DOI:10.22337/2587-9618-2022-18-3-78-85

CONTROL OF A NONLOCAL IN TIME FINITE ELEMENT MODEL OF THE DYNAMIC BEHAVIOR OF A COMPOSITE BEAM BASED ON THE RESULTS OF A NUMERICAL EXPERIMENT

***Vladimir N. Sidorov* [1,2], *Elena P. Detina* [1], *Elena S. Badina* [1,2,3]**

[1] Russian University of Transport (MIIT), Moscow, RUSSIA
[2] Moscow State University of Civil Engineering, Moscow, RUSSIA
[3] Institute of Applied Mechanics of Russian Academy of Sciences, Moscow, RUSSIA

Abstract. The article presents numerical methods for controlling the parameters of temporal nonlocality of computer models of rod structures made of composite materials. The finite element method is the most widely used numerical method for solving practical problems of the analysis of mechanical systems. A nonlocal in time internal damping model is integrated into the algorithm of this method. The one-dimensional model of the Euler-Bernoulli beam is presented in the article. The equilibrium equation of a moving mechanical system is solved numerically using an implicit scheme. In the article the damping matrix obtained from the condition of stationarity of the total deformation energy was used. The article presents the study of non-local in time damping model properties. The model is integrated into the finite element method. The non-local model is algorithmized and programmed in the MATLAB software package.

Keywords: nonlocal in time damping, damping with memory, composite material, Euler-Bernoulli beam vibration, equilibrium equation, least squares technique, finite element analysis, iterative implicit scheme, structural dynamics

УПРАВЛЕНИЕ НЕЛОКАЛЬНОЙ ВО ВРЕМЕНИ КОНЕЧНО-ЭЛЕМЕНТНОЙ МОДЕЛЬЮ ДИНАМИЧЕСКОГО ПОВЕДЕНИЯ БАЛКИ ИЗ КОМПОЗИТА ПО РЕЗУЛЬТАТАМ ЧИСЛЕННОГО ЭКСПЕРИМЕНТА

***В.Н. Сидоров* [1,2], *Е.С. Бадьина* [1,2,3], *Е.П. Детина* [2]**

[1] Российский университет транспорта (МИИТ), Москва, РОССИЯ
[2] Национальный исследовательский Московский государственный строительный университет, Москва, РОССИЯ
[3] Институт прикладной механики Российской академии наук, Москва, РОССИЯ

Аннотация. В статье представлены численные методики управления параметрами временной нелокальности расчетных моделей стержневых конструкций из композиционных материалов. Нелокальная во времени модель демпфирования интегрирована в алгоритм метода конечных элементов – наиболее широко применяемого численного метода при решении практических задач анализа механических систем. В работе рассматривается одномерная модель балки Эйлера-Бернулли. Численное решение уравнения равновесия расчётной модели конструкции в движении выполняется по неявной схеме. При этом матрица демпфирования получена из условия стационарности полной энергии деформирования движущейся механической системы. В статье приведены результаты исследования нелокальной во времени расчетной модели, реализованной в среде MATLAB.

Ключевые слова: демпфирование нелокальное во времени, демпфирование с памятью, композитный материал, колебания балок Эйлера-Бернулли, уравнение равновесия, метод наименьших квадратов, метод конечных элементов, неявная схема, динамика механических систем

INTRODUCTION

Today, the models of oscillatory processes taking place in mechanical systems can be modeled in various ways. Also the computational mathematical and algorithmic apparatus has been significantly developed. Despite this, scientists devote a special place to the issue of adequate modeling of the damping properties of structures made of structurally complex materials. For example, the papers [1, 2] present ideas of damping kernels created as a linear combination of decreasing functions. In works [3, 4, 5] study the issues of constructing the effective characteristics of a layered composite material, the layers of which are viscoelastic.

In this article, we present the results of modeling of the damping properties of structures made of structurally complex materials, built on the assumption that the material has nonlocal in time properties of internal damping. In [6, 7, 8] the matrix form of the modified equation of motion of mechanical systems was studied:

$$M \cdot \ddot{\bar{V}}(t_{i+1}) + \alpha \cdot D \cdot \dot{\bar{V}}(t_{i+1}) \cdot \\ \cdot (1-\alpha) \cdot \left(\int_{t_0}^{t_i} G(t_i - \tau)\dot{\bar{V}}(\tau) d\tau \right) \cdot \\ \cdot K \cdot \bar{V}(t_{i+1}) = F(t_{i+1}), \qquad (1)$$

the integral term endows the classic computational model with the time nonlocality. Here $0 < \alpha < 1$ is the temporal nonlocality weight coefficient [7]; t_0 – initial time of the oscillatory process. Matrices of masses *M*, damping *D* and stiffness *K* of the computer model are developed from the condition of a minimum change in the total energy of a mechanical system deformed in motion [6, 8, 9]. When $\alpha = 1$ – the model preserves the locality of the time component. The function $G(t_i - \tau)$ in equation (1) is usually called the damping kernel function [7]. The Gaussian curve was taken as the damping kernel into a solution:

$$G(t_i - \tau) = e^{-\mu^2 (t_i - \tau)^2} \qquad (2)$$

the parameter $\mu > 0$ in (2) characterizes the area of nonlocal properties of the time component, while regardless of the value of μ, $\int_{t_0}^{t} \frac{2\mu}{\sqrt{\pi}} \cdot e^{-\mu^2(t-\tau)^2} d\tau = 1$; $\tau\, [sec]$ – all moments of time preceding the considered component of the time axis t_i. Reducing the value of the parameter μ increases the level of nonlocality along the time axis of the model**.** Such a statement of the problem endows the damping forces in the calculation model with the property of "memory" (hereinafter, this property of the model will be called "damping with memory"). Thus, in the numerical calculation of the structure according to the implicit scheme in the term (1), responsible for damping with memory, the values of the rates of change of displacements and deformations are taken into account not only at the previous computational step t_i, but also at all previous time steps up to t_i.

The use of the kernel of the internal damping operator (2) can be attributed to the mathematical idealization of the description of the distribution of the "memory" of the composite in time, which is generally not based on the features of the microstructure of the material. To use the constructed model in practical calculations, it must be calibrated based on the data of a physical or alternative, for example numerical experiment. In this case, the parameter μ in (2) becomes the main control parameter of the considered computational model, which sets the degree of nonlocality.

MODEL CALIBRATION TECHNIQUES

As an example, consider the oscillations of a beam made from the composition material. The beam is rigidly fixed at the edges and loaded with an instantly applied uniformly distributed load. The general physical and mechanical parameters of the design and the values of the momentarily applied load are presented in Table 1.

Table 1

General parameters of a fiberglass vinyl ester beam.
Young's of elasticity [Pa]: E=1720000; Beam length [m]: L=12; Material density [t/m^3]: ρ=1.9; Beam cross-sectional area (constant along the entire length) [m^2]: A=0.06; Moment of inertia [m^4]: I=4.5000e-04; Instantaneously applied load [N/m^2]: q=-1.

To emphasize the necessity and benefit of further studies of the nonlocal in time model of damping properties of composite materials, the comparison of the results of equation (1) solution in a local statement (for $\alpha = 1$) with experimental data is presented. The fig.1 shows the time history of vertical displacements of the middle node of the beam. The solid line shows the numerical solution of the problem based on a one-dimensional local in time computational model; dotted line - data obtained as a result of a numerical experiment implemented in the finite element software package SIMULIA Abaqus (structurally complex properties of the composition were taken into account using an orthotropic material model).

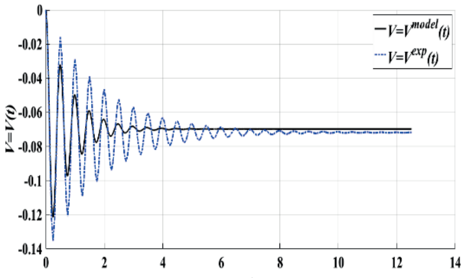


Figure 1. Vertical displacement of the middle node of the oscillating beam made of a composite material: $\bar{V}^{exp}(t)$ – experimental curve; $\bar{V}^{model}(t)$ – is a time-local curve

Based on the simulation results, it was concluded that the computer model, local in time, approximates the (dynamic) oscillatory process inside a structure made of a structurally complex material with a reliability that is not sufficient for further application in design justification process. Calculations using isotropic or local one-dimensional models for composites give a significant error, which is unacceptable in the calculation of such structures subjected to dynamic effects.

The article presents two main approaches (methodologies) to determining the optimal value of the parameter μ for a non-local in time one-dimensional model of the dynamic behavior of a structure made of composite material.

In this work the following indices are used:

$\bar{V}^{exp}(t)$ – displacement vector obtained as a result of a numerical experiment implemented in the software package SIMULIA Abaqus;

$\bar{V}^{model}(t)$ – the displacement vector obtained as a result of solving equation (1), according to the implicit scheme by the modified Newmark method [10];

$v^{synth}(t)$ – a curve synthesizing the values of the displacement vector obtained as a result of a numerical experiment.

$f^{error}(\mu)$ – a sum of squared deviations of vector elements $\bar{V}^{model}(t)$ to $\bar{V}^{exp}(t)$.

Methodology 1. Direct construction of a search model for the optimal nonlocality parameter of the dynamic properties of the composite by the least squares method (LSM).This technique provides an automated search for the value of the parameter μ, in which the sum of the squared deviations $f^{error}(\mu)$ takes the minimum value. Fig. 2 shows the dependency graph $f^{error}(\mu)$, showing the behavior of the non-local model (1) in a fairly wide range of parameter values $1 \leq \mu \leq 200$ (the value of μ was not further increased in this work). In Figure 3, we have localized the range of μ values relative to $f^{error}(\mu)$ smallest value.

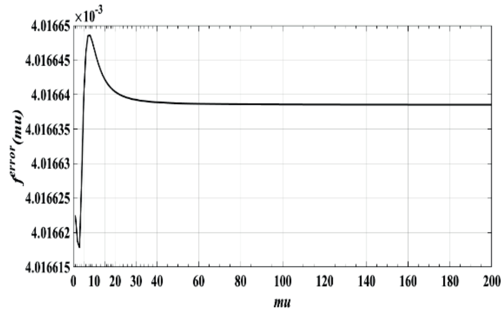


Figure 2. Dependence of f^{error} on μ. The ordinate shows the summation of the squared deviations $f^{error}(\mu)$

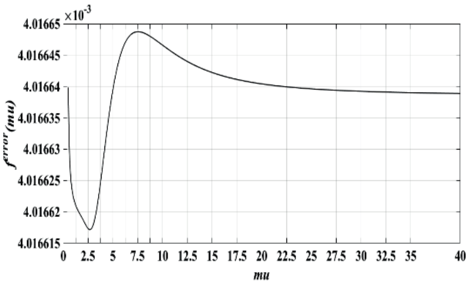


Figure 3. Dependence of f^{error} on μ, localized in the region of extreme values of μ: the abscissa shows the values of the parameter μ in the range $1 \leq \mu \leq 40$, the ordinate shows the sum of squared deviations $f^{error}(\mu)$

As a result of processing the data of the LSM experiment, we obtain $\bar{V}^{model}(t)$ with the minimum value $f^{error}(t)$, corresponding to the value $\mu = 2{,}68601$. Below, in Figure 4, there are two graphs of the vertical displacement of the middle section of an oscillating beam made of composite material: $\bar{V}^{exp}(t)$ – experimental displacement data; $\bar{V}^{model}(t)$ – displacement data based on a calibrated time-nonlocal damping model at $\mu = 2{,}68601$ sec.

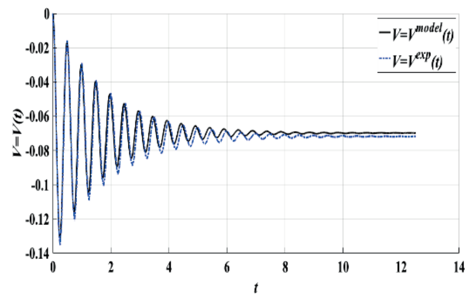


Figure 4. Time history of the composite beam middle node vertical displacement: $\bar{V}^{exp}(t)$ – experimental curve; $\bar{V}^{model}(t)$ – calibrated non-local in time curve at $\mu = 2{,}68601$ sec.

As can be seen from the fig 4., the non-local in time computational model, approximates the oscillatory process of an element made of a structurally complex material with sufficient reliability. The result of the search for the optimal nonlocality parameter value for a one-dimensional composite beam, shows the efficiency of the constructed model. However, this technique seems to be computationally difficult. Below we describe the developed alternative technique, which is simpler both in terms of computational costs and the search for the optimal value of the nonlocality parameter μ for a composite material.

Methodology 2. Search for the optimal value of the nonlocality parameter based on the least squares method using a synthesizing curve.

Under the synthesizing curve, $v^{synth}(t)$, we will mean some analytical (interpolating) curve approximating the experimental data with a satisfactory accuracy, $\bar{V}^{exp}(t)$. Below is an algorithm for constructing an expression for such a curve by the least squares method in the form of the polynomial of the fourth degree:

$$v^{synth}(t) = a_0 + a_1 \cdot t + \\ + a_2 \cdot t^2 + a_3 \cdot t^3 + a_4 \cdot t^4, \quad (3)$$

$\{a_j\}_{j=1}^{4}$ – the desired polynomial coefficients calculated in comparison with the values of $\bar{V}^{exp}(t)$.

Representing (3) in matrix form, we find the coefficients a_j by the least squares method from the conditions for the minimum sum of squared deviations of the values $\bar{V}^{exp}(t)$ from the desired curve $v^{synth}(t)$:

$$F(a_0, a_1, a_2, a_3, a_4) = \\ = \min \sum_{i=1}^{N} \left(\bar{V}^{exp}(t_i) - v^{synth}(t_i) \right)^2, \quad (4)$$

N – number of nodes taken along the time axis t.

Below, the minimum condition is expressed in partial derivatives $\frac{\partial F}{\partial a_j}$ equated to zero, written as a system, where $j = 0,1,2,3,4$ – is the number of the required coefficient a_j.

$$\begin{cases} a_4 \cdot \sum_{i=1}^{N} t_i^4 + a_3 \cdot \sum_{i=1}^{N} t_i^3 + a_2 \cdot \sum_{i=1}^{N} t_i^2 + a_1 \cdot \sum_{i=1}^{N} t_i + a_0 \cdot \sum_{i=1}^{N} 1 = \sum_{i=1}^{N} \bar{V}^{exp} \\ a_4 \cdot \sum_{i=1}^{N} t_i^5 + a_3 \cdot \sum_{i=1}^{N} t_i^4 + a_2 \cdot \sum_{i=1}^{N} t_i^3 + a_1 \cdot \sum_{i=1}^{N} t_i^2 + a_0 \cdot \sum_{i=1}^{N} t_i = \sum_{i=1}^{N} \bar{V}^{exp} \cdot t_i \\ a_4 \cdot \sum_{i=1}^{N} t_i^6 + a_3 \cdot \sum_{i=1}^{N} t_i^5 + a_2 \cdot \sum_{i=1}^{N} t_i^4 + a_1 \cdot \sum_{i=1}^{N} t_i^3 + a_0 \cdot \sum_{i=1}^{N} t_i^2 = \sum_{i=1}^{N} \bar{V}^{exp} \cdot t_i^2 \\ a_4 \cdot \sum_{i=1}^{N} t_i^7 + a_3 \cdot \sum_{i=1}^{N} t_i^6 + a_2 \cdot \sum_{i=1}^{N} t_i^5 + a_1 \cdot \sum_{i=1}^{N} t_i^4 + a_0 \cdot \sum_{i=1}^{N} t_i^3 = \sum_{i=1}^{N} \bar{V}^{exp} \cdot t_i^3 \\ a_4 \cdot \sum_{i=1}^{N} t_i^8 + a_3 \cdot \sum_{i=1}^{N} t_i^7 + a_2 \cdot \sum_{i=1}^{N} t_i^6 + a_1 \cdot \sum_{i=1}^{N} t_i^5 + a_0 \cdot \sum_{i=1}^{N} t_i^4 = \sum_{i=1}^{N} \bar{V}^{exp} \cdot t_i^4 \end{cases}$$

Here t_i – time coordinate; N – the number of points taken on the time axis. The system of equations (4) is represented in matrix form:

$$Koef \cdot \bar{a} = \bar{s}, \qquad (5)$$

Then

$$Koef = \begin{pmatrix} \sum_{i=1}^{N} t_i^4 & \cdots & \sum_{i=1}^{N} 1 \\ \vdots & \ddots & \vdots \\ \sum_{i=1}^{N} t_i^8 & \cdots & \sum_{i=1}^{N} t_i^4 \end{pmatrix},$$

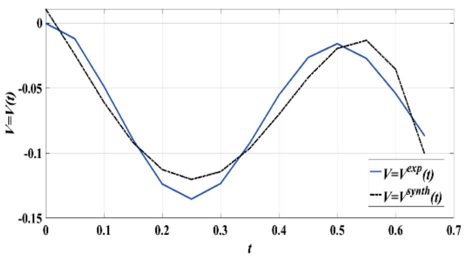
$$\bar{a} = \begin{pmatrix} a_0 \\ \vdots \\ a_4 \end{pmatrix}, \qquad (6)$$

$$\bar{s} = \begin{pmatrix} \sum_{i=1}^{N} \bar{V}^{exp}(t_i) \\ \vdots \\ \sum_{i=1}^{N} \bar{V}^{exp}(t_i) \cdot t_i^4 \end{pmatrix},$$

as well as the solution of equation (5):

$$\bar{a} = Koef^{-1} \cdot \bar{s}, \qquad (7)$$

The Fig. 5 graphically presents a comparison of the experimental values of dynamic vertical displacements of the middle node of the beam FE model $\bar{V}^{exp}(t)$, with the displacement values obtained using the synthesizing curve $v^{synth}(t)$.



<u>Figure 5.</u> Graphical comparison of the numerical experimental values of the middle node vertical displacements $\bar{V}^{exp}(t)$, with the displacement values obtained using the synthesizing curve $v^{synth}(t)$ for the time period $T = 0.65$ sec.

The Fig. 5 shows some difference between the values of the experiment and the model values, which indicates the possibility of developing the presented methodology by clarifying the appropriate type of curve.

The main advantage of using the synthesizing curve lies in the "simplicity" of its further application in identifying the optimal value of the temporal nonlocality parameter of the model μ. This is also determined by the fact that the derivatives of such a synthesizing function can be represented as a functional dependence. This greatly simplifies the calculation and results analysis at the model calibration stage.

Let us substitute the expressions of the synthesizing curve $v^{synth}(t_i)$ and its derivatives $v'^{\,synth}(t_i), v''^{\,synth}(t_i)$ of the first and second order into equation (1):

$$\begin{gathered} M \cdot \left(12a_4t_i^2 + 6a_3t_i + 2a_2\right) + \\ +\alpha \cdot D \cdot \left(4a_4t_i^3 + 3a_3t_i^2 + 2a_2t_i + a_1\right) + \\ +(1-\alpha) \cdot D \cdot \int \frac{2\mu}{\sqrt{\pi}} e^{-\mu^2(t_i-\tau)^2} \cdot (4a_4(t_i - \\ \tau)^3 + 3a_3(t_i-\tau)^2 + 2a_2(t_i-\tau) + \\ +a_1)d\tau + K \cdot \left(a_4t_i^4 + a_3t_i^3 + a_2t_i^2 + \right. \\ \left. +a_1t_i + a_0\right) = F(t_i). \end{gathered} \qquad (8)$$

Now we transform equation (8) in such a way that the terms containing the desired parameter μ, are located to the left side, and those free from it are to the right:

$$\begin{gathered} \frac{(1-\alpha)2\mu}{\sqrt{\pi}} \cdot D \cdot \int_{t_0}^{t_i} e^{-\mu^2(t_i-\tau)^2} (4a_4(t_i-\tau)^3 + \\ +3a_3(t_i-\tau)^2 + 2a_2(t_i-\tau) + a_1)d\tau = \\ = F(t_i) - M \cdot \left(12a_4t_i^2 + 6a_3t_i + 2a_2\right) - \\ -\alpha \cdot D \cdot \left(4a_4t_i^3 + 3a_3t_i^2 + 2a_2t_i + a_1\right) - \\ -K \cdot \left(a_4t_i^4 + a_3t_i^3 + a_2t_i^2 + a_1t_i + a_0\right). \end{gathered} \qquad (9)$$

Let us denote in (9) by $R(t_i)$ the "effective load" vector calculated for the i moment of time:

$$\begin{gathered} R(t_i) = F(t_i) - \\ -M \cdot \left(12a_4t_i^2 + 6a_3t_i + 2a_2\right) - \\ -\alpha D \cdot \left(4a_4t_i^3 + 3a_3t_i^2 + 2a_2t_i + a_1\right) - \\ -K \cdot \left(a_4t_i^4 + a_3t_i^3 + a_2t_i^2 + a_1t_i + a_0\right). \end{gathered} \qquad (10)$$

Then the total effective load vector will look like:

$$R = \sum_{i=1}^{N} R(t_i). \tag{11}$$

Now we denote by $Z(\mu, t_i)$, the integral operator in (8) - (9), calculated for the i-th moment of time and containing the nonlocality parameter of the model μ:

$$Z(\mu, t_i) = \int_{t_0}^{t_i} e^{-\mu^2(t_i-\tau)^2} (4a_4(t_i-\tau)^3 + \\ +3a_3(t_i-\tau)^2 + 2a_2(t_i-\tau) + a_1)d\tau, \tag{12}$$

τ – time parameter characterizing the moments of time preceding the moment t_i.
Then the total integral operator takes the form:

$$Z(\mu) = \sum_{i=1}^{N} Z(\mu, t_i). \tag{13}$$

Substituting expressions (11) and (13) into (9), we obtain an equation for an unknown quantity (the nonlocality parameter of the model μ):

$$\frac{2\mu}{\sqrt{\pi}} \cdot Z(\mu) = \frac{1}{1-\alpha} \cdot D^{-1} \cdot R. \tag{14}$$

The solution of equation (14) is algorithmized and performed in accordance with the stated method 2 and programmed in MATLAB. As a result of the numerical solution (14), we obtained the value of the nonlocality parameter in the middle node of the composite beam, equal to $\mu = 2.78328$ sec. Figure 6 shows a graph of the values of the vector $\bar{V}^{model}(t)$, of to $\mu = 2.78328$ sec, in comparison with the experimental values $\bar{V}^{exp}(t)$.

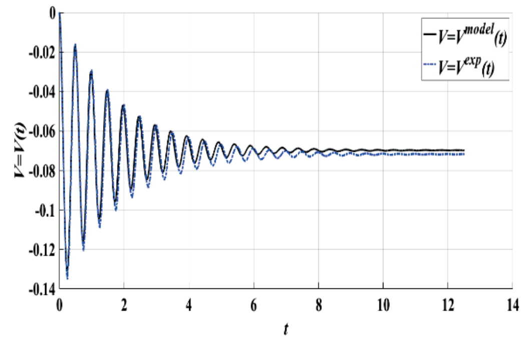


Figure 6. Vertical displacement of the composite beam middle node: $\bar{V}^{exp}(t)$ – experimental curve; $\bar{V}^{model}(t)$ – calibrated non-local in time curve at $\mu = 2.78328$ sec.

In Table 2 comparison of the calibration results of the non-local model by the two methods described with the results of the local model is presented. The average relative error is calculated by the formula $\frac{100\%}{N} \cdot \sum_{i=1}^{N} \left| \frac{\bar{V}^{exp}(t_i) - \bar{V}^{model}(t_i)}{\bar{V}^{exp}(t_i)} \right|$, where N=251 – is the number of nodal points taken along the time axis.

Table 2.

	The value of the non-locality parameter μ, [sec]	Relative calculation error, [%]
Local model	-	44,08
Non-local model calibrated by methodology 1	2.68601	4.52
Non-local model calibrated by methodology 2	2.78328	4.52

CONCLUSION

A non-local damping model applied to dynamic calculations of structures made of composite materials gives a result with a smaller relative calculation error in comparison with the experimental results than a local one.

Two techniques have been developed for controlling a non-local model of nonlocal in time damping properties according to experimental data. Those techniques are presented in the article on the example of a specific composite sample analysis. When rounding the error to a hundredth of a percent, both techniques give a result with the same reliability of calculations. To date, technique 1 seems to be basic, while technique 2 is more promising in terms of algorithmization of the process of modeling the temporal nonlocality of damping properties of composite materials. The choice of synthesizing curves suitable for real experimental data is a matter for a separate study.

ACKNOWLEDGEMENT

This research has been supported by the Russian Science Foundation (Project No 21-19-00634).

REFERENCES

1. **Oleynik O.A., Shamaev A.S., Yosifian G.A.**, 1992, Mathematical Problems in Elasticity and Homogenization. Elsevier, North-Holland.
2. **Bardzokas D.I., Zobnin A.I.**, 2003, Mathematical Modelling of Physical Processes in Composite Materials of Periodical Structures. URSS, Moscow.
3. **Shamaev A.S., Shumilova V.V.**, 2016, Asymptotic behavior of the spectrum of one-dimensional vibrations in a layered medium consisting of elastic and kelvin–voigt viscoelastic materials. Proceedings of the Steklov Institute of Mathematics 295(1), 202-212.
4. **Shamaev A. S., Shumilova V. V.**, 2016, Homogenization of the equations of state for a heterogeneous layered medium consisting of two creep materials. Proceedings of the Steklov Institute of Mathematics 295(1), 213-224.
5. **Yang X.J.**, 2019, General Fractional Derivatives: Theory, Methods and Applications. CRC Press, New York.
6. **Sidorov V.N., Badina E.S**, 2021, The Finite Element Method in Problems of Stability and Vibrations of Bar Structures. Examples of calculations in Mathcad and MATLAB. ASV Publishing House, Moscow.
7. **Sidorov V.N., Badina E.S**, 2021, Non-local damping models in dynamic calculations of structures made of composite materials. Civil engineering, №. 9, p. 66-70.
8. **Sidorov V.N., Badina E.S., Detina E.P.**, 2021, Nonlocal in time model of material damping in composite structural elements dynamic analysis. International Journal for Computational Civil and Structural Engineering, 17(4):14-21.
9. **Zenkevich O.**, 1975, Finite element methods in engineering. Mir, Moscow.
10. **Bathe K. J., Wilson E.L.,** 1976, Numerical methods in finite element analysis. Prentice Hall, New York.

СПИСОК ЛИТЕРАТУРЫ

1. **Oleynik O.A., Shamaev A.S., Yosifian G.A.**, 1992, Mathematical Problems in Elasticity and Homogenization. Elsevier, North-Holland.
2. **Bardzokas D.I., Zobnin A.I.**, 2003, Mathematical Modelling of Physical Processes in Composite Materials of Periodical Structures. URSS, Moscow.
3. **Shamaev A.S., Shumilova V.V.**, 2016, Asymptotic behavior of the spectrum of one-dimensional vibrations in a layered medium consisting of elastic and kelvin–voigt viscoelastic materials. Proceedings of the Steklov Institute of Mathematics 295(1), 202-212.
4. **Shamaev A. S., Shumilova V. V.**, 2016, Homogenization of the equations of state for a heterogeneous layered medium consisting of two creep materials. Proceedings of the Steklov Institute of Mathematics 295(1), 213-224.

5. **Yang X. J.**, 2019, General Fractional Derivatives: Theory, Methods and Applications. CRC Press, New York.
6. **Сидоров В.Н., Бадьина Е.С.**, 2021, Метод конечных элементов в задачах устойчивости и колебаний стержневых конструкций. Примеры расчетов в Mathcad и MATLAB. Издательство АСВ, Москва.
7. **Сидоров В.Н., Бадьина Е.С.**, 2021, Нелокальные модели демпфирования в динамических расчетах конструкций из композитных материалов. Промышленное и гражданское строительство, №. 9, р. 66-70.
8. **Sidorov V.N., Badina E.S., Detina E.P.**, 2021, Nonlocal in time model of material damping in composite structural elements dynamic analysis. International Journal for Computational Civil and Structural Engineering, 17(4):14-21.
9. **Зенкевич О.**, 1975, Метод конечных элементов в технике. Мир, Москва.
10. **Bathe K. J., Wilson E.L.,** 1976, Numerical methods in finite element analysis. Prentice Hall, New York.

Vladimir N. Sidorov, Corresponding Member of Russian Academy of Architecture and Construction Science, Professor, Dr.Sc, Head of the Department of Computer Science and Applied Mathematics, National Research University Moscow State University of Civil Engineering, Professor of «Building Structures, Buildings and Facilities» Department, Institute of Railway Track, Construction and Structures, Russian University of Transport (MIIT), Professor of Department «Engineering Structures and Numerical Mechanics», Perm National Research Polytechnic University; 127994, Russia, Moscow, Obraztsova st., 9, b. 9, phone: +74956814381, e-mail: sidorov.vladimir@gmail.com.

Elena S. Badina, Ph.D, Associate Professor of «Computer Aided Design» Department, Institute of Railway Track, Construction and Structures, Russian University of Transport (MIIT), Senior Researcher at the Scientific and Educational Center for Computer Modeling of Unique Buildings, Structures and Complexes of the Moscow State University of Civil Engineering, Senior Researcher at the Department of Mechanics of Structured and Heterogeneous Environment of the Institute of Applied Mechanics of the Russian Academy of Sciences;
127994, Russia, Moscow, Obraztsova st., 9, b. 9, phone: +74956092116, e-mail: shepitko-es@mail.ru.

Elena P. Detina, Research Engineer, Department of Analytical Fundamental Scientific Research on the Dynamics of Building Structures, Scientific and Educational Center for Computer Modeling of Unique Buildings, Structures and Complexes (REC KM), Lecturer at the Department of Applied Mathematics and Informatics, Moscow State University of Civil Engineering (NRU MGSU); 129337, Russia, Moscow, Yaroslavskoe shosse, 26, phone +74957819988, e-mail: detinaep@mgsu.ru

Сидоров Владимир Николаевич, член-корреспондент РААСН, профессор, доктор технических наук, заведующий кафедрой информатики и прикладной математики Национального исследовательского Московского государственного строительного университета, профессор кафедры «Строительные конструкции, здания и сооружения» Института пути, строительства и сооружений Российского университета транспорта (МИИТа); 127994, Россия, г. Москва, ул. Образцова, д.9, стр. 9, телефон: +74956814381, e-mail: sidorov.vladimir@gmail.com.

Бадьина Елена Сергеевна, кандидат технических наук, доцент кафедры «Системы автоматизированного проектирования» Института пути, строительства и сооружений Российского университета транспорта (МИИТа), старший научный сотрудник Научно-образовательного центра компьютерного моделирования уникальных зданий, сооружений и комплексов Московского государственного строительного университета, старший научный сотрудник Отдела механики структурированной и гетерогенной среды Института прикладной механики Российской академии наук,
127994, Россия, г. Москва, ул. Образцова, д.9, стр. 9, телефон: +74956092116, e-mail: shepitko-es@mail.ru.

Детина Елена Петровна, инженер-исследователь отдела аналитических фундаментальных научных исследований по динамике строительных конструкций Научно-образовательного центра компьютерного моделирования уникальных зданий, сооружений и комплексов (НОЦ КМ), преподаватель кафедры Прикладной математики и информатики Московского государственного строительного университета (НИУ МГСУ); 129337, Россия, г. Москва, Ярославское шоссе, д. 26, телефон +74957819988, e-mail: detinaep@mgsu.ru