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NUMERICAL SIMULATION OF GAS ATOM COORDINATE DISPERSION IN A MATHEMATICAL MODEL OF DEEP BLAST COMPACTION FOR SUBSIDENCE SOILS

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Abstract: within the framework of mathematical modeling of geological systems, applied inverse problems arise that require solutions. This paper presents approaches to constructing solutions (approximate and explicitly analytical) of boundary value problems describing the compaction of subsidence soils by the method of deep explosions. Numerical simulation of the dispersion of the coordinates of the gas atoms formed in the subsidence soil as a result of a deep explosion of a concentrated charge is carried out. Approximate solutions of the problem are constructed for soils with characteristic properties of isotropy and anisotropy for cases of complete absorption of gas atoms by the surrounding soil and complete reflection from it.

Keywords: subsidence soil, loess, soil compaction, deep explosion, mathematical modeling, numerical modeling, dispersion of atomic coordinates

ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ ДИСПЕРСИИ КООРДИНАТ АТОМОВ ГАЗА В МАТЕМАТИЧЕСКОЙ МОДЕЛИ УПЛОТНЕНИЯ ПРОСАДОЧНЫХ ГРУНТОВ ГЛУБИННЫМИ ВЗРЫВАМИ

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Аннотация. В рамках математического моделирования геологических систем возникают прикладные обратные задачи, требующие решения. В настоящей работе представлены подходы построения решений (приближенных и в явном аналитическом виде) краевых задач, описывающих уплотнение просадочных грунтов методом глубинных взрывов. Проведено численное моделирование дисперсии координат атомов газа, образующегося в просадочном грунте, в результате глубинного взрыва сосредоточенного заряда. Построены приближённые решения поставленной задачи для грунтов с характерными свойствами изотропности и анизотропности для случаев полного поглощения атомов газа окружающим его грунтом и полного отражения от него.

Ключевые слова: просадочный грунт, лёсс, уплотнение грунта, глубинный взрыв, математическое моделирование, численное моделирование, дисперсия координат атомов

INTRODUCTION

Engineering-geological design of buildings and structures requires rational and time-consuming numerical calculations of the density and deformation characteristics of soil. This is especially relevant on subsidence loess soils, which are rather mysterious and debatable by origin Quaternary rocks. Subsidence soil strata are widespread (more than 17%) on the territory of Russia. Their greatest concentration (about 80%) is fixed in the North Caucasus, the South of Russia, Siberia, Yakutia, etc. [1, 2]. There are different methods of soil compaction [3]. Here we shall consider a method of soil compaction by means of deep explosions of the concentrated charges. This method of compaction of subsidence soils in its practical implementation demonstrates low production costs and economic efficiency [4].

A DIRECT PROBLEM

Changes in values of the average density of the compacted soil skeleton q per unit time t at a point (x_1, x_2, x_3) in a given space from a concentrated source of explosive charge is described as an initial boundary problem, typical for the case of complete absorption of gas atoms by the surrounding soil (compaction of subsiding soil is realized) [5].

$$\frac{\partial q}{\partial t} + U\frac{\partial q}{\partial x} = \frac{\partial}{\partial x}K_x\frac{\partial q}{\partial x} + \frac{\partial}{\partial y}K_y\frac{\partial q}{\partial y} + \frac{\partial}{\partial z}K_z\frac{\partial q}{\partial z} + f, \quad t \in [t_0, T] \tag{1}$$

with initial conditions

$$q(t_0, x, y, z) = Q\delta\left(x - x^0\right)\delta\left(y - y^0\right)\delta\left(z - z^0\right), \tag{2}$$

and boundary conditions

$$q(t, x, y, z)\Big|_{z=z^0} = 0, \; t > t_0. \tag{3}$$

In the case of complete reflection of the gas atoms from the surrounding ground, when the ground ejection to the surface is realized, the changes of mean ground density values q are described by the initial-boundary problem (1), (2),

$$K_{zz}\frac{\partial q}{\partial z}\Big|_{z=z^0} = 0, \; t > t_0, \tag{4}$$

where $Q = const > 0$, $\delta(x)$ is the Dirac δ-function, $z^0 = H$ is the depth of the concentrated explosive charge, $U = const$ is the horizontal transfer rate (along the axis Ox), K_x, K_y, K_z is the diffusion coefficients.

The explosive gas source function is represented as

$$f(t, x, y, z) = Q \cdot R(t, x, y, z) = Q\delta(t - t_0)\delta\left(x - x^0\right) \times \delta\left(y - y^0\right)\delta(z - H). \tag{5}$$

Suppose that U, K_x, K_y, K_z are continuous functions of the argument z:

$$U = U(z),$$
$$K_x = K_x(z), \; K_y = K_y(z), \; K_z = K_z(z)$$

In addition to equation (1) to calculate the average values of the compacted soil skeleton density, it is also possible to use the function [6, 7]

$$q'(t, x, y, z) = \frac{Q}{(2\pi)^{3/2}\sigma_x(t)\sigma_y(t)\sigma_y(t)} \times \exp\left\{-\left[\frac{\left(x - \overline{U}t\right)^2}{2\sigma_x^2(t)} + \frac{y^2}{2\sigma_y^2(t)} + \frac{(z - H)^2}{2\sigma_z^2(t)}\right]\right\}, \tag{6}$$

which is a Gaussian function of the density distributions of the soil skeleton, where $\sigma_x^2(t)$, $\sigma_y^2(t)$, $\sigma_z^2(t)$ are the dispersion coordinate changes of the gas atoms in the compacted soil, respectively along the axes Ox, Oy, Oz at the time t; $\sigma_x^2(t)$, $\sigma_y^2(t)$, $\sigma_z^2(t)$ are functions continuously differentiable by the argument t, $t \geq 0$.

According to [8], for $t \to \infty$

$$\frac{\sigma_x^2(t)}{t} \to \sigma_x^2, \; \frac{\sigma_y^2(t)}{t} \to \sigma_y^2, \; \frac{\sigma_z^2(t)}{t} \to \sigma_z^2,$$

where $\sigma_x^2 > 0$, $\sigma_y^2 > 0$, $\sigma_z^2 > 0$ are certain constants. Consequently, the following approximations are valid for $t \to \infty$:

$$\sigma_x^2(t)=\sigma_x^2 t,\ \sigma_y^2(t)=\sigma_y^2 t,\ \sigma_z^2(t)=\sigma_z^2 t. \quad (7)$$

The density of the compacted soil skeleton $q'(t,x,y,z)$, given by the Gaussian (6), when the approximate equations (7) are fulfilled, satisfies the equation

$$\frac{\partial q'}{\partial t}+\overline{U}\frac{\partial q'}{\partial x}-\frac{1}{2}\sigma_x^2\frac{\partial^2 q'}{\partial x^2}-\frac{1}{2}\sigma_y^2\frac{\partial^2 q'}{\partial y^2}-\frac{1}{2}\sigma_z^2\frac{\partial^2 q'}{\partial z^2}=0$$

and the initial condition (2). Let us rewrite equation (1) as

$$\begin{aligned}&\frac{\partial q}{\partial t}+\overline{U}\frac{\partial q}{\partial x}-\frac{1}{2}\sigma_x^2\frac{\partial^2 q}{\partial x^2}-\frac{1}{2}\sigma_y^2\frac{\partial^2 q}{\partial y^2}-\\&-\frac{1}{2}\sigma_z^2\frac{\partial^2 q}{\partial z^2}+\left(U(z)-\overline{U}\right)\frac{\partial q}{\partial x}=\\&=\frac{\partial}{\partial x}\left(K_x(z)-\frac{1}{2}\sigma_x^2\right)\frac{\partial q}{\partial x}+\\&+\frac{\partial}{\partial y}\left(K_y(z)-\frac{1}{2}\sigma_y^2\right)\frac{\partial q}{\partial y}+\\&+\frac{\partial}{\partial z}\left(K_z(z)-\frac{1}{2}\sigma_z^2\right)\frac{\partial q}{\partial z}+f.\end{aligned} \quad (8)$$

Obviously, the smaller the difference

$$\left(U(z)-\overline{U}\right),\ \left(K_x(z)-\frac{1}{2}\sigma_x^2\right),\ \left(K_y(z)-\frac{1}{2}\sigma_y^2\right),\ \left(K_z(z)-\frac{1}{2}\sigma_z^2\right)$$

differs from 0, the less q', given by the Gaussian (6), will differ from the exact solution q (if only q continuously depends on the coefficients of equation (1)). In order to $\overline{U}$, σ_x^2, σ_y^2, σ_z^2 have the smallest scattering on the interval of $[0,h]$ from $U(z)$, $K_x(z)$, $K_y(z)$, $K_z(z)$, where h is the depth of penetration of the gas atoms into the surrounding ground, it is sufficient to fulfill the conditions:

$$\begin{aligned}&\int_0^h\left(U(z)-\overline{U}\right)^2 dz\to\min_{\overline{U}},\\&\int_0^h\left(K_x(z)-\frac{1}{2}\sigma_x^2\right)^2 dz\to\min_{\sigma_x^2},\\&\int_0^h\left(K_y(z)-\frac{1}{2}\sigma_y^2\right)^2 dz\to\min_{\sigma_y^2},\\&\int_0^h\left(K_z(z)-\frac{1}{2}\sigma_z^2\right)^2 dz\to\min_{\sigma_z^2},\end{aligned} \quad (9)$$

then $\overline{U}$, σ_x^2, σ_y^2, σ_z^2, from the conditions (9), will take the form:

$$\begin{aligned}&\overline{U}=\frac{1}{2}\int_0^h U(z)dz,\ \sigma_x^2=\frac{2}{h}\int_0^h K_x(z)dz,\\&\sigma_y^2=\frac{2}{h}\int_0^h K_y(z)dz,\ \sigma_z^2=\frac{2}{h}\int_0^h K_z(z)dz.\end{aligned} \quad (10)$$

Consequently, the Gaussians $q'(t,x,y,z)$, given by expression (6), where $\sigma_x(t)$, $\sigma_y(t)$, $\sigma_z(t)$ are described by relations (7), (10), can be taken as an approximate solution of problems (1), (2), (3) and (1), (2), (4).

INVERSE PROBLEM FORMULATION

According to the known average values of the skeleton density $q_1(t,x,y,z)$ of the soil compacted by the method of deep explosion of the concentrated explosive source under the condition of complete gas atom reflection from the surrounding ground (release to the surface), or the average values of the skeleton density $q_2(t,x,y,z)$ of the compacted soil under the condition of complete gas absorption by the surrounding ground (compacting of the sagging soil), as well as by the given depth of placement of explosive H and the known values of explosive charge power Q, $\sigma_x^2(t)$, $\sigma_y^2(t)$ are the dispersions of coordinates of gas atoms in the surrounding soil, respectively along the axes Ox, Oy, to determine the unknown values $\sigma_z^2(t)$ dispersions of coordinates of gas atoms in the compacted soil along the axis Oz at the time of t.

NUMERICAL SIMULATION

Build an approximate solution of the inverse problem on the basis of the apparatus of numerical methods for solving transcendental equations. Let us apply the method of simple iteration [9, 10]. Consider two cases, according to the statement of the problem, for $q_1(t,x,y,z)$ and $q_2(t,x,y,z)$.

1) numerical solution of the applied problem by the simple iteration method (under the assumption that there is a complete reflection of the gas atoms by the surrounding ground, the surface emission is realized). Assuming that the interval $[a,b]$ is separated (a and b estimated, e.g., by selection or graphical method), which contains the sought root σ_z of the equation

$$q_1(t,x,y,z) = \frac{Q}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_z t^3}\times \\ \times\exp\left\{-\left(\frac{(x-\overline{U}t)^2}{2\sigma_x^2 t}+\frac{y^2}{2\sigma_y^2 t}\right)\right\}\times \\ \times\left[\exp\left\{-\frac{(z-H)^2}{2\sigma_z^2 t}\right\}+ +\exp\left\{-\frac{(z+H)^2}{2\sigma_z^2 t}\right\}\right]=0. \tag{11}$$

If the investigated geological medium has characteristic properties of anisotropy, the equation (11) is rewritten in the following form

$$\sigma_z = \frac{Q}{q_1(t,x,y,z)(2\pi)^{3/2}\sigma_x\sigma_y t^3}\times \\ \times\exp\left\{-\left(\frac{(x-\overline{U}t)^2}{2\sigma_x^2 t}+\frac{y^2}{2\sigma_y^2 t}\right)\right\}\times \\ \times\left[\exp\left\{-\frac{(z-H)^2}{2\sigma_z^2 t}\right\}+ +\exp\left\{-\frac{(z+H)^2}{2\sigma_z^2 t}\right\}\right],$$

Iterative successive approximations to the desired root will be defined according to representation:

$$\sigma_z^{(n+1)} = \frac{Q}{q_1(t,x,y,z)(2\pi)^{3/2}\sigma_x\sigma_y t^3}\times \\ \times\exp\left\{-\left(\frac{(x-\overline{U}t)^2}{2\sigma_x^2 t}+\frac{y^2}{2\sigma_y^2 t}\right)\right\}\times \\ \times\left[\exp\left\{-\frac{(z-H)^2}{2\left(\sigma_z^{(n)}\right)^2 t}\right\}+ +\exp\left\{-\frac{(z+H)^2}{2\left(\sigma_z^{(n)}\right)^2 t}\right\}\right]. \tag{12}$$

If the investigated geological medium has isotropic properties, then $\sigma_z=\sigma_y=\sigma_x$. Therefore, the equation (11) can be represented as follows

$$\sigma_z = \sqrt[3]{\frac{Q}{q_1(t,x,y,z)(2\pi)^{3/2}\sigma_z^2 t^3}}\times \\ \times\sqrt[3]{\exp\left\{-\left(\frac{(x-\overline{U}t)^2}{2\sigma_z^2 t}+\frac{y^2}{2\sigma_z^2 t}\right)\right\}}\times$$

$$\times\sqrt[3]{\left[\exp\left\{-\frac{(z-H)^2}{2\sigma_z^2 t}\right\}+\exp\left\{-\frac{(z+H)^2}{2\sigma_z^2 t}\right\}\right]}.$$

The iterative successive approximations to the desired root in this case will be determined instead of formula (12) by the following representation:

$$\sigma_z^{(n+1)}=\sqrt[3]{\frac{Q}{q_1(t,x,y,z)(2\pi)^{3/2}\left(\sigma_z^{(n)}\right)^2 t^3}}\times$$
$$\times\sqrt[3]{\exp\left\{-\left(\frac{\left(x-\bar{U}t\right)^2}{2\left(\sigma_z^{(n)}\right)^2 t}+\frac{y^2}{2\left(\sigma_z^{(n)}\right)^2 t}\right)\right\}}\times$$
$$\times\sqrt[3]{\left[\begin{array}{l}\exp\left\{-\dfrac{(z-H)^2}{2\left(\sigma_z^{(n)}\right)^2 t}\right\}+\\+\exp\left\{-\dfrac{(z+H)^2}{2\left(\sigma_z^{(n)}\right)^2 t}\right\}\end{array}\right]}. \qquad (13)$$

From the segment of the root separation $[a,b]$, we arbitrarily determine the first approximation $\sigma_z^{(1)}$ to the desired root (the first iteration).

The criterion for the end of the computational search for the solution of the equation is the fulfillment of the inequality

$$\left|\sigma_z^{(n)}-\sigma_z^{(n-1)}\right|<\varepsilon, \qquad (14)$$

where ε is the desired accuracy of calculations.

2) numerical construction of the solution of the set problem by the method of simple iteration for the case of complete absorption of gas atoms by the surrounding soil (realization of compaction of subsidence thickness). Assuming that the interval $[a,b]$ (as above, a andи b estimated by fitting method or graphical method) containing the required root of the equation

$$q_2(t,x,y,z)=\frac{Q}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_z t^3}\times$$
$$\times\exp\left\{-\left(\frac{\left(x-\bar{U}t\right)^2}{2\sigma_x^2 t}+\frac{y^2}{2\sigma_y^2 t}\right)\right\}\times \qquad (15)$$
$$\times\left[\begin{array}{l}\exp\left\{-\dfrac{(z-H)^2}{2\sigma_z^2 t}\right\}-\\-\exp\left\{-\dfrac{(z+H)^2}{2\sigma_z^2 t}\right\}\end{array}\right]=0.$$

Given that the studied geological medium has characteristic properties of anisotropy, let us transform equation (15) to the form:

$$\sigma_z=\frac{Q}{q_2(t,x,y,z)(2\pi)^{3/2}\sigma_x\sigma_y t^3}\times$$
$$\times\exp\left\{-\left(\frac{\left(x-\bar{U}t\right)^2}{2\sigma_x^2 t}+\frac{y^2}{2\sigma_y^2 t}\right)\right\}\times$$
$$\times\left[\exp\left\{-\frac{(z-H)^2}{2\sigma_z^2 t}\right\}-\exp\left\{-\frac{(z+H)^2}{2\sigma_z^2 t}\right\}\right].$$

The iterative process of successive approximations to the desired root will be numerically determined by the formula

$$\sigma_z^{(n+1)}=\frac{Q}{q_2(t,x,y,z)(2\pi)^{3/2}\sigma_x\sigma_y t^3}\times$$
$$\times\exp\left\{-\left(\frac{\left(x-\bar{U}t\right)^2}{2\sigma_x^2 t}+\frac{y^2}{2\sigma_y^2 t}\right)\right\}\times$$
$$\times\left[\begin{array}{l}\exp\left\{-\dfrac{(z-H)^2}{2\left(\sigma_z^{(n)}\right)^2 t}\right\}-\\-\exp\left\{-\dfrac{(z+H)^2}{2\left(\sigma_z^{(n)}\right)^2 t}\right\}\end{array}\right]. \qquad (16)$$

If the investigated geological medium has characteristic properties of isotropy, then $\sigma_z = \sigma_y = \sigma_x$. Consequently, the equation (15) is transformed to the form

$$\sigma_z = \sqrt[3]{\frac{Q}{q_2(t,x,y,z)(2\pi)^{3/2}\,\sigma_z^2 t^3}} \times \sqrt[3]{\exp\left\{-\left(\frac{\left(x-\overline{U}t\right)^2}{2\sigma_z^2 t}+\frac{y^2}{2\sigma_z^2 t}\right)\right\}} \times \sqrt[3]{\left[\exp\left\{-\frac{(z-H)^2}{2\sigma_z^2 t}\right\}-\exp\left\{-\frac{(z+H)^2}{2\sigma_z^2 t}\right\}\right]}.$$

The iterative sequence of approximations to the required root instead of (16) will be constructed according to the formula

$$\sigma_z^{(n+1)} = \sqrt[3]{\frac{Q}{q_2(t,x,y,z)(2\pi)^{3/2}\left(\sigma_z^{(n)}\right)^2 t^3}} \times \sqrt[3]{\exp\left\{-\left(\frac{\left(x-\overline{U}t\right)^2}{2\left(\sigma_z^{(n)}\right)^2 t}+\frac{y^2}{2\left(\sigma_z^{(n)}\right)^2 t}\right)\right\}} \times \sqrt[3]{\left[\begin{array}{l}\exp\left\{-\dfrac{(z-H)^2}{2\left(\sigma_z^{(n)}\right)^2 t}\right\}- \\ -\exp\left\{-\dfrac{(z+H)^2}{2\left(\sigma_z^{(n)}\right)^2 t}\right\}\end{array}\right]}. \qquad (17)$$

The first approximation $\sigma_z^{(1)}$ to the required root (first iteration) is any σ_z value, which belongs to the root separation interval $[a,b]$.

The criterion for ending the computational process of finding a numerical solution to equation (15) is the fulfillment of the inequality

$$\left|\sigma_z^{(n)} - \sigma_z^{(n-1)}\right| < \varepsilon,$$

where ε is the desired accuracy of calculations.

CONCLUSIONS

The results of this study are analytical and numerical (iterative) representations of dispersion changes in the coordinates of gas atoms $\sigma_z^2(t)$ in the ground compacted by the deep blast method. The cases of complete absorption of gas atoms by the surrounding ground and the surface ejection of the ground during the explosion have been described. Isotropic and anisotropic properties of the compacted soil have been considered. The obtained expressions allow to perform computational experiments with a given accuracy.

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