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ASSESSMENT OF THE INFLUENCE OF THE ROTATIONAL COMPONENTS OF SEISMIC ACTION ON THE SSS OF A MULTISTOREY REINFORCED CONCRETE BUILDING

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Abstract. The urgent task of earthquake-resistant construction is to improve the methods of calculating the seismic effect. Currently, most calculations for seismic effects are made taking into account only the translational components of the earthquake. However, seismic soil rotations make a tangible contribution to the dynamic response of structures that are sensitive to the wave effect of seismic impact. Modern studies show that building structures have a spatial nature of work, and in calculations for seismic loads, in addition to three translational components, it is necessary to consider the impact on buildings and structures from additional three rotational components. The purpose of this work is to assess the influence of rotational components on the stress-strain state of a multielement reinforced concrete building. In the course of the work, calculations of a reinforced concrete building were made without taking into account and taking into account the rotational components of the seismic impact, based on the results of which a comparative analysis was made. The calculation for the earthquake was performed by an explicit direct dynamic method of central differences in the LS-DYNA software in six settings: the action of the translational component is set along the X axis; the action of the translational component is set along the X axis and rotational component – relative to the Y axis; the action of the translational component is set along the Y axis; the action of the translational component is set along the Y axis and rotational component – relative to the X axis; the impact of three translational components; the action of three translational and three rotational components. As a result of a comparative analysis of the obtained displacements and stresses, it was concluded that taking into account rotational components does not make a tangible contribution to the SSS of the building under consideration.

Keywords: Accounting For Rotational Components, Method Of Central Differences, Earthquake-Resistant Construction, Nonlinear Dynamic Methods.

ОЦЕНКА ВЛИЯНИЯ РОТАЦИОННЫХ КОМПОНЕНТ СЕЙСМИЧЕСКОГО ВОЗДЕЙСТВИЯ НА НДС МНОГОЭТАЖНОГО ЖЕЛЕЗОБЕТОННОГО ЗДАНИЯ

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Аннотация. Актуальной задачей сейсмостойкого строительства является совершенствование методов расчёта на сейсмическое воздействие. В настоящее время большинство расчетов на сейсмические воздействия производятся с учетом только поступательных компонент землетрясения. Однако сейсмические ротации грунта дают ощутимый вклад в динамическую реакцию сооружений, чувствительных к волновому эффекту сейсмического воздействия Современные исследования показывают, что строительным конструкциям присущ пространственный характер работы, а при расчётах на сейсмические нагрузки помимо трёх поступательных компонент, необходимо рассмотрение воздействия, оказываемого на здания и сооружения от дополнительных трёх ротационных компонент. Целью данной работы является оценка влияния ротационных компонент на напряженно-деформированное состояние многоэлементного железобетонного здания. В ходе работы были произведены расчеты железобетонного здания без учёта и с учётом ротационных компонент сейсмического воздействия, по результатам которых был произведен сравнительный анализ. Расчет на землетрясение был произведен явным прямым динамическим методом центральных разностей в ПК LS-DYNA в шести постановках: воздействие поступательной компоненты задано вдоль оси X; воздействие поступательной компоненты задано вдоль оси X и ротационной относительно оси Y; воздействие поступательной компоненты задано вдоль оси Y; воздействие поступательной компоненты задано вдоль оси Y и ротационной относительно оси X; воздействие трёх поступательных компонент; воздействие трёх поступательных и трёх ротационных компонент. В результате сравнительного анализа полученных перемещений и напряжений был получен вывод о том, что учёт ротационных компонент не вносит ощутимого вклада в НДС рассматриваемого здания.

Ключевые слова: учет компонентов вращения, метод центральных разностей, сейсмостойкие конструкции, нелинейные динамические методы.

1. INTRODUCTION

About a third of the population of the Russian Federation lives in seismically active regions, while earthquakes rank third after typhoons and floods in terms of damage caused. Hundreds and thousands of people die during destructive earthquakes. Reducing the negative consequences of earthquakes is possible thanks to the approaches of earthquake-resistant construction, one of the tasks of which is to determine and study the processes of interaction between buildings and their foundations. Currently, when calculating for an earthquake, only translational components are taken into account, while a number of works of scientists, in particular Yu P Nazarov [1–4], shows that when designing in seismic regions for all buildings and structures, it is also necessary to take into account rotational components seismic impact.

The first mention of rotational vibrations of the base area was the monograph by A G Nazarov [1]. The monograph specifies that the considered elementary area of the base has six degrees of freedom, which consist of three translational and three rotational, from which it follows that the vibrations made by the elementary platform are decomposed into translational and rotational. Ten years later, the first analytical description of rotational vibrations was made by N M Newmark [5], where the relationship between the torsion of the structure and the rotational component of the seismic action was shown. A number of works by N M Newmark [5-7] laid the foundation for further studies of the properties of fields of seismic activity. As a result of research by V V Lee and M D Trifunac [8], the possibility of obtaining synthetic rotational accelerograms using exact physical dependencies describing the propagation of elastic waves was stated, a description of one of the methods for calculating the rotational components was made. The accelerograms obtained by this method have realistic properties and resemble the real rotational motion of the soil foundation.

The purpose of this study is to assess the influence of rotational component on the stress-strain state of a reinforced concrete building.

2. MATERIALS AND METHODS

The dynamic method used in this work is based on the equation of motion, which has the following form:

$$M \cdot \ddot{u} + C\,\dot{u} + K \cdot u = f^{a}, \qquad (1)$$

where:

- $\boldsymbol{M}$ – system mass matrix;
- $\boldsymbol{C}$ – damping matrix;
- $\boldsymbol{K}$ – system stiffness matrix;
- $\boldsymbol{f^a}$ – vector of external influences.

The explicit method of central differences is used in the work. The solution of the differential equation (1) is reduced to a system of nonlinear differential equations at each step of time integration. Integration of the equation of motion can be performed both according to explicit and implicit schemes, but the most effective method is considered to be the explicit method of central differences implemented in the LS-DYNA software and used in this work.

Let each time step $\Delta t_i = \Delta t$, then the expressions for the central differences of the derivatives of the displacement at the time t_i can be written in the following form:

$$\dot{u}_i = \frac{u_{i+1} - u_{i-1}}{2\Delta t}; \quad \ddot{u}_i = \frac{u_{i+1} - 2u_i + u_{i-1}}{(\Delta t)^2}. \quad (2)$$

Substituting equation (2) in (1) for a linear elastic system, we get:

$$m\frac{u_{i+1} - 2u_i + u_{i-1}}{(\Delta t)^2} + c\frac{u_{i+1} - u_{i-1}}{2\Delta t} + ku_i = p_i, \quad (3)$$

where it is assumed that u_i and u_{i-1} are known from the solution for the previous time steps.
Moving these known ones to the right side, we get the following equation:

$$\left[\frac{m}{(\Delta t)^2} + \frac{c}{2\Delta t}\right]u_{i+1} = p_i - \left[\frac{m}{(\Delta t)^2} - \frac{c}{2\Delta t}\right]u_{i-1} - \left[k - \frac{2m}{(\Delta t)^2}\right]u_i. \quad (4)$$

Let us replace with variables:

$$\tilde{k} = \frac{m}{(\Delta t)^2} + \frac{c}{2\Delta t}; \quad \tilde{p}_i = p_i - \left[\frac{m}{(\Delta t)^2} - \frac{c}{2\Delta t}\right]u_{i-1} - \left[k - \frac{2m}{(\Delta t)^2}\right]u_i. \quad (5)$$

Then (5) can be written in the following form:

$$\tilde{k}u_{i+1} = \tilde{p}_i. \quad (6)$$

The method of finding u_{i+1} at a time point $i+1$ based on the equilibrium of equation (1) at time i without using the same equation at a time $i+1$ is called explicit. To calculate u_{-1}, we use formula (2) for $i = 0$ and we get:

$$\dot{u}_0 = \frac{u_1 - u_{-1}}{2\Delta t}; \quad \ddot{u}_0 = \frac{u_1 - 2u_0 + u_{-1}}{(\Delta t)^2}. \quad (7)$$

$$u_{-1} = u_0 - \Delta t(\dot{u}_0) + \frac{(\Delta t)^2}{2}\ddot{u}_0. \quad (8)$$

$$\ddot{u}_0 = \frac{p_0 - c\dot{u}_0 - ku_0}{m}. \quad (9)$$

The object of the study is a monolithic reinforced concrete structure with a cross arrangement of girders. The height of the building is 8 floors (24 m). The dimensions of the structure in the plan are 20.4 m in width and length. Vertical load-bearing elements are presented in the form of columns located with a pitch of 6 m in one direction and 9 m in the other (Fig. 1). In the course of the development of the scheme, the following sections and thicknesses were adopted: a square section of 0.4 × 0.4 (h) m for columns, a rectangular section of 0.4 × 0.5 (h) m for girders, the thickness of the floors was taken equal to 0.2 m. The design parameters of the material are taken as for concrete B25: modulus of elasticity E = $3 \cdot 10^{10}$ Pa, density ρ = 2500 kg/m^3, attenuation coefficient c = 0,05 of the critical, Poisson's ratio = 0.2. In addition to its own weight, all slabs are subject to a vertical force equal to 200 kg·s/m^2.
The calculation of the structure for seismic action was carried out in the LS-DYNA software, where an explicit method of integrating the equation of motion was implemented. Columns and girders were modelled with finite elements of the BEAM type, slabs – with flat finite elements of the SHELL type. The material type is set as ELASTIC. Constant vertical loads are specified with the BODY_Z function. All nodes of intersections of girders, columns and slabs are rigid, and the finite element mesh of girders and slabs is joint. The step of dividing all finite elements is taken equal to 0.3 m. In total, the model contains 46400 finite elements, 4784 of which are bar (BEAM), and 41616 are flat four-node (SHELL).
Modelling of the seismic impact was carried out with the assumption that the structure under study is located on some absolutely rigid platform (Fig. 2) for which the dependences of accelerations/velocities/displacements on time are set. All rotational components are calculated according to the integral model. Damping was simulated in proportion to the strain rate.

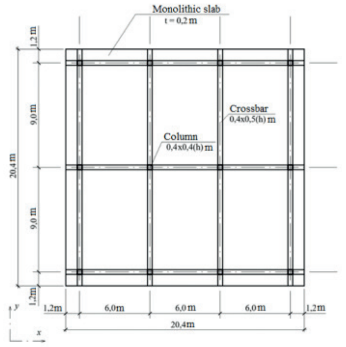


Figure 1. Constructive floor plan of the building under study

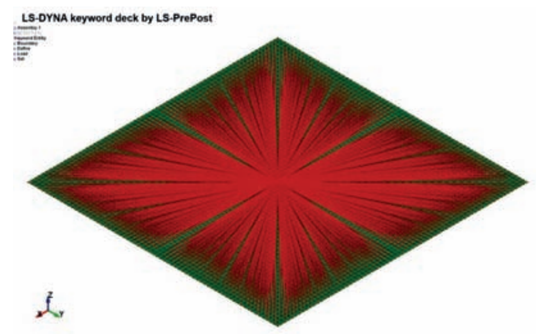


Figure 2. General view of the platform model in the design scheme

All nodes of the platform model in Fig. 2 are combined into one absolutely rigid nodal body with a central main node for which the diagrams of the dependence of translational and rotational accelerations on time were set.

As the analysed calculation results, the work considers nodal displacements of nodes numbered 13071 (central), 15383 (extreme) relative to the centre of the platform located on the upper slab and the forces in the lower finite element (B 9253) of the extreme corner column. A general view of the design scheme of the building under study in the LS-DYNA software is shown in Fig. 3.

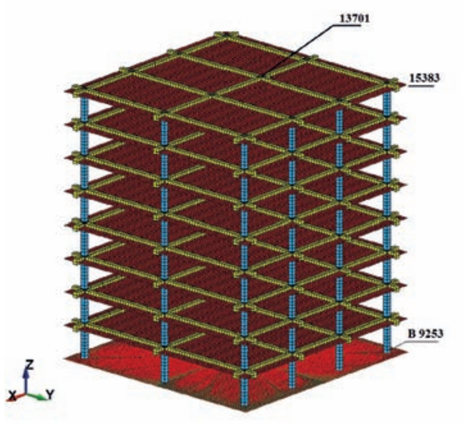


Figure 3. General view of the design scheme

2.1 Calculation from accounting the rotational component relative to the Y axis

In the course of the work, a calculation was carried out taking into account the rotational component relative to the Y axis, which included the following calculations:

1. Taking into account one translational component the action of which is directed along the X axis.

2.Taking into account two components, where one is translational along the X axis, and the other arises from the translational one along the X axis, rotational component — relative to the Y axis. Accelerograms of a given seismic action are shown in Fig. 4-5 for translational and rotational actions, respectively.

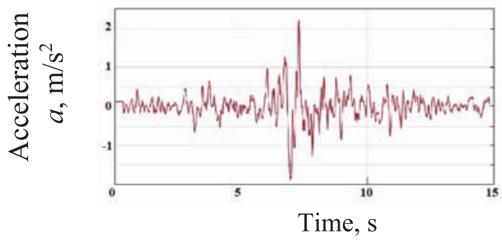


Figure 4. Accelerogram of translational motion along the X-axis

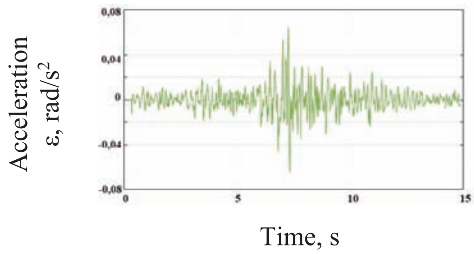


Figure 5. Accelerogram of rotational movement with respect to the Y-axis from the action ax

2.2 Calculation from accounting the rotational component relative to the X axis

In the course of the work, a calculation was also carried out taking into account the rotational component relative to the X axis, which included the following calculations:
1. Taking into account one translational component the action of which is directed along the Y axis.
2. Taking into account two components, where one is translational along the Y axis, and the other arises from the translational one along the Y axis, rotational component — relative to the X axis. Accelerograms of a given seismic action are shown in Fig. 6-7 for translational and rotational actions, respectively.

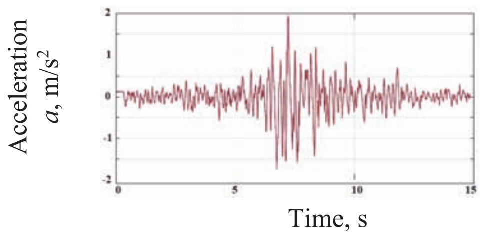


Figure 6. Accelerogram of translational motion along the Y-axis

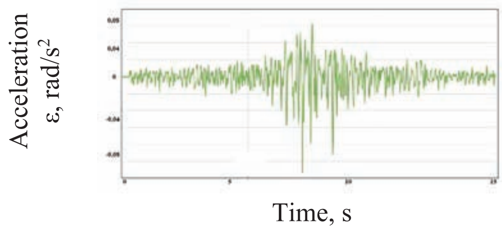


Figure 7. Accelerogram of rotational movement with respect to the X-axis from the action a_y

2.3 Calculation from accounting the six rotational components

Also in the course of the work, a calculation was made for a six-component seismic action, which included the following calculations:
1. Taking into account three translational components the action of which is directed along three mutually perpendicular axes.
2. Taking into account six components, where three are translational, and the other three are rotational and arise from translational.
Accelerograms of a given seismic action are shown in Fig. 8-10 for translational and in Fig. 11-13 for rotational actions.

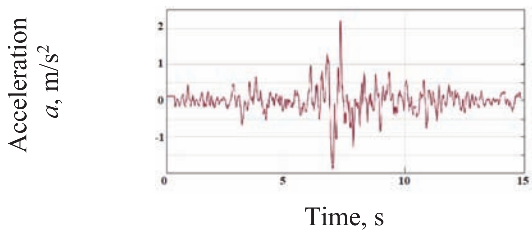


Figure 8. Accelerogram of translational motion along the X-axis

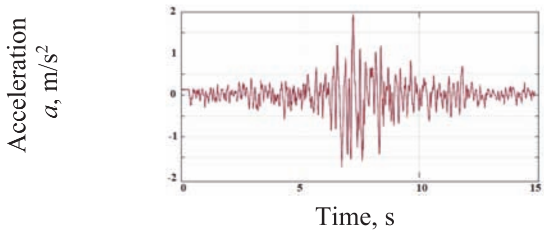


Figure 9. Accelerogram of translational motion along the Y-axis

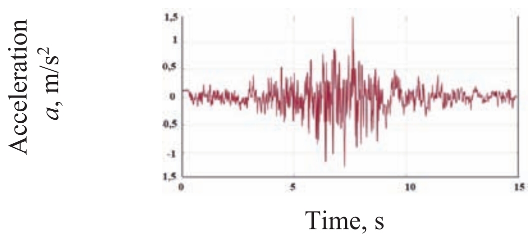


Figure 10. Accelerogram of translational motion along the Z-axis

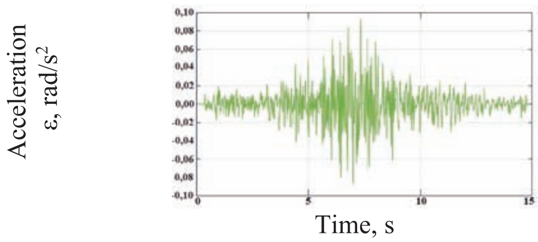
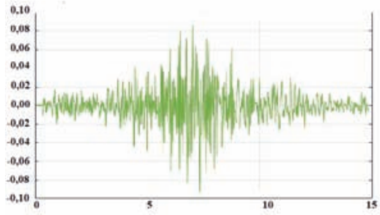


Figure 11. Accelerogram of rotational movement with respect to the X-axis from the action a_y, a_z

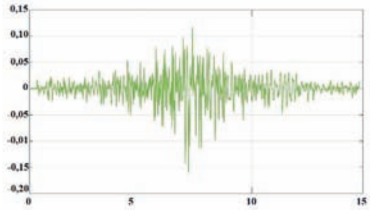
Acceleration ε, rad/s²



Time, s

Figure 12. Accelerogram of rotational movement with respect to the Y-axis from the action a_x, a_z

Acceleration ε, rad/s²



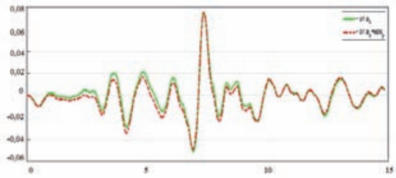
Time, s

Figure 13. Accelerogram of rotational movement with respect to the Z-axis from the action a_y, a_x

3. RESULTS

Fig.14–16 reflect the results (for calculation from item 2.1) obtained, the comparison of the maximum and minimum values of which are given in Table 1. Fig.17–19 reflect the results (for calculation from item 2.2) obtained, the comparison of the maximum and minimum values of which are given in Table 2. Fig.20–26 reflect the results (for calculation from item 2.3) obtained, the comparison of the maximum and minimum values of which are given in Table 3.

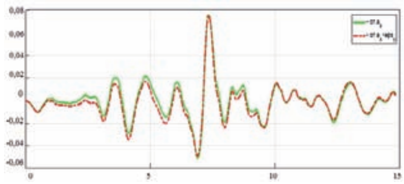
Displacement, m



Time, s

Figure 14. Graphical comparison of displacement values of node No 13071 along the X axis

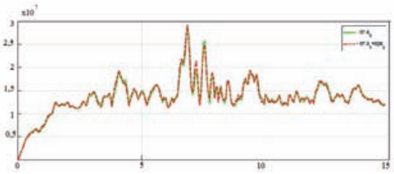
Displacement, m



Time, s

Figure 15. Graphical comparison of displacement values of node No 15383 along the X axis

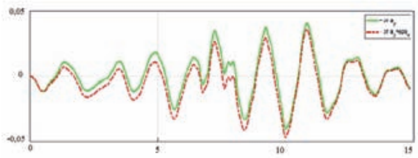
Von Mises stresses, Pa



Time, s

Figure 16. Graphical comparison of von Mises stresses of element B9253

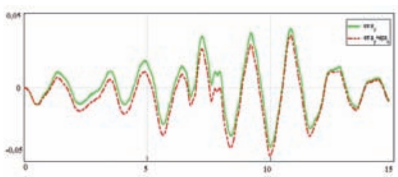
Displacement, m



Time, s

Figure 17. Graphical comparison of displacement values of node No 13071 along the Y axis

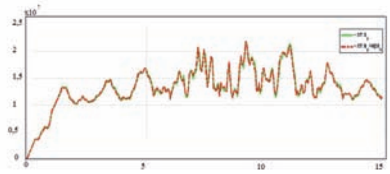
Displacement, m



Time, s

Figure 18. Graphical comparison of displacement values of node No 15383 along the Y axis

Von Mises stresses, Pa



Time, s

Figure 19. Graphical comparison of von Mises stresses of element B9253

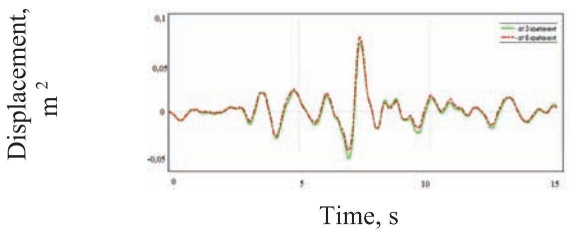


Figure 20. *Graphical comparison of displacement values of node No 13071 along the X axis*

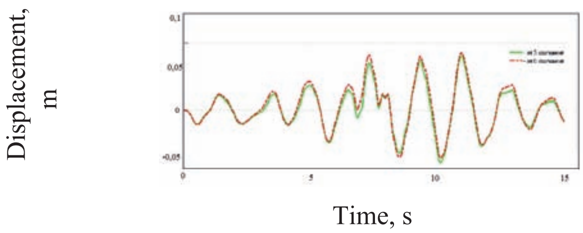


Figure 21. *Graphical comparison of displacement values of node No 13071 along the Y axis*

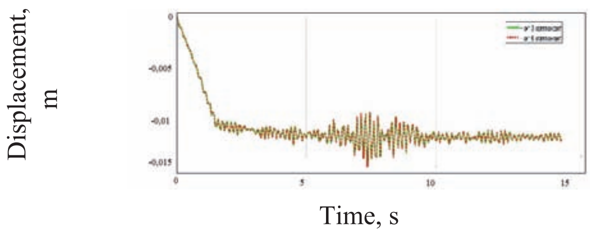


Figure 22. *Graphical comparison of displacement values of node No 13071 along the Z axis*

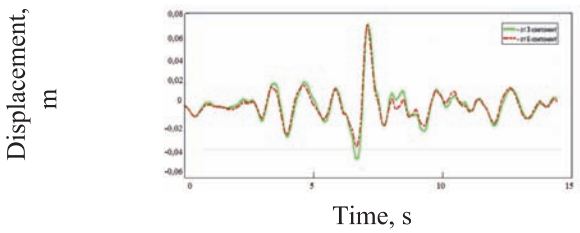


Figure 23. *Graphical comparison of displacement values of node No 15383 along the X axis*

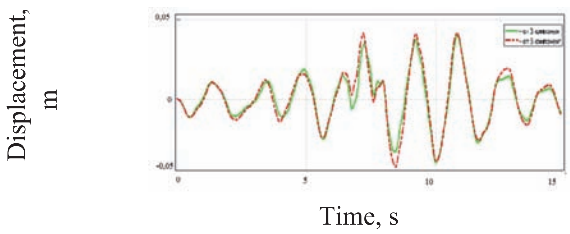


Figure 24. *Graphical comparison of displacement values of node No 15383 along the Y axis*

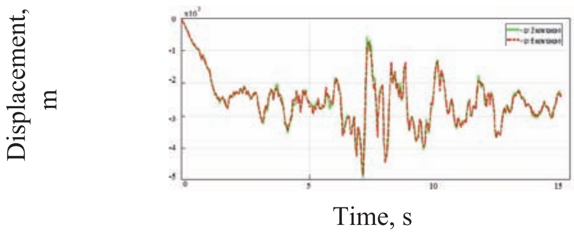


Figure 25. *Graphical comparison of displacement values of node No 15383 along the Z axis*

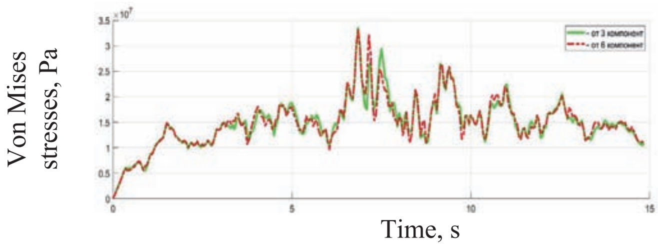


Figure 26. *Graphical comparison of von Mises stresses of element B9253*

Table 1. Comparison of the maximum and minimum values of displacements and stresses from accounting ε_y

		From the action a_y	From the action $a_y + \varepsilon_x$	Difference
Node 13071	u_y^{max}, m	$4{,}02 \cdot 10^{-2}$	$3{,}56 \cdot 10^{-2}$	11,44 %
	u_y^{min}, m	$-4{,}05 \cdot 10^{-2}$	$-4{,}64 \cdot 10^{-2}$	12,72 %
Node 15383	u_y^{max}, m	$4{,}05 \cdot 10^{-2}$	$3{,}57 \cdot 10^{-2}$	13,45 %
	u_y^{min}, m	$-4{,}04 \cdot 10^{-2}$	$-4{,}64 \cdot 10^{-2}$	12,93 %
Element 9253	σ_M^{max}, Pa	$2{,}17 \cdot 10^{7}$	$2{,}18 \cdot 10^{7}$	0,49 %

Table 2. **Comparison of the maximum and minimum values of displacements and stresses from accounting** ε_x

		From the action a_x	From the action $a_x+\varepsilon_y$	Difference
Node 13071	u_x^{max}, m	$7{,}48\cdot10^{-2}$	$7{,}47\cdot10^{-2}$	0,13 %
	u_x^{min}, m	$-5{,}11\cdot10^{-2}$	$-4{,}91\cdot10^{-2}$	4,07 %
Node 15383	u_x^{max}, m	$7{,}52\cdot10^{-2}$	$7{,}51\cdot10^{-2}$	0,13 %
	u_x^{min}, m	$-5{,}10\cdot10^{-2}$	$-4{,}90\cdot10^{-2}$	4,08 %
Element 9253	σ_M^{max}, Pa	$2{,}88\cdot10^{7}$	$2{,}91\cdot10^{7}$	0,95 %

Table 3. **Comparison of the maximum and minimum values of displacements and stresses from taking into account three and six rotational components**

		3 components	6 components	Difference
Node 13071	u_x^{max}, m	$7{,}48\cdot10^{-2}$	$8{,}01\cdot10^{-2}$	7,09 %
	u_x^{min}, m	$-5{,}11\cdot10^{-2}$	$-4{,}23\cdot10^{-2}$	20,80 %
	u_y^{max}, m	$4{,}02\cdot10^{-2}$	$4{,}28\cdot10^{-2}$	6,07 %
	u_y^{min}, m	$-4{,}05\cdot10^{-2}$	$-3{,}66\cdot10^{-2}$	10,66 %
	u_z^{max}, m	$-9{,}27\cdot10^{-4}$	$-9{,}28\cdot10^{-4}$	0,11 %
	u_z^{min}, m	$-1{,}47\cdot10^{-2}$	$-1{,}48\cdot10^{-2}$	0,67 %
Node 15383	u_x^{max}, m	$7{,}49\cdot10^{-2}$	$7{,}37\cdot10^{-2}$	1,63 %
	u_x^{min}, m	$-5{,}14\cdot10^{-2}$	$-3{,}76\cdot10^{-2}$	26,7 %
	u_y^{max}, m	$3{,}98\cdot10^{-2}$	$4{,}15\cdot10^{-2}$	4,09 %
	u_y^{min}, m	$-4{,}02\cdot10^{-2}$	$-4{,}27\cdot10^{-2}$	5,85 %
	u_z^{max}, m	$-5{,}82\cdot10^{-4}$	$-7{,}58\cdot10^{-4}$	23,22 %
	u_z^{min}, m	$-4{,}88\cdot10^{-3}$	$-4{,}80\cdot10^{-3}$	1,67 %
Element 9253	σ_M^{max}, Pa	$3{,}35\cdot10^{7}$	$3{,}33\cdot10^{7}$	0,60 %

4. CONCLUSION

For the buildings considered in the work of a relatively simple geometric shape that are not related to structures of increased number of storeys, taking into account the rotational components of the seismic effect leads to a decrease in the horizontal displacements of the upper mark to 27%, an increase in vertical displacements to 23% and an increase in stresses by 0.6%.

The results of the study are agreement with similar works [1-4] and allow us to conclude that the influence of the rotational components on the stress-strain state of the building under study is insignificant.

The calculation for a monolithic reinforced concrete 8-floor building, taking into account the rotational components of the seismic effect, was carried out in the LS-DYNA software and according to the data obtained in Fig. 14–26 and Tables 1–3, we can conclude that for buildings of relatively simple geometric shape that are not related to structures of increased number of storeys, with significant plan symmetry, when calculating the seismic effect, it is possible to neglect the effect of the rotational component, since they do not suffer from such dangerous phenomena as swinging about two horizontal axes or twisting relative to the vertical one, and the effect of rotational components on such structures does not make a tangible contribution to the stress-strain state.

However, this does not mean that the same conclusion can be drawn for high-rise buildings and structures of complex geometric shapes, significant asymmetry and developed in plan. Thus, further work on the study of the spatial work of buildings involves a detailed study based on modern calculation methods, which should ensure the required reliability and safety of building structures designed in seismic regions.

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