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MODEL OF THERMOMECHANICAL VIBRATIONS OF CURRENT-CARRYING CONDUCTORS

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Abstract. In the operation practice of overhead power transmission lines (OHL), the phenomenon of "galloping" of conductors is well known – vibrations with frequencies of ~ 1 Hz and with amplitudes of the order of the static sag [1, 2]. This phenomenon is observed, as a rule, when the symmetry of the conductor section is violated due to icy deposits, which gives the conductor some aerodynamic efficiency. However, this model does not explain all the observed cases of galloping. In this regard, it is advisable to pay attention to the little-known experience of Academician Abram F. Ioffe, who experimentally discovered the self-excitation of a current-carrying conductor – a stretched string that heats up when connected to an electrical circuit. Solving this issue can significantly expand the understanding of the nature of conductor galloping and open up new ways to fend off this phenomenon, which poses a danger to the stability of the functioning of energy systems. This requires a mathematical model of the OHL conductor describing the interaction of mechanical and thermal processes. The purpose of this work is to construct the simplest version of this model, on the basis of which the condition of self-excitation of thermomechanical self-excitation of real OHL conductors can be justified.

Keywords: power transmission, sagging conductor, heat generation, heat transfer, thermomechanical processes, vibrations, self-excitation, galloping

МОДЕЛЬ ТЕРМОМЕХАНИЧЕСКИХ КОЛЕБАНИЙ ТОКОНЕСУЩИХ ПРОВОДНИКОВ

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Аннотация. В практике эксплуатации воздушных линий электропередачи (ВЛЭ) известен феномен «пляски» проводов – колебания с частотами ~1 Гц и с амплитудами порядка стрелы статического провисания провода [1, 2], наблюдаемые, как правило, при потере симметрии сечения провода вследствие гололедных отложений, что придает проводу некоторое аэродинамическое качество. Однако эта модель не объясняет всех наблюдаемых случаев пляски. В связи с этим целесообразно обратить внимание на малоизвестный опыт академика А.Ф. Иоффе, экспериментально обнаружившего самовозбуждение токонесущего проводника – натянутой струны, нагревающейся при включении в электрическую цепь. Решение этого вопроса может существенно расширить представления о природе пляски проводов и открыть новые пути парирования этого феномена, представляющего опасность для стабильности функционирования энергетических систем. Для этого необходима математическая модель провода ВЛЭ, описывающая взаимодействие механических и тепловых процессов. Целью данной работы является построение наиболее простого варианта этой модели, на базе которого может быть обосновано условие самовозбуждения термомеханического самовозбуждения реальных проводов ВЛЭ.

Ключевые слова: электропередача, провисающий провод, тепловыделение, теплоотдача, термомеханические процессы, колебания, самовозбуждение, галопирование

INTRODUCTION

Open mechanical oscillatory systems interacting with energy sources and the environment are widely known in technology. These include, for example, relay thermoregulators that include current only in a given range; their mode of operation is relaxation self–oscillation. In thermistor generators, self-oscillations do not occur as a result of circuit interruption, but due to the dependence of resistance on temperature. A special class of thermomechanical self-oscillations was experimentally discovered by A.F. Ioffe, who demonstrated, as an illustration for a lecture course in physics, the self-excitation of a conductor – a stretched string that heats up when it is connected to a DC electric circuit.

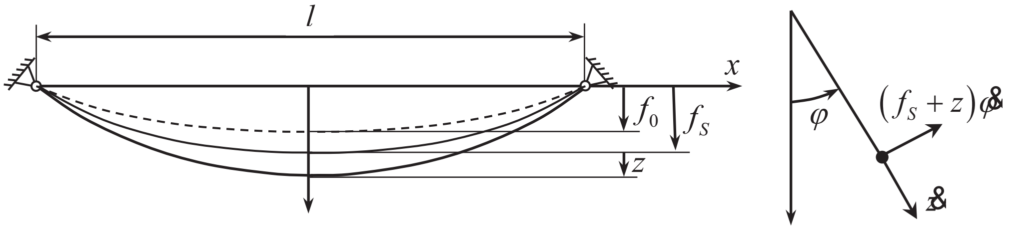

Figure 1. Kinematic parameters of three conductor states

The works devoted to the construction of a theory explaining this phenomenon are [3-7]. It is shown that self-oscillations in such systems can be caused by the interaction of a number of thermomechanical factors: Joule heat release, thermal and deformation changes in the electrical resistance of the conductor (thermoresistive effect), dependence of heat transfer to the medium on the vibration amplitude. In [7] there are indications of a repetition of the experience of A.F. Ioffe. Of practical interest is the question of whether the concept of thermomechanical vibrations can serve as an explanation for the phenomenon of conductor galloping – low-frequency vibrations of overhead power line (OHL) conductors with frequencies of ~ 1 Hz and with amplitudes of the order of static sag. The authors of the cited works have made such an assumption, but it has not yet received convincing confirmation: there is no transfer of the effect, modeled theoretically and observed in a laboratory model, to the full-scale OHL conductors. The purpose of this work is to build, if possible, an elementary model suitable for assessing the conditions for self-excitation of vibrations and transferring them to the actual operating conditions of OHL.

1. MODEL OF CONDUCTOR VIBRATION

The conductor is considered as an elastic flexible heavy thread in a homogeneous field of gravity (Figure 1). With a static sag that is small compared to the span length $\left(f_S << l\right)$, the curvature and tension of the conductor can be considered constant, and the conductor configuration in the equilibrium state is a parabola $y_S = f_S\left(1-4x^2/l^2\right) = f_S\sigma\left(x\right)$.

It is assumed that the shape of vertical oscillations relative to the static equilibrium position coincides with the shape function $\sigma\left(x\right)$: $y\left(t,s\right)-y_S\left(s\right)=z\left(t\right)\sigma\left(x\right)$. The state parameters are: the sag variation $z\left(t\right)$, the angle of deviation of the conductor plane from the vertical $\varphi\left(t\right)$, temperature $\theta\left(t\right)$. Ambient air temperature θ_B is at the moment of self-excitation of vibrations.

There are three conductor states:

- natural:

$$f = f_0,\ \varphi = 0,\ \theta = \theta_0,$$

when there are no deformations (θ_0 – installation temperature before static deformations occur in it);
- stationary:

$$f = f_S,\ \varphi = 0,\ \theta = \theta_S,$$

when the deformations and temperature correspond to equilibrium with the electric voltage switched on and stationary heat release Q_S;
- perturbed (vibration mode):

$$f = f_S + z,\ \varphi \neq 0,\ \theta = \theta_S + \eta,$$

when perturbations are imposed on static deformations and temperature z, η, due to changes in configuration, heat release $Q_V(t)$ and heat transfer.
The lengths of the conductor in the natural, static and perturbed states are respectively equal to:

$$L_0 = l\left(1 + \frac{8f_0^2}{3l^2}\right),\ L_S = l\left(1 + \frac{8f_S^2}{3l^2}\right),$$
$$L_t = l\left[1 + \frac{8(f_S + z)^2}{3l^2}\right].$$

Static and total deformations are:

$$\varepsilon_S = \frac{8}{3l^2}\left(f_S^2 - f_0^2\right),$$
$$8\%= \frac{8}{3l^2}\left[(f_S + z)^2 - f_0^2\right] = \qquad (1)$$
$$= \varepsilon_S + \frac{8}{3l^2}\left(z^2 + 2zf_S\right) = \varepsilon_S + \varepsilon.$$

When describing coupled thermomechanical vibrations, the role of the elastic potential W passes to the free energy [8]:

$$F = W - 3k\alpha_T\theta e,$$

where $k = E/3(1-2\nu)$ is modulus of volume elasticity, e is volume strain, α_T is coefficient of linear thermal expansion. Expressing the volume strain in terms of the elongation strain by the formula $e = (1-2\nu)\varepsilon$, find the free energy expression for the entire volume of the conductor:

$$F = Bl\left[\frac{1}{2}(\varepsilon_S + \varepsilon)^2 - \alpha_T(\theta_S + \eta - \theta_0)(\varepsilon_S + \varepsilon)\right] +$$
$$+ \frac{2}{3}mgl\left[(f + z)(1 - \cos\varphi) - z - f_S + f_0\right].$$

Here $B = ES$ is tensile conductor stiffness; $S = \pi a^2$ – effective conductor cross section with section radius a; m – linear mass.
Kinetic energy is

$$K = m\frac{V_0^2}{2}\int_{-l/2}^{l/2}\sigma^2(x)dx = \frac{4}{15}mlV_0^2,$$

where $V_0 = \sqrt{\dot{z}^2 + (f_S + z)^2\dot{\phi}^2}$ – the speed of the midpoint of the span.
Let's limit ourselves to taking into account the aerodynamic drag. With normal flow, the linear aerodynamic force is equal to $F = -\rho_B \boldsymbol{V}|\boldsymbol{V}|C_D a$, where ρ_B, $\boldsymbol{V}$ – density and velocity vector of incoming air. The drag coefficient for a circular cylinder in the current range of Reynolds numbers $\mathrm{Re} = 10^3 - 10^5$ changes slightly and can be taken equal to $C_D = 1{,}2$.
Dissipative function [9] is

$$\Phi = \rho_B C_D a\frac{|V_0|^3}{3}\int_{-l/2}^{l/2}\sigma^3(x)dx = \frac{16}{105}\rho_B C_D al|V_0|^3.$$

We write the vibration equations in the Lagrange form:

$$\frac{8}{15}ml\left[\ddot{z} - (f + z)\dot{\phi}^2\right] + \frac{16}{35}\rho_в alc_L V_0\dot{z} +$$

$$+Bl[\varepsilon+\varepsilon_S-\alpha_T(\theta_S-\theta_0)-\alpha_T\eta]\frac{d\varepsilon}{dz}-$$
$$-\frac{2}{3}mgl+\frac{2}{3}mgl(1-cos\,\phi)=0,$$
$$\frac{8}{15}ml\left[\ddot{\phi}(z+f)^2+2\dot{\phi}\dot{z}(z+f)\right]+$$
$$+\frac{16}{35}\rho_B alc_l V_0(f_S+z)^2\dot{\phi}^2+$$
$$+\tfrac{2}{3}mgl(z+f)\,sin\,\phi=0. \quad (2)$$

2. CONDUCTOR TEMPERATURE

The conductor heating occurs due to Joule heat generation, the power of which is uniform in volume. When the conductor oscillates, it acquires a velocity relative to the air, as a result of which heat transfer occurs, uneven on the surface. Taking into account the unevenness would dramatically complicate the model, so the heat transfer is taken into account according to Newton's law with an averaged heat transfer coefficient over the cross section and over the span length. In this approximation, the temperature distribution is axisymmetric and constant along the conductor length, and the first law of thermodynamics takes the form [8, 10]:

$$\rho c\frac{\partial\eta}{\partial t}+3k\alpha_T\theta_0\frac{de}{dt}=$$
$$=\lambda\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial(\theta_S+\eta-\theta_0)}{\partial r}\right)+Q_S+Q_V(t), \quad (3)$$

where ρ and c – density and specific heat capacity of the conductor.

The stationary component of the temperature satisfies the equation and the boundary conditions:

$$\lambda\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial(\theta_S-\theta_0)}{\partial r}\right)+Q_S=0;$$
$$\left.\frac{\partial(\theta_S-\theta_0)}{\partial r}\right|_0=0, \quad (4)$$
$$\left.\frac{\partial(\theta_S-\theta_0)}{\partial r}\right|_a+\alpha\left.(\theta_S-\theta_0-\theta_B)\right|_a=0.$$

Passing, as before, from e to ε and taking into account (4), we transform (3) to the form

$$\rho c\frac{\partial\eta}{\partial t}+E\alpha_T\theta_0\frac{d\varepsilon}{dt}=$$
$$=\lambda\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\eta}{\partial r}\right)+Q_V(t). \quad (5)$$

Let us apply to both parts of this equation the averaging over the cross-sectional area:

$$\rho c\int_0^a\frac{\partial\eta}{\partial t}rdr=\lambda\int_0^a\frac{\partial}{\partial r}\left(r\frac{\partial\eta}{\partial r}\right)dr+$$
$$+\int_0^a Q_V rdr-\int_0^a\alpha_T E\theta_0\,\dot{\varepsilon}rdr.$$

On the surface $r=a$, Newton's heat transfer condition must be satisfied both for the total temperature $\theta_S-\theta_0+\eta$ and for its stationary component; hence it follows that the variable component must satisfy the condition $-\lambda\partial\eta/\partial r=\alpha\eta$. Taking this into account and introducing the cross-sectional average temperature $\tilde{\eta}=2a^{-2}\int_0^a r\eta dr$, we arrive at an equation for its change:

$$\rho c\dot{\tilde{\eta}}=Q_V-\frac{2\alpha}{a}\tilde{\eta}-\alpha_T E\tilde{\theta}_0\dot{\varepsilon}. \quad (6)$$

Here, omitting for brevity a rather elementary justification, the result is used: for the Fourier numbers $\mathrm{Fo}=\lambda/\rho ca^2\omega$, characteristic of the modes of dancing of the OHL, the amplitude of the periodic temperature component is almost constant over the conductor cross section, which gave reason to replace the surface temperature $\eta(a)$ in (6) with its average value $\tilde{\eta}$.

3. HEAT RELEASE AND HEAT TRANSFER

The power of heat release in the span of a conductor with electrical resistance R is equal to $W^+=U^2/R=I^2R$. In the general case, a change in resistance changes the current and voltage simultaneously, but two limiting cases can be distinguished: a circuit with a stabilized voltage and a circuit with a stabilized current.

In the first case, as the resistance increases, the heat release decreases, and in the second case, it increases. Small fluctuations in resistance cannot change the current in the OHL conductors, which is determined by the large load resistance, so the second option is of practical importance $W^{+} = I^{2}R(1+\cos 2\omega_{e}t)$, where I – effective current, ω_{e} – its frequency. Since the oscillation frequency of the conductor is small compared to ω_{e}, we can neglect the variable component in the expression for W^{+}.

The change in conductor resistance R depends on the change in length, cross-sectional area (tensoresistive effect [11]), and the temperature dependence of resistivity:

$$\frac{dR}{R} = \left(\frac{d\rho_e}{\rho_e} + \frac{dl}{l} - \frac{dS}{S}\right);$$
$$\frac{dl}{l} = \varepsilon + \alpha_T\eta,\ \frac{d\rho_e}{\rho_e} = \beta_1\eta,$$
$$\frac{dS}{S} = 2\left(\alpha_T\eta - \nu\varepsilon\right);$$

where ρ_e – resistivity, $\beta_1 \approx 4.10^{-3}\text{K}^{-1}$ – coefficient of temperature dependence of electrical resistivity, $\beta_2 = 1 + 2\nu$ – coefficient characterizing the tensoresistive effect, ν – Poisson's ratio of conductor material. As a result, taking into account that $\alpha_T \ll \beta_1$, we find

$$R = 4\rho_e\left(\beta_1\theta + \beta_2\varepsilon\right)l/\pi d^2,$$

and the variable component of the heat release power per unit volume of the conductor is (hereinafter, the tilde sign above the variable η is omitted):

$$Q_V\left(t\right) = 16\frac{I^2\rho_e}{\pi^2 d^4}\left(\beta_1\eta + \beta_2\varepsilon\right). \tag{7}$$

Heat transfer depends on the speed of blowing with ambient air and on the presence of icy deposits on the surface of the wire. Let's look at the first of these factors. Heat transfer from the wire surface is taken into account according to Newton's law. The heat transfer coefficient is determined by the Nusselt number $\alpha = \text{Nu}\cdot\lambda_B/d$, depending on the Reynolds and Grashof numbers

$$\text{Re} = Vd/\nu_B,$$
$$\text{Gr} = g\beta_B d^3\left(\eta_S - \eta_B\right)/\nu_B^2.$$

Here: $\lambda_B, \nu_B, \beta_B$ are coefficient of thermal conductivity, kinematic viscosity and the coefficient of volumetric thermal expansion of air, d – conductor diameter. Approximation of the data contained in [12, 13] leads to the following relationship (at a constant flow rate):

$$\alpha = 0{,}47\frac{\lambda_B}{d}\sqrt{\text{Re} + \sqrt{\text{Gr}/2}}. \tag{8}$$

We assume that this dependence is also true for the instantaneous values of the Reynolds number with its periodic change (quasi-stationary model), and the conductor speed during vibrations will be replaced by its average value over the span $\tilde{V} = 2V_0/3$. At low velocities $\tilde{V}$, which are characteristic of the initial stage of soft self-excitation of oscillations ($\text{Re} \ll \sqrt{\text{Gr}/2}$), the determining factor is the free convection effect and $\alpha = 0.39\sqrt[4]{\text{Gr}}\,\lambda_B/d$.

The reason for the dance is traditionally associated with the icing of the wire [1, 2], as a result of which its cross section becomes similar to a wing profile, and the excitation of the dance is likened to a flutter. However, practice shows [1] that dancing can also occur with a uniform deposition of a thin layer of ice, in which the section does not acquire an aerodynamic quality. From the standpoint of the thermomechanical model, the role of the ice sheath may be to change the thermal regime of the wire. In this regard, it is necessary to assess the degree of this influence.

Taking into account the significant uncertainty in the shape of ice deposits, for evaluation we consider a simplified auxiliary problem, replacing the axisymmetric model with a flat one: an infinite plate with a thickness of $2a$ (analogous to a wire) contacts with plates with a thickness δ (analogous to an ice shell). Considering, due to symmetry, half of the region and placing the origin of coordinates on the contact surface, we write down the equations of heat conduction:

$$\dot{T}_1 = \chi_1 T_1'' + \frac{Q}{\rho_1 c_1} e^{i\omega t} \; (-a \le x \le 0);$$
$$\dot{T}_2 = \chi_2 T_2'' \;\; (0 < x < \delta).$$

Assuming $T_1 = e^{i\omega t}\eta_1(x),\; T_2 = e^{i\omega t}\eta_2(x)$, we arrive at the equations for the amplitudes

$$-i\omega\eta_1 + \chi_1\eta_1'' + q = 0, \; -i\omega\eta_2 + \chi_2\eta_2'' = 0;$$

$\chi_{1,2} = \lambda_{1,2} / \rho_{1,2}c_{1,2}$ – coefficients of thermal conductivity of the conductor and ice, $q = Q/\rho_1 c_1$. The solutions of the equations have the form of heat waves $\eta_{1,2} = \exp\left(\pm\nu_{1,2}x\right)$, where $\nu_{1,2} = \sqrt{i\omega/\chi_{1,2}}$. The characteristic values of the thermal parameters of ice are [14, 16]:

$\rho_2 :\;$ 200 kg/m^3, $c_2 :\;$ 2 kJ/kg·K,
$\lambda_2 :\;$ 0.1 W/m·K (granular frost),
$\rho_2 :\;$ 900 kg/m^3, $c_2 :\;$ 2 kJ/kg·K,
$\lambda_2 :\;$ 2.2 W/m·K (ice).

At vibration frequencies ~1 Hz the value $|\nu_2| \approx 10^3 - 10^4$ 1/m, $|\nu_1| \approx 250$. It follows that the variable temperature component does not penetrate deeply into the ice shell and attenuates at a distance of less than 1 mm from the wire surface. This allows us to represent the temperature amplitudes in the form:

$$\eta_1 = \left| -\frac{iq}{\omega} + A_1 e^{\nu_1 x} + A_2 e^{-\nu_1 x} \right|, \quad \eta_2 = B\left|e^{-\nu_2 x}\right|,$$

under boundary conditions:

$$\eta'(-a) = 0, \;\; \lambda_1\eta_1'(0) = \lambda_2\eta_2'(0),$$
$$\eta_1(0) = \eta_2(0).$$

Determining the constants, we find

$$\eta_1 \approx -\frac{q}{\omega}\left(1 - \frac{\varepsilon}{1+\varepsilon}e^{|\nu_1|x} - \frac{\varepsilon}{1+\varepsilon}e^{-|\nu_1|(2a+x)}\right).$$

Taking into account that $\varepsilon = \nu_2\lambda_2/\nu_1\lambda_1 = 0.01 \div 0.1$, as well as the estimate $\nu_1 a \approx 2 \div 5$, we will make up for the interface $x = 0$ the ratio of the type of heat transfer condition from the conductor to the icy shell $-\lambda_1\eta_1' = \alpha_{ecv}\eta_1$, from which follows an approximate estimate of the equivalent heat transfer coefficient:

$$\alpha_{экв} = -\lambda_1 \frac{\eta_1'}{\eta_1}(0) \approx \lambda_1\nu_1\varepsilon = \lambda_2\nu_2. \qquad (9)$$

Therefore, for an iced wire, the variable temperature can be calculated in the same way as for a bare conductor when using the value α_{ecv} as the heat transfer coefficient in equation (6). Note that for $\nu_2\delta > 2$, which is already achieved at $\delta > 1$ mm, the heat transfer with respect to the variable temperature does not depend on the speed of blowing the conductor with the air flow.

4. RESOLUTION SYSTEM OF EQUATIONS AND SIMILARITY PARAMETERS

Let's move on to compiling a resolving system of equations. Let us transform (2), substituting expressions for deformations (1) into them and passing to dimensionless variables:

$$q=\frac{z}{f_S},\ \ \overline{t}=t/\tau,\ \ \tau=\sqrt{\frac{4f_S}{5g}},\ \ \vartheta=\frac{\eta}{\theta_S}. \quad (10)$$

Note that the accepted time scale differs by only 10% from the oscillation period of a mathematical pendulum with length f_S. Let us represent the heat transfer coefficient (8) as

$$\alpha=\lambda_B\mu(q,\varphi)/d\text{, где }\mu(\dot{q},\dot{\phi})=\sqrt{\alpha_1 v(\dot{q},\dot{\phi})+\alpha_2^2},$$

$$v(\dot{q},\dot{\phi})=\sqrt{\dot{q}^2+(1+q)^2\dot{\phi}^2},,$$

$$\alpha_1=0.44f_S a/v_B\tau,\ \ \alpha_2^2=0.15\sqrt{Gr}.$$

Considering that in static equilibrium the tension in the conductor $T=B(\varepsilon_S-\varepsilon_T)$, where $\varepsilon_T=\alpha_T(\theta_S-\theta_0)$, is related to the sag by the ratio $8Tf_S=mgl^2$, we finally obtain

$$\ddot{q}+\varsigma v(\dot{q},\dot{\phi})\dot{q}-(1+q)\dot{\phi}^2+ +(1+2\beta)q+\beta(3q^2+q^3)+1- -\cos\phi-\gamma\vartheta(1+q)=0, \quad (11)$$

$$(1+q)\ddot{\phi}+\varsigma v(\dot{q},\dot{\phi})(1+q)\dot{\phi}+ +2\dot{q}\dot{\phi}+\sin\phi=0, \quad (12)$$

Substituting (7), (8) or (9) into equation (6), taking into account expression (1) for strains and passing to the previously accepted dimensionless values, we finally obtain:

$$\dot{\vartheta}=\vartheta\xi_1+\xi_2(q^2+2q)- -\chi\dot{q}(1+q)-\vartheta\Delta\mu(\dot{q},\dot{\phi}). \quad (13)$$

The parameters ς, β, γ, ξ_1, ξ_2, ϑ are determined by the formulas (14) below.

The first and second terms in the right part determine the change in the heat output power due to the dependence of the electrical resistance on temperature and on the conductor deformation (strain-resistive effect); the third term describes thermoelastic connectivity – cooling by increasing the deformation (downward movement) and heating by decreasing it (upward movement); the last term determines the heat transfer due to the conductor movement relative to the air during vibrations.

Thus, the system is described by equations (11)–(13) and a set of dimensionless similarity parameters:

$$\begin{gathered}\alpha_1=0.22\frac{f_S d}{v_B\tau},\ \ \alpha_2^2=0.15\sqrt{Gr},\\ \beta=\frac{64Bf_S^3}{3mgl^4},\ \ \xi=16\frac{I^2\rho_e\tau}{\rho c\theta_S\pi^2 d^4},\\ \xi_1=\xi\theta_S\beta_1,\ \ \xi_2=\frac{8f_S^2}{3l^2}\xi\beta_2,\\ \Delta=4\frac{\lambda_B\tau}{d^2\rho c},\ \ \chi=\frac{16f_S^2}{3l^2}\frac{\alpha_T E\theta_0}{\rho c\theta_S},\\ \gamma=\frac{8f_S\alpha_T\theta_S B}{mgl^2}=\frac{\varepsilon_T}{\varepsilon_S-\varepsilon_T},\\ \varsigma=\frac{3\rho_в f_S d c_L}{7m}.\end{gathered} \quad (14)$$

In the presence of icy deposits with the parameters indicated above λ_2, ρ_2, c_2 heat transfer to air is replaced by heat transfer to ice and parameter Δ in (14) is taken as $\Delta=4\tau\sqrt[4]{1+2\beta}\sqrt{\lambda_2\rho_2 c_2}/\rho cd$ and the parameter μ in (13) is assumed to be 1.

It is advisable to express the similarity parameters (14) in terms of operational and easily measured values in the experiment, fixing the design parameters and characteristics of aluminum as the predominant conductor material:

$$\begin{gathered}\alpha_1=5.5\cdot10^4 d\sqrt{f_S},\ \ \alpha_2=44\sqrt[4]{d^3(\theta_S-\theta_B)},\\ \beta=1.09\cdot10^4\frac{f_S^3}{l^4},\ \ \xi=5.33\cdot10^{-15}\frac{I^2\sqrt{f_S}}{\theta_S d^4},\\ \xi_1=21.3\cdot10^{-18}\frac{I^2\sqrt{f_S}}{d^4},\\ \xi_2=22.7\cdot10^{-15}\frac{I^2\sqrt{f_S^5}}{d^4 l^2\theta_S},\\ \Delta=1.6\cdot10^{-8}\frac{\sqrt{f_S}}{d^2},\ \ \chi=3.69\frac{f_S^2\theta_0}{l^2\theta_S},\\ \gamma=490\frac{f_S\theta_S}{l^2},\ \ \varsigma=0.67\frac{f_S d}{m};\end{gathered} \quad (15)$$

in the presence of ice

$$\Delta = \Delta_0 \frac{\sqrt{f_S}}{d}, \quad \Delta_0 = 10^{-4} - 10^{-3}.$$

In an experimental study of the excitation of vibrations on a laboratory model, it is most convenient to vary the measurable operational parameters of the model: the conductor temperature before turning on the current θ_0, current I, temperature θ_S and sag f_S in the heated state, leaving the design parameters of the model unchanged.

Under the conditions of a laboratory experiment, the air temperature θ_B and the conductor temperature in the natural (installation) state θ_0 are naturally considered to be the same. Therefore, the entire set of coefficients in (15) is expressed in terms of current I, sag f_S, and stationary temperature θ_S. In this case, it is advisable to empirically establish the dependence of the temperature and the sagging arrow on the current. This will allow, in the experimental study of self-excitation, to express all similarity parameters (14), (15) related to a given physical model, through a single and easily adjustable quantity – the current.

CONCLUSION

The resulting system of equations and a set of dimensionless similarity parameters are intended for the primary analytical analysis of the conditions for self-excitation of thermomechanical oscillations on a laboratory scale model and for transferring the results to natural conductors of overhead lines. In the future, it is planned to use the results obtained on the analytical model to build a detailed model that more fully takes into account the features of overhead power lines and their operating conditions.

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REFERENCES

1. **Yakovlev L.V.** Plyaska provodov na vozdushnyh liniyah elektroperedachi i sposoby bor'by s neyu / Prilozhenie k zhurnalu «Energetik» [Galloping of overhead power lines conductors and ways to deal with it / Appendix to the magazine "Energetik"], Iss. 11 (47). Moscow, NTF «Energoprogress» Publ., 2002. 96 p. (in Russian).
2. **Alexandrov G.P.** (ed.) Proektirovanie linij elektroperedachi sverhvysokogo napryazheniya [Design of ultra-high voltage power transmission lines]. St. Petersburg, "Energoatomizdat" Publ., 1993. 368 p. (in Russian).
3. **Landa P.S.** Nelinejnye kolebaniya i volny [Nonlinear vibrations and waves]. Moscow, Nauka-Fizmatlit Publ., 1997. 495 p. (in Russian).
4. **Babitsky V.I., Landa P.S.** Avtokolebaniya v sistemah s inercionnym vozbuzhdeniem [Self-vibrations in systems with inertial excitation] // Dokl. USSR Academy of Sciences, 1982, Vol. 266, No. 5. Pp. 1087–1089. (in Russian).
5. **Penner D.I., Duboshinsky Ya.B., Duboshinsky D.B., Petrosov V.A., Porotnikov A.A.** Parametricheskie termomekhanicheskie kolebaniya [Parametric thermomechanical vibrations]. In book: Nekotorye voprosy vozbuzhdeniya nezatuhayushchih kolebanij [Some issues of excitation of undamped oscillations]. Vladimir, VGPI Publ., 1974. Pp. 168–183 (in Russian).
6. **Galkin Yu.V., Duboshinsky D.B., Vermel A.S., Penner D.I.** Vertikal'nye termomekhanicheskie kolebaniya [Vertical thermomechanical vibrations]. Ibid. Pp. 150–158. (in Russian).

7. **Feldshteyn V.A.** Termomekhanicheskie kolebaniya tokonesushchih provodnikov [Thermomechanical vibrations of current-carrying conductors] // Journal of Applied Mechanics and Technical Physics, 2017, Vol. 58, No. 6. Pp. 158–166. (in Russian). DOI: 10.15372/PMTF20170615
8. **Lurie A.I.** Teoriya uprugosti [Theory of elasticity]. Moscow, "Nauka" Publ., 1970. 940 p. (in Russian).
9. **Lurie A.I.** Analiticheskaya mekhanika [Analytical mechanics]. Moscow, GIFML Publ., 1961. 824 p. (in Russian).
10. **Sneddon I.N., Berry D.S.** Klassicheskaya teoriya uprugosti [Classical theory of elasticity]. Moscow, "Fizmatgiz" Publ., 1961. 219 p. (in Russian).
11. **Klokova N.P.** Tenzorezistory: teoriya, metody rascheta, razrabotki [Tensoresistors: theory, calculation methods, development]. Moscow, "Mashinostroenie" Publ., 1990. 224 p. (in Russian).
12. **Wang H.** Osnovnye formuly i dannye po teploobmenu dlya inzhenerov. Spravochnik [Basic formulas and heat exchange data for engineers. Guide]. Moscow, "Atomizdat" Publ., 1979. 216 p. (in Russian).
13. Metodika rascheta predel'nyh tokovyh nagruzok po usloviyam sohraneniya mekhanicheskoj prochnosti provodov i dopustimyh gabaritov vozdushnyh linij / Standart organizacii «FSK EES» 56947007-29.240.55.143-2013 [The method of calculating the maximum current loads under the conditions of maintaining the mechanical strength of wires and the permissible dimensions of overhead lines / The standard of FGC UES 56947007-29.240.55.143-2013]. Moscow, "FGC UES" Publ., 2013. 67 p. (in Russian).
14. **Osokin N.I., Sosnovsky A.V., Chernov R.A.** Koefficient teploprovodnosti snega i ego izmenchivost' [Coefficient of thermal conductivity of snow and its variability] // Cryosphere of the Earth. 2017, Vol. XXI, No. 3. – Pp. 60-68. (in Russian).
15. GOST 839-80. Provoda neizolirovannye dlya vozdushnyh linij elektroperedachi. Tekhnicheskie usloviya [GOST 839-80. Non-insulated wires for overhead power lines. Technical conditions]. Moscow, Branch of IPK Standards Publ., Printing House "Moskovsky Pechatnik", 2002. 21 p. (in Russian).
16. Rukovodstvo po raschyotu rezhimov plavki gololeda na grozozashchitnom trose so vstroennym opticheskim kabelem (OKGT) i primeneniyu raspredelyonnogo kontrolya temperatury OKGT v rezhime plavki / Standart organizacii «FSK EES» 56947007-29.060.50.122-2012 [Guidelines for the calculation of ice melting modes on a lightning-proof cable with a built-in optical cable (OCGT) and the use of distributed temperature control of OCGT in the melting mode / The standard of FGC UES 56947007-29.060.50.122-2012]. Moscow, "FGC UES" Publl., 2012. 119 p. (in Russian).

СПИСОК ЛИТЕРАТУРЫ

1. **Яковлев Л.В.** Пляска проводов на воздушных линиях электропередачи и способы борьбы с нею / Приложение к журналу «Энергетик», выпуск 11 (47). – М.: Изд-во НТФ «Энергопрогресс», 2002. – 96 стр.
2. **Александров Г.П.** (ред.). Проектирование линий электропередачи сверхвысокого напряжения. – С-Пб.: Изд-во «Энергоатомиздат», 1993. – 368 с.
3. **Ланда П.С.** Нелинейные колебания и волны. – М.: Изд-во «Наука. Физматлит», 1997. – 495 с.
4. **Бабицкий В.И., Ланда П.С.** Автоколебания в системах с инерционным возбуждением // ДАН СССР. 1982. Т. 266. № 5. – С. 1087–1089.
5. **Пеннер Д.И., Дубошинский Я.Б., Дубошинский Д.Б., Петросов В.А., Поротников А.А.** Параметрические термо-

механические колебания. / В кн. Некоторые вопросы возбуждения незатухающих колебаний. – Владимир: Изд-во ВГПИ, 1974. – С. 168–183.
6. **Галкин Ю.В., Дубошинский Д.Б., Вермель А.С., Пеннер Д.И.** Вертикальные термомеханические колебания / Там же. – С. 150–158.
7. **Фельдштейн В.А.** Термомеханические колебания токонесущих проводников // ПМТФ. 2007. Т. 58. № 6. – С. 158–166.
8. **Лурье А.И.** Теория упругости. – М.: Изд-во «Наука», 1970. – 940 с.
9. **Лурье А.И.** Аналитическая механика. – М.: Изд-во ГИФМЛ, 1961. – 824 с.
10. **Снеддон И.Н., Берри Д.С.** Классическая теория упругости. – М.: Изд-во «Физматгиз», 1961. – 219 с.
11. **Клокова Н.П.** Тензорезисторы: теория, методы расчета, разработки. – М.: Изд-во «Машиностроение». 1990. – 224 с.
12. **Уонг Х.** Основные формулы и данные по теплообмену для инженеров. Справочник. – М.: Изд-во «Атомиздат», 1979. – 216 с.
13. Методика расчета предельных токовых нагрузок по условиям сохранения механической прочности проводов и допустимых габаритов воздушных линий / Стандарт организации «ФСК ЕЭС» 56947007-29.240.55.143-2013. – М.: Изд-во ОАО «ФСК ЕЭС». 2013. – 67 с.
14. **Осокин Н.И, Сосновский А.В., Чернов Р.А.** Коэффициент теплопроводности снега и его изменчивость // Криосфера Земли. 2017. Т. XXI. № 3. – С. 60–68. DOI: 10.21782/KZ1560-7496-2017-3(60-68)
15. ГОСТ 839-80. Провода неизолированные для воздушных линий электропередачи. Технические условия. – М.: Филиал ИПК Изд-во стандартов, типография «Московский печатник». 2002. – 21 с.
16. Руководство по расчёту режимов плавки гололеда на грозозащитном тросе со встроенным оптическим кабелем (ОКГТ) и применению распределённого контроля температуры ОКГТ в режиме плавки / Стандарт организации «ФСК ЕЭС» 56947007-29.060.50.122-2012. – М.: Изд-во ОАО «ФСК ЕЭС». 2012. – 119 с.

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