

# ANALYSIS OF THE FILTRATION PROBLEM BY BITWISE SEARCH METHOD

*Liudmila I. Kuzmina<sup>1</sup>, Yuri V. Osipov<sup>2</sup>, Maxim D. Astakhov<sup>2</sup>*

<sup>1</sup> HSE University, Moscow, RUSSIA

<sup>2</sup> Moscow State University of Civil Engineering, Moscow, RUSSIA

**Abstract.** During the construction of ground and underground structures, filtration of a liquid grout in loose soil makes it possible to strengthen the foundation and create underground waterproof partitions. A one-dimensional problem of filtering a bidisperse suspension in a homogeneous porous medium with size-exclusion particle capture mechanism is considered. The article is devoted to the calculation of the exact solution of the problem given as the upper limit of the integral with a singularity. The proposed bitwise search method for calculating integrals makes it possible to smooth out fluctuations of the solution near the singularity. Partial and total retention profiles are analyzed.

**Keywords:** deep bed filtration, bidisperse suspension, retention profiles, exact solution, bitwise search method

# ИССЛЕДОВАНИЕ ЗАДАЧИ ФИЛЬТРАЦИИ МЕТОДОМ ПОРАЗРЯДНОГО ПОИСКА

*Л.И. Кузьмина<sup>1</sup>, Ю.В. Осипов<sup>2</sup>, М.Д. Астахов<sup>2</sup>*

<sup>1</sup> Национальный исследовательский университет «Высшая школа экономики», Москва, РОССИЯ

<sup>2</sup> Национальный исследовательский Московский государственный строительный университет, Москва, РОССИЯ

**Аннотация.** При строительстве наземных и подземных сооружений фильтрация жидкого раствора укрепителя в рыхлом грунте позволяет укрепить фундамент и создавать подземные водонепроницаемые перегородки. Рассматривается одномерная задача фильтрации бидисперсной суспензии в однородной пористой среде с размерным механизмом захвата частиц. Статья посвящена вычислению точного решения задачи, заданного в виде верхнего предела интеграла с особенностью. Предлагаемый метод поразрядного поиска для вычисления интегралов позволяет сгладить колебания решения вблизи особенности. Исследуются частичные и полные профили концентрации осажденных частиц.

**Ключевые слова:** глубинная фильтрация, бидисперсная суспензия, профили осадка, точное решение, метод поразрядного поиска

## 1. INTRODUCTION

Filtration problems are relevant for many areas of science and technology. In construction problems, modeling the filtration of the smallest particles in a porous medium allows one to study the properties of soils and analyze the possibilities of strengthening loose soil to create a solid foundation [1–4].

The carrier fluid flows through the porous medium and carries the fine solids of suspension.

During the filtration process, some particles are retained and form a deposit. There are many different mechanisms of particle capture, which are determined by the physicochemical properties of the porous medium, liquid, and suspension particles, as well as the geometry of the porous structure [5–7]. If the particle sizes and pore cross sections are of the same order, the prevailing particle capture mechanism is size-exclusion: suspended particles are transported through wide pores and get stuck at the entrance

of narrow pores [8–10]. Suppose that the Newtonian fluid is incompressible, the suspended particles move at the same speed as the carrier fluid, the retained particles are stationary and cannot be knocked out of the porous frame by the fluid or suspended particles. The standard one-dimensional mathematical model of filtration of a monodisperse suspension in a homogeneous porous medium includes the balance equation for the masses of suspended and retained particles and the kinetic equation for the growth of the particles retained concentration [11–14]. The equations are considered in a dimensionless form: the concentrations of suspended and retained particles are normalized by dividing by the concentration of the suspension at the inlet of the porous medium, the length of the porous sample is taken as a unit, and the unit of time is the period of passage of a suspended particle through the porous medium from inlet to outlet. The dimensionless velocity of particles and carrier fluid is equal to 1.

When filtering in a porous medium, the deposit is unevenly distributed. The distribution of retained particles is given by the retention profile - the concentration of deposited particles at a fixed time, which depends on the coordinate. For a monodisperse suspension, the retention profile decreases monotonically: it is maximum at the inlet of the porous medium and minimum at the outlet.

If the suspension contains suspended particles of two different sizes, then the mass balance and kinetic equations of deposit growth are written separately for each type of particles. The connection between the equations for different particles is carried out by a single filtration function, which is included in both kinetic equations and depends on a linear combination of partial retained concentrations. When filtering a bidisperse suspension, the profile of large particles decreases monotonically, while the profile of small particles is nonmonotone: near the inlet of a porous medium, the profile increases, reaches a maximum, and then decreases monotonically. As time increases, the maximum point moves away from the inlet. The

monotonicity or nonmonotonicity of the total retention profile depends on the model parameters. The retention profiles of a bidisperse suspension were studied numerically in [15, 16], and an analytical solution was obtained in [17]. Filtration problems, as a rule, do not have an exact analytical solution. In many works devoted to the numerical solution of filtration problems, the finite difference method is used [18–20]. Calculation using an explicit difference scheme allows you to quickly make calculations, but the presence of discontinuities significantly complicates finding a solution. If an exact solution in an implicit closed form or its asymptotics is known, it is used to numerically calculate the solution in an explicit form [21, 22]. For the problem of filtering a bidisperse suspension in a porous medium, an exact implicit solution is obtained. The solution is given in the form of integrals with variable limits, the integrand has a singularity. Finding the value of the integral near the singularity is a difficult computational problem. In this article, bitwise search method for calculation of a solution is used [23]. This method makes it possible to smooth out fluctuations in the solution that arise when calculating integrals with singularities using standard methods. The results of calculations for solving the filtration problem by the standard method and by the bitwise search method are presented. The profiles of the total retained concentration and partial retained concentrations of particles of the same size are obtained and analyzed.

## 2. MATHEMATICAL MODEL

In the domain  $\Omega = \{x \geq 0, t \geq 0\}$  consider the system

$$\frac{\partial c_i}{\partial t} + \frac{\partial c_i}{\partial x} + \frac{\partial s_i}{\partial t} = 0, \quad (1)$$

$$\frac{\partial s_i}{\partial t} = (1-b)\lambda_i c_i, \quad b = B_1 c_1^0 s_1 + B_2 c_2^0 s_2, \quad i = 1, 2. \quad (2)$$

Here  $\lambda_i, B_i, c_i^0$  are positive constants,  $\lambda_1 > \lambda_2$ ,  $c_1^0 + c_2^0 = 1$ . The unknowns  $c_i, s_i, i=1,2$  are the suspended and retained particles concentrations, respectively,  $b$  is the concentration of occupied sites,  $B_i$  is the individual area that an attached particle occupies at the rock surface, and  $c_i^0$  are the particle concentrations in the injected suspension.

For the uniqueness of the solution of problem (1), (2), the conditions are set at the inlet of the porous medium and at the initial moment:

$$x=0: c_i = c_i^0, \quad t=0: c_i = 0, s_i = 0, \quad i=1,2. \quad (3)$$

The solutions  $c_1(x,t), c_2(x,t)$  have a discontinuity on the characteristic line  $t=x$ ,

because the initial and boundary conditions do not match at the origin. The line  $t=x$  is the concentration front  $\Gamma$  of the suspended and retained particles which divides the interior of the domain  $\Omega$  into two zones. In the domain  $\Omega_0 = \{x > 0, 0 < t < x\}$  the problem has a zero solution; in the domain  $\Omega_1 = \{x > 0, t > x\}$  the solution is positive. In the domain  $\Omega_1$  the solutions  $c_1(x,t), c_2(x,t)$  are related by the formulae

$$c_1 = c_1^0 \left( \frac{c_2}{c_2^0} \right)^{\lambda_1/\lambda_2}, \quad c_2 = c_2^0 \left( \frac{c_1}{c_1^0} \right)^{\lambda_2/\lambda_1} \quad (4)$$

and are given in implicit form

$$\begin{aligned} \int_{c_1^-}^{c_1} \frac{dc}{c \left( B_1 c_1^0 (c_1^0 - c) + B_2 c_2^0 \left( c_2^0 - c_2^0 \left( \frac{c}{c_1^0} \right)^{\lambda_2/\lambda_1} \right) \right)} &= \lambda_1(t-x), \\ \int_{c_2^-}^{c_2} \frac{dc}{c \left( B_1 c_1^0 \left( c_1^0 - c_1^0 \left( \frac{c}{c_2^0} \right)^{\lambda_1/\lambda_2} \right) + B_2 c_2^0 (c_2^0 - c) \right)} &= \lambda_2(t-x), \end{aligned} \quad (5)$$

where

$$c_i^- = c_i(x, x) = c_i^0 e^{-\lambda_i x} \quad (6)$$

is the solution on the concentration front  $\Gamma$ .

For known  $c_1(x,t), c_2(x,t)$  concentrations of retained particles are given by the formula

$$s_i = \frac{c_i - c_i^-}{B_1 c_1^0 (c_1^0 - c_1^-) + B_2 c_2^0 (c_2^0 - c_2^-)}, \quad i=1,2. \quad (7)$$

In particular, at the inlet  $x=0$  solution (7) takes the form

$$\begin{aligned} s_i^0(t) &= \frac{\lambda_i c_i^0}{B} (1 - e^{-Bt}), \quad i=1,2, \\ B &= \lambda_1 B_1 (c_1^0)^2 + \lambda_2 B_2 (c_2^0)^2. \end{aligned} \quad (8)$$

Consider the properties of retention profiles given by formula (7) at fixed time  $t$ .

- The partial retention profile  $s_1(x,t)$  decreases monotonically for all  $t > x$ ;

- The partial retention profile  $s_2(x,t)$  decreases monotonically for  $x < t < t_0$  and increases monotonically for  $t > t_0$ ;

-The total retention profile  $s(x,t)$  decreases monotonically for all  $t > x$  if  $B_1 c_1^0 < B_2 c_2^0$ ; decreases monotonically at  $x < t < T_0$  and increases monotonically at  $t > T_0$  if  $B_1 c_1^0 > B_2 c_2^0$ .

So, any non-monotonic retention profile has a maximum point. As time increases, the maximum point shifts from the inlet of the porous medium.

### 3. BITWISE SEARCH METHOD

Obtained exact solution makes it possible to perform calculations using formulae (5) without a numerical solution of the original problem (1)–(3). Calculation of the implicit solution – the upper limit of integration in integrals (5) was performed by the bitwise search method [24, 25]. Finding the solution numerically is complicated by the fact that the upper limit of integration is close to the singularity of the integrand. Calculations by standard methods lead to oscillations of the solution and increase of an error, since the derivative solutions are limited. To obtain a smooth solution, the bitwise search method was chosen as one of the direct search methods that does not use derivatives in calculations. With a large error in calculating the values of the function, the bitwise search method makes it possible to avoid an excessive number of iterations.

The profile is constructed by calculating the profiles of suspended particles concentrations  $c_1(x, t)$ ,  $c_2(x, t)$  at each point of the porous medium at a fixed time  $t$ . Formulae (4) set a relation for concentrations and make it possible to implement two approaches to computation of a solution. You can use one profile as a basic one, calculating it by formula (5), and obtain the second profile algebraically by formula (4). If we calculate both profiles by formulae (5), algebraic formulae (4) can be used to check the accuracy of the solutions found.

The input parameters of the program are:

- $x\_step$  – step by  $x$ ,
- $t$  – time for which the solution profile is calculated,
- accuracy – obtained accuracy of the solution  $c_n$ ,
- $calc$  – list of profiles, which are calculated by formulae (5).

Denote the following variables:

- $Need_n$  – the right side of equation (5), the value of the integral,
- $Depth$  – the current bit of the search for  $c_n$ , called the search depth,

- $S$  – the calculated value of the integral at the current iteration,
- $Res_n$  – the last obtained profile value,
- $Best$  – a pair of variables:
  - $Best[0] = Delta$  – deviation between the values of the integral  $S$  and  $Need$ ,
  - $Best[1] = c_n$  – the calculated solution,
- Plus – flag storing the search direction:
  - true (1) – in the direction of increase of  $c_n$ ,
  - false (0) – in the direction of decrease of  $c_n$ .

The first step is to calculate the right side  $Need_n = \lambda_n(t - x)$  of the integral (5), then the condition  $Need > 0$  is checked, which means that the solution behind the front  $t > x$  is positive. If  $Need \leq 0$ , the point  $x$  belongs to the front and  $c_n = c_n^-$ . Otherwise, we proceed to the calculation for one of the basic profiles.

The search depth (Depth) for the next step in  $x$  is calculated using the linear interpolation formula

$$Depth = -\log_{10} |Res_n - c_n| - 1.$$

The search depth determines in which bit the next value of  $c_n$  can be obtained, and reduce the number of iterations of the algorithm.

Then it is checked that the upper integration limit cannot be less than the lower one:  $Res_n \geq c_n^-$ , otherwise  $Res_n = c_n^-$ .

The condition  $Res_n \leq c_n^0$  is also checked, which means that the concentration cannot exceed the initial concentration of the suspension, otherwise  $Res_n = c_n^-$ .

Let's start the calculation of  $c_n$  at a given point  $x$ , setting  $Res_n$  as the initial value. The algorithm includes the following steps.

1. Calculate the integral on the left side of formula (5) with the current value  $c_n$  using the Simpson method.
2. Calculate  $Delta = |S - Need|$ .

3. If  $\Delta \leq \text{Best}[0]$ , then  $\text{Best} = \{\Delta, c_n\}$ .
4. If  $\Delta \leq 10^{-(\text{accuracy})}$ , then the value of the integral is found with the required accuracy, go to step 1 with the next value of  $x$ .
5. If  $S > \text{Need}$  and  $\text{Plus} = \text{true}$ , change the search direction ( $\text{Plus} = \text{false}$ ) and increase the search bit ( $\text{Depth} += 1$ ).
6. If  $S < \text{Need}$  and  $\text{Plus} = \text{false}$ , change the search direction ( $\text{Plus} = \text{true}$ ) and increase the search bit ( $\text{Depth} += 1$ ).
7. If  $\text{Depth} > \text{accuracy}$ , then the specified accuracy of the solution is reached, go to step 1 with the next value of  $x$ .
8. Let's take the next step
 
$$\begin{cases} c_n = c_n + 10^{-\text{Depth}}, & \text{if } \text{Plus} = \text{false}, \\ c_n = c_n - 10^{-\text{Depth}}, & \text{if } \text{Plus} = \text{true}. \end{cases}$$
9. Check that the value of  $c_n$  does not go beyond the limits of the interval  $[c_n^-, c_n^0]$ .

If  $c_n \notin [c_n^-, c_n^0]$ , then take a step back

$$\begin{cases} c_n = c_n - 10^{-\text{Depth}}, & \text{if } \text{Plus} = \text{false}, \\ c_n = c_n + 10^{-\text{Depth}}, & \text{if } \text{Plus} = \text{true}. \end{cases}$$

and increase  $\text{Depth} += 1$ , then return to step 8.

10. Increase  $x$  by one step and go to step 1.

At the end of the loop, the result of the calculation is  $\text{Best}[1]$ .

Dependent profile is calculated by formula (4). If both profiles were calculated by formulae (5), then with the help of (4) the calculation error is determined - the discrepancy between the values of the same profiles is calculated.

#### 4. NUMERICAL SIMULATION

Figure 1 shows the profiles of partial and total concentration of retained particles at different time for the parameters  $\lambda_1 = 25$ ,  $\lambda_2 = 5$ ,  $B_1 = 0.125$ ,  $B_2 = 0.025$ ,  $c_1^0 = 0.5$ ,  $c_2^0 = 0.5$ .

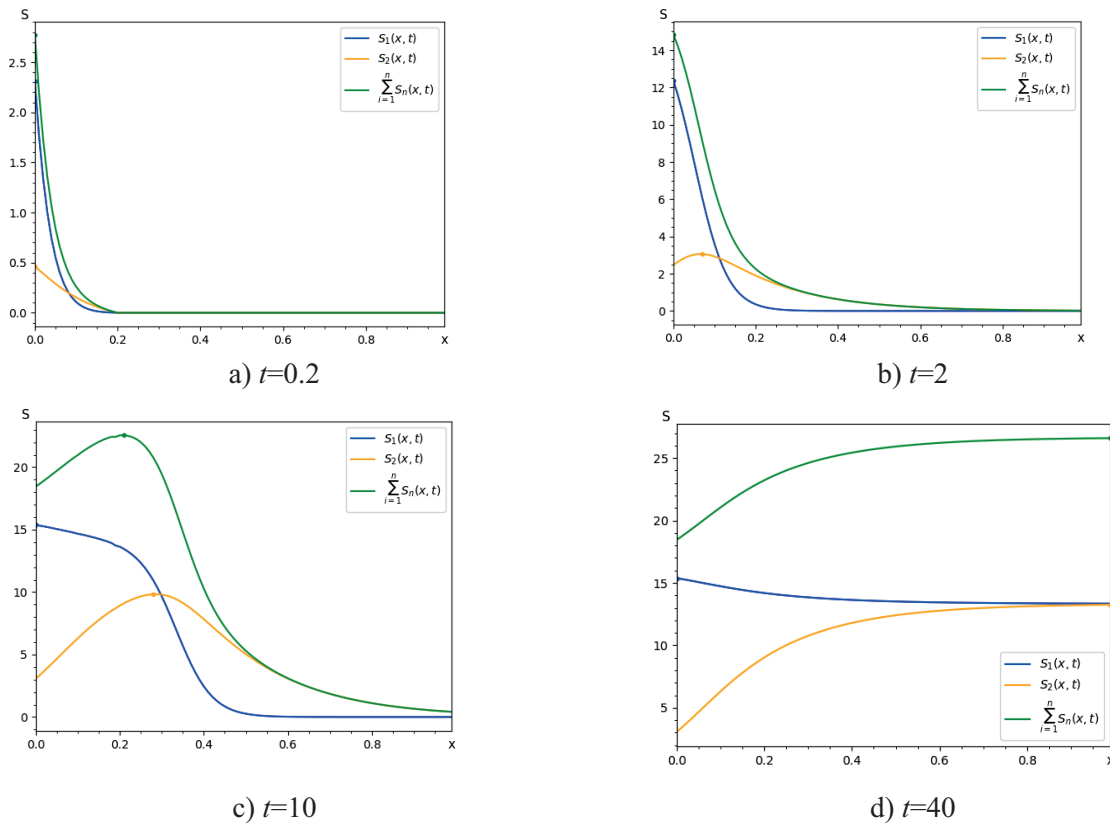


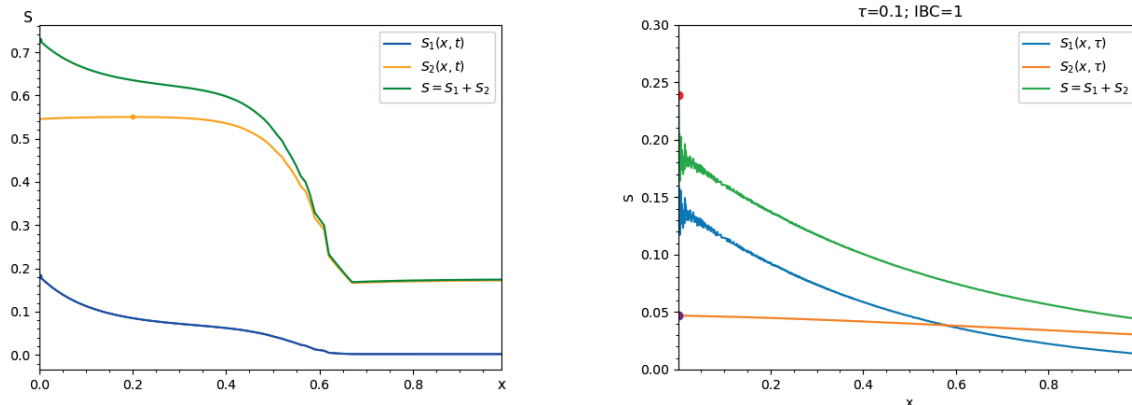
Figure 1. Retained concentration profiles



According to Fig. 1 profiles of large and small particles are separated at a long time. Large particles are mainly deposited near the inlet of the porous medium, while small particles are deposited near the outlet. The monotonicity of

the total sediment profile depends on the parameters of the problem.

The graphs of solutions obtained without the procedure for smoothing oscillations have kinks and oscillating nonmonotonic sections that are unacceptable in smooth solutions (Fig. 2).



*Figure 2. Unacceptable non-smooth retention profiles*

## 5. CONCLUSION

For the filtration model of bidisperse suspension in a homogeneous porous medium

- Implicit exact solutions are found in the form of integrals with singularities.
- Bitwise search method is used to calculate the smoothed solution.
- The algorithm of the bitwise search method is described.
- Smooth numerical solutions are obtained.
- Retention profiles of total and partial deposit concentrations are constructed.

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*Liudmila I. Kuzmina*, Candidate of Physical and Mathematical Sciences, Associate Professor, Department of Applied Mathematics, National Research University Higher School of Economics, 101000, Russia, Moscow, Myasnitskaya st., 20, tel. +7(495) 77295 90 \* 15219, e-mail: lkuzmina@hse.ru.

*Yuri V. Osipov*, Candidate of Physical and Mathematical Sciences, Associate Professor, Department of Computer Science and Applied Mathematics, Moscow State University of Civil Engineering, 129337, Russia, Moscow, Yaroslavskoe Shosse, 26, tel. +7(499)1835994, e-mail: yuri-osipov@mail.ru.

*Maxim D. Astakhov*, 3rd year bachelor's degree student of the direction of training 09.03.02 of the Institute IDTMC of the National Research Moscow State University of Civil Engineering; 129337, Russia, Moscow, Yaroslavskoe shosse, 26; tel. +7(499)1834674; e-mail: work@astakhovmd.ru.

*Кузьмина Людмила Ивановна*, доцент, кандидат физико-математических наук, Департамент прикладной математики, Национальный исследовательский университет «Высшая школа экономики»; 101000, г. Москва, ул. Мясницкая, д. 20, тел. +7(495) 77295 90 \*15219; e-mail: lkuzmina@hse.ru.

*Осипов Юрий Викторович*, доцент, кандидат физико-математических наук, кафедра информатики и прикладной математики Национального исследовательского Московского государственного строительного университета; 129337, Россия, г. Москва, Ярославское шоссе, д. 26; тел. +7(499)1835994; e-mail: yuri-osipov@mail.ru.

*Астахов Максим Денисович*, студент 3 курса бакалавриата направления подготовки 09.03.02 института ИЦТМС Национального исследовательского Московского государственного строительного университета; 129337, Россия, г. Москва, Ярославское шоссе, д. 26; тел. +7(499)1834674; e-mail: work@astakhovmd.ru.