

FORMATION OF COMPUTATIONAL SCHEMES OF ADDITIONAL TARGETED CONSTRAINTS THAT REGULATE THE FREQUENCY SPECTRUM OF NATURAL OSCILLATIONS OF ELASTIC SYSTEMS WITH A FINITE NUMBER OF DEGREES OF MASS FREEDOM, THE DIRECTIONS OF MOVEMENT OF WHICH ARE PARALLEL, BUT DO NOT LIE IN THE SAME PLANE PART 1: THEORETICAL FOUNDATIONS

Leonid S. Lyakhovich¹, Pavel A. Akimov²

¹ Tomsk State University of Architecture and Civil Engineering, Tomsk, RUSSIA

² National Research Moscow State University of Civil Engineering, Moscow, RUSSIA

Abstract: For some elastic systems with a finite number of degrees of freedom of masses, in which the directions of mass movement are parallel and lie in the same plane (for example, rods), special methods have been developed for creating additional constraints, the introduction of each of which purposefully increases the value of only one natural frequency and does not change any from the natural modes. The method of forming a matrix of additional stiffness coefficients that characterize such targeted constraint in this problem can also be applied when solving a similar problem for elastic systems with a finite number of degrees of mass freedom, in which the directions of mass movement are parallel, but do not lie in the same plane (for example, plates). At the same time, for such systems, only the requirements for the design schemes of additional targeted constraints are formulated, and not the methods for their creation. The distinctive paper proposes an approach that allows researcher to create computational schemes for additional targeted constraints for such systems. A variant of the computational scheme, represented by a rod system with one degree of activity, is considered. Some special properties of such targeted constraints are revealed. When forming the computational scheme, the material consumption for creating a constraint is minimized, and design restrictions are taken into account. Particular attention is paid to the modification of the computational scheme of the constraint, when, during its formation, rods appear that “pass” through the original system.

Keywords: natural frequency, natural modes, generalized additional targeted constraint, stiffness coefficients

ФОРМИРОВАНИЕ РАСЧЕТНЫХ СХЕМ ДОПОЛНИТЕЛЬНЫХ СВЯЗЕЙ, ПРИЦЕЛЬНО РЕГУЛИРУЮЩИХ СПЕКТР ЧАСТОТ СОБСТВЕННЫХ КОЛЕБАНИЙ УПРУГИХ СИСТЕМ С КОНЕЧНЫМ ЧИСЛОМ СТЕПЕНЕЙ СВОБОДЫ МАСС, У КОТОРЫХ НАПРАВЛЕНИЯ ДВИЖЕНИЯ ПАРАЛЛЕЛЬНЫ, НО НЕ ЛЕЖАТ В ОДНОЙ ПЛОСКОСТИ ЧАСТЬ 1: ТЕОРЕТИЧЕСКИЕ ОСНОВЫ

Л.С. Ляхович¹, П.А. Акимов²

¹ Томский государственный архитектурно-строительный университет, г. Томск, РОССИЯ

² Национальный исследовательский Московский государственный строительный университет,
г. Москва, РОССИЯ

Аннотация: Для некоторых упругих систем с конечным числом степеней свободы масс, у которых направления движения масс параллельны и лежат в одной плоскости, (например, стержни) разработаны

методы создания дополнительных связей, введение каждой из которых прицельно увеличивает значение только одной собственной частоты и не изменяет ни одну из форм собственных колебаний. Метод формирования матрицы дополнительных коэффициентов жесткости, характеризующих в этой задаче такую прицельную связь, может быть применен и при решении аналогичной задачи для упругих систем с конечным числом степеней свободы масс, у которых направления движения масс параллельны, но не лежат в одной плоскости (например, пластины). Вместе с тем для таких систем сформулированы лишь требования к расчетным схемам дополнительных прицельных связей, а не методы их создания. В данной статье предлагается подход, позволяющий создавать расчетные схемы дополнительных прицельных связей и для таких систем. Рассмотрен вариант расчетной схемы связи, представленный стержневой системой с одной степенью активности. Выявлены некоторые особые свойства таких прицельных связей. При формировании расчетной схемы выполняется минимизация расхода материала на создание связи, учитываются конструктивные ограничения. Особое внимание уделено модификации расчетной схемы связи, когда при ее формировании появляются стержни, «проходящие» сквозь исходную систему.

Ключевые слова: частота собственных колебаний, форма собственных колебаний, обобщенная прицельная дополнительная связь, коэффициенты жесткости

As is known [1, 2, 3, 4, 5, 6], introduction of generalized targeted constraints is one of the methods for freeing a given interval of the natural frequency spectrum from one or more of natural frequencies.

Original solutions of problems of forming a matrix of additional stiffness and creating on the basis of this matrix of the computational scheme of the corresponding targeted constraints are presented in [1, 2, 3, 4]. These solutions are based on the displacement method for systems with a finite number of degrees of freedom of masses, in which the directions of mass movement are parallel and lie in the same plane.

It was shown in [5, 6] that the method of forming a matrix of additional stiffness coefficients can also be applied in solving a similar problem for elastic systems with a finite number of mass degrees of freedom, in which the directions of mass movement are parallel, but do not lie in the same plane. Besides, the requirements for the computational schemes of additional targeted constraints were formulated in [5, 6] in relation to this problem.

Let us give the order of formation of the matrix of additional stiffness in relation to the considering systems.

In the papers mentioned above, the main system of the displacement method [7] was chosen, which was obtained by introducing linear relations in the direction of mass movement. For example, for the plate [8, 9] shown in Figure 1a, the main system is shown in Figure 1b.

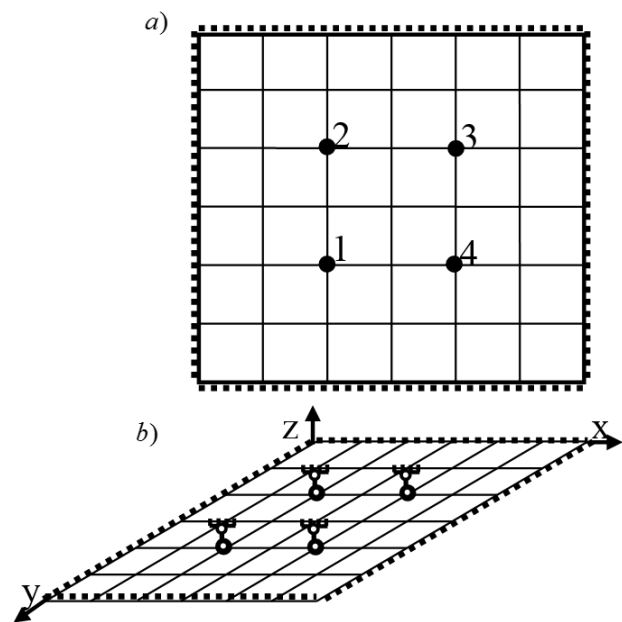


Figure 1. Sample of structure.

The displacement method equations were written in the conventional form for systems with a finite number of degrees of freedom:

$$\left\{ \begin{aligned} &(r[1,1] + m[1]\omega^2)v[1,j] + r[1,2]v[2,j] + \\ &\quad + \dots + r[1,q]*v[q,j] + \dots + r[1,n]*v[n,j] = 0 \\ &r[2,1]v[1,j] + (r[2,2] + m[2]\omega^2)v[2,j] + \\ &\quad + \dots + r[2,q]*v[q,j] + \dots + r[2,n]*v[n,j] = 0 \\ &..... \\ &r[n,1]v[1,j] + r[n,2]v[2,j] + \\ &\quad + \dots + r[n,q]v[q,j] + \\ &\quad + \dots + (r[n,n] + m[n]*\omega^2)*v[n,j] = 0 \end{aligned} \right. \quad (1)$$

In (1) the values $r[i, k]$ form a matrix of stiffness coefficients $A = \|r[i, k]\|$; $m[i]$ are the mass values, forming a diagonal matrix $M = \|m[i]\|$; ω is the frequency of natural oscillations of the system; $v[k, j]$ are displacements in the direction of mass movement in the j -th natural mode ($j = 1, 2, \dots, q, \dots, n$) (forms of natural oscillations). Equation roots

$$|A - \omega^2 M| = 0 \quad (2)$$

determine the frequency spectrum of natural oscillations of the system

$$\omega[1], \omega[2], \dots, \omega[q-1], \omega[q], \omega[q+1], \dots, \omega[n] \quad (3)$$

It is shown that the creation of generalized targeted constraint that increases only one frequency of natural oscillations (for example, $\omega[q]$) to a predetermined value and does not change any of the natural modes and the values of the remaining frequencies of the spectrum is based on the formation of a matrix of additional stiffness coefficients:

$$A_0 = A_{SO} A_S = A_{SO} \|a_0[i, k]\|_{i,k=1}^n, \quad (4)$$

where we have

$$A_S = \|a_0[i, k]\|_{i,k=1}^n. \quad (5)$$

The matrix A_0 must have special properties. If the introduced constraint is “targeted” at the (q) -th frequency of natural oscillations, then the stiffness coefficients $\|a_0[i, k]\|_{i,k=1}^n$ should be orthogonal to the coordinates of the natural modes of the remaining $(n-1)$ frequencies of the spectrum, that is

$$\sum_{k=1}^n a_0[i, k] M[k, j] = 0, \quad (6)$$

$$i = 1, 2, \dots, n, \quad j = 1, 2, \dots, (q-1), (q+1), \dots, n.$$

With respect to the (q) -th natural frequency, at which the introduced constraints is “targeted”, the coefficients are not orthogonal, that is,

$$\sum_{k=1}^n a_0[i, k] v_{\omega}[k, q] \neq 0, \quad (i = 1, 2, \dots, n). \quad (7)$$

It can be shown that conditions (6) and (7) will be satisfied by the coefficients

$$a_0[i, k] = m[i] m[k] v_{\omega}[i, q] v_{\omega}[k, q]. \quad (8)$$

The value of the multiplier is defined as the root of the equation

$$|(A - \omega_s^2 M) + A_{SO} A_S| = 0. \quad (9)$$

Since the (q) -th natural mode of the original system remains its natural mode after the introduction of the targeted constraint and at a frequency ω_s , the factor A_{SO} can be found as

$$A_{SO} = \frac{-\sum_{i=1}^n \sum_{k=1}^n (a[i, k] - \omega_s^2 m[i, k]) v_{\omega}[i, q] v_{\omega}[k, q]}{\sum_{i=1}^n \sum_{k=1}^n a_0[i, k] v_{\omega}[i, q] v_{\omega}[k, q]}. \quad (10)$$

The result of solving the equation

$$|(A + A_{SO} A_S) - \omega^2 M| = 0. \quad (11)$$

must confirm that the natural modes have not changed, and the “targeted” frequency has increased to ω_s .

The support device, to which the matrix of additional stiffness coefficients A_0 will correspond, must provide the ratio between the nodal displacements the same as between the coordinates of the (q) -th natural mode of the original system. It is shown that such a ratio will be realized if the additional support system transfers forces

to the nodes of the main rod system, the ratios between which are proportional to the values

$$R_0[i] = m[i]v[i, (q)]. \quad (12)$$

In [5, 6], the requirements for the computational schemes of additional targeted constraints were formulated in relation to systems with a finite number of degrees of freedom of masses, in which the directions of mass movement are parallel, but do not lie in the same plane.

For such systems, the generalized targeted constraint should correspond to the matrix of additional stiffness coefficients A_0 (4). If the computational scheme of constraint is represented by a variant of the hinged-rod system, then it should be once statically indeterminate, in the nodes of the system where the masses are located, vertical members are installed in the direction of movement of the masses, and the prestress of any one constraint member causes forces $Nst[i](i = 1, \dots, n)$ in these vertical members, the relationship between which are proportional to the ratios between the forces $R_0[i]$ (11). In this case, the constraint structure should not have any connections with the original system, except for vertical members installed in the nodes where the masses are located.

It was noted in [5, 6] that the computational schemes of generalized targeted constraint that meet the above requirements are multivariant and depend on the geometry of the original system, the location of the masses, and some other characteristics of the considering object.

In particular, it is possible to accept the computational scheme of the targeted constraint in the form of a once statically indeterminate hinge-rod system, the geometry of which is determined both by the lengths of the main vertical members installed in the nodes in the direction of mass movement $lst[i](i = 1, 2, \dots, n)$ and by the given lengths of additional rods $ld[k](k = 1, 2, \dots, n_1)$, that have no connections with the original system.

Then, after forming the matrix of additional stiffnesses A_0 (4), computing the values A_{so}

(10) and $R_0[i]$ (12), the problem is reduced to finding the lengths $lst[i](i = 1, 2, \dots, n)$ from the conditions for the occurrence in the main vertical members of forces $Nst[i](i = 1, \dots, n)$, the ratios between which will be proportional to the ratios between the forces $R_0[i](i = 1, \dots, n)$.

Below, one of the options for finding the lengths of the rods will be presented, which determine the geometry of the targeted constraint, in which the necessary ratios between the forces in the main vertical members are provided.

The considering variant is based on methods for minimizing the square of the difference between the forces arising in the main vertical members $Nst[i](i = 1, \dots, n)$ in the process of forming the targeted constraint and the values $R_0[i](i = 1, \dots, n)$. Thus, we have the problem of minimization of the function

$$fo = \sum_{i=1}^{i=n} (Nst[i] - R_0[i])^2. \quad (13)$$

in the parameter space $lst[i](i = 1, 2, \dots, n)$.

In accordance with the above requirements, a computational scheme of the targeted constraint is created in the form of a once statically indeterminate hinge-rod system, in which the length of one of the main racks (for example, g -th) is specified. Let's call this vertical member the base one $lst[g] = lst0[g]$. According to the design conditions, the lengths of additional rods $ld[k](k = 1, 2, \dots, n_1)$ are selected, which do not vary during the formation of the scheme of the targeted constraint. The initial values of the remaining variable lengths

$$\bar{lst}[i](i = 1, 2, \dots, (g-1), (g+1), \dots, n)$$

are also set. These actions determine the initial geometry of the computational scheme of the targeted constraint. Since the computational scheme of the targeted constraint is once statically indeterminate, the force in one of the main vertical members (for example, in q -th) is set. .

The method of searching for the minimum of the objective (target) function (13) in the space of variable lengths (the steepest descent, random search, etc.) is selected.

Let us consider an algorithm for implementing actions to form a computational scheme for targeted constraint.

We recommend application of special algorithm, presented below.

1. In accordance with the chosen method of searching for the minimum of the objective function, increments to variable lengths

$$lst[i] = \bar{lst}[i] + \Delta l (i = 1, 2, \dots, (g-1), (g+1), \dots, n)$$

are set and the geometry of the computational scheme of the targeted constraint is updated.

2. A system of equilibrium equations is constructed in order to determine the forces in the rods of targeted constraint. Since it was assumed that $Nst[q] = R_0[q]$, now the number of unknown forces in the rods of the targeted constraint is equal to the number of equilibrium equations.

3. Unknown forces in the rods of the targeted constraint are determined from the equations of equilibrium with allowance for $Nst[q] = R_0[q]$.

4. The value of the objective function is computed

$$fo = \sum_{i=1}^{i=n} (Nst[i] - R_0[i])^2.$$

5. If we have $fo > OOO$, then in accordance with the chosen method of searching for the minimum of the objective function (12), the increments $\Delta l[i]$ are corrected (values $\Delta l[i]$ and ratios between them are changed). We have

$$\bar{lst}[i] = lst[i] (i = 1, 2, \dots, (g-1), (g+1), \dots, n).$$

Then the transition to the third step (item 3) is made. and the process continues.

6. If the value fo is less than a preselected small value OOO , then the process ends, and the computational scheme of the targeted constraint is formed with a given error (OOO) provided

that the length of the base rack is accepted $lst[g] = lst0[g]$.

The cross-sectional areas of the rods of targeted constraint are found from the condition that its stiffness coincides with the stiffness determined by matrix $A_0 = A_{s0} \cdot A_s = A_{s0} \cdot \|a_0[i, k]\|_{i,k=1}^n$ (4).

Targeted constraint is constructed on the basis of forces $R_0[i]$, which correspond to the forces in the main vertical members $Nst[i] = R_0[i] (i = 1, 2, \dots, n)$, in additional vertical members $Nd[k] (k = 1, 2, \dots, n1)$ and in the rods of the belt of constraint $N_p[j] (j = 1, 2, \dots, n2)$.

The derivation of the dependency that determines the area of the cross-sections of the rods of targeted constraint for the systems, in which the directions of mass movement are parallel and lie in the same plane, is given in [4]. This dependence with allowance for some modifications, can also be applied to the system under consideration

$$A_{s0} \left(\sum_{i=1}^n \frac{N_{st}^2[i] \cdot |lst[i]|}{E \cdot F_{st}[i]} + \sum_{j=1}^{n2} \frac{N_p^2[j] \cdot l_p[j]}{E \cdot F_p[j]} + \sum_{k=1}^{n1} \frac{N_d^2[k] \cdot ld[k]}{E \cdot F_d[k]} \right) = 1, \quad (14)$$

where

$$F_{st}[i] = F \cdot \alpha[i]; \quad F_p[j] = F \cdot \beta[j]; \quad F_d[k] = F \cdot \gamma[k] \quad (15)$$

are respectively, the cross-sectional area of the vertical members, belts and additional rods of the targeted constraint; E is the modulus of elasticity of the material of the rods. The coefficients $\alpha[i]$, $\beta[j]$ and $\gamma[k]$ determine the ratios between the cross-sectional areas in the rods of targeted constraint.

The value F is determined by dependence

$$F = A_{s0} \left(\sum_{i=1}^n \frac{N_{st}^2[i] \cdot |lst[i]|}{E \cdot \alpha[i]} + \sum_{j=1}^{n2} \frac{N_p^2[j] \cdot l_p[j]}{E \cdot \beta[j]} + \sum_{k=1}^{n1} \frac{N_d^2[k] \cdot l_p[k]}{E \cdot \gamma[k]} \right), \quad (16)$$

The length of the base vertical member

$$lst[g] = lst0[g]$$

and the values $\alpha[i]$, $\beta[j]$ and $\gamma[k]$ depending on the design conditions, can either be set or found by minimizing the volume of material of the targeted constraint.

If the volume of material of the targeted constraint is minimized, then the objective function (volume of material of the targeted constraint V_{sv}) has the form:

$$V_{sv} = F \cdot \left\{ \sum_{i=1}^n \alpha[i] \cdot |lst[i]| + \sum_{j=1}^{n2} \beta[j] \cdot l_p[j] + \sum_{k=1}^{n1} \gamma[k] \cdot ld[k] \right\}. \quad (17)$$

When constructing the computational scheme of the targeted constraint, the values of some variable lengths may turn out to be negative. Therefore, absolute values of variable lengths $|lst[i]|$ are introduced into (14), (16) and (17).

When constructing the computational scheme of the targeted constraint and minimizing the function (17), the limitations of the variable values can be taken into account. Restrictions on the values of $\alpha[i]$, $\beta[j]$ and $\gamma[k]$ are related to the conditions of strength, stiffness, and stability of the rods. These restrictions are not considered in the distinctive paper. The restrictions on the lengths of the main vertical members can be written in the following form:

$$l_{\max} \geq lst[i] \geq l_{\min}, (i = 1, 2, \dots, n), \quad (18)$$

where $lst[i]$ are the lengths of the main vertical members; $l_{\min}$ and $l_{\max}$ are respectively their allowable minimum and maximum values. Since the ratios between the forces

$$Nst[i] = R_0[i] (i = 1, 2, \dots, n)$$

do not change during the construction of the targeted constraint at $f_o \leq OOO$, the ratios between the lengths of the variable values do not change when the length of the base vertical member changes. This circumstance allows us to attribute restrictions (18) to one variable length – the length of the base vertical member $lst[g] = lst0[g]$. If for $f_o \leq OOO$ among the main vertical members the largest length is equal to $lst[k1]$, and the smallest is equal to $lst[k2]$, then, denoting $\chi1 = lst[g]/lst[k1]$ and $\chi2 = lst[g]/lst[k2]$, expression (18) can be rewritten as:

$$l_{\max} 0 \geq lst0[g] \geq l_{\min} 0, \quad (19)$$

where we have

$$l_{\max} 0 = l_{\max}^* \chi1; \quad l_{\min} 0 = l_{\min}^* \chi2.$$

Now, when searching for the minimum of function (13), the range of acceptable values $lst0[g]$ is determined by dependence (19).

Constraints in the form (18), (19) are used provided that the signs of the lengths of all main vertical members are positive. If the signs of the lengths of all main vertical members are negative, then the sign of the coordinate in the direction of the vertical members is reversed.

There are cases in construction of computational scheme of the targeted constraint, when the values of the lengths of some main vertical members turn out to be positive, while others are negative. Structurally, such a scheme requires an ideally free “passage” of a part of the rods of targeted constraint “through” the original system, which is almost unrealizable. In these cases, the targeted constraint should be shifted in the direction of movement of the masses in a positive or negative direction by an amount at which the values of all the lengths of the main vertical members will be of the same sign.

Let us designate by $lst[i]_{\max}$ the largest length among the “positive vertical members” at $f_o \leq OOO$, and by $lst[k]_{\min}$ the largest absolute value among the “negative lengths”.

If the targeted constraint is moved in a positive direction, then the shift value must be greater than

$$Z_v = lst[k]_{\min} + l_{\min}.$$

Obviously, in this case, the lengths of all vertical members will be “positive”. In this case, the vertical member of the smallest length will be in the node where the vertical member was with $lst[k]_{\min}$. Now the length of the vertical member in this node will be equal to $l_{\min}$. The longest length of vertical member will be at the node where the vertical member was with $lst[i]_{\max}$. Now the length of the vertical member in this node will be equal to

$$lst[i]_{\max} + lst[k]_{\min} + l_{\min}.$$

If the targeted constraint is moved in a negative direction, then the shift value must be greater than

$$Z_N = lst[i]_{\max} + l_{\min}.$$

Obviously, in this case, the lengths of all vertical members will be “negative”. In this case, the vertical member with the smallest absolute value of the length will be in the node where the vertical member was with $lst[i]_{\max}$. Now the absolute value of the length of the vertical member in this node will be equal to $l_{\min}$. The largest absolute length of the vertical member will be at the node where the vertical member was with $lst[k]_{\min}$. Now the absolute value of the vertical member length in this node will be equal to

$$lst[i]_{\max} + lst[k]_{\min} + l_{\min}.$$

In these cases, the restrictions on the lengths of the vertical members take the form

$$l_{\max} \geq (lst[i]_{\max} + lst[k]_{\min} + l_{\min}). \quad (20)$$

As noted above, at $f_0 \leq 000$, the ratios between the lengths of the variable quantities do not change. This circumstance allows us to attribute restrictions (20) to one variable length – the length of the base vertical member $lst[g] = lst0[g]$. Using (20) we get

$$l_{\max} - l_{\min} \geq lst[i]_{\max} + lst[k]_{\min}$$

or

$$\frac{l_{\max} - l_{\min}}{lst0[g]} \geq \frac{lst[i]_{\max}}{lst0[g]} + \frac{lst[k]_{\min}}{lst0[g]}. \quad (21)$$

Since the ratios

$$\frac{lst[i]_{\max}}{lst0[g]} \quad \text{and} \quad \frac{lst[k]_{\min}}{lst0[g]}$$

remain constant when the length $lst0[g]$ changes, then we have

$$\frac{lst[i]_{\max}}{lst0[g]} + \frac{lst[k]_{\min}}{lst0[g]} = \frac{lst[i]_{\max} + lst[k]_{\min}}{lst0[g]} = \frac{1}{\chi^3} \quad (22)$$

where

$$\chi^3 = \frac{lst0[g]}{lst[i]_{\max} + lst[k]_{\min}}. \quad (23)$$

remains constant when changing $lst0[g]$.

Thus, constraint (20) can be represented as:

$$(l_{\max} - l_{\min}) * \chi^3 \geq lst0[g]. \quad (24)$$

Now, when searching for the minimum of function (17), the range of acceptable values $lst0[g]$ for cases where the lengths of the main vertical member turn out to be of different signs is determined by dependence (24).

The choice of χ^3 , χ^2 and χ does not affect the computational scheme of the targeted constraint, but only affects the values of the

cross-sectional areas of its rods. The value of the length of the base vertical member affects both the geometry of the computational scheme of the targeted constraint and the cross-sectional areas of its rods.

Let's consider the procedure for implementing actions to minimize the volume of material of targeted constraint.

If the values l , A and E are set according to the design conditions, then, after determining the initial values of the cross-sectional areas of the rods of targeted constraint, we can determine the length of the base vertical member, at which the objective function (17) takes minimum value in the range of permissible values of this length ((19) or (24)). It can be done by the above algorithm and one of the variants of the one-dimensional search method.

If the values l , A and E are also determined from the conditions of the minimum material of the targeted constraint, then in this case one of the variants of the method of successive approximations can be used. The initial values l , A and E are preliminarily selected, and the initial values of the cross-sectional areas of the rods of targeted constraint are determined. Each approximation of the method consists of two successive steps:

1. On the basis of the algorithm given above and the one-dimensional search method, with known l , A and E the length of the base vertical member is determined, at which the objective function V_{SV} (17) takes the minimum value in the range of allowable length values $l \in [l_0, l_1]$ ((19) or (24)).

2. One of the methods for finding the minimum of the objective function (17) (the steepest descent, random search, and others) in the space of variable values A , E and l continues the process of minimizing the function (17).

Approximations of the method (the first and the second steps) are repeated until the difference between the weight functions (17) of two neighboring approximations becomes less than a sufficiently small preselected value.

REFERENCES

1. **Nudelman Ya.L., Lyakhovich L.S., Giterman D.M.** O Naibolee Podatlivykh Svjazzh Naibol'shej Zhestkosti [About the Most Pliable Bonds of the Greatest Rigidity]. // Issues of applied mechanics and mathematics. Tomsk, TSU Publishing House, 1981, pp. 113-126 (in Russian).
2. **Giterman D.M., Lyakhovich L.S., Nudelman Ya.L.** Algoritm Sozdaniya Rezonansno-Bezopasnykh Zon pri Pomoshhi Nalozhenija Dopolnitel'nykh Svjazej [Algorithm for Creation of Resonantly Safe Zones by Applying Additional Constraints]. // "Dynamics and strength of machines", Vol. 39. Kharkov, "Vishcha shkola", 1984, pp. 63-69 (in Russian).
3. **Lyakhovich L.S., Maletkin O.Yu.** O Pricel'nom Regulirovanii Sobstvennykh Chastot Uprugih Sistem [On Targeted Control of Natural Frequencies of Elastic Systems]. // Izvestiya vuzov. Construction and architecture, 1990, No. 1, pp. 113-117 (in Russian).
4. **Lyakhovich L.S.** Osobyje Svojstva Optimal'nykh Sistem i Osnovnye Napravlenija ih Realizacii v Metodah Rascheta Sooruzhenij [Special Properties of Optimal Systems and the Main Directions of Their Implementation in the Methods of Calculation of Structures]. Tomsk, Tomsk State University of Architecture and Construction, 2009, 372 pages (in Russian).
5. **Lyakhovich L.S., Akimov P.A.** Aimed control of the frequency spectrum of eigen-vibrations of elastic plates with a finite number of degrees of freedom of masses by superimposing additional constraints. // International Journal for Computational Civil and Structural Engineering, 2021, Volume 17, Issue 2, pp. 76-82.
6. **Akimov P.A., Lyakhovich L.S.** Pricel'noe Regulirovanie Spektra Chastot Sobstvennykh Kolebanij Uprugih Plastin s Konechnym Chislom Stepenej Svobody Mass Putem Vvedeniya Dopolnitel'nykh Obobshhennykh Svjazej i Obobshhennykh Kinematicheskikh

Ustrojstv [Precision Control for Eigen-Frequency of Elastic Plates with Finite Number of Mass Degrees of Freedom by Using Additional Generalized Connections and Kinematic devices]. // Vestnik Tomskogo gosudarstvennogo arkhitekturno-stroitel'nogo universiteta. Journal of Construction and Architecture, 2021, Volume 23, Issue 4, pp. 57-68 (in Russian).

7. **Akimov P.A., Lyakhovich L.S.** Aimed Control of the Frequency Spectrum of Eigenvibrations of Elastic Plates with a Finite Number of Degrees of Mass Freedom by Introducing Additional Generalized Kinematic Devices. // International Journal for Computational Civil and Structural Engineering, 2021, Volume 17, Issue 4, pp. 181-187.
8. **Shitikova M.V., Krusser A.I.** Force Driven Vibrations of Nonlinear Plates on a Viscoelastic Winkler Foundation Under the Harmonic Moving Load. // International Journal for Computational Civil and Structural Engineering, 2021, Volume 17, Issue 4, pp. 161-180.
9. **Akimov P.A., Mozgaleva M.L., Kaytukov T.B.** Numerical Solution of the Problem of Isotropic Plate Analysis with the Use of B-Spline Discrete-Continual Finite Element Method. // International Journal for Computational Civil and Structural Engineering, 2020, Volume 16, Issue 4, pp. 14-28.
10. **Manuylov G.A., Kosytsyn S.B., Grudtsyna I.E.** Influence of Buckling Forms Interaction of Stiffened Plate Bearing Capacity. // International Journal for Computational Civil and Structural Engineering, 2020, Volume 16, Issue 2, pp. 83-93.
11. **Mozgaleva M.L., Akimov P.A., Kaytukov T.B.** Localization of Solution of the Problem of Isotropic Plate Analysis with the Use of B-Spline Discrete-Continual Finite Element Method. // International Journal for Computational Civil and Structural Engineering, 2021, Volume 17, Issue 1, pp. 55-74.

СПИСОК ЛИТЕРАТУРЫ

1. **Нудельман Я.Л., Ляхович Л.С., Гитерман Д.М.** О наиболее податливых связях наибольшей жесткости. // Вопросы прикладной механики и математики. – Томск: Издательство ТГУ, 1981, с. 113-126.
2. **Гитерман Д.М., Ляхович Л.С., Нудельман Я.Л.** Алгоритм создания резонансно-безопасных зон при помощи наложения дополнительных связей. // Динамика и прочность машин», Вып. 39. – Харьков: «Вища школа», 1984, с. 63-69.
3. **Ляхович Л.С., Малеткин О.Ю.** О прицельном регулировании собственных частот упругих систем. // Известия вузов. Строительство и архитектура, 1990, №1, с. 113-117.
4. **Ляхович Л.С.** Особые свойства оптимальных систем и основные направления их реализации в методах расчета сооружений. – Томск: Издательство Томского государственного архитектурно-строительного университета, 2009. – 372 с.
5. **Lyakhovich L.S., Akimov P.A.** Aimed control of the frequency spectrum of eigenvibrations of elastic plates with a finite number of degrees of freedom of masses by superimposing additional constraints. // International Journal for Computational Civil and Structural Engineering, 2021, Volume 17, Issue 2, pp. 76-82.
6. **Акимов П.А., Ляхович Л.С.** Прицельное регулирование спектра частот собственных колебаний упругих пластин с конечным числом степеней свободы масс путем введения дополнительных обобщенных связей и обобщенных кинематических устройств. // Вестник Томского государственного архитектурно-строительного университета, 2021, Том 23, №4, с. 57-67.
7. **Akimov P.A., Lyakhovich L.S.** Aimed Control of the Frequency Spectrum of Eigenvibrations of Elastic Plates with a Finite Number of Degrees of Mass Freedom by

- Introducing Additional Generalized Kinematic Devices. // International Journal for Computational Civil and Structural Engineering, 2021, Volume 17, Issue 4, pp. 181-187.
8. **Shitikova M.V., Krusser A.I.** Force Driven Vibrations of Nonlinear Plates on a Viscoelastic Winkler Foundation Under the Harmonic Moving Load. // International Journal for Computational Civil and Structural Engineering, 2021, Volume 17, Issue 4, pp. 161-180.
9. **Akimov P.A., Mozgaleva M.L., Kaytukov T.B.** Numerical Solution of the Problem of Isotropic Plate Analysis with the Use of B-Spline Discrete-Continual Finite Element Method. // International Journal for Computational Civil and Structural Engineering, 2020, Volume 16, Issue 4, pp. 14-28.
10. **Manuylov G.A., Kosytsyn S.B., Grudtsyna I.E.** Influence of Buckling Forms Interaction of Stiffened Plate Bearing Capacity. // International Journal for Computational Civil and Structural Engineering, 2020, Volume 16, Issue 2, pp. 83-93.
11. **Mozgaleva M.L., Akimov P.A., Kaytukov T.B.** Localization of Solution of the Problem of Isotropic Plate Analysis with the Use of B-Spline Discrete-Continual Finite Element Method. // International Journal for Computational Civil and Structural Engineering, 2021, Volume 17, Issue 1, pp. 55-74.

Leonid S. Lyakhovich, Full Member of the Russian Academy of Architecture and Construction Sciences, Professor, DSc, Head of Department of Structural Mechanics, Tomsk State University of Architecture and Building; 2, Solyanaya St., 2, Tomsk, 634003, Russia;
E-mail: lls@tsuab.ru

Pavel A. Akimov, Full Member of the Russian Academy of Architecture and Construction Sciences, Professor, Dr.Sc.; Rector of National Research Moscow State University of Civil Engineering; 26, Yaroslavskoe Shosse, Moscow, 129337, Russia; phone +7(495) 651-81-85;
Email: AkimovPA@mgsu.ru, pavel.akimov@gmail.com.

Ляхович Леонид Семенович, академик РААСН, профессор, доктор технических наук, профессор кафедры строительной механики, Томский государственный архитектурно-строительный университет; 634003, Россия, г. Томск, Соляная пл. 2;
E-mail: lls@tsuab.ru

Акимов Павел Алексеевич, академик РААСН, профессор, доктор технических наук; ректор Национального исследовательского Московского государственного строительного университета; 129337, г. Москва, Ярославское шоссе, д. 26; тел. +7(495) 651-81-85;
Email: AkimovPA@mgsu.ru, pavel.akimov@gmail.com.