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THE MODEL OF FREE SPREADING A FLOW RAPID BEHIND A RECTANGULAR PIPE

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Abstract. The article considers the free spreading of a turbulent stationary two-dimensional open potential water flow into a wide diverting riverbed behind a non-pressure pipe of a rectangular section. A system of nonlinear partial differential equations of motion has been adopted as the mathematical model of the flow in the physical plane. When moving to the plane of the velocity hodograph, the nonlinear system of equations is transformed into a linear system with respect to partial derivatives. Using the obtained system of equations, various problems along the flow of two-dimensional water streams have been solved analytically. The paper determines the flow kinetics parameter τ and the angle θ characterizing the direction of the local flow velocity vector at the intersection points of an arbitrary equipotential and an arbitrary current line. The X, Y coordinates of these points are found. The peculiarities of changing the angle θ during the transition of the vertical front of the X_D are taken into account. Article proposes a module for the transition from a two-dimensional water flow model to a one-dimensional one. This module is necessary for using the laws of flow resistance and taking into account the resistance forces. The model proposed in this paper is a development of analytical methods for calculating potential flows with previously unknown boundaries and before the flow expands. It allows determining the entire range of geometric and kinematic parameters of the flow with an error not exceeding 10%. The adequacy of the model for all flow parameters improves the accuracy of previously existing methods. This allows the designers of road culverts to increase its reliability.

Keywords: mathematical model, two-dimensional flow, motion equations, resistance forces, flow energy equations, line flow, hydrodynamic pressure, flow spread parameters, free spreading of the flow into the diverting riverbed.

МОДЕЛЬ СВОБОДНОГО РАСТЕКАНИЯ БУРНОГО ПОТОКА ЗА ПРЯМОУГОЛЬНОЙ ТРУБОЙ

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Аннотация. Рассматривается свободное растекание бурного, стационарного, двухмерного в плане, открытого, потенциального, водного потока в широкое отводящее русло за безнапорной трубой прямоугольного сечения. Математическая модель потока в физической плоскости описывается в виде системы нелинейных дифференциальных уравнений движения в частных производных. При переходе в плоскость годографа скорости нелинейная система уравнений трансформируется в линейную систему относительно частных производных. Пользуясь полученной системой уравнений аналитически решены различные задачи по течению двухмерных водных потоков. Определены параметр кинетичности τ потока и угол θ, характеризующий направление вектора местной скорости потока в точках пересечения произвольной эквипотенциали и произвольной линии тока. Найдены координаты X, Y этих точек. Учтены особенности изменения угла θ при переходе вертикального фронта $X_Д$. Предложен модуль перехода от двумерной в плане модели течения водного потока к одномерной. Этот модуль необходим для использования законов сопротивления потоку и учету сил сопротивления. Модель, предложенная в работе представляет собой развитие аналитических методов расчёта потенциальных потоков с заранее неизвестными границами и до расширения потока $\beta = 7 \div 10$. Позволяет с погрешностью не превышающую 10% определять весь спектр геометрических и кинематических параметров потока. Адекватность модели по всем параметрам потока до расширения β = 7÷10 улучшает адекватность по ранее существующим методам, что позволяет проектировщикам ГТС дорожных водопропускных сооружений повышать их надёжность.

Ключевые слова: математическая модель, двухмерный в плане водный поток, уравнения движения, силы сопротивления, уравнения энергии потока, линия тока, гидродинамический напор, параметры растекания потока, свободное растекание потока в отводящее русло.

INTRODUCTION

This work is devoted to the development of I.A. Sherenkov's ideas [1] for solving a practical problem concerning the turbulent flow behind the voluntary flow into a wide discharge channel. The flow parameters behind the non-pressure pipe are necessary for the road water conduit structures' design under roads and railways [2]. On the basis of the characteristic theory, I.A. Sherenkov carried out the solution to the problem [3, 4] and supplemented it with the analytical studies.

I.A. Sherenkov developed a theory for calculating the flow parameters, based on which he proposed a universal schedule as it was shown in the work [5]. Previously, this theory provided a powerful impetus for the culverts' calculation development. However, it turned out to be very approximate due to the graphical methods of the characteristic theory use, borrowed from the gas dynamics [6]. Thereby, t**he purpose of this work** is development of the same problem solution based on the analytical methods.

1. RESEARCH METHODS

1.1. Equations of water flow motion in the physical plane of its streaming.

The motion basic equations of a two-dimensional plan water flow have the form [7, 8]:

$$\begin{cases} X-\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}-T_x=\dfrac{du_x}{dt}; \\ Y-\dfrac{1}{\rho}\dfrac{\partial p}{\partial y}-T_y=\dfrac{du_y}{dt}; \\ Z-\dfrac{1}{\rho}\dfrac{\partial p}{\partial z}-T_z=\dfrac{du_z}{dt}, \end{cases} \tag{1}$$

where X, Y, Z are the volumetric force components; T_x, T_y, T_z are the components of resistance forces per liquid mass unit; ρ denotes liquid density; p is the local pressure.

For the steady-state water flow with vertical axis direction z and the action of a single volume force (gravity) in liquid, the system (1) takes the following form [9, 10]:

$$\begin{cases} -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}-T_x=u_x\dfrac{\partial u_x}{\partial x}+u_y\dfrac{\partial u_x}{\partial y}+u_z\dfrac{\partial u_x}{\partial z}; \\ -\dfrac{1}{\rho}\dfrac{\partial p}{\partial y}-T_y=u_x\dfrac{\partial u_y}{\partial x}+u_y\dfrac{\partial u_y}{\partial y}+u_z\dfrac{\partial u_y}{\partial z}; \\ -\dfrac{1}{\rho}\dfrac{\partial p}{\partial z}-T_z=u_x\dfrac{\partial u_z}{\partial x}+u_y\dfrac{\partial u_z}{\partial y}+u_z\dfrac{\partial u_z}{\partial z}. \end{cases} \tag{2}$$

Without considering the resistance forces to the flow

$$T_x=T_y=T_z=0$$

and taking into account:

$$u_z=0;$$

$$\frac{\partial u_z}{\partial x}=\frac{\partial u_z}{\partial y}=\frac{\partial u_z}{\partial z}=0; \quad \frac{\partial u_x}{\partial z}=\frac{\partial u_y}{\partial z}=0$$

the system (2) transforms to the following one:

$$\begin{cases} -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}-T_x=u_x\dfrac{\partial u_x}{\partial x}+u_y\dfrac{\partial u_x}{\partial y}+u_z\dfrac{\partial u_x}{\partial z}; \\ -\dfrac{1}{\rho}\dfrac{\partial p}{\partial y}-T_y=u_x\dfrac{\partial u_y}{\partial x}+u_y\dfrac{\partial u_y}{\partial y}+u_z\dfrac{\partial u_y}{\partial z}; \\ -g-\dfrac{1}{\rho}\dfrac{\partial p}{\partial z}=0. \end{cases} \tag{3}$$

From the third equation of the system (3), it follows:

$$p = -\gamma z + f(x, y),$$

where $f(x, y)$ denotes the arbitrary function. Since $z = z_n$, $p = p_n = \text{const}$ on a free surface, we arrive to the hydrostatic law of pressure distribution on the vertical:

$$p - p_n = \gamma(z_n - z).$$

Denoting the watercourse bottom coordinate with z_0, we get:

$$\frac{\partial p}{\partial x} = \gamma\frac{\partial}{\partial x}\left(z_0 + h\right), \quad \frac{\partial p}{\partial y} = \gamma\frac{\partial}{\partial y}\left(z_0 + h\right)\cdot$$

The Eqs. (3) under these conditions is written in the form:

$$\begin{cases} u_x\dfrac{\partial u_x}{\partial x} + u_y\dfrac{\partial u_x}{\partial y} = -\mathrm{g}\dfrac{\partial}{\partial x}(z_0 + h); \\ u_x\dfrac{\partial u_y}{\partial x} + u_y\dfrac{\partial u_y}{\partial y} = -\mathrm{g}\dfrac{\partial}{\partial y}(z_0 + h). \end{cases}$$

Having supplemented this system of equations with the continuity equation for a two-dimensional flow

$$\frac{\partial}{\partial x}(hu_x) + \frac{\partial}{\partial y}(hu_y) = 0$$

we obtain the system of equations:

$$\begin{cases} u_x\dfrac{\partial u_x}{\partial x} + u_y\dfrac{\partial u_x}{\partial y} = -\mathrm{g}\dfrac{\partial}{\partial x}(z_0 + h); \\ u_x\dfrac{\partial u_y}{\partial x} + u_y\dfrac{\partial u_y}{\partial y} = -\mathrm{g}\dfrac{\partial}{\partial y}(z_0 + h); \\ \dfrac{\partial}{\partial x}(hu_x) + \dfrac{\partial}{\partial y}(hu_y) = 0. \end{cases} \tag{4}$$

In case of the discharge channel horizontal bottom z_0, Eqs. (4) can be written as follows:

$$\begin{cases} u_x\dfrac{\partial u_x}{\partial x} + u_y\dfrac{\partial u_x}{\partial y} + \mathrm{g}\dfrac{\partial h}{\partial x} = 0; \\ u_x\dfrac{\partial u_y}{\partial x} + u_y\dfrac{\partial u_y}{\partial y} + \mathrm{g}\dfrac{\partial h}{\partial y} = 0; \\ \dfrac{\partial}{\partial x}(hu_x) + \dfrac{\partial}{\partial y}(hu_y) = 0. \end{cases} \tag{5}$$

The system of partial differential equations (5) describes the two-dimensional flow in terms of open stationary flows in a horizontal conduit without taking into account the flow resistance forces. This system is a system of essentially nonlinear equations isolated with respect to the unknown functions:

$$u_x = u_x(x, y); \quad u_y = u_y(x, y); \quad h = h(x, y).$$

Introducing an additional condition for the flow potentiality:

$$\Omega = \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} = 0,$$

where Ω is the velocity vortex for a two-dimensional flow, we are convinced that there exists such a potential function $\phi = \phi(x, y)$ that:

$$u_x = \frac{\partial \phi}{\partial x}; \quad u_y = \frac{\partial \phi}{\partial y}.$$

In this particular case, the system (5) is reduced to the form:

$$\begin{cases} \dfrac{u_x^2 + u_y^2}{2\mathrm{g}} + h = H_0; \\ \dfrac{\partial}{\partial x}(hu_x) + \dfrac{\partial}{\partial y}(hu_y) = 0; \\ u_x = \dfrac{\partial \phi}{\partial x}; \quad u_y = \dfrac{\partial \phi}{\partial y}, \end{cases} \tag{6}$$

where H_0 is a constant for the whole flow.

The system of equations is the main one for solving the problem of planned flows directly in the flow physical plane.

1.2. The flow equations in the velocity hodograph plane.

The system of equations (6) accepts a transition to the velocity hodograph plane as suggested by S.A. Chaplygin for the gas flows study [11, 12]. As a result of this transformation, the nonlinear system of equations (6) transformed into a linear system with respect to partial derivatives.

Introducing the squared velocity coefficient

$$t = \frac{V^2}{2gH_0},$$

where $V^2 = u_x^2 + u_y^2$; angle "θ" characterizing the local flow velocity vector direction; current function $\psi = \psi(\tau,\theta)$; potential function $\phi = \phi(\tau,\theta)$, we obtain a system of equations in the velocity hodograph plane $\Gamma(\tau,\theta)$

$$\begin{cases} \dfrac{\partial\phi}{\partial\tau} = \dfrac{h_0}{2H_0}\dfrac{3\tau-1}{\tau(1-\tau)^2}\dfrac{\partial\psi}{\partial\theta}; \\ \dfrac{\partial\phi}{\partial\theta} = 2\dfrac{h_0}{H_0}\dfrac{\tau}{(1-\tau)}\dfrac{\partial\psi}{\partial\tau}, \end{cases} \tag{7}$$

where τ, θ are the independent variables.

Wherein:

$$h = H_0(1-t); \quad V = t^{\frac{1}{2}}\sqrt{2gH_0}, \ \frac{1}{3} < \tau \le 1;$$

$$H_0 = h_0 + \frac{V_0^2}{2g},$$

where h_0, V_0 are define depth and flow velocity at some characteristic point.

The system solution (7) is reduced to solving the following equation

$$\frac{\partial}{\partial\tau}\left(\frac{2\tau}{1-\tau}\frac{\partial\psi}{\partial\tau}\right) + \frac{1-3\tau}{2\tau(1-\tau)^2}\frac{\partial^2\psi}{\partial\theta^2} = 0.$$

Moreover, there is a complex differential link between the planes $\Phi(x,y)$ and $\Gamma(\tau,\theta)$ [8]:

$$d(x+iy) = \left[d\phi + i\frac{h_0}{H_0(1-\tau)}d\psi\right]\frac{e^{i\theta}}{\tau^{\frac{1}{2}}\sqrt{2gH_0}}, \tag{8}$$

where $i = \sqrt{-1}$ - is an imaginary unit; x, y are the coordinates of a liquid flow particle in terms of its flow; τ, θ are the independent variables of a liquid particle flow in the velocity hodograph plane; $\phi = \phi(\tau,\theta)$ is a potential function; $\psi = \psi(\tau,\theta)$ is a stream function.

Eqs. (7) makes it possible solving the boundary problem of flow spreading first in the velocity hodograph plane $\Gamma(\tau,\theta)$, and then using the relation (8) to obtain a solution to the problem in the physical plane $\Phi(x,y)$.

In the works [7, 8], a whole spectrum of the system's analytical solutions (7) has been obtained. However, it was proved that the system solution (7)

$$\psi = A\frac{\sin\theta}{\tau^{\frac{1}{2}}}; \ \phi = A\frac{h_0}{H_0}\frac{\cos\theta}{\tau^{\frac{1}{2}}(1-\tau)} \tag{9}$$

is the solution providing sufficient adequacy to the real flow for the flow outlet vicinity in the range from the rectangular pipe up to the flow expansion:

$$\beta = \frac{y}{b/2} = 7 \div 10,$$

where b is the culvert width; y defines the transverse coordinate of the extreme streamline, see Fig.1.

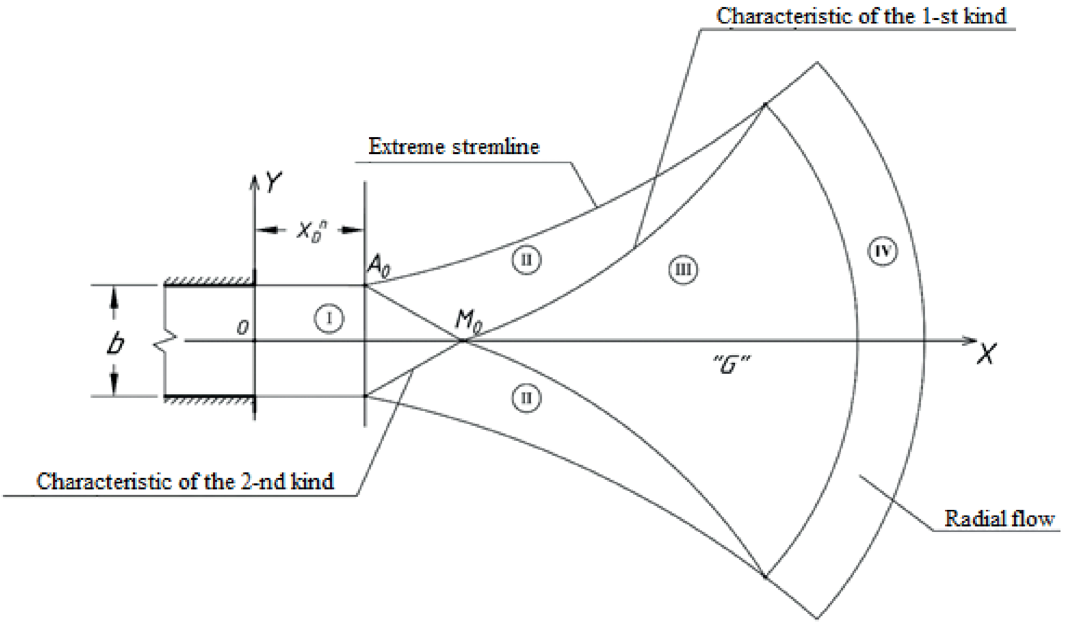

Figure 1. The plan of the flow

2. THE PROBLEM SOLUTION IN THE VELOCITY HODOGRAPH PLANE

Solving the boundary problem in the velocity hodograph plane means determining the constant "A" in the formulas (9) and determining the flow parameters τ, θ at the intersection points of an arbitrary streamline and an arbitrary equipotential.

2.1. The constant A definition.

The extreme streamline cuts off 50% of the application rate (referred to «h_0») from the longitudinal symmetry axis. Therefore:

$$\psi = \frac{V_0 b}{2} = A\frac{\sin\theta}{\tau^{1/2}}.$$

At infinity $\tau = 1;\ \theta = \theta_{\max}$. Then

$$A = \frac{V_0 b}{2\sin\theta}.$$

The angle $\theta_{\max}$ is determined by the formulas following from the characteristic theory [3, 4]:

$$\theta_{\max} = C_1 + (\sqrt{3}-1)\frac{\pi}{2};$$

$$C_1 = arctg\sqrt{\frac{3\tau_0 - 1}{1-\tau_0}} - \sqrt{3}arctg\left(\frac{1}{\sqrt{3}}\sqrt{\frac{3\tau_0-1}{1-\tau_0}}\right).$$

2.2. Defining the flow parameters τ, θ at the arbitrary streamline intersection points with an arbitrary equipotential.

According to the equations (9), the following equalities are completed along the arbitrary streamline:

$$\begin{cases} \dfrac{\sin\theta}{\tau^{1/2}} = K\sin\theta_{\max}; \\ \dfrac{\cos\theta}{\tau^{1/2}(1-\tau)} = \dfrac{1}{\tau_{oc}^{1/2}(1-\tau_{oc})}, \end{cases} \tag{10}$$

where $K \in [0,1]$ is the flow coefficient along the streamline under consideration; τ_{oc} is the parameter "τ" at the intersection of the equipotential under consideration with the longitudinal symmetry flow axis.

The system of equations (10) is solved analytically with the standard methods by reducing it to solving a cubic equation [13]. In this case, the equation root should satisfy the condition:

$$\tau_0 < \tau \le 1 .$$

Next, it is necessary to move to the physical region of the streams with known parameters τ, θ at the intersection point of an arbitrary line flow with an arbitrary equipotential.

3. DETERMINATION OF THE X, Y COORDINATES IN THE PLANE $\Phi(x, y)$ CORRESPONDING TO THE POINT τ, θ IN THE VELOCITY HODOGRAPH PLANE $\Gamma(\tau, \theta)$.

When inspecting the arbitrary streamline, starting from the culvert, one can reach the given point with the coordinates τ, θ. Therefore, we consider $d\psi = 0$ in the equation (8). Moreover,

$$d\left(x + iy\right) = d\phi \frac{e^{i\theta}}{\tau^{1/2}\sqrt{2\mathrm{g}H_0}} \tag{11}$$

follows from (8).
Separating the variables in Eq. (11), we obtain:

$$\begin{cases} dx = \dfrac{d\phi\cos\theta}{\tau^{1/2}\sqrt{2\mathrm{g}H_0}}; \\ dy = \dfrac{d\phi\sin\theta}{\tau^{1/2}\sqrt{2\mathrm{g}H_0}}. \end{cases} \tag{12}$$

Eqs. (12) confirm that along the streamline:

$$\frac{dy}{dx} = \mathrm{tg}\theta .$$

The velocity vector along the streamline is inclined with angle “θ” to the flow symmetry axis *Ox*, i.e., it is tangential to streamline.

From the first equation of the system (10), we determine:

$$\sin\theta = K\tau^{1/2}\sin\theta_{\max} . \tag{13}$$

Substituting the right-hand part of Eq. (13) instead of $\sin\theta$ into the second equation of the system (12), we obtain:

$$dY = \frac{d\phi K\tau^{1/2}\sin\theta_{\max}}{\tau^{1/2}\sqrt{2\mathrm{g}H_0}} = \frac{d\phi K\sin\theta_{\max}}{\sqrt{2\mathrm{g}H_0}} . \tag{14}$$

Additional information from the experimental research.
It was experimentally revealed in the work [12-17] that there is a vertical front with a length “*X*D” along the flow symmetry axis, along which the flow parameters do not change (Fig.1).
There is an abrupt change in angle “θ” from zero to $\theta_{\text{н}}$ after point “*K*”. “*X*D” length was determined by processing more than a hundred experiments. The following formula of which has been obtained:

$$X_{\mathrm{D}} = \mathrm{trunc}\left[\frac{\sqrt{F_0 - 1}}{\sin\theta_{\max}(F_0 + 2)}h_0\right] + 1,$$

where $F_0 = \dfrac{V_0^2}{\mathrm{g}h_0}$ is the Froude number at the flow outlet from the pipe; h_0 denotes flow depth at the pipe outlet; $\theta_{\max}$ is the maximum flow angle; values of X_{D}, h_0 are determined experimentally and given in cm.
Integrating the Eq. (14), we obtain:

$$Y_{\mathrm{M}} - Y_{\mathrm{T}} = \frac{K\sin\theta_{\max}}{\sqrt{2\mathrm{g}H_0}}\left(\phi_{\mathrm{M}} - \phi_{\mathrm{T}}\right), \tag{15}$$

where $$\phi_{\mathrm{M}} = A\frac{h_0}{H_0}\frac{\cos\theta_{\mathrm{M}}}{\tau_{\mathrm{M}}^{1/2}(1 - \tau_{\mathrm{M}})};$$

$$\phi_{\mathrm{T}} = A\frac{h_0}{H_0}\frac{\cos\theta_{\mathrm{T}}}{\tau_{\mathrm{T}}^{1/2}(1-\tau_{\mathrm{T}})};$$

$\theta_{\mathrm{T}}, \tau_{\mathrm{T}}$ are the flow parameters defining the initial equipotential.
The system's first equation integration (12) leads to the following one:

$$X_{\mathrm{M}} = X_{\mathrm{D}} + X_{\mathrm{T}} + \frac{Ah_0}{H_0\sqrt{2\mathrm{g}H_0}}\left[J_1 - K^*J_2 - K^*J_3\right]\Bigg|_{\tau_{\mathrm{T}}}^{\tau_{\mathrm{M}}},$$

where $K^* = \sin^2\theta^{\mathrm{T}}_{\max}$, $\theta^{\mathrm{T}}_{\max}$ defines the maximum angle of the flow spreading along the selected streamline;

$$J_1 = \frac{1+\tau}{\tau(1-\tau)} + \ln\frac{\tau}{1-\tau};$$

$$J_2 = \frac{2}{1-\tau} + \ln\frac{1-\tau}{\tau};\ J_3 = \ln\frac{\tau}{1-\tau}\ . \qquad (16)$$

In a particular case from Eq. (15) and Eq. (16), it follows that, along the flow symmetry axis, the relationship between X and τ takes the form:

$$X = X_{\mathrm{D}} + \tilde{X}_{\mathrm{D}} +$$
$$+\frac{Ah_0}{H_0\sqrt{2\mathrm{g}H_0}}\left[\frac{1+\tau}{\tau(1-\tau)} - \ln\frac{1-\tau}{\tau} - \frac{1+\tau_0}{\tau_0(1-\tau_0)} + \ln\frac{1-\tau_0}{\tau_0}\right],$$

where $\tilde{X}_{\mathrm{D}} = \frac{b}{2}\mathrm{tg}\frac{\theta_k}{2}$. The angle θ_k and the parameter τ_k are determined from the system:

$$\begin{cases} \dfrac{\sin\theta_k}{\tau_k^{1/2}} = \sin\theta_{\max}; \\ \dfrac{\cos\theta_k}{\tau_k^{1/2}(1-\tau_k)} = \dfrac{1}{\tau_0^{1/2}(1-\tau_0)}. \end{cases}$$

Along the extreme streamline, it follows from Eq. (15) and Eq. (16) that:

$$X = X_{\mathrm{D}} +$$
$$+\frac{Ah_0}{H_0\sqrt{2\mathrm{g}H_0}}\left[\frac{1+\tau}{\tau(1-\tau)} - \frac{2\sin^2\theta_{\max}}{1-\tau} - \ln\frac{1-\tau}{\tau} - \quad (17)\right.$$
$$\left.-\frac{1+\tau_k}{\tau_k(1-\tau_k)} + \ln\frac{1-\tau_k}{\tau_k} + \frac{2\sin^2\theta_{\max}}{1-\tau_k}\right];$$

$$Y = \frac{b}{2} + A\frac{h_0\sin^2\theta_{\max}}{H_0\sqrt{2\mathrm{g}H_0}}\left[\frac{\cos\theta}{\tau^{1/2}(1-\tau)} - \frac{\cos\theta_k}{\tau_k^{1/2}(1-\tau_k)}\right]. \quad (18)$$

We determined depths and flow rates with the known parameter "τ" according to the formulas:

$$h = H_0(1-\tau);\quad V = \tau^{1/2}\sqrt{2\mathrm{g}H_0}\ .$$

4. THE TRANSITION FROM A TWO-DIMENSIONAL MODEL TO A ONE-DIMENSIONAL

This module is necessary to use the flow resistance laws and takes into account the resistance forces.
We use the flow rate conservation equation

$$Q = BVH,$$

where

$$V = \tau^{1/2}\sqrt{2\mathrm{g}H_0};\quad h = H_0(1-\tau)$$

or in other form:

$$Q = BH_0(1-\tau)\sqrt{2\mathrm{g}H_0}\,\tau^{1/2} \qquad (19)$$

Assuming the known width of the flow $B = 2Y$, where Y is taken from a two-dimensional flow model, it is possible to determine the parameter "τ_{cp}" from the solution of the cubic equation:

$$(1-\tau_{\mathrm{cp}})\tau_{\mathrm{cp}}^{1/2} = \frac{Q}{2YH_0\sqrt{2\mathrm{g}H_0}}.$$

For $\tau_0 \le \tau_{\text{cp}} \le 1$, the root of τ_{cp} can be determined from the equation:

$$(1-\tau_{\text{cp}})^2 \tau_{\text{cp}} = \frac{Q^2}{4Y^2 H_0^3 2gH_0}.$$

We will further define the parameters of a conditionally one-dimensional flow with the characteristics:

$$h_{\text{cp}} = H_0(1-\tau_{\text{cp}}); \quad V_{\text{cp}} = \tau_{\text{cp}}^{1/2}\sqrt{2gH_0}\,. \quad (20)$$

The corresponding *X* coordinate is determined by equation (17). Therefore, the parameter "τ" is determined for the abscissa "*X*" given. Further, *Y*(τ) is determined from equation (18). And the average depths and velocities $h_{\text{cp}}, V_{\text{cp}}$ in the considered non-pressure flow line are determined by formulas (20).

THE RESULTS OF THE STUDY

1. The model proposed in the work represents the analytical methods' development for calculating the potential flows with the previously unknown boundaries. This allows determining the entire range of geometric and kinematic flow parameters with an error not exceeding 10% up to the flow expansion $\beta = 7 \div 10$.
2. The model adequacy for all flow parameters up to the flow expansion $\beta = 7 \div 10$ improves the accuracy of the previously existing methods, which allows the designers of the road culverts' hydraulic structures to increase its reliability using the results of the structure fastening designers' work.
3. This model can be applied as an initial model for calculating the real flows behind the culverts taking into account the fluid resistance.
4. The authors propose an additional module to take into account the fluid resistance.
5. The transition to a one-dimensional model allow us to take into account the action of the fluid resistance and recalculate the flow parameters required by the designers of the hydraulic structure. The paper indicates only the direction for possible consideration of the fluid resistance, which will be further developed and detailed.
6. The package of applied programs is available at the Department of General Engineering Disciplines of the Platov South-Russian State Polytechnic University.

CONCLUSIONS

The model proposed in the paper represents the development of analytical methods for calculating potential flows with previously unknown boundaries and before the flow $\beta = 7 \div 10$ expands. It allows determining the entire range of geometric and kinematic parameters of the flow with an error not exceeding 10%. The substantiation of this position is confirmed by experimental experiments and numerical calculations given in [8, 12].

The adequacy of the model in all parameters of the flow before $\beta = 7 \div 10$ expansion improves the accuracy of previously existing methods, which allows the designers of the hydraulic structure of road culverts to increase its reliability using the results of the work of the designers of the fastening of the structure.

The article proposes a module of transition from a two-dimensional water flow model to a one-dimensional one. This module is necessary for using the laws of fluid resistance and takes into account the fluid resistance.

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