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THE EFFECT OF DRAW-OFF ON FILTRATION REGIME OF EARTH-FILL DAM

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Abstract. The article discusses the solution to the problem of transient seepage in relation to cases of rapid drawdown or draw-off, which may be caused by technological necessity or an emergency. This paper provides an overview of research and calculation methods for the problem under consideration. Using the PLAXIS software package, a numerical solution of the filtration problem for a homogeneous soil dam with a drainage prism is obtained. This problem has been repeatedly solved using other methods and computational programs. Comparison of the obtained results with those performed earlier by other methods showed their reliability and good comparability. The obtained results of solving the filtration problem at the next stage of the study of the structure operability can be used to assess the filtration strength of the soils of the dam and the stability of the upstream slope.

Keywords: drawdown, draw-off, filtration flow, depression surface, filtration gradient, slope stability.

ВЛИЯНИЕ СРАБОТКИ ВОДОХРАНИЛИЩА НА ФИЛЬТРАЦИОННЫЙ РЕЖИМ ГРУНТОВОЙ ПЛОТИНЫ

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Аннотация. В статье рассматривается решение задачи неустановившейся фильтрации применительно к случаям быстрой сработки водохранилищ гидроузлов, что может быть вызвано технологической необходимостью или аварийной ситуацией. Дается обзор исследований и методов расчета рассматриваемой задачи. С использованием программного комплекса PLAXIS получено численное решение фильтрационной задачи для грунтовой однородной плотины с дренажной призмой. Данная задача ранее неоднократно решалась с использованием других методов и вычислительных программ. Сравнение полученных результатов с выполненными ранее другими методами показали их достоверность и хорошую сопоставимость. Полученные результаты решения фильтрационной задачи на следующем этапе изучения работоспособности конструкции могут быть использованы для оценки фильтрационной прочности грунтов плотины и устойчивости верхового откоса.

Ключевые слова: сработка водохранилища, фильтрационный поток, депрессионная поверхность, градиент фильтрации, устойчивость откоса.

INTRODUCTION

The operation of earth-fill dams often requires drawdown or draw-off. It can be caused by the technological scheme of the work of a hydraulic structure - the use of water for various purposes, for example, power generation, irrigation or water supply. Also, draw-off can be caused by the need to reduce the current water head in the event of an emergency situation in the hydraulic structure. Such a decrease in the water level, which sometimes occurs at a sufficiently high speed, causes a change in the filtration regime of the earth-fill dam: the position of the depression surface, the direction and magnitude of the velocities and filtration gradients change. These changes can lead to negative consequences: the occurrence of filtration deformations, a decrease in stability and the collapse of the slope due to the action of the emerging hydrodynamic filtration loads [1-4].

In the practice of hydraulic engineering, there are cases of accidents at structures caused by a rapid drawdown or draw-off [5-7]. As an example, a massive slope collapse at the 116 m high San Luis dam in California occurred in 1981 as a result of lowering the upstream level by 55 m in 120 days during the dry season [5, 6]. A 340 m wide landslide brought down an array of soil with a volume of 310000 m^3 into the reservoir. After repair work, during which 1 100 000 m^3 of soil was additionally poured into the body of the dam, the dam's operability was restored.

An important issue in the design of soil dams is the prediction of the behavior of the structure with changes in water level and ensuring its stability and safety [8, 9]. The solution to this problem has two parts:

1) Solution of transient seepage problem, which makes it possible to determine changes in the position of the depression surface, filtration head, filtration gradients and velocities, distribution of filtration hydrodynamic load in the upper prism of a soil dam.

2) Assessment of the stability of the dam slope, taking into account the results of solving the filtration problem.

In this paper, we consider the solution of transient seepage problem. The paper provides an overview of the development and current state of the issue in solving such problems. The results of the numerical solution of a transient seepage problem for a earth-fill dam with a rapid drawdown or draw-off using the PLAXIS software package are presented. Comparison of the obtained results with the available analytical solution, the results of physical modeling and numerical solution using the FILTR program is given.

METHODS

The solution of the considered filtration problem is based on the theory of transient motion of an incompressible fluid in a non-deformable porous medium. The porous medium is completely saturated with liquid. For the first time, general differential equations describing the process of transient seepage were formulated by N.E. Zhukovsky [10], but due to the complexity of the solution, these equations were used much later. Hydraulic methods based on the Boussinesq equation [11] have received significant development in the practice of calculating transient seepage problems:

$$\frac{\partial H}{\partial t} = \frac{kh_a}{\mu}\left(\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2}\right) + \frac{\varepsilon}{\mu}, \qquad (1)$$

where H – head, t – time period, k – coefficient of filtration, h_m – average head, μ – water absorption coefficient, ε – soil porosity.

The main assumption adopted in hydraulic theory is the constancy of horizontal filtration velocities along the vertical section.

Boussinesq's hypothesis about the constancy of horizontal velocities along the vertical section of the filtration flow was tested in the works of Musket (1949) (Muskat) [12, 13], and later V. M. Shestakov [14], which showed its inaccuracy.

The main results of the considered filtration calculations are the determination of the position of the depression surface and the values of the head gradients. Many researchers in the late 30s - early 40s of the last century [11] adopted an approximate scheme, according to which it was believed that during the fall of the water level in the head water, the depression curve does not have time to change its position and is in the initial position. Analyzing the picture of filtration movement in the dam slope obtained under this assumption, R. Muller [11] came to the conclusion that the average head gradient near the slope is approximately equal to Sin β (where β is the angle of inclination of the slope surface to the horizon) and is directed along the slope surface. However, further research by numerous authors showed that such a scheme often overestimates filtration gradients and forces and is acceptable for low-permeability soils. The problems associated with the calculations of transient seepage found their development in the works of V. I. Aravin [15], N.

N. Bindeman [16], N. N. Verigin [17], V. M. Dombrovsky [18], G. Kamensky [19], L.S. Leybenson [20], V.S. Lukyanova [21], P.Ya. Polubarinova-Kochina [3, 4], I.A. Charny [22], V.M. Shestakova [14, 23, 24], Musket (1949) (Muskat) [13], H.R. Cedergreen [1], E. Reinius [25] and others.

All studies of transient seepage problems presented above were carried out under certain restrictions, which is caused by the imperfection of the methods used. A homogeneous area was considered, a constant rate of reservoir drawdown was taken, and a set of structures for which the proposed methods could be applied was limited.

Today, a qualitatively higher level of solving such problems is achieved using numerical methods and, first of all, using the finite element method (FEM) [26, 27]. The solution of the filtration problem in most of the used computational programs and complexes is reduced to solving the basic differential equation of the theory of filtration using the known boundary and initial conditions:

$$\frac{\partial}{\partial x}\left(K_x\frac{\partial H}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial H}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial H}{\partial z}\right)-\mu\frac{\partial H}{\partial t}=0 \qquad (2)$$

where $H=f(x,y,z,t)$ is the required head function in the computational domain, changing in time t; Kx, Ky, Kz - filtration coefficients in the directions of the coordinate axes X, Y, Z; μ - coefficient of soil water loss.

It is assumed that the movement of water between the soil particles occurs when the pores are completely filled with water under the action of a hydraulic gradient from an area with higher pressure to an area with lower pressure. The ability of the soil to conduct a saturated water flow under the influence of a hydraulic pressure gradient is characterized by a filtration coefficient. By value, it is equal to the velocity of the filtration flow with an equal unit of the hydraulic pressure gradient. The problem is to determine the position of the free (depression) flow surface with such a formulation of the task. This is solved by excluding finite elements from the computational domain, for which the value of the filtration head obtained from the solution turns out to be less than their altitude position [26, 27].

In the considered filtration problems, the water level in the head water changes, as a result of which the pores that are not completely filled with water are saturated (when the level rises) or water outflows from them (when the level decreases). These issues (in relation to porous soils) were considered in the works of famous scientists in the field of soil hydrology [28]. The concept of the basic hydrophysical characteristic was introduced as a characteristic of the water-holding capacity of the soil [28]. There are many models that characterize the hydraulic behavior of unsaturated soils [29-31].

The PLAXIS software package used in this work includes modules for performing calculations of steady-state groundwater filtration. The PlaxFlow module allows calculations of transient groundwater filtration. Filtration parameters include the water permeability of the soil (in a saturated state), as well as models and filtration parameters in an unsaturated zone. One of the most common and proven models is used – the Van Genuchten model [31], the basic equation of which relates water saturation to head as follows:

$$S(\phi_p)=S_{res}+(S_{sat}-S_{res})\left[1+(g_a|\phi_p|)^{g_n}\right]^{g_c} \qquad (3)$$

where $\phi_p=-\frac{p_w}{\gamma_w}$ и p_w- pore suction pressure, γ_w - unit weight of pore liquid, S_{res} – residual water saturation, which characterizes the part of the liquid remaining in the pores even with high suction, S_{sat} – may be less than one, because pores can be occupied by gas, g_a – a parameter that characterizes the amount of gas that has penetrated into the soil, g_n – parameter characterizing the rate of fluid loss from the soil, $g_c = \frac{(1-g_n)}{g_n}$ - parameter used in the general Van Genuchten equation.

Further, the paper presents some results of numerical solutions of transient seepage problems performed using the PLAXIS program and their comparison with analytical solutions, results of physical and numerical modeling using the FILTR program [32].

RESEARCH OBJECT

To evaluate the numerical solution of the problem of transient seepage when the water level of the head water changes, obtained using the PLAXIS software package, a test problem was solved, which has solutions by various methods [11, 14, 32]. A homogeneous sand dam with a drainage prism was considered (Fig. 1). The height of the dam is 24.0 m, the upstream slope was laid - 1: 3.0, downstream slope - 1: 1.85. In the downstream fill, drainage is arranged in the form of a drainage fill with a height of 5.0 m. The initial water level of the head water is H_1 = 22.0 m, the water level of the tailwater is constant and equal to h_2 = 3.0 m. The sand filtration coefficient is k = 1.92 m/day, water absorption coefficient μ = 0.2. The drawdown rate is v = 1.0 m/day.

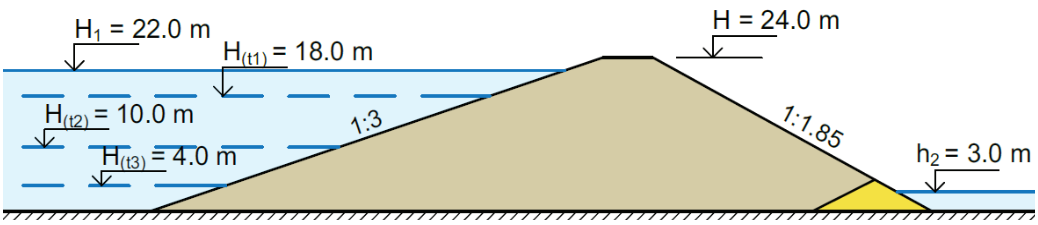

Figure 1. Schematic diagram of an earth dam with a drainage banquet

The moments of time τ = 4 days (drawdown by 4 meters, the permanent water level 18.0 m), τ = 12 days (drawdown by 12 meters, the permanent water level 10.0 m) and τ = 18 days (drawdown by 18 meters, the permanent water level 4.0 m).

RESULTS

The comparison results for the position of the depression surface in the design sections are presented in Fig. 2 and Table 1.

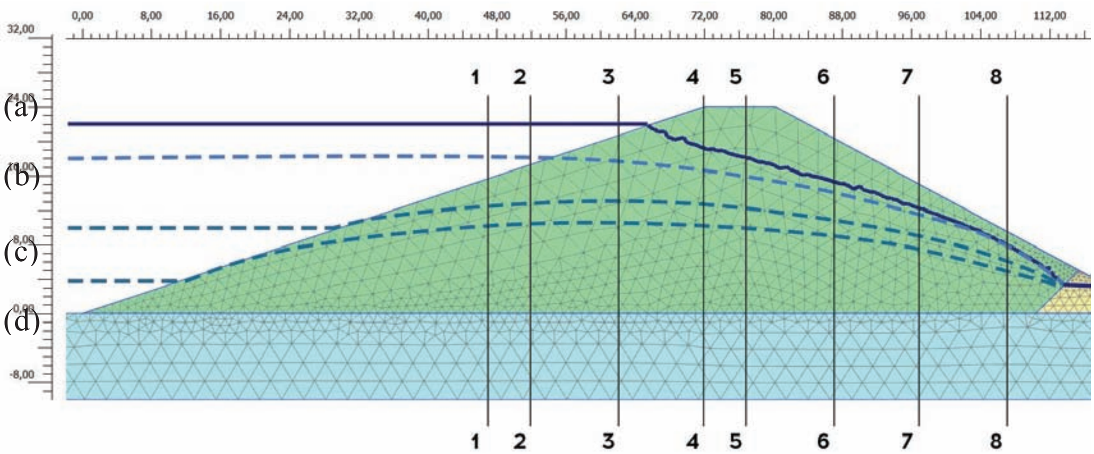

Figure 2. Position of depression curves, where (a) is the normal water level, (b), (c), (d) is the position at times τ = 4, 12, 18 days

A fairly good comparability of the results obtained by all methods can be noted. So, at the moment of time $\tau = 4$ days, the difference in the altitude of the points on the depression surface is in the interval (0.1-0.5) m, which does not exceed 3% of the headwater level.

Table 1. Results for the position of the depression surface in the design sections

Time	*Calculation method*	*Coordinates of the surface points of the depression curve*							
		Design sections							
		1-1	*2-2*	*3-3*	*4-4*	*5-5*	*6-6*	*7-7*	*8-8*
T= 4 days	hydraulic integrator V.S. Lukyanov		18.0	18.0	17.1	16.0	14.3	11.9	8.5
	V.M.Shestakov's method		18.0	17.3	16.8	15.7	13.9	11.6	7.9
	FILTR		18.0	17.8	17.0	16.1	14.4	11.7	7.5
	Plaxis		18,0	17.7	16.7	15.9	14.2	11.6	7.6
T= 12 days	hydraulic integrator V.S. Lukyanov	12.3	13.0	13.5	12.8	12.3	11.2	9.7	7.2
	V.M.Shestakov's method	13.0	13.9	14.0	13.7	13.0	12.0	10.0	7.8
	FILTR	13.1	13.3	13.5	13.0	12.5	11.4	9.3	6.2
	Plaxis	12.6	12.9	13.2	12.9	12.4	10.7	8.9	6.0
T=18 days	hydraulic integrator V.S. Lukyanov	9.5	10.0	10.6	10.5	10.2	9.2	8.2	6.2
	V.M.Shestakov's method	11.6	11.9	12.0	11.5	11.0	10.0	8.9	6.0
	FILTR	10.8	11.0	11.1	10.9	10.5	9.5	8.0	5.5
	Plaxis	10.2	10.4	10.5	10.2	9.7	8.9	7.4	5.1

At the moment of time $\tau = 18$ days, the difference in results at some points increases to 1.5 m (PLAXIS - Shestakov's method in sections 2-2, 3-3). It is possible to note a slight excess of the results for the second and third calculated points in time according to the method of V.M. Shestakov with other methods and good comparability of numerical solutions and modeling on a hydraulic integrator.

Figure 3 shows the distribution of filtration heads for the design points in time, obtained from the solution of the transient seepage problem using the PLAXIS software package.

The obtained pictures of the filtration head distribution allow us to estimate the dynamics of changes in the direction of movement of the filtration flow depending on the water level in the head water. For the moment of time $\tau = 4$ days (Fig. 3, b), in an insignificant part of the upstream slope above the drawdown mark of 18.0 m, there was an outflow of water and drainage of the soil. Movement in the entire calculated area below the depression surface is directed from the upstream slope to the downstream fill side. By the moment of time $\tau = 12$ days (Fig. 3, c) in the center of the soil massif of the dam, an area with increased head is formed relative to the area adjacent to the upstream slope of the dam. This indicates the occurrence of the movement of the filtration flow towards the upstream slope of the dam.

With a further decrease in the water level (Fig. 3, d), the head distribution does not change and the movement of water from the center to the upstream slope continues.

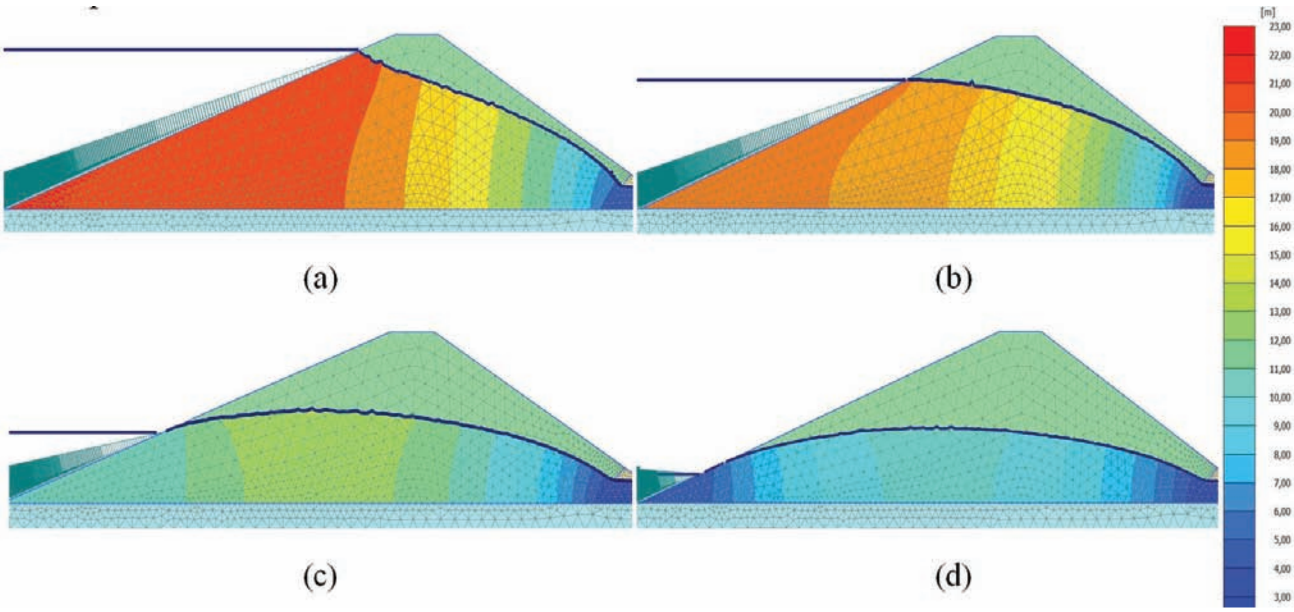

Figure 3. Distribution of filtration head, where (a) - the normal water level, (b), (c), (d) is the position at times τ = 4, 12, 18 days

The occurrence of a filtration flow directed towards the head water during the drawdown negatively affects the stability of the slope [7-9]. To assess the stability, it is necessary to know the distribution over the calculated area of the volumetric hydrodynamic load, which is directly proportional to the filtration gradient. As a result of solving the considered problem using the PLAXIS calculation complex, distributions of filtration gradients were obtained at calculated time points. Table 2 shows the values of the components of the filtration gradients and their comparison with the results of calculations in FILTR. Fig. 4 shows the results of calculations in the form of a distribution of horizontal components of the filtration gradient.

Table 2. Results of calculations of the components of the filtration gradient

Time	*Software package*	*Component of the filtration gradient*			
		upstream slope		*downstream slope*	
		horizontal, i_x	*vertical, i_y*	*horizontal, i_x*	*vertical, i_y*
normal water level	FILTR	-0.01	0.02	-0.52	0.72
	Plaxis	-0.001	0.02	-0.6	0.55
T= 4 days	FILTR	0.04	-0.12	-0.43	0.71
	Plaxis	0.02	-0.17	-0.57	0.51
T= 12 days	FILTR	0.31	0.07	-0.33	0.59
	Plaxis	0.25	-0.1	-0.39	0.34
T= 18 days	FILTR	0.32	0.09	-0.26	0.46
	Plaxis	0.31	-0.1	-0.29	0.23

The results of calculations in two computing complexes showed a fairly good comparability of the results. Thus, at the initial moment of time (Fig. 4, a), the horizontal components of the filtration gradient change quite smoothly from a value close to zero near the upstream slope to a value of ~ -0.52 at the outlet of the filtration flow into the drainage prism (the "-" sign corresponds to the direction of the horizontal component of the velocity towards the downstream fill). Throughout the massif, the vertical component of the gradient is positive, which corresponds to the direction of the vector of the vertical component of the velocity to the base of the dam. At the outlet of the filtration flow into the drainage prism, its value reaches 0.72.

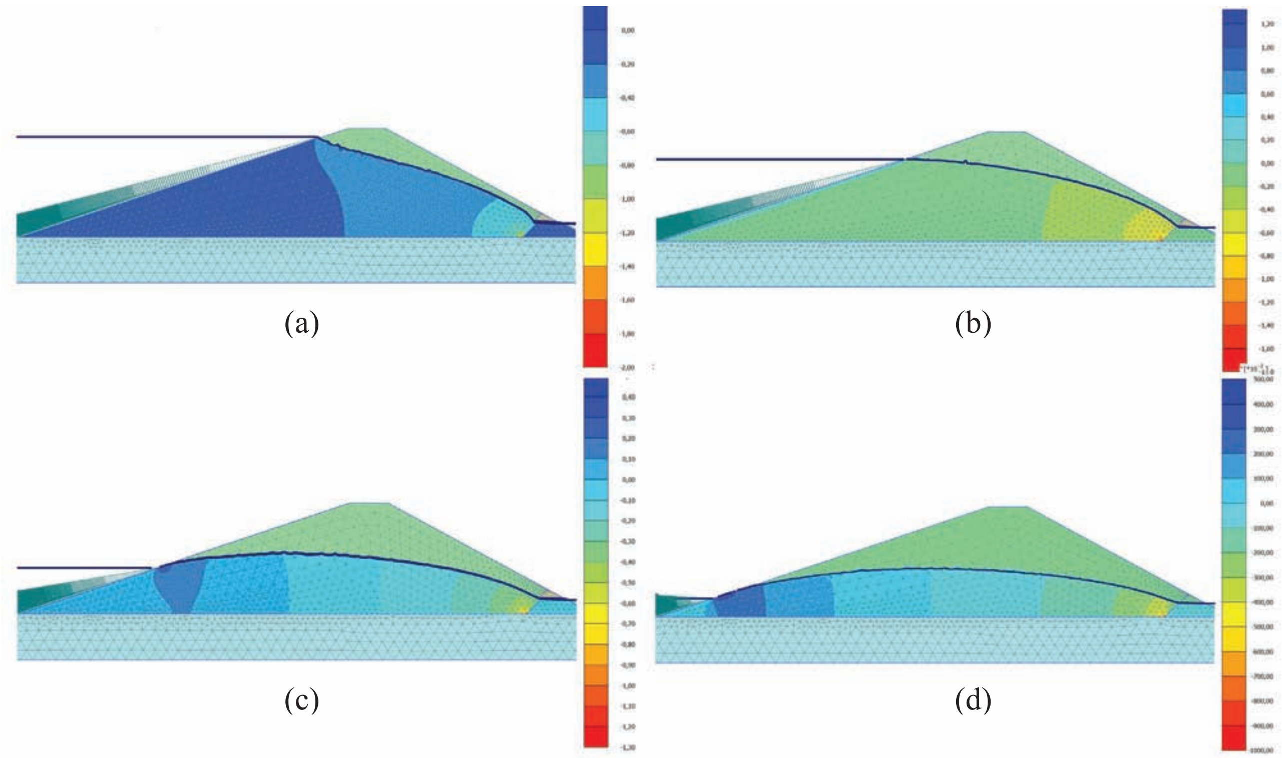

Figure 4. Distribution of horizontal components of the filtration gradient in the Plaxis software package where (a) - the normal water level, (b), (c), (d) is the position at times τ = 4, 12, 18 days

The obtained distributions of filtration gradients for other calculated cases (Fig. 4, b, c, d) are qualitatively and quantitatively comparable (Table 2). The values and places of occurrence of the maximum gradients are revealed: at the surface of the upper slope near the exit point of the depression surface and at the inlet of the filtration flow into the drainage prism.

The results of the numerical solution of the transient seepage problem can be further used to assess the stability of the slopes of a earth dam.

CONCLUSIONS

1. The numerical solution of the transient seepage problem obtained using PLAXIS is quite reliable. This is confirmed by good comparability with the results of other solution methods: on a hydraulic integrator, according to V.S. Shestakov's method.
2. A good agreement was obtained between the results of the numerical solution using the FILTR software (using the "classical" solution of the basic differential equation of the filtration theory) and the PLAXIS software (using the

Van Genuchten soil water absorption-water loss model).

3. The used technique and computational complex allows obtaining a reliable detailed picture of changes in the position of the depression curve, filtration gradients and velocities with a decrease in the water level.

4. The obtained results of solving the transient seepage problems are necessary to assess the filtration strength of the elements of ground dams and to check the stability of the slopes of the construction, since in the case of rapid drawdown or draw-off, additional hydrodynamic forces that worsen it arise.

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