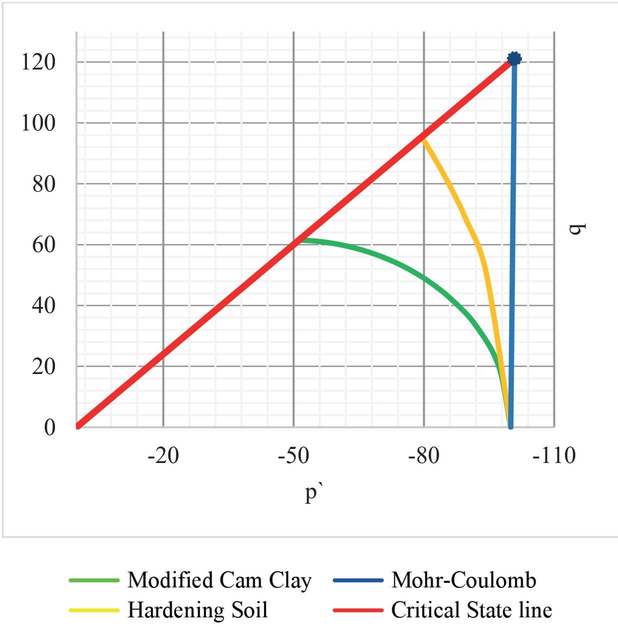

Figure 10. Paths of effective stresses under undrained loading

Table 2. Soil parameters for models

Model	Strength parameters	Deformability parameters
Mohr-Coulomb	$\phi' = 30^o, c' = 0$	$E' = 30MPa$
Modified Cam Clay	$M = 1.2$	$\lambda = 0.005, \kappa = 0.0018$
Hardenin g Soil	$\phi' = 30^o, c' = 0$	$E_{oed}^{ref} = 30MPa, E_{50}^{ref} = 30MPa, E_{ur}^{ref} = 150MPa, m = 0.5, p_{ref} = 0.1MPa$

3. RESULTS AND DISCUSSION

The authors performed mathematical modeling of paths of undrained deviatoric loading for:

- ideal elastic-plastic body (Mohr-Coulomb Model);
- Modified Cam Clay models.

Also, during virtual triaxial tests in the Soil Test module, paths were built for the Mohr-Coulomb, Modified Cam Clay, Hardeming Soil models.

Mathematical modeling of an ideal elastic-plastic material shows that with an undrained deviatoric loading, there is no deviation of the loading path from the vertical. In this case, one can observe a deviation of the path with a decrease in the mean effective stress [15], [16] in real samples of normally consolidated soil. For example, figure 10 presents the effective stress paths for an undrained triaxial test of kaolin clays.

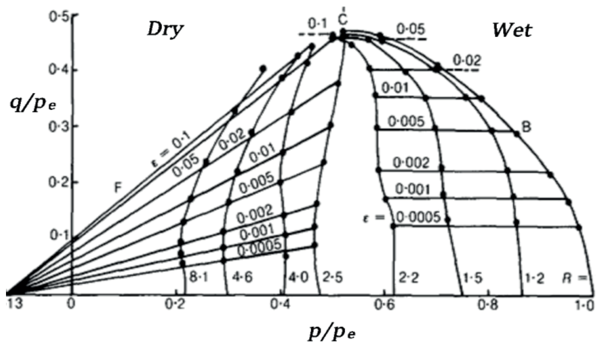

Figure 11. Effective Stress Paths for an Undrained Triaxial Test on Kaolin Clays [17]

At the same time, the graph clearly shows the relationship between the curvature of the stress path and the degree of overconsolidation of the soil.

In addition, the Modified Cam Clay model was considered. Comparison of the results of mathematical modeling with the behavior of the Modified Cam Clay model in the Soil Test shows the complete convergence of the analytical loading path with the test results. This indicates the correctness of the mathematical modeling. Figure 12 presents the results of the comparison. From the point of view of mathematical modeling, the path of undrained deviatoric loading depends on the shape of the cup and on how the elastic and plastic components of deformations were described. Thus, if the shape of the yield surface, the law of plastic deformation (associated or non-associated) and the relationship between stresses and strains are known, one can obtain the path of effective stresses.

The Modified Cam Clay model describes the relationship between volumetric strains and mean stress, using an elliptical yield surface and the associated plastic flow law. This means that the shear deformations will depend only on the calculated volumetric ones, which can greatly distort the deformation pattern when shear deformation prevails (for example, when analyzing pits).

The mathematical proven for the Cam Clay model is similar to the Modified Cam Clay model, except for the different "cup" functions. The Cam Clay model was not considered in this article due to its absence in numerical simulation software packages. This is due to the impossibility of differentiating the function of the yield surface of the model at one point.

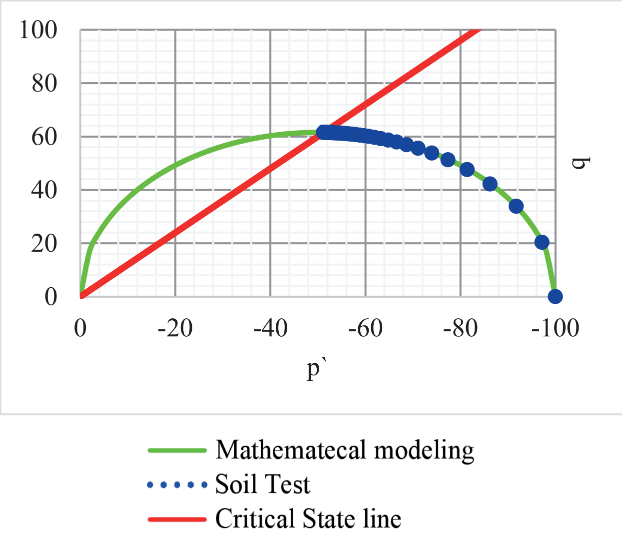

Figure 12. Comparison of analytical calculation results with Soil Test results

Mathematical modeling of the Hardening Soil model is hampered by the lack of an unambiguous definition of the parameters (H is the model hardening parameter, α is the "cup" shape parameter) [3], which do not allow to reproduce the path of undrained deviatoric loading analytically.

Virtual soil tests in the Soil Test complex allow you to evaluate the performance of various soil models with similar characteristics.

Table provides the values of the coefficients A_f calculated for different soil models.

Table 1. Coefficient Values of the coefficient A_f for different soil models

Soil model	Coefficient A_f
Mohr-Coulomb	0,33
Hardening Soil	0,55
Modified Cam Clay	1,13

Depending on the chosen model, the description of plastic deformation is fundamentally different. This affects to the path of deviatoric loading and the resulting resistance to undrained shear.

Considering the curvature of the path, the value of the coefficient A, mathematically derived by Skempton, is no longer a constant value (as for ideal elastic-plastic behavior), but a function. Specific failure values A_f for different types of soils were obtained by Skempton during a series of triaxial consolidated-undrained tests of some types of soils [4] (Table 4).

For numerical simulation of problems that consider the undrained behavior of soils, it is necessary either to introduce the resistance to undrained shear directly, or to obtain a "true" effective stress path.

After the appearance of cup models, the path of undrained deviatoric soil loading became mathematically tied to the function of the cup [18]. Numerical modeling of undrained behavior of soils requires consideration of specific soil models.

Table 2. Values of the coefficient A_f [4]

Type of clay soil	Values of the coefficient A_f
Clays with high sensitivity	*0.75 to 1.5*
Normally consolidated clays	*0.5 to 1*
Compacted sandy clays	*0.25 to 0.75*
Lightly over-consolidated clays	*0 to 0.5*
Compacted clay-gravels	*-0.25 to 0.25*
Heavily over-consolidated clays	*-0.5 to 0*

Thus, "true" path of effective stresses under undrained deviatoric loading can be obtained when conducting consolidated-undrained triaxial soil tests at the stage of engineering surveys. The designer can obtain specific values A_f for soils analyzing path data. Knowing the values of the Skempton coefficient obtained from laboratory tests and the theoretical values of A_f, one can choose a soil model that adequately describes the undrained behavior.

The correct choice of the soil model will allow not to overestimate (which can lead to loss of stability of the excavation wall) and not to underestimate the resistance to undrained shear (which can lead to excessive reserves and, accordingly, an increase in the cost of design solutions).

In known soil models, undrained shear resistance C_u is affected by over consolidation, stiffness, and effective strength parameters. This can be represented as a dependency of the form:

$$\frac{c_u}{\sigma'_{v0}} = f(\phi', OCR, C_c, C_s) \quad (35)$$

where σ'_{v0} is the effective vertical stress;

OCR is the soil consolidation factor;

C_s is the elastic component of soil skeleton compressibility.

To obtain correct data on soil compaction, stiffness and undrained strength, it is necessary to conduct laboratory and field tests using appropriate correlations and available geotechnical testing experience in the region [19], [20].

For the soil conditions of St. Petersburg, a special viscoelastic-plastic model with an independent hardening mechanism was developed. Figure 13 presents the model construction scheme.

This model does not belong to the "cup" models. The construction of the loading surfaces was carried out directly based on the results of testing the soils of St. Petersburg.

The deviation of isolines of zero volumetric plastic deformations is explained by the dilatancy phenomenon and, at the same time, it is mentioned that the isolines are approximated by a straight vertical line, referring to the lack of knowledge of the dilatancy phenomenon in soft clay soils of St. Petersburg [6].

The different behavior of soil models under undrained deviatoric loading makes the choice of the model especially important when calculating excavations in an undrained setting. For further comparison, it is advisable to consider specialized models for shale calculations in an undrained formulation under various loading paths, for example, NGI-ADP.

4. CONCLUSION

An accurate description of the path of effective soil stresses using the theory of elasticity does not correspond to real soil tests. This circumstance limits the use of ideal elastic-plastic models in undrained calculations.

Using similar initial parameters in various nonlinear soil models, one can obtain fundamentally different functions of plastic volumetric deformation and, as a result, paths of undrained deviatoric loading. This leads to completely different Skempton coefficient A_f and undrained shear strength values.

An analytical method for determining the Skempton coefficient A_f for the Modified Cam Clay model has been presented in the article;

The paper provides recommendations for assigning a soil model for undrained calculations based on the Skempton coefficient A_f, which can be obtained by processing the results of consolidated undrained triaxial soil tests;

The obtained solutions allow more accurate designing the pits of great depth in soft water-saturated soils.

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