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DEFORMATION MODEL AND ALGORITHM FOR CALCULATION OF REINFORCED CONCRETE STRUCTURES OF ROUND CROSS-SECTION UNDER TORSION WITH BENDING

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Abstract. Despite a fairly long period of research and a significant number of publications around the world on the problem of the complex resistance of reinforced concrete, the existing calculation models still remain far from perfect. This is especially true for structures with a non-rectangular cross section. The article presents a version of the model and an algorithm for the analytical calculation of reinforced concrete structures of a circular cross section in torsion with bending, which most fully reflects the specifics of the power resistance of such structures. The model takes into account all the components of external forces in a rod element of a circular cross section, the spatial nature of cracks, with the combined action of moments, various cases of the location of the compressed concrete zone, depending on the ratio of the acting forces in the calculated structure. For a spatial crack, calculated sections are taken in the form of diagonal large and small ellipses and a spatial surface bounded by concave and convex spatial parabolas. In compressed and stretched concrete, a broken section of three sections is considered, two in the form of longitudinal trapezoid and the third, middle section in the form of a small ellipse rotated at an angle to the longitudinal axis of the structure. The obtained analytical dependencies allow one to determine interconnected design parameters, such as stresses in the concrete of the compressed zone, the height of the compressed concrete, stresses in the longitudinal and transverse reinforcement, deformations in concrete and reinforcement, the length of the projection of a spatial inclined crack, and others. The deformation model and algorithm can be used in the design of reinforced concrete structures of circular and annular cross-section, working in bending with torsion.

Keywords: reinforced concrete, circular section, calculation scheme, bending moment, torsion, spatial crack, dangerous spatial crack, governing equations.

ДЕФОРМАЦИОННАЯ МОДЕЛЬ И АЛГОРИТМ РАСЧЕТА ЖЕЛЕЗОБЕТОННЫХ КОНСТРУКЦИЙ КРУГЛОГО ПОПЕРЕЧНОГО СЕЧЕНИЯ ПРИ КРУЧЕНИИ С ИЗГИБОМ

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Аннотация. Несмотря на достаточно продолжительный срок исследований и значительное количество публикаций во всем мире по проблеме сложного сопротивления железобетона существующие расчетные модели до настоящего времени остаются далеко не совершенными. Особенно это относится к конструкциям непрямоугольного поперечного сечения. В статье представлен вариант модели и алгоритм аналитического расчета железобетонных конструкций круглого поперечного сечения при кручении с изгибом наиболее полно отражающий специфику силового сопротивления таких конструкций. Модель учитывает все составляющие внешних усилий в стержневом элементе круглого поперечного сечения, пространственный характер трещин, при совместном действии моментов, различные случаи расположения сжатой зоны бетона в зависимости от соотношения действующих усилий в рассчитываемой конструкции. Для пространственной трещины приняты расчетные сечения в виде диагонального большого и малого эллипсов и пространственной поверхности ограниченной вогнутой и выпуклой пространственными параболами. В сжатом и растянутом бетоне рассмотрено ломанное сечение из трех участков, – два в виде продольных трапеции и третий, средний участок в виде малого эллипса, повернутого под углом к продольной оси конструкции. Полученные аналитические зависимости позволяют определять связанные между собой расчетные параметры, такие как напряжения в бетоне сжатой зоны, высоту сжатого бетона, напряжения в продольной и поперечной арматуре, деформации в бетоне и арматуре, длину проекции пространственной наклонной трещины и другие. Деформационная модель и алгоритм могут быть использованы при проектировании железобетонных конструкций круглого и кольцевого поперечного сечения, работающих на изгиб с кручением.

Ключевые слова: железобетонные конструкции, круглое сечение, расчетная схема, прочность, изгибающий момент, крутящий момент, пространственная трещина, разрешающие уравнения.

INTRODUCTION

It is known that the rational and safe design of building structures is largely determined by the availability of effective and relatively simple and understandable methods for their calculation. This is especially true for critical structures of buildings experiencing a complex stress state, which undoubtedly include structures that work on the simultaneous action of bending with torsion. Until now, this is one of the most complex and little-studied problems of the theory of reinforced concrete, since it is applied and used in domestic [10, 15, 18, 19, 23, 24] and foreign [2, 5–9, 13, 16, 17, 20, 21] research and in regulatory documents up to the present time, the methods remain extremely conditional and do not reflect the complex resistance of reinforced concrete structures at all levels of loading under such effects. Numerical solutions using software systems based on the finite element method and other numerical methods do not reflect the physics and all the specifics of the deformation of such structures, and the results of the solutions obtained are not unambiguous and are largely determined by the qualifications of the engineer. Known analytical solutions usually consider the calculation of structures of rectangular [2, 5, 7, 9, 13, 18, 20], and more recently box-shaped [26] sections and do not investigate the specifics of the calculation of structures of circular and annular sections. At the same time, structures with these cross-sectional shapes are often used in such critical structures as, for example, the stiffening core of high-rise buildings, bridge supports, cable cars, and the efficiency of their design solutions depends on the accuracy of the calculation. Therefore, the development of analytical methods for calculating building structures made of reinforced concrete, fiber- and steel-reinforced concrete and other similar multicomponent conglomerates remains in demand not only for verifying software systems and developing regulatory documents, but also when designing building objects that use fundamentally new design solutions and technologies that have not passed verification in the practice of construction and operation.

Therefore, the purpose of this study was to build a deformation model and an algorithm for calculating the complex resistance of reinforced concrete structures of a circular cross section under the combined action of torsional and bending moments, which most fully reflects the physical features of the force deformation of such structures.

Deformation model. In the development of studies [25], in which, to determine the calculated forces in reinforced concrete structures of the considered cross section, a general calculated scheme with spatial sections was proposed and the corresponding resolving equilibrium equations and deformation equations were compiled that determine the stress-strain state in such structures under the considered effects on Based on the specification of these constitutive equations, here is an algorithm for determining all the calculated parameters used in this model.

It is assumed that the calculated reinforced concrete structure is conditionally divided into two blocks and in the space of these blocks there is a spatial crack limited along the length of the structure by normal cross sections 1-1 at the beginning and 3-3 at the end of the projection of this crack. The area of the beam covered by the crack, being projected onto the side surface of the structure, can be described by a large ellipse (Figure 1) with a projection length equal to **c**.

a)

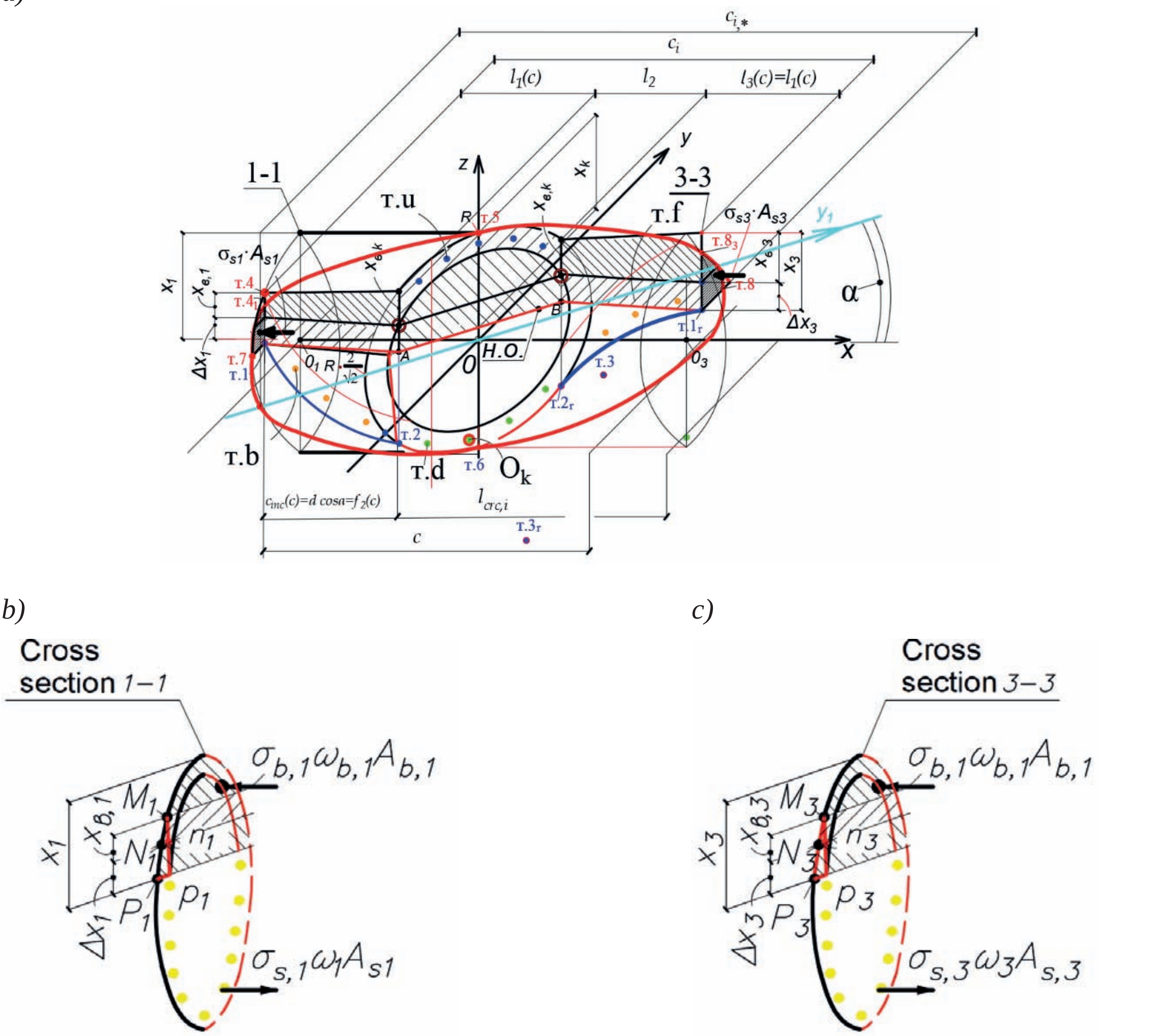

Figure 1. Calculatied scheme of a reinforced concrete element of a circular cross section under the action of bending and torque moments: a – block with small and large ellipses between normal sections I-I and 3-3; b, c – diagram of forces, compressed and stretched zones in section I-I and 3-3 respectively

In the compressed concrete of the design section within the projection of the inclined crack there are three characteristic sections with dimensions l_1, l_2, l_3, – longitudinal sections (l_1 and l_3), as well as a cross section of a small ellipse at a longitudinal distance l_2 (see Figure 1). In the stretched concrete there are the same three sections l_1, l_2, l_3, but in addition to the ellipse for the section l_2, there is also a parabola envelope in the sections l_1 and l_3.
The small ellipse equation for a spatial section is:

$$\frac{y^2}{2R^2}+\frac{z^2}{R^2}=1. \tag{1}$$

Then in stretched concrete in the first section we have a spatial curve $f_{par1,2,3}(x,y,z)$ and in the second section a spatial parabola $f_{par1_r,2_r,3_r}(x,y,z)$.
To construct the first spatial parabola $f_{par1,2,3}(x,y,z)$ we find the coordinates (x,y,z) of point 1 (T.1) along the axis x ($x=-l_1-0.5l_2$), the coordinate along the axis z ($z=R-x_1$) and, belonging to the circle in section 1-1, the coordinate along the axis y:

$$y=\pm\sqrt{R^2-(R-x_1)^2}\,. \tag{2}$$

Next, we find the coordinates (x,y,z) of point 2 (T. 2) along the x axis ($x=-0.5l_2$), the z coordinate ($z=-R+x_k-\Delta x_1-x_{b,k}$) and the y coordinate.
Let us write down the equation of the first spatial (in the form of a propeller curve) parabola:

$$f_{par1,2,3}(x,y,z)=\sqrt{z_1(x)^2+y_1(x)^2}. \tag{3}$$

where $z_1(x)^2$ and $y_1(x)$ – parabolas along respective coordinate axes.
The construction of the second spatial parabola $f_{par1_r,2_r,3_r}(x,y,z)$ is similar to the above scheme the auxiliary plane curve is defined $y_2(x)$ and the equation of the second spatial parabola is obtained accordingly:

$$f_{par1_r,2_r,3_r}(x,y,z)=\sqrt{z_2(x)^2+y_2(x)^2}\,. \tag{4}$$

CALCULATION ALGORITHM

In accordance with the accepted design scheme of the reinforced concrete structure of the beam of round cross section under the action of bending and torque moments and transverse forces, the most dangerous are spatial sections located at the support with maximum torque, bending moments and transverse forces.
In this scheme, the first block is separated by the cross section I-I passing at the end of the spatial crack, is in equilibrium under the action of external forces applied to it from the side of the support and internal forces arising at the place of the cross section.
When evaluating the complex resistance of the beam under consideration, the design scheme shown in Figure 1 is implemented in the following order.
1. The main one is an arbitrary vertical section k, passing through the end of the front of the spatial crack, in which the intensity of deformations is taken $\varepsilon_{i,u}=\varepsilon_{b,u}$.
2. We calculate the value of shear deformations $\varepsilon_{q,u}$ (Figure 2):

$$\varepsilon_{q,u}=2\cos\beta\sin\beta\left(\varepsilon_{1,u}-\varepsilon_{3,u}\right), \tag{5}$$

where angle β determines direction of main deformation of concrete shortening in vertical section *k*.
3. Using the diagram of the dependence proposed in [23] $a/h_0-\cos\beta$ (where *a* – is the distance from the support to the edge of the crack, h_0 – the working height of the section) the value of the shear stress limit is calculated R_q:

$$R_q = \tau = \tau_{z1,x1} = \frac{\sigma_{i,u}}{\varepsilon_{i,u}} \frac{1}{2[1+\mu(\lambda)]} \varepsilon_{q,u}. \quad (6)$$

4. For point 1 of state diagrams [23] of compressed zone concrete in the section under consideration using known dependencies of solid deformable body mechanics, strains and stresses are successively calculated:

$$\varepsilon_{x,1} = \varepsilon_{1,u} \cos^2\beta + \varepsilon_{3,u} \sin^2\beta; \quad (7)$$

$$\varepsilon_{z,1} = \varepsilon_{1,u} \sin^2\beta + \varepsilon_{3,u} \cos^2\beta; \quad (8)$$

$$\sigma_{x,1} = \frac{\sigma_{i,u}}{\varepsilon_{i,u}} \frac{1}{1-\mu^2(\lambda)} \left[\varepsilon_{x,1} + \mu(\lambda)\varepsilon_{z,1}\right]; \quad (9)$$

$$\sigma_{z,1} = \frac{\sigma_{i,u}}{\varepsilon_{i,u}} \frac{1}{1-\mu^2(\lambda)} \left[\varepsilon_{z,1} + \mu(\lambda)\varepsilon_{x,1}\right]. \quad (10)$$

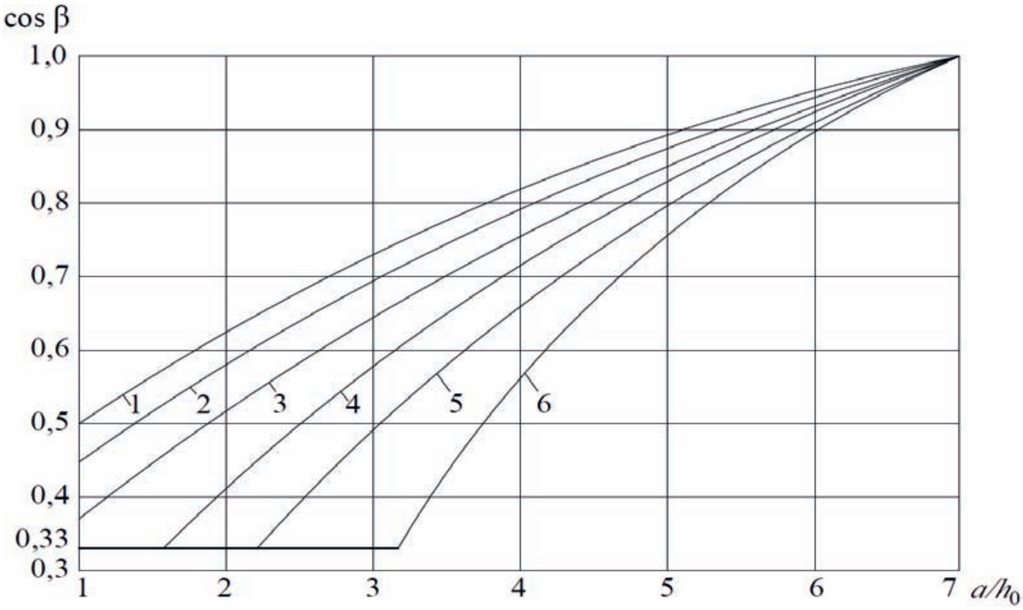

Figure 2. Diagram $a/h_0 - \cos\beta$: 1 - for ratio T/M = 0.1; 2 - for ratio T/M = 0.25; 3 - for ratio T/M = 0.5; 4 - for ratio T/M = 0.75; 5 - for ratio T/M = 1.0; 6 - for ratio T/M > 1.0

5. The transition from stresses in compressed concrete on the inclined site of the section under consideration to stresses on the normal site is carried out according to the known formulas of mechanics of the solid deformable body:

$$\sigma_{x,1} \cos^2\alpha + \sigma_{z,1} \sin^2\alpha + \tau_{x1z1} \sin 2\alpha; \quad (11)$$

$$\tau_{xz,k} = \frac{\sigma_{x,1} - \sigma_{z,1}}{2} \sin 2\alpha - \tau_{z1,x1} \cos 2\alpha; \quad (12)$$

$$\sigma_{bx,u,k} = \frac{\sigma_{x,k}}{\cos\alpha}; \quad (13)$$

$$\tau_{xz,u} = \tau_{xz,u} = \frac{\tau_{xz,k}}{\cos\alpha}; \quad (14)$$

$$\tau_{yx,k} = \sqrt{R_q^2 - \tau_{xz,k}^{\ 2}}. \quad (15)$$

Here α – is the angle between the cross section normal to the longitudinal axis and the section *k-k* in question (see Figure 1).

In practical calculations, the direction of the main deformations of compressed concrete shortening (angle β) in the vertical section passing through the end of the front of the spatial crack can be determined using a graph of dependence $a/h_0 - \cos\beta$ (see Figure 2).

6. From equilibrium equation of projections of all forces acting in section I-I on axis x ($\sum X = 0$), height of compressed zone of concrete in normal section (unknown x):

$$\sigma_{bI,u} b_1 \varphi(x_k, x) x - \sigma_{s,I} m A_{s,I} = 0. \quad (16)$$

Knowing the circumference radius of the circular cross section, the area of the compressed zone of concrete in this sections is calculated $A_{b,I}$ and then the height of the compressed zone x of concrete in this section is calculated.

From the hypothesis of proportionality of longitudinal deformations there are stresses in longitudinal working reinforcement $\sigma_{s,I}$ in section I-I:

$$\sigma_{s,I} = \frac{\varphi_{10,*}\sigma_{bu,x,I} \cdot E_s(\lambda)}{E_b(\lambda)} \cdot \frac{h_0 - x}{x} + \sigma_0 \le R_{s,I} . \quad (17)$$

If condition (17) is not met, voltage $\sigma_{s,I}$ is taken equal to R_s.

Normal shortening deformations along the x axis in compressed concrete at various points of design section k-k and in section I-I can be found on the condition of their proportionality to limit deformations $\varepsilon_{bu,k,rig,x}$ at the extreme point of section k-k (see Figure 1a):

– from right point (rig) to section I-I (similar to section 3-3)

$$\varepsilon_{b,I} = \frac{\varepsilon_{bu,k,rig,x} \cdot a}{a - l_1}; \quad (18)$$

– from right point (rig) to middle point (b$_k$)

$$\varepsilon_{b,k,x} = \frac{\varepsilon_{bu,k,rig,x} \cdot a_{m,b}}{a - l_1} = \frac{\varepsilon_{bu,k,rig,x} \cdot \left[a - \left(l_1 + \frac{\sqrt{2}}{2} l_2 \cdot \frac{1}{2} - \eta_{hor,b} \cdot l_2 \right) \right]}{a - l_1}; \quad (19)$$

– from right point (rig) to left point (lef)

$$\varepsilon_{bu,k,lef,x} = \frac{\varepsilon_{bu,k,rig,x} \cdot \left[a - (l_1 + l_2) \right]}{a - l_1} . \quad (20)$$

7. From the equilibrium equation of torques of all internal and external forces acting in section I-I, relative to the axis perpendicular to this section and passing through the point of application of equal forces in concrete of compressed zone ($\sum T = 0$), shear stresses from torque and transverse force (unknown $\tau_{T,I}$ and $\tau_{Q,I}$), are determined, where they are directed in opposite directions, and the ratio between them is taken equal to the ratio T_1 / Q_1 :

$$\omega_+ \omega_{1+} \tau_{\sum +} \frac{b_1}{2} b' x + \omega_- \omega_{1-} \tau_{\sum -} \frac{b_1}{2} b'' x - T_1 = 0; \quad (21)$$

$$\tau_{\sum +} = \tau_{T,I} \pm \tau_{Q,I}; \quad (22)$$

$$\frac{\tau_{T,I}}{\tau_{Q,I}} = \frac{T_1}{Q_I} . \quad (23)$$

Here b' (b'') – distance in radius from the center of gravity of the section to the arc of the section contour on the side where shear stresses $\tau_{T,I}$ and $\tau_{Q,I}$ are directed to one side (directed to opposite sides).

For total shear stresses calculated by the formula (22) the condition must be met:

$$\tau_{\sum +} \le \tau_{pl} . \quad (24)$$

Here $\tau_{pl} = 1.1 R_{bt}$.

The second support block is separated from the reinforced concrete element by a spatial section formed by a spiral-shaped crack and a vertical section passing along the compressed zone of concrete through the end of the front of the spatial crack (see Figure 1).

For a circular cross section, the torque in section 1-1 will be:

$$M_{t,I} = \tau_{pl,u} \cdot \lambda x_1 \cdot b_{cir} \cdot \varphi_{cir,\tau} \cdot (z_1 - 0.5 \cdot \lambda x_1) + 0.5 \cdot \tau_{pl,u} \cdot (x_1 - \lambda x_1) \cdot b_{cir} \cdot \left[0.5 \cdot \lambda x_1 + \frac{1}{3} \cdot (x_1 - \lambda x_1) \right] + Q_s (h_0 - z_1) . \quad (25)$$

From the equilibrium equation of projections of internal and external forces acting in section I-I on the axis Y ($\sum Y = 0$):

$$-\tau_{pl,x} \cdot x \cdot b - \tau_{pl,x} \cdot k_{Q,m} \cdot (h_0 - x) \cdot b + K_M \cdot R_{sup} = 0. \qquad (26)$$

Here, it is assumed that the load forces in the working reinforcement on average between the cracks in the cross section I-I are zero.

The lateral force perceived by the compressed zone concrete will be:

$$Q_{I,b} = \tau_{pl,x} \cdot x \cdot b. \qquad (27)$$

In turn, the transverse force perceived by the concrete of the stretched zone will be (Figure 4):

$$Q_{II,T} = \tau_{pl,x} \cdot k_{Q,m} (h_0 - x) \cdot b. \qquad (28)$$

On the other hand, the value of the transverse force as part of the entire transverse force perceived by the design section is

$$Q_{II,T} = Q - Q_{I,b}. \qquad (28)$$

The equation (26) can be used to determine a parameter $k_{Q,m}$, hat takes into account the presence of adjacent spatial cracks on the stressed-strain state of the stretched zone in the middle (between the cracks) of the calculated cross section I-I:

$$k_{Q,m} = \frac{K_M \cdot R_{sup} - \tau_{pl,x} \cdot x \cdot b}{\tau_{pl,x} (h_0 - x) \cdot b}. \qquad (29)$$

8. From the equilibrium equation of bending moments of all internal and external forces acting in section I-I ($\sum M_B = 0$), a generalized reference reaction (unknown R_{sup}) is determined:

$$\sigma_{s,I} m A_{s,I} \left[h_{1,0} - \varphi(x_k, x) x \right] - M_I - R_{sup} a_1 = 0 \qquad (30)$$

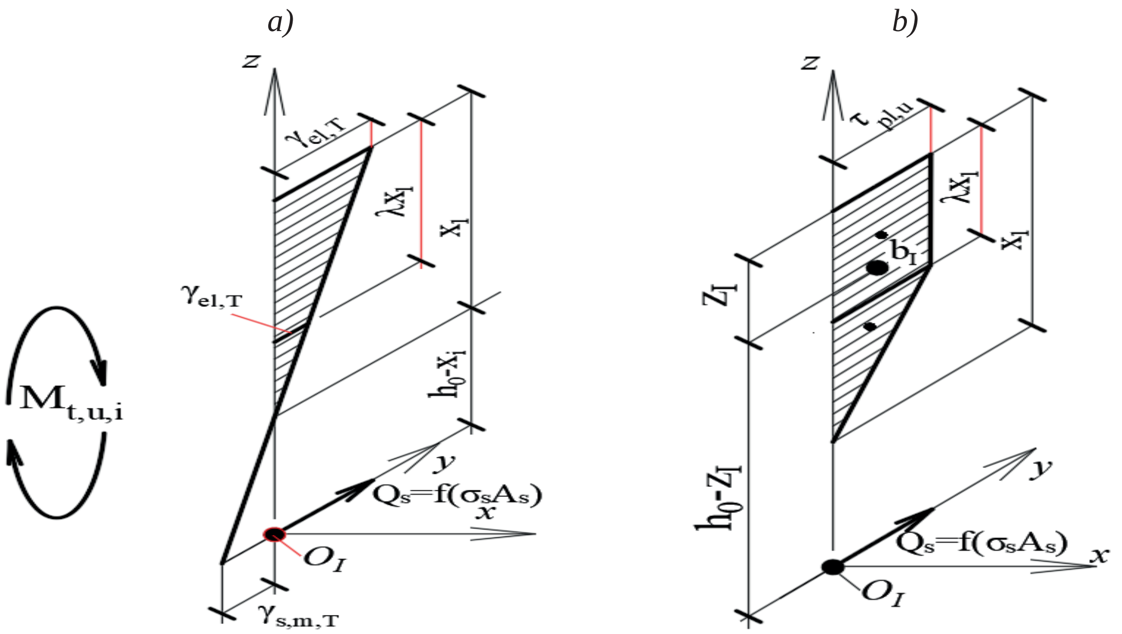

Figure 3. Schemes of distribution of shear strains and shear stresses in the cross section I-I (3-3)

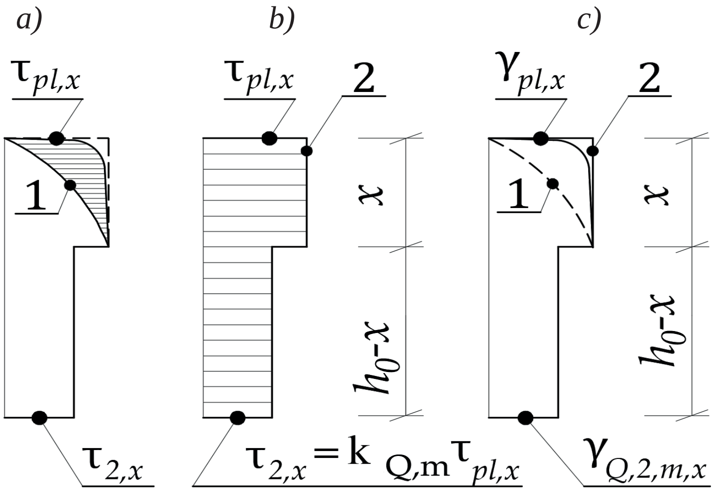

Figure 4. Diagram of transverse shear stresses τ_Q in medium cross sections I–I (3-3)

Where $\varphi(x_k, x)$ – coefficient of completeness of stress graphic in compressed concrete. For the limit states of the first group, the value of this coefficient can be taken to be 0.75); m is the number of rods of longitudinal working reinforcement in the section under consideration.

For a circular cross section from the equilibrium equation of moments of internal and external forces in section I-I relative to the y axis passing through the point of application of equal forces O_I in the stretched reinforcement ($\sum M_{O,I}=0$), we obtain the unknown $M_{bend,I}$:

$$M_{bend,I} = R_{sup,I,M} \cdot a_{m,I};\ \varphi_{10,*} \cdot \sigma_{bu,x,I} A_{b,I} [h_0 - \varphi_{z,cir} \cdot x] + +m \cdot R_{sc,I,up} \cdot \omega_{up,cir} \cdot A_{sc,up} \left(h_0 - a'_s\right) + \sum R_{sc,I,i,lef} \cdot \omega_{c,cir} \cdot A_{sc,I,i,lef} \left(h_0 - a'_{s,i,lef}\right) + + \sum R_{sc,I,i,rig} \cdot \omega_{c,cir} \cdot A_{sc,I,i,rig} \left(h_0 - a'_{s,i,rig}\right) - \sum R_{s,I,i,lef} \cdot \omega_{cir} \cdot A_{s,I,i,lef} \left(a_{s,i,lef} - a_{s,d}\right) - - \sum R_{s,I,i,rig} \cdot \omega_{cir} \cdot A_{s,I,i,rig} \left(a_{s,i,rig} - a_{s,d}\right) - K_M K_{pr,M} \cdot R_{sup,I} - R_{sup,I} \cdot a = 0, \quad (31)$$

and then lateral force from internal forces:

$$Q_I = R_{sup,I,M} = \frac{M_{bend,I}}{a_{m,I}} = \frac{M_{bend,I}}{a + K_M K_{pr,M}}. \quad (32)$$

9. From the condition of zero equality of the sum of projections of all forces on the x axis (∑X=0, block II, see Figure 1) acting in space section k, an unknown parameter is determined - the height of the compressed zone in section k ($x_{B,k}$) :

$$-\sigma_{s,k} \cdot m_d \cdot A_{s,d} + \sum R_{sc,up,i} \cdot \omega_{up} \cdot A_{sc,up,i} - \sum R_{s,i,rig} \cdot \omega_{rig} \cdot A_{s,i,rig} - \sum R_{s,i,lef} \cdot \omega_{lef} \cdot A_{s,i,lef} + + \sum R_{sc,i,rig} \cdot \omega_{c,rig} \cdot A_{sc,i,rig} + \sum R_{sc,i,lef} \cdot \omega_{c,lef} \cdot A_{sc,i,lef} - \varphi_{10} \tau_{xy,u,Mt} \cdot A_{b,l_1} + \varphi_{10} \tau_{xy,u,Mt} \cdot A_{b,l_3} - \sigma_{b,1,rig} \cdot A_{b,*} - \sigma_{sc,1,rig} \cdot A_{sc,1,rig} - \sigma_{b,3,lef} \cdot A_{b,**} - \sigma_{sc,3,lef} \cdot A_{sc,3,lef} - - X_{b,k} \cdot A_{b,k,x_B} - \varphi_{10} \tau_{xy,u,Mt} \cdot A_{b,l_1,ad} + \varphi_{10} \tau_{xy,u,Mt} \cdot A_{b,l_3,ad} - \sigma_{b,1,rig} \cdot A_{b,*,cir,ad} - \sigma_{b,3,lef} \cdot A_{b,**,cir,ad} - X_{b,k} \cdot A_{b,k,core} = 0. \quad (33)$$

The following restrictions must be taken into account for $x_{B,k}$:

$$0.1h_0 \le x_{B,k} \le 0.3h_0; \quad (34)$$

$$x_{B,k} \le x_k. \quad (35)$$

In equation (34), $X_{b,k}$ – projection of the resulting directions in the grain of concrete on the axis x; A_{b,l_1} – area of compressed concrete on the site l_1; A_{b,l_3} – area of compressed concrete on the site l_3; $A_{b,*,cir}$ – concrete area of the right sector of broken section (sector with height $x_{B,1}$); $A_{b,**,cir}$ – concrete area of the left sector of broken section (sector with height $x_{B,3}$); $A_{b,l_1,ad}$ – concrete area with height Δx_1 at the site l_1; $A_{b,l_3,ad}$ – concrete area with height Δx_3 at the site l_3; $A_{b,*,cir,ad}$ – area of compressed concrete of part of the right sector with height Δx_1; $A_{b,**,cir,ad}$ – compressed concrete area of the left sector part with height Δx_3; $A_{b,k,core}$ – area of part of compressed zone of concrete in section k of small ellipse enclosed between points AB and section of arc of ellipse spaced from surface of small ellipse by height $x_{B,k}$ (see figure 1); A_{b,k,x_B} – area of part of compressed zone of concrete limited by sections of arcs of ellipses in section k-k, spaced from each other at a distance $x_{B,k}$; $\sum_{k=1}^{n_{rig}} R_{s,i,rig} \cdot \omega_{rig} \cdot A_{s,i,rig}$ ($\sum_{k=1}^{m_{rig}} R_{sw,i,rig} \cdot \omega_{rig} \cdot A_{sw,i,rig}$) – forces in stretched longitudinal reinforcement (transverse clamps) located in the area of the RH circuit of the first section of the spatial parabola $f_{par1,2,3}(x,y,z)$ (site l_1); $\sum_{k=1}^{m_{lef}} R_{sw,i,lef} \cdot \omega_{lef} \cdot A_{sw,i,lef}$ ($\sum_{k=1}^{n_{lef}} R_{s,i,lef} \cdot \omega_{lef} \cdot A_{s,i,lef}$) – forces in the stretched longitudinal reinforcement (transverse clamps) located in the area of the left contour of the second section of the spatial parabola $f_{par1_r,2_r,3_r}(x,y,z)$ (site l_3); h_0 – section working height, $\varphi_{7,lef}$, $\varphi_{7,rig}$, $\varphi_{8,lef}$, $\varphi_{8,rig}$, – parameters that take into account the components of the "heating" effect in the working reinforcement.

To determine the projection of component stresses in compressed concrete $X_{b,k}$ the x axis is previously calculated as the following unknown in the section k-k.

Deformations of longitudinal reinforcement in the left and right parts of the section $\varepsilon_{s,k,rig,x}$ and $\varepsilon_{s,k,lef,x}$ on condition of their proportionality in section x and section k-k:

$$\varepsilon_{s,k} = \frac{\varepsilon_{s,I} \cdot a_{m,s}}{a} = \frac{\varepsilon_{s,I} \cdot \left(a - \frac{h}{2h+b} \cdot c - \frac{2h+b}{c} \cdot 0.5 \cdot b \right)}{a}; \quad (36)$$

$$\varepsilon_{s,k,rig,x} = \frac{\varepsilon_{s,I} \cdot (a - l_1)}{a}; \quad (37)$$

$$\varepsilon_{s,k,lef,x} = \frac{\varepsilon_{s,k,rig,x} \cdot [a - (l_1 + l_2)]}{a - l_1}. \quad (38)$$

The relationship between angular deformations is recorded:

$$\gamma_{zx,pl} = k_* \cdot \gamma_{zx,Mt,el}; \quad (39)$$

where $k_* = M_t / M_{t,crc}$. In the first approximation for the crack torque we take $M_{t,crc} = 0{,}4M_t$.

We also take limit values of tangent stresses $\tau_{zx,pl} = R_q = R_{ch}$.

Knowing the angular deformations from the torque, the values of the tangent stresses from the action of the transverse force are determined

$$\tau_{zx,Q} = \tau_{zx} - \tau_{zx,Mt}; \quad (40)$$

$$k_{**} = \frac{\tau_{yx,Mt}}{\tau_{zx,Mt}} = \frac{\tau_{yx}}{\tau_{zx}}; \quad (41)$$

For the middle point (b_k) of the compressed zone of the broken design section k-k, the projection of the component stresses in the compressed concrete $X_{b,k}$ on the x axis in the section l_2 is calculated by the formula:

$$X_{b,k} = \frac{\varphi_{z,1} \cdot \sigma_{bu,x,I} \cdot \varphi_{1,*} \cdot \left[a - \left(l_1 + \frac{\sqrt{2}}{2} l_2 \cdot \frac{1}{2} - \eta_{hor,b} \cdot l_2 \right) \right]}{a} + \left(\varphi_{12} \cdot \tau_{zx,u,Q} \pm \varphi_{10} \cdot \tau_{xy,u,Mt} \right) \cdot \varphi_{3,*}; \quad (42)$$

The height of the compressed zone x_k in the section k-k between the cross sections I-I and III-III (see Figure 1) can be found from a linear proportion;

$$x_k = \frac{x_1 + x_3}{2}, \quad (43)$$

where x_1– is the height of the compressed zone in section I-I; x_3 – is the height of the compressed zone in section III-III.

By calculating the height of the compressed zone of concrete $x_{B,k}$ in section k-k from equation (34) and using the conditions of proportionality between the heights of the compressed zone in sections k-k and 1-1, the following ratios are written to determine the height of the compressed zone in section 1-1:

$$Ч_1 = \frac{x_{B,k}}{x_{B,k} + \Delta x_1}. \quad (44)$$

$$\Delta x_1 = x_k - x_1. \quad (46)$$

$$\frac{x_{B,k}}{x_{B,k} + \Delta x_1} = Ч_1 = \frac{x_{B,1}}{x_{B,1} + \Delta x_1}$$

from which is calculated

$$x_{B,1} = \frac{Ч_1 \cdot \Delta x_1}{Ч_1 - 1}. \quad (45)$$

Similarly, the height of the compressed zone is in another normal section $x_{B,3}$ (see Figure 1):

10. Intensity of load in clamps on the right side of round cross section is determined from equality of zero of sum of projections of all forces acting in space section k-k on z axis (∑Z=0, refer to Figure 1):

$$\begin{aligned} q_{sw,rig} = \frac{tg\alpha}{h}\Big[& q_{sw,lef} \cdot \eta_q \cdot \frac{h}{tg\alpha_0} - \varphi_{7,*} R_s \sum \omega_{*,cir} A_{s,*,cir} - \varphi_{7,*,rig} R_s \sum \omega_{rig,*,cir} A_{s,rig} + \\ & + \varphi_{7,*,lef} R_s \sum \omega_{lef,*} A_{s,lef} - Q + R_{sup} + \left(\varphi_{11} \tau_{zy,u,Mt} + \varphi_9 \tau_{zy,u,Q} \right) \cdot A_{b,l_1} + \\ & + \left(\varphi_{11} \tau_{zy,u,Mt} - \varphi_9 \tau_{zy,u,Q} \right) \cdot A_{b,l_3} + \\ & \left(\varphi_{14} \cdot \tau_{zx,u,Mt} + \varphi_{12} \cdot \tau_{zx,u,Q} \right) \cdot A_{b,*,cir} + \\ & \left(\varphi_{14} \cdot \tau_{zx,u,Mt} - \varphi_{12} \cdot \tau_{zx,u,Q} \right) \cdot A_{b,**,cir} + Z_{b,k} \cdot A_{b,k,x_B} - \left(\varphi_{11} \tau_{zy,u,Mt} + \varphi_9 \tau_{zy,u,Q} \right) \cdot A_{b,l_1,ad} + \\ & \left(\varphi_{11} \tau_{zy,u,Mt} - \varphi_9 \tau_{zy,u,Q} \right) \cdot A_{b,l_3,ad} + \left(\varphi_{14} \cdot \tau_{zx,u,Mt} + \varphi_{12} \cdot \tau_{zx,u,Q} \right) \cdot A_{b,*,cir,ad} + \\ & + \left(\varphi_{14} \cdot \tau_{zx,u,Mt} - \varphi_{12} \cdot \tau_{zx,u,Q} \right) \cdot A_{b,**,cir,ad} + Z_{b,k} \cdot A_{b,k,core}. \end{aligned} \quad (46)$$

Here $\varphi_{7,lef}$, $\varphi_{7,rig}$ – parameters that take into account the components of the "heating" effect in the reinforcement. At each iteration step, these parameters are taken into account as constants, not as functions and are determined based on the second level model [25]; Q – the transverse force of the section from the support to the section k-k.

For the middle point (b_k) of the compressed zone of the broken design section k-k the projection of the component stresses in the compressed concrete on the z axis in the section l_2 is calculated by the formula:

$$Z_{b,k} = \left(\varphi_{12} \cdot \tau_{zx,u,Q} \pm \varphi_{14} \cdot \tau_{zx,u,Mt}\right) \cdot \frac{\sqrt{2}}{2} + \left(\varphi_9 \cdot \tau_{zy,u,Q} \pm \varphi_{11} \cdot \tau_{zy,u,Mt}\right) \cdot \frac{\sqrt{2}}{2} \quad (47)$$

Here, the components of tangent stresses in compressed concrete are calculated by analogy as in para. 9, with replacement by $\tau_{zx,u,Q}$ by $\tau_{zx,ad,Q}$, $\tau_{zx,u,Mt}$ by $\tau_{zy,ad,Q}$, $\tau_{zy,u,Mt}$ by $\tau_{zy,ad,Mt}$ etc.

The load intensity in the clamps on the left side of the round cross section $q_{sw,lef}$ is based on the ratio:

$$q_{sw,lef} = q_{sw,rig} - Ч_{11,*} \cdot \tau_Q; \quad (48)$$

It is assumed that the projections of inclined cracks on the left and right sides of the circular section are approximately the same $c_1 \approx c_2$ and then the value of the coefficient $Ч_{11,*} = Ч_{11} \cdot b$, where b is taken equal to the radius of the cross section of the calculated structure. Values of tangent stresses taking into account the action of transverse force on the left or right of section are performed with plus-minus sign ($\pm\tau_Q$) respectively.

At that, condition check is performed for load intensity in clamps $q_{sw,lef}$:

$$\frac{n \cdot R_{bt} \cdot A_{sw}}{u_s} \le q_{sw,lef} \le \frac{0.8 \cdot R_{sw} \cdot A_{sw}}{u_s} - Ч_{11,*} \cdot \tau_Q. \quad (49)$$

Values of parameters included in formula (51) are calculated by formulas:

$$\sigma_{sw} = \frac{E_{sw}}{E_b} \cdot \sigma_{bt} = n \cdot R_{bt};$$

$$\frac{n \cdot R_{bt} \cdot A_{sw}}{u_s} = q_{sw,lef,min}; \quad (50)$$

$$Ч_{11} \cdot \tau_Q = \psi_{Q,*} \cdot \tau_{pl,u} = \psi_{Q,*} \cdot R_{ch}\left(\frac{c}{h_0}\right); \quad (51)$$

At the same time, tangent stresses $\tau_{Q,k}$ are based on the condition of proportionality of relations of stresses and forces in section k and in section I-I:

$$\frac{\tau_{Q,I}}{\tau_{Q,k}} = \frac{Q_1}{Q_{k,m}}. \quad (52)$$

Here $\tau_{Q,1}$ and $\tau_{Q,k}$ – tangent stresses due to the action of the transverse force in the normal section I-I and in the center of the compressed zone of the spatial section k- k, respectively; Q_1 and $Q_{k,m}$ – a transverse force acting in the normal section I-I and in the center of the compressed zone of the space section k-k, respectively.

11. Stresses in longitudinal reinforcement σ_s are determined from the equation of moments of all internal and external forces ($\sum M_B = 0$), acting in the vertical longitudinal plane relative to the axis Z, passing through the point of application of equal forces in the concrete of the compressed zone:

$$\sigma_s m A_s \cdot \left(h_{1,0} - 0.5x_k\right) - M_k - R_{sup} a_{1,m} = 0 \quad (53)$$

This checks the condition:

$$\sigma_s \le m_{a3} R_s. \quad (54)$$

If this condition is not met, then σ_s we take it equal to $m_{a3}R_s$, where $m_{a3} = l_x / l_{anc}$), l_x – distance from the beginning of the stress transfer zone in the pre-stressed reinforcement to the section under consideration; If there is no prestress in the design, the factor $m_{a3} = 1$.

The bending moment, as a function of the calculated parameters of the section k-k, is from the static condition of equal to zero moments of all internal and external forces acting in the vertical longitudinal plane relative to the y axis passing through the point of application of equal forces in the concrete of the compressed zone b_k ($\sum M_{b,k}$=0,see Figure 1) $M_{bend,k}(\sigma_{s,k}, \sigma_{s,rig}, \sigma_{s,lef}, h_0, x_{B,k}, A_{s,d}, R_{s,rig}, R_{s,lef} A_s, \omega_{*,cir}, \varphi_{5,*}, \varphi_{7,*}, \varphi_9, \varphi_{11}, q_{sw,rig}, q_{sw,lef}, A_{b,*,cir}, A_{b,**,cir}, l_1, l_2, l_3, \tau_{zy,u,Q}, \tau_{zy,u,Mt}, x_{B,k}, x_{B,1}, x_{B,3}, X_{b,k})$.

This constraint is detailed in [25].

The bending moment corresponding to the level of crack formation is from the following constraint

$$R_{bt} = \frac{M_{crc,k}}{0.85W_{x,k}} = \frac{M_{crc,k}}{0.85W_0 \cdot \gamma}; \tag{55}$$

Here, $R_{bt} = 2.2 МПа$ - design tensile strength of concrete; $\gamma = 2$ - a coefficient taken for a circular section equal to two; $W_{bend} = \pi D^3 / 32$.

The generalized reference reaction corresponding to the moment of crack formation is accepted $R_{sup,crc} = 0{,}4\ R_{sup,u}$.

From here, the transverse force from internal forces in the design section is determined:

$$Q_k = R_{sup,k,M} = \frac{M_{bend,k}}{a_{m,k}} = \frac{M_{bend,k}}{a_{m,b}(c) + K_M K_{pr,M}}. \tag{56}$$

12. Intensity of load in clamps located in lower stretched zone of round cross section is determined from equality of sum of projections of all forces acting in spatial section k- k on y axis equal to zero ($\sum Y$=0, refer to Figure 1) we have:

$$q_{sw,\sigma} = \frac{1}{\sqrt{l_2^2 + 4R^2}} \cdot \Big[-\varphi_{8,*,rig} R_s \sum \omega_{rig,cir} A_{s,rig} - -\varphi_{8,*,lef} R_s \sum \omega_{lef,cir} A_{s,lef} - \varphi_{8,*} \cdot R_s \sum \omega_* A_s +$$
$$+ \varphi_{13} \cdot \tau_{yx,u,Mt} \cdot A_{b,*} + \varphi_{13} \cdot \tau_{yx,u,Mt} \cdot A_{b,**} + +Y_{b,k} \cdot x_{B,k} \cdot \sqrt{l_2^2 + b^2} + \varphi_{13} \cdot \tau_{yx,u,Mt} \cdot A_{b,*,ad} +$$
$$+ \varphi_{13} \cdot \tau_{yx,u,Mt} \cdot A_{b,**,ad} + Y_{b,k} \cdot A_{b,k,x_B} + Y_{b,k} \cdot A_{b,k,core} \Big]. \tag{57}$$

Here, φ_{8*} – parameters, which take into account components of "heating" effect of reinforcement. At each iteration step, these parameters are taken into account as constants, not as functions, and are determined based on the second level model [25];

For the middle point (b_k) of the compressed zone of the broken design section k-k, the projection of the component stresses in the compressed concrete on the y axis in the section l_2 is calculated by formula:

$$Y_{b,k} = \varphi_9 \cdot \tau_{zy,u,Q} \cdot \frac{\sqrt{2}}{2} \pm \varphi_{13} \cdot \tau_{yx,u,Mt} \cdot \frac{\sqrt{2}}{2}. \tag{58}$$

Here, the components of tangent stresses in compressed concrete, by analogy with formula (49) with the replacement of the corresponding components of tangent stresses $\tau_{zy,ad,Q}$ instead of $\tau_{zy,u,Q}$ and $\tau_{yx,ad,Mt}$ instead of $\tau_{yx,u,Mt}$,

$\tau_{zy,ad,Q} = \tau_{zy,u,Q} - \tau_{zy,crc,Q}$;

$\tau_{yx,ad,Mt} = \tau_{yx,u,Mt} - \tau_{yx,crc,Mt}$.

13. The torque, as a function of the calculated parameters of the section k-k, is from the static condition of equal to zero moments of all internal and external forces acting in the vertical transverse plane relative to the x axis passing through the point of application of equal forces in the concrete of the compressed zone b_k ($\sum T_{b,k}$=0, see Figure 1) $M_{t,k}(q_{sw,\sigma}, q_{sw,rig}, q_{sw,lef}, \eta_q, \eta_{hor,b}, \varphi_{7,*}, \varphi_{8,*}, R_s, A_s, x_{B,k}, h_0, A_{b,l_1}, A_{b,l_2}, A_{b,l_3}, \varphi_{11}, \varphi_9, \tau_{yz,u,Mt}, \tau_{yz,u,Q}, A_{b,*,cir}, A_{b,**,cir}, \eta_{hor,b})$.

This constraint is detailed in [25].

14. The length of the projection of the spatial crack is determined using the function $f(x,y,z)$ for the diagonal large ellipse of the circular cross-section structure introduced into the design scheme, with a smaller $b=R$ and larger diagonal $a=\frac{l(c)+R\sqrt{2}+l(c)}{2cos\alpha}= =\frac{2l(c)+R\sqrt{2}}{2\,cos\,\alpha}$), respectively:

$$\frac{(y-y_0)^2}{(\frac{2l(c)+R\sqrt{2}}{2\,cos\,\alpha})^2}+\frac{(z-z_0)^2}{R^2}=1. \qquad (59)$$

Based on this, the size of the spatial crack for projecting it onto the horizontal axis is:

$$c=l_1(c)+l_2+l_3(c)=\frac{2yR}{\sqrt{(R^2-z^2)}}cos\,\alpha \le R\cdot 3\sqrt{2}. \qquad (60)$$

where, $l_1(c)=l_3(c)=d\,cos\,\alpha=d\cdot cos\,45°=d\frac{\sqrt{2}}{2}$; $z=0$; $y=\frac{0.5l_2}{cos\,\alpha}=\frac{0.5R\sqrt{2}}{cos\,\alpha}=\frac{R\sqrt{2}}{2\,cos\,\alpha}$.

$$c=l_1(c)+l_2+l_3(c)=\frac{2yR}{\sqrt{(R^2-z^2)}}cos\,\alpha=\frac{2R^2\sqrt{2}}{2R}=R\sqrt{2}\le R\cdot 3\sqrt{2}\;\; R\sqrt{2}\le R\cdot 3\sqrt{2}$$

$$R\cdot 1.41\le R\cdot 4.24$$

As a result, for the length boundary of the spatial crack, we can write:

$$1.41R\le c\le 4.24R.$$

In addition, for the length of the spatial crack, the limitation of the existing standards must be checked - no more than c_0, determined by the formula of paragraph 8.1.9 SP 63.13330.2018:

$$c_0=\sqrt{\frac{R_sA_{s,I}(2h+b)}{q_{sw,1}}}, \qquad (61)$$

and take into account the limitation $c\le 2h+b$.

CONCLUSION

1. A block analytical calculated model and an algorithm for estimating the complex resistance of a reinforced concrete structure of a circular cross section from the action of bending with torsion, with modeling of the calculated sections by small and large ellipses and modeling of the calculation spatial crack by sections of specially constructed spatial parabolas.

2. Straining model equations for determination of unknown bending $M_{bend,k}$ and torque moment M_t, height of compressed zone of concrete $x_{B,k}$, deformations $\varepsilon_{s,k,rig,x}$, $\varepsilon_{s,k,lef,x}$ and stresses $\sigma_{s,k,rig,x}$, $\sigma_{s,k,lef,x}$ in reinforcement on the left and right of design section, intensity of load in clamps located respectively on the left and right side of design section $q_{sw,lef}$ $q_{sw,rig}$ and intensity of load in clamps located in lower stretched zone of section $q_{sw,\sigma}$. obtained using physical ratios for concrete and reinforcement and static conditions in spatial design section.

3. In the spatial section k, for the block cut off by a complex section passing along a spiral-shaped crack in the compressed zone, all reinforcement is taken into account, falling into this cross section and "heating" effect "in the stretched longitudinal and transverse reinforcement, falling into this spatial section, as well as normal and tangent stresses, located on sections normal to longitudinal axis at the distance x from support and that, as bending

moments increase, height of compressed zone of concrete in section k between first and third round normal cross sections decreases.

4. For dangerous spatial crack length of projection of this crack is found With projection on horizontal axis of diagonal large ellipse of function $f(x,y,z)$ with smaller diagonal $b = R$ and ellipse with larger diagonal $a = l_1(c) + l_2 + l_3(c)$ for which restriction is accepted $1.41R \le c \le 4.24R$.

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