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MODELING OF SEISMIC WAVES STRESSES IN A HALF-PLANE WITH A VERTICAL CAVITY FILLED WITH WATER (THE RATIO OF WIDTH TO HEIGHT IS ONE TO TEN)

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Abstract. The problem of mathematical modeling of unsteady seismic waves in an elastic half-plane with a vertical rectangular cavity filled with water is considered. The problem of modeling problems of the transition period is an actual scientific problem. A quasi-regular approach is proposed to solve a system of linear ordinary differential equations of the second order in displacements with initial conditions and to approximate the region under study. The method is based on the schemes: a point, a line and a plane. An algorithm and a set of programs for solving flat (two-dimensional) problems that allow obtaining a stress-strain state in complex objects have been developed. To assess the reliability of the developed methodology, algorithm and software package, the problem of the effect of a plane longitudinal wave in the form of a Heaviside function on an elastic half-plane was solved. The numerical solution corresponds quantitatively to the analytical solution. The problem of mathematical modeling of unsteady elastic stress waves in a half-plane with a cavity filled with water (the ratio of width to height is one to ten) under seismic influence is solved. A system of equations consisting of 8016008 unknowns is solved. Contour stresses and components of the stress tensor are obtained in the characteristic areas of the problem under study. A cavity filled with water, with a width-to-height ratio of one to ten, reduces the amount of elastic contour stress.

Keywords: wave theory of seismic safety, wave propagation, elastic half-plane, Heaviside function, vertical rectangular cavity, water medium, contour stresses

МОДЕЛИРОВАНИЕ СЕЙСМИЧЕСКИХ ВОЛН НАПРЯЖЕНИЙ В ПОЛУПЛОСКОСТИ С ВЕРТИКАЛЬНОЙ ПОЛОСТЬЮ ЗАПОЛНЕННОЙ ВОДОЙ (СООТНОШЕНИЕ ШИРИНЫ К ВЫСОТЕ ОДИН К ДЕСЯТИ)

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Аннотация. Рассматривается задача о математическом моделировании нестационарных сейсмических волн в упругой полуплоскости с вертикальной прямоугольной полостью, заполненной водой. Проблема моделирования задач переходного периода является актуальной научной задачей. Предложен квазирегулярный подход к решению системы линейных обыкновенных дифференциальных уравнений второго порядка в перемещениях с начальными условиями и к аппроксимации исследуемой области. Методика основывается на схемах: точка, линия и плоскость. Разработаны алгоритм и комплекс программ для решения плоских (двумерных) задач, которые позволяют получать напряженно-деформированное состояние в сложных объектах. Для оценки достоверности разработанной методики, алгоритма и комплекса программ была решена задача о воздействии плоской продольной волны в виде функции Хевисайда на упругую полуплоскость. Численное решение количественно соответствует аналитическому решению. Решена задача о математическом моделировании нестационарных упругих волн напряжений в полуплоскости с полостью заполненной водой (соотношение ширины к высоте один к десяти) при сейсмическом воздействии. Решается система уравнений из 8016008 неизвестных. В характерных областях исследуемой задачи получены контурные напряжения и компоненты тензора напряжений. Полость заполненной водой, с соотношением ширины к высоте один к десяти, уменьшает величину упругого контурного напряжения.

Ключевые слова: волновая теория сейсмической безопасности, распространение волн, упругая полуплоскость, функция Хевисайда, вертикальная прямоугольная полость, водная среда, контурные напряжения

1. STATEMENT OF THE PROBLEM OF NONSTATIONARY WAVE EFFECTS IN DEFORMABLE BODIES

Unsteady elastic stress waves propagating in a deformable body interact with each other [1–8, 15–16, 18–29, 31].

After several passes and reflection of stress waves in the body, the process of propagation of disturbances becomes steady, the body is in oscillatory motion [1–8, 15–16, 18–29, 31].

The formulation of some problems of deformable solid mechanics is given in the following works [1–31].

In [9–13], some information is given about the formulation, analysis and technology for developing optimal algorithms for numerical modeling of structural mechanics problems.

The application of the considered numerical method, algorithm and software package for solving non-stationary wave problems in deformable bodies is given in the following works [7–8, 18 –29, 31].

Verification (evaluation of accuracy and reliability) of the considered numerical method, algorithm and software package is given in the following works [7–8, 18–21, 23–29, 31].

To solve the problem of modeling elastic unsteady stress waves in deformable regions of complex shape, we consider a certain body Γ in a rectangular cartesian coordinate system XOY, to which at the initial moment of time $t = 0$ a mechanical non-stationary pulse effect is reported [1, 3–5, 7–8, 18–19].

Suppose that a certain body Γ is made of a homogeneous isotropic material obeying the elastic Hooke law for small elastic deformations [1, 3–5, 7–8, 18–19].

The exact equations of the two-dimensional (plane stress state) dynamic theory of elasticity have the following form [1, 3–5, 7–8, 18–19]

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = \rho \frac{\partial^2 u}{\partial t^2},$$

$$\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2}, \ (x, y) \in \Gamma,$$

$$\sigma_x = \rho C_p^2 \varepsilon_x + \rho (C_p^2 - 2C_s^2) \varepsilon_y,$$

$$\sigma_y = \rho C_p^2 \varepsilon_y + \rho (C_p^2 - 2C_s^2) \varepsilon_x,$$

$$\tau_{xy} = \rho C_s^2 \gamma_{xy},$$

$$\varepsilon_x = \frac{\partial u}{\partial x}, \ \varepsilon_y = \frac{\partial v}{\partial y},$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \ (x, y) \in (\Gamma \cup S), \quad (1)$$

where: σ_x, σ_y и τ_{xy} – components of the elastic stress tensor; ε_x, ε_y и γ_{xy} – components of the elastic strain tensor; u and v – components of the vector of elastic displacements along the axes OX and OY accordingly; ρ – material density; $C_p = \sqrt{\frac{E}{\rho(1-\nu^2)}}$ – the velocity of the longitudinal elastic wave; $C_s = \sqrt{\frac{E}{2\rho(1+\nu)}}$ – the velocity of the transverse elastic wave; ν – Poisson's ratio; E – modulus of elasticity; S $(S_1 \bigcup S_2)$ – the boundary contour of the body Γ.

System (1) in the area occupied by the body Γ, should integrate under initial and boundary conditions [1, 3–5, 7–8, 18–19].

2. DEVELOPMENT OF THE METHODOLOGY AND ALGORITHM

To solve a two-dimensional plane dynamic problem of the theory of elasticity with initial and boundary conditions (1), we use the finite element method in displacements [7–8, 19]. The problem is solved by the method of end-to-end counting, without highlighting gaps [7–8, 18–19].

The main relations of the finite element method are obtained using the principle of possible displacements [7–8, 18–19].

Taking into account the definition of the stiffness matrix, the inertia vector and the vector of external forces for the body Γ, we write down the approximate value of the equation of motion in the theory of elasticity [7–8, 18–19]

$$\overline{H}\ddot{\vec{\Phi}} + \overline{K}\vec{\Phi} = \vec{R},$$
$$\vec{\Phi}\big|_{t=0} = \vec{\Phi}_0, \; \dot{\vec{\Phi}}\big|_{t=0} = \dot{\vec{\Phi}}_0 \qquad (2)$$

where: $\overline{H}$ – diagonal inertia matrix; $\overline{\mathrm{K}}$ – stiffness matrix; $\vec{\Phi}$ – vector of nodal elastic displacements; $\dot{\vec{\Phi}}$ – vector of nodal elastic displacement velocities; $\ddot{\vec{\Phi}}$ – vector of nodal elastic accelerations; $\vec{R}$ – vector of external nodal elastic forces.

Thus, using the finite element method, a linear problem with initial and boundary conditions (1) was led to a linear Cauchy problem (2).

We determine the elastic contour stress at the boundary of the region free from loads [7–8, 18–19].

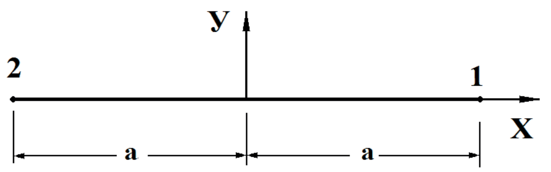


Figure 1. Contour end element with two node points

Using the degeneracy of a rectangular finite element with four nodal points, we obtain a contour finite element with two nodal points (fig. 1).

When turning the axis x on corner α counterclockwise, we get an elastic contour stress σ_k in the center of gravity of a contour finite element with two nodal points [7–8, 18–19]

$$\sigma_k = (E/(2a(1-\nu^2)))((u_1 - u_2)\cos\alpha +$$
$$+ (v_1 - v_2)\sin\alpha) \qquad (3)$$

To integrate equation (2) with a finite element version of the Galerkin method, we reduce it to the following form [7–8, 18–19]

$$\overline{H}\frac{d}{dt}\dot{\vec{\Phi}} + \overline{K}\vec{\Phi} = \vec{R}, \; \frac{d}{dt}\vec{\Phi} = \dot{\vec{\Phi}}. \qquad (4)$$

Integrating the relation (4) over the time coordinate using a finite-element version of the Galerkin method, we obtain an explicit two-layer scheme for internal and boundary node points [7–8, 18–19]

$$\dot{\vec{\Phi}}_{i+1} = \dot{\vec{\Phi}}_i + \Delta t \overline{H}^{-1}(-\overline{K}\vec{\Phi}_i + \vec{R}_i),$$
$$\vec{\Phi}_{i+1} = \vec{\Phi}_i + \Delta t \dot{\vec{\Phi}}_{i+1}. \qquad (5)$$

The main relations of the finite element method in displacements are obtained using the principle of possible displacements and a finite element version of the Galerkin method [7–8, 18–19].

The general theory of numerical equations of mathematical physics requires for this purpose the imposition of certain conditions on the ratio of steps along the time coordinate Δt and by spatial coordinates, namely [7–8, 18–19]

$$\Delta t = k\frac{\min \Delta l_i}{C_p} \quad (i = 1,\ 2,\ 3, ...), \qquad (6)$$

where: Δl – the length of the side of the end element.

The results of the numerical experiment showed that at $k = 0{,}5$ the stability of the explicit two-layer scheme for internal and boundary node points on quasi-regular grids is ensured [7–8, 18–19].

For the study area consisting of materials with different physical properties, the minimum step along the time coordinate is selected (6).

On the basis of the finite element method in displacements, a technique is developed, an algorithm is developed and a set of programs is compiled for solving two-dimensional wave problems of the dynamic theory of elasticity [7–8, 18–19].

3. LONGITUDINAL WAVES IN AN ELASTIC HALF-PLANE WHEN EXPOSED AS A HEAVISIDE FUNCTION

The problem of the effect of a flat longitudinal wave in the form of a Heaviside function (fig. 3) on an elastic half-plane (fig. 2) is considered to assess the physical reliability and mathematical accuracy [7–8, 18–19].

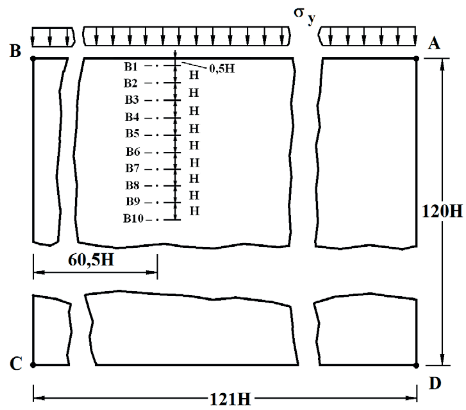


Figure 2. Statement of the problem of propagation of plane longitudinal waves in the form of a Heaviside function in an elastic half-plane

The calculations were carried out for the following units of measurement: kilogram-force (kgf); centimeter (cm); second (s). The following assumptions were made for switching to other units of measurement: 1 kgf/cm^2 ≈ 0,1 MPa; 1 kgf s^2/cm^4 ≈ 10^9 kg/m^3.

On the boundary of the half-plane AB (fig. 2) a normal voltage σ_y is applied, which at $0 \le \mathrm{n} \le 11$ ($n = t/\Delta t$) changes linearly from 0 to P, and at $n \ge 11$ is equal to P ($P = \sigma_0$, $\sigma_0 = $-0,1 MPa (-1 kgf/cm^2)). Boundary conditions for a contour $BCDA$ on $t > 0$ $u = v = \dot{u} = \dot{v} = 0$. Reflected waves from the contour $BCDA$ they do not reach the studied points when $0 \le n \le 100$.

The calculations were carried out with the following initial data: $H = \Delta x = \Delta y$; $\Delta t =$ 1,393·10^{-6} s; E = 3,15·10^4 MPa (3,15·10^5 kgf/cm^2); $\nu =$ 0,2; $\rho =$ 0,255·10^4 kg/m^3 (0,255·10^{-5} kgf s^2/cm^4); $C_p =$ 3587 m/s; $C_s =$ 2269 m/s.

The studied computational domain has 14762 nodal points. A system of equations consisting of 59048 unknowns is solved.

The calculation results are obtained at characteristic points $B1$ - B10 (fig. 2).

As an example, a change in the normal voltage is given $\bar{\sigma}_y$ ($\bar{\sigma}_y = \sigma_y / |\sigma_0|$) (fig. 4) in time n at the point $B1$ (1 – numerical solution; 2 – analytical solution).

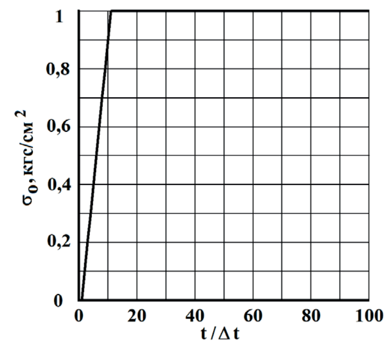


Figure 3. Impact in the form of a Heaviside function

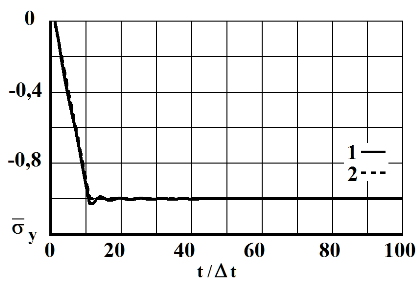


Figure 4. Change in elastic normal stress $\bar{\sigma}_y$ (the problem of propagation of plane longitudinal waves in the form of a Heaviside function in an elastic half-plane) in time $t/\Delta t$ at the point $B1$: 1 – numerical solution; 2 – analytical solution

In this case, you can use the conditions on the plane wave front, which are described in the paper [5].

At the front of a plane longitudinal wave, there are the following analytical dependences for a plane stress state $\sigma_y = -|\sigma_0|$. From here we see that the exact solution of the problem corresponds to the impact σ_0 (fig. 3).

4. MODELING OF STRESS WAVES IN A HALF-PLANE WITH A LIQUID-FILLED CAVITY (THE RATIO OF WIDTH TO HEIGHT IS ONE TO TEN) IN CASE OF SEISMIC IMPACT

The problem of the impact of a plane longitudinal unsteady seismic wave (fig. 6) parallel to the free surface of an elastic half-plane, with a cavity filled with water (the ratio of width to height is one to ten) is considered (fig. 5).

The problem under consideration was solved for the first time by V.K. Musayev using the developed methodology, algorithm and software package [7–8, 18–19].

The calculations were carried out for the following units of measurement: kilogram-force (kgf); centimeter (cm); second (s). The following assumptions were made for switching to other units of measurement: 1 kgf/cm^2 ≈ 0,1 MPa; 1 kgf s^2/cm^4 ≈ 10^9 kg/m^3.

From a point F parallel to the free surface $ABEFG$ voltage normal applied (fig. 5), which on $0 \leq n \leq 11$ ($n = t/\Delta t$) changes linearly from 0 before P, and when $n \geq 11$ is equal to P ($P = \sigma_0, \sigma_0 = 0{,}1$ MPa (1 kgf/cm^2)).

Boundary conditions for a contour $GHIA$ on $t > 0$ $u = v = \dot{u} = \dot{v} = 0$.

Reflected waves from the contour $GHIA$ they do not reach the studied points when $0 \leq n \leq 1000$.

Contour $ABEFG$ free from loads, except for the point F.

The calculations were carried out with the following initial data.

For the region $ABCDEFGHI$: $H = \Delta x = \Delta y$; $\Delta t = 1{,}393 \cdot 10^{-6}$ s; $E = 3{,}15 \cdot 10^4$ MPa ($3{,}15 \cdot 10^5$ kgf/cm^2); $\nu = 0{,}2$; $\rho = 0{,}255 \cdot 10^4$ kg/m^3 ($0{,}255 \cdot 10^{-5}$ kgf s^2/cm^4); C_p = 3587 m/s; $C_s = 2269$ m/s.

For the region $BEDC$: $H = \Delta x = \Delta y$; Δt = $3{,}268 \cdot 10^{-6}$ s; ρ= $1{,}045 \cdot 10^3$ кг/м^3 ($1{,}045 \cdot 10^{-6}$ кгс с^2/см^4); C_p= 1530 м/с.

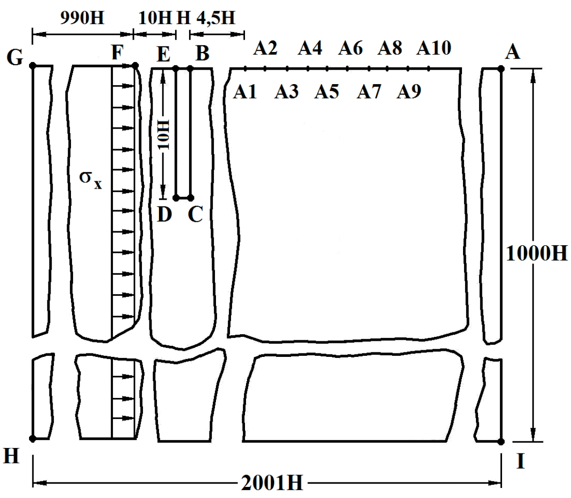


Figure 5. Statement of the problem of the effect of a plane longitudinal seismic wave on an elastic half-plane with a cavity filled with water (the ratio of width to height is one to ten)

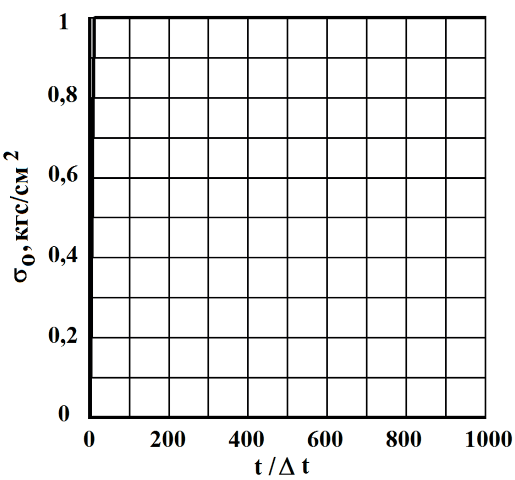
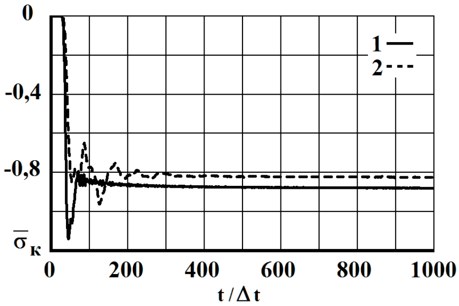
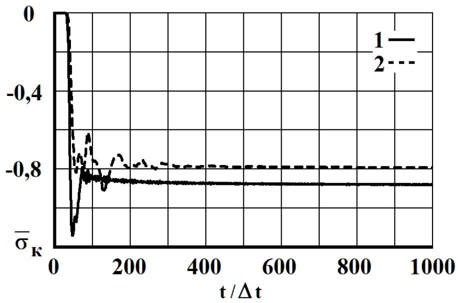


Figure 6. Impact in the form of a Heaviside function



<u>Figure 7.</u> Changing the elastic contour stress $\overline{\sigma}_k$ in time $t/\Delta t$ at the point $A1$: 1 – in a problem without a cavity; 2 – in the problem with a cavity filled with water (the ratio of width to height is one to ten)



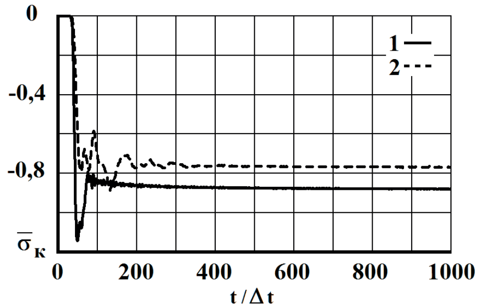
<u>Figure 8.</u> Changing the elastic contour stress $\overline{\sigma}_k$ in time $t/\Delta t$ at the point $A2$: 1 – in a problem without a cavity; 2 – in the problem with a cavity filled with water (the ratio of width to height is one to ten)

When calculating, the minimum time step is taken $\Delta t = 1{,}393 \cdot 10^{-6}$ s.

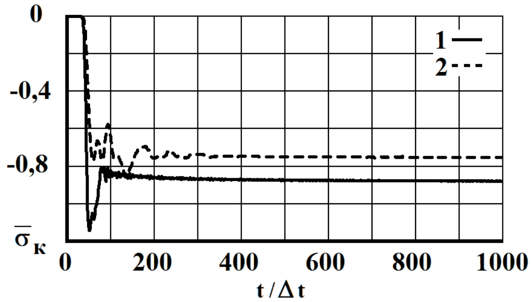
At the boundary of materials with different properties, the conditions of continuity of displacements are assumed.

The studied computational domain has 2004002 nodal points. A system of equations consisting of 8016008 unknowns is solved.

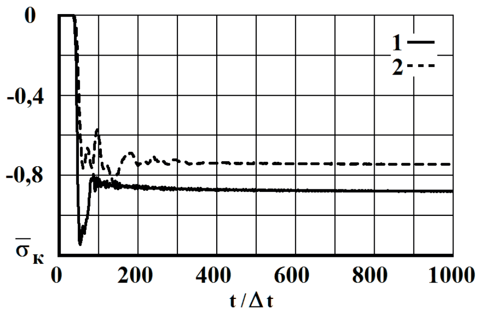
As an example, fig. 7-11 shows the change in the elastic contour stress $\overline{\sigma}_k$ ($\overline{\sigma}_k = \sigma_k / |\sigma_0|$) in time n in points $A1$ - A5 (рис. 5), located on the free surface of an elastic half-plane: 1 – in the problem without a cavity; 2 – in the problem with a cavity filled with water (the ratio of width to height is one to ten).



<u>Figure 9.</u> Changing the elastic contour stress $\overline{\sigma}_k$ in time $t/\Delta t$ at the point $A3$: 1 – in a problem without a cavity; 2 – in the problem with a cavity filled with water (the ratio of width to height is one to ten)



<u>Figure 10.</u> Changing the elastic contour stress $\overline{\sigma}_k$ in time $t/\Delta t$ at the point $A4$: 1 – in a problem without a cavity; 2 – in the problem with a cavity filled with water (the ratio of width to height is one to ten)



<u>Figure 11.</u> Changing the elastic contour stress $\overline{\sigma}_k$ in time $t/\Delta t$ at the point $A5$: 1 – in a problem without a cavity; 2 – in the problem with a cavity filled with water (the ratio of width to height is one to ten)

The distance between the points: $A1$ and $A2$ is H; $A2$ and $A3$ are H; $A3$ and $A4$ are H; $A4$ and $A5$ are H; $A5$ and $A6$ are H; $A6$ and $A7$ are H; $A7$ and $A8$ are H; $A8$ and $A9$ are H; $A9$ and $A10$ are H)

5. CONCLUSIONS

1. On the basis of the finite element method, a methodology, an algorithm and a set of programs for solving linear two-dimensional plane problems have been developed, which allow solving complex problems with non-stationary wave effects on complex objects. The main relations of the finite element method are obtained using the principle of possible displacements. The elasticity matrix is expressed in terms of the velocity of longitudinal waves, the velocity of transverse waves and the density.
2. A linear dynamic problem with initial and boundary conditions in the form of partial differential equations, for solving problems under wave effects, using the finite element method in displacements, is reduced to a system of linear ordinary differential equations with initial conditions, which is solved by an explicit two-layer scheme.
3. To predict the seismic safety of an object, under non-stationary wave effects, numerical modeling of the equations of mechanics of a deformable solid is used. A method, algorithm and a set of programs for solving linear two-dimensional (flat) problems for solving problems of safety in terms of bearing capacity (strength) in multiphase deformable bodies under non-stationary wave influences have been developed.
4. The area under study is divided by spatial variables into triangular and rectangular finite elements of the first order. According to the time variable, the area under study is divided into linear finite elements of the first order. Two displacements and two velocities of displacements at the node of the finite element are taken as the main unknowns.
5. A system with an infinite number of unknowns is reduced to a system with a finite number of unknowns. A quasi-regular approach is proposed to solve a system of linear ordinary differential equations of the second order in displacements with initial conditions and to approximate the region under study. The method is based on the schemes: a point, a line and a plane.
6. The problem of the effect of a plane longitudinal wave in the form of a Heaviside function on an elastic half-plane is solved. The computational domain under study has 14762 nodal points and 14520 finite elements. A system of equations consisting of 59048 unknowns is solved. A comparison was made with the results of the analytical solution, which showed that the discrepancy for the maximum compressive elastic normal stress $\overline{\sigma}_y$ is 2,8 %.
7. The problem of mathematical modeling of unsteady elastic stress waves in a half-plane with a cavity filled with water (the ratio of width to height is one to ten) under seismic influence is solved. At the boundary of materials with different properties, the conditions of continuity of displacements are assumed. The studied computational domain has 2004002 nodal points. A system of equations consisting of 8016008 unknowns is solved. A cavity filled with water, with a width-to-height ratio of one to ten, reduces the value of the elastic contour stress on the free surface of the elastic half-plane under non-stationary wave seismic influences.

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