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LOCAL STABILITY AND NATURAL MOTIONS OF THE MULTI-FACE DOME ROD STRUCTURE

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Abstract: The research object is a cyclically symmetric construction of a two-tier dome in the form of a convex polyhedron. Load and deflection critical parameters were determined for this construction pyramidal element. The behavior features of the conservative system in the dome's central assembly vertical displacement critical region value analysis has been carried out. The elastic rod model fluctuating and construction deviations from its equilibrium state reaction have been researched. System's behavior at natural motions and nonlinear restoring force is refined on the base of the findings carried out.

Keywords: polyhedral dome, rod structure, snap-through, restoring force, motion equation, elliptic integral, modular angel, phase plane

ЛОКАЛЬНАЯ УСТОЙЧИВОСТЬ И СВОБОДНЫЕ КОЛЕБАНИЯ СТЕРЖНЕВОЙ КОНСТРУКЦИИ МНОГОГРАННОГО КУПОЛА

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Аннотация: В качестве объекта исследования рассматривается циклически симметричная конструкция двухъярусного купола в форме выпуклого многогранника, для пирамидального элемента которой определены параметры критической нагрузки и прогиба. Выполнен анализ особенностей поведения консервативной системы в окрестности критического значения вертикального перемещения центрального узла купола. Исследован процесс колебаний упругой стержневой модели и изучена реакция конструкции на отклонения от ее равновесного состояния. На основании полученных результатов уточняется поведение системы при свободных колебаниях и нелинейной восстанавливающей силе.

Ключевые слова: многогранный купол, стержневая конструкция, прощелкивание, восстанавливающая сила, уравнение движения, эллиптический интеграл, модулярный угол, фазовая плоскость

The equilibrium stability rigorous definition considered as a mechanical system motion particular case was firstly given in the A.M. Lyapunov's work[1]. Rod structures in the form of convex polyhedron are trended to the snap-through according to the sustainability research majority were carried out on the static criterion base [2,3,4].The system fluctuation problem of this type in their possible equilibrium states region in the case when value nodal load goes up to the critical level, requires both theoretical and experimental researches, since the solution of this actual problem is still far from its complete solution. The considered problem has become more acute recently since high sensitivity of the shell type long span rod structure to the snap-through of the both individual and multiply connected assemblies was identified as a result of major disasters.

Circumscribed according to the 320-hedron scheme sphere of spatial rod construction in the form of a two-tier part of a polyhedral dome

whole surface section is chosen as a research object (Fig. 1). A vertical force P is placed at the vertex of the pentagonal pyramidal element. Force P is placed at the joint 1, in which five symmetrical inclined rods 1-2 are connected (Fig. 2).
The inclination rod angel α in comparison with its initial value α_0 changes. Two-tier dome pentagonal circle is assumed inextensible according to the $l_0 \sin\alpha_0 = l \sin\alpha = a$.

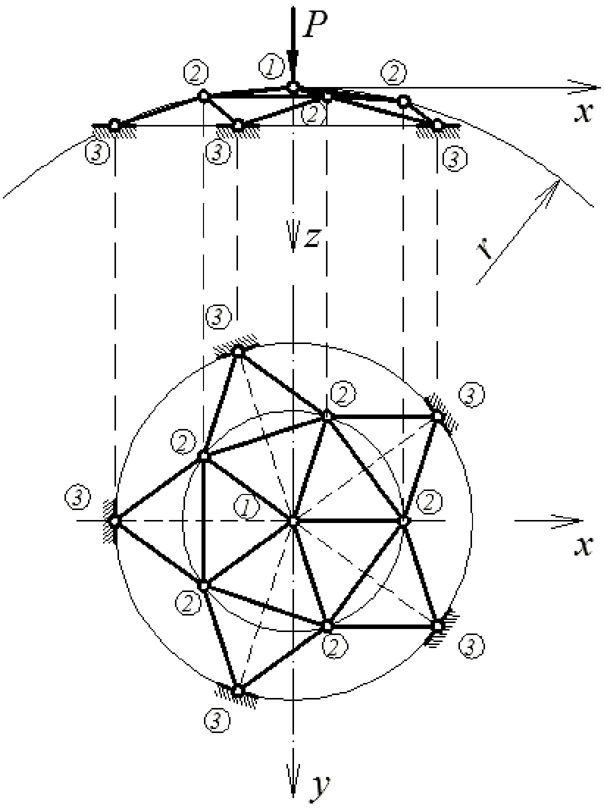


Figure 1. Spatial rod construction in the form of two-tier polyhedral dome

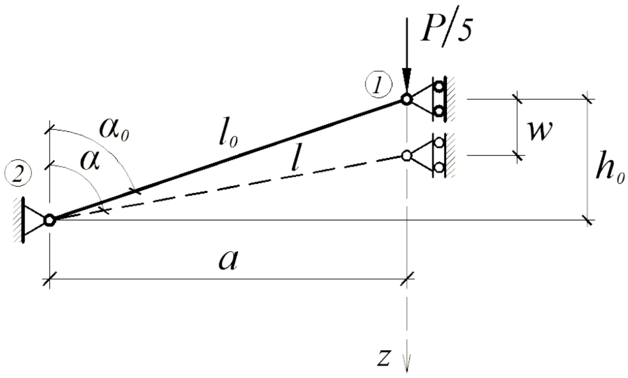


Figure 2. Pentagonal pyramid rod elements design model

Rod structure behavior under the load P_1 and P_2 the forces placed at the joint 1 and 2 have to be analyzed in general case to determine possibility to add the upper tier pentagonal circle non-deformability restriction. The nodal load uneven distribution influence on the local stability of the considered rod system effect must be considered. This distribution is set according to the load transferring area calculated for this joint type on the horizontal domed surface projection for the central joint 1 and five peripheral joints 2. For example, in this case, the ratio of the joint forces is $P_2 / P_1 = 1{,}34$. This situation is clearly depicted on the system equilibrium diagram (Fig. 3), $P_1^* = P_1 / (5EF)$; $\eta_{12} = \eta_1 - \eta_2 = (w_1 - w_2)/l_{12}$ is accepted.
Let us denote that central joint snap-through load level increased by 15% under the force P_2 (curve 1) in comparison with the case when the circle is inextensible (curve 2).

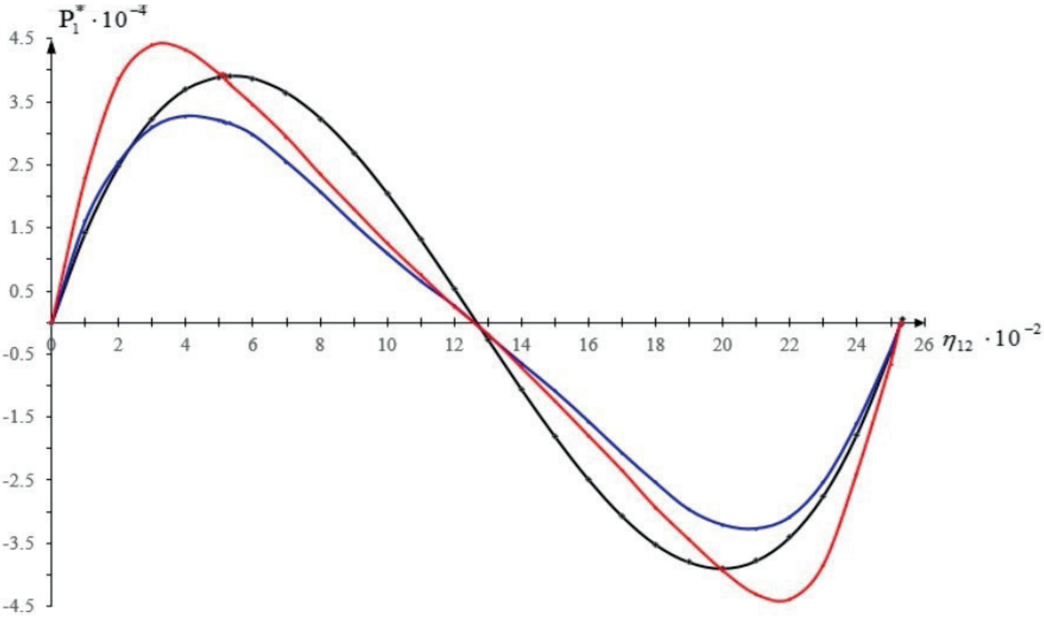


Figure 3. Equilibrium states diagram for a two-tier part rod construction in the form of a 320-hedron

If the coefficient $k = 5P_2 / P_1$ is added then the formula for the critical load changes to [5]:

$$P_{1,cr.} = \frac{10\sqrt{3}}{9} \frac{EF}{1+k} \cos^3 \gamma_{23} . \quad (1)$$

The expression (1) is written as follows in the case of non-deformability pentagonal circle:

$$P_{1,cr.} = \frac{5\sqrt{3}}{9} EF \cos^3 \gamma_{12} . \quad (2)$$

Equating the right-hand sides of the expressions (1) and (2) to each other, the result is:

$$k = 2(\cos\gamma_{23} / \cos\gamma_{12})^3 - 1 , \quad (3)$$

whence it follows that $k = 0{,}0723$ and $P_2 = 0{,}014 P_1$. In other words, tensile forces take place in the pentagonal circle rod elements only in the case when $P_2 < 0{,}014 P_1$

The curve 3 shows rod structure behavior in case when both radial displacement of the nodes 2 in the in the horizontal circle plane and the vertical displacement of the nodes 1,2 are considered. In this case the critical load level is decreased by 11% in comparison with its pentagonal circle non-deformability value.

The curves' 2 and 3 almost complete coincidence is clearly observed in the parameter values interval η_{12} $0 < \eta_{12} < 3 \cdot 10^{-3}$.

In this case, the relationship between the force P and the vertical displacement w is expressed:

$$P = 5EF\left(1 - \frac{w}{a}\tan\alpha_0\right)\left[\frac{1}{\sqrt{\tan^2\alpha_0 + \left(1 - \frac{w}{a}\tan\alpha_0\right)^2}} - \cos\alpha_0\right] \quad (4)$$

After adding a new variable $\xi = 1 - \zeta$,where $\zeta = \frac{w}{a}\tan\alpha_0$, the functional relationship $P = P(\xi)$ is:

$$\hat{P} = \xi\left(\frac{1}{\sqrt{\tan^2\alpha_0 + \xi^2}} - \cos\alpha_0\right). \qquad (5)$$

$\hat{P} = P/(5EF)$ is a nondimensional load parameter.

Let us use the condition to determine critical load

$$\frac{d\hat{P}}{d\xi} = 0,$$

and write it in the following manner:

$$\frac{d\hat{P}}{d\xi} = \left(\tan^2\alpha_0 + \xi^2\right)^{-1/2} - $$
$$-\cos\alpha_0 - \xi^2\left(\tan^2\alpha_0 + \xi^2\right)^{-3/2} = 0 \qquad (6)$$

Assuming that $\tan^2\alpha_0 + \xi^2 = z^2$, we determine:

$$z^3_{кр} = \tan^3\alpha_0(\sin\alpha_0)^{-1};$$

$$z_{кр} = \tan\alpha_0(\sin\alpha_0)^{-1/3}. \qquad (7)$$

The critical load parameter value $\hat{P}_c$ is equal to

$$\hat{P}_{cr} = \left(1 - \sin^{2/3}\alpha_0\right)^{3/2}. \qquad (8)$$

Firstly to determine the value corresponding to the moment when the joint load reaches its critical level we must note that:

$$\xi^2 = \tan^2\alpha_0 \sin^{-2/3}\alpha_0\left(1 - \sin^{2/3}\alpha_0\right). \qquad (9)$$

On the other hand the equation must be $\xi^2 = (1-\varsigma)^2$ must be.

Equating the right-hand sides of the last two equations to each other, the result is quadratic equation for the nondimensional displacement parameter

$$\varsigma^2 - 2\varsigma + 1 - \tan^2\alpha_0 \sin^{-2/3}\alpha_0\left(1 - \sin^{2/3}\alpha_0\right) = 0. \qquad (10)$$

The solution of the equation (10) is:

$$\varsigma_{кр} = 1 - \tan\alpha_0\left(\sin^{-2/3}\alpha_0 - 1\right)^{1/2} \qquad (11)$$

for the pyramidal element with 320-hedron spatial configuration α_0= 82.2°. According to this, the numerical values of the trigonometric functions in the initial position of the rod system are:

$\sin\alpha_0$=0,991950; $\cos\alpha_0$=0,126591;
$\tan\alpha_0$=7,83586.

The vertical displacement nondimensional parameter required value can be found using the values obtained from the expression (11),

$$\varsigma_{cr} = 1 - 0{,}575 = 0{,}425 = 17/40.$$

The rod system equilibrium states diagram, constructed with the dependence (4), is shown in Fig. 4.

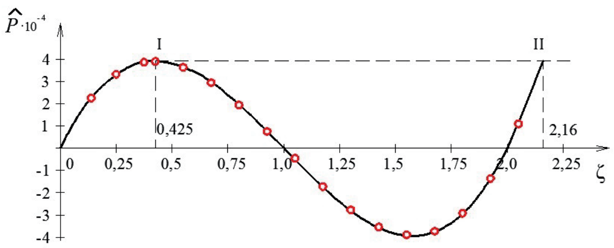


Figure 4. Dependency diagram for a pyramidal element of a rod construction

As we can see, when the maximum load level is reached, the system jumps from the equilibrium state I to the position II. The circles show the equilibrium states diagram for the pyramidal element of a two-tier dome. The stability analysis of this element was carried out in [5] by using the energy method.

Snap-through of the rod construction is not allowed. Let us analyze the stability of its equilibrium state when it is near critical value of the pentagonal pyramidal 320-hedron element top vertical displacement. In other words, let us choose a point on the system equilibrium states curve where the inequation $\varsigma = 2/5 < \varsigma_{cr}$ is valid. The nondimensional displacement parameter value corresponding to this displacement is equal to $\hat{P}(2/5) = 0{,}393 \cdot 10^{-3}$. As for the initial forces of axial compression in the rods, they are determined by the equation

$$S_0 = -\frac{8}{9} EF \cos^2 \alpha \quad (12)$$

The rod structure of two-tier dome in the form of the 320-hedron fluctuating problem solution is considered only taking account of the fluctuations along the mass m vertical axis held by five meridional direction rods in the regular pentagonal pyramid central joint.

Fluctuating at the initial displacement $(x_0)_{t=0} = x_0$ and without initial speed $(\dot{x}_0)_{t=0} = 0$ at dome pyramidal element central joint deviation from the equilibrium by slight displacement x in the upward vertical mass m motion equation has the following form [6]:

$$\ddot{x} + p^2 f(x) = 0 \quad (13)$$

Here $f(x)$ means

$$f(x) = 2x + 27\frac{\tan\alpha}{a}x^2 + 9\frac{\tan^2\alpha}{a^2}x^3 \quad (14)$$

In its turn, $p^2 f(x)$ is nothing other than the restoring force related to the unit mass in the x function displacement.

In this case $\cot\alpha = (3/5)\cot\alpha_0 = 0{,}076571$, it follows that, $\tan\alpha = 13{,}0598$ and $\cos\alpha = 0{,}076348$. The calculation for the p^2 is : $p^2 = 0{,}8216 g/a$.

Mass motion time from the extreme position $(x = x_0)$ to the position $(x = 0)$, when the system goes back to its initial state is determined by the following formula[7]:

$$\tau = -\int_{x_0}^{0} \frac{dx}{\sqrt{2p^2 \int_x^{x_0} f(x)dx}} = \int_0^{x_0} \frac{dx}{\sqrt{2p^2 \int_x^{x_0} f(x)dx}} \quad (15)$$

Integral radicand according to (14) is:

$$\int_x^{x_0} f(x)dx = \int_x^{x_0} \left(2x + 27\frac{\tan\alpha}{a}x^2 + 9\frac{\tan^2\alpha}{a^2}x^3 \right) dx \quad (16)$$

As a result of integration and substitution in (15), we have

$$\tau = \frac{\sqrt{2}}{2p} \int_0^1 \frac{dz}{\sqrt{(1-z^2) + 6\gamma(1-z^3) + \gamma^2(1-z^4)}} \quad (17)$$

Here is: $z = \frac{x}{x_0}$, $\gamma = \frac{3}{2}\frac{x_0}{a}\tan\alpha$.

Further differential $\frac{dz}{\sqrt{R}}$ must be transformed to obtain polynomial under the radical that doesn't contain the variable z uneven degree. Assuming that $u = \sqrt{1-z^2}$ and making a substitution, we obtain $dz = -\frac{u}{\sqrt{1-u^2}} du$

$$\tau = \frac{\sqrt{2}}{2p}\int_0^1 \frac{du}{\sqrt{1-u^2}\sqrt{1+9\gamma+2\gamma^2-\gamma(3+\gamma)u^2}} . \quad (18)$$

In this case we have [8]:

$$\int_0^x \frac{dt}{\sqrt{a^2-t^2}\sqrt{c^2-t^2}} = \frac{1}{c}F\left(\frac{a}{c},\varphi\right) . \quad (19)$$

Here $F\left(\frac{a}{c},\varphi\right)$ is the first kind elliptic integral. According to the collating (18) with the standard integral (19) we conclude that

$$a^2 = 1; \quad c^2 = \frac{1+9\gamma+2\gamma^2}{\gamma(3+\gamma)},$$

$$\sin\varphi = \frac{x}{a} = 1; \quad \varphi = \frac{\pi}{2}.$$

To find the numerical value of γ quantity, let us use the equation $x_0 \tan\alpha_0 / a = 1/40$. As $\tan\alpha_0 = 3\tan\alpha/5$, we determine:

$$\frac{x_0}{a}\tan\alpha = \frac{1}{24}; \quad \gamma = \frac{3}{2}\frac{x_0}{a}\tan\alpha = \frac{1}{16}.$$

The calculations results:

$$c^2 = \frac{1+\gamma(9+2\gamma)}{\gamma(3+\gamma)} = 8{,}20408; \quad c = 2{,}8643;$$

$$\frac{1}{c} = 0{,}3491 .$$

Let us find the integral $F(\theta)$ in the case when the amplitude is equal to $\varphi = \pi/2$ and the modulus is equal to $k = 0{,}3491$. The modular angle $\theta = 20{,}43^\circ$ will be determined from the equation $\sin\theta = 0{,}3491$.

The 20° angel corresponds to $F(20^\circ) = 1{,}62003$ and 21° angel corresponds to $F(21^\circ) = 1{,}62523$ according to the table [9] and the amplitude $\varphi = \pi/2$. Assuming that the $F(\theta)$ integral increment is proportional to the angle θ, the result is:

$$\frac{F(20{,}43^\circ) - F(20^\circ)}{F(21^\circ) - F(20^\circ)} = 0{,}43 .$$

Whence it follows that $F(20{,}43^\circ) = 1{,}6275$. $\tau = 0{,}918/p$ is to be found with the expression (18).

When deriving the solution numerically, we note that the expression (14) contains the summand x in the first degree and further the motion differential equation (13) is to be written in a slightly modified form:

$$\ddot{\eta} + \bar{p}^2(\eta + \delta_i) = 0 . \quad (20)$$

Here is: $\bar{p}^2 = 2p^2$; $\eta = \frac{x}{a}\tan\alpha$ and δ_i means that:

$$\delta_i = \frac{9}{2}\left(3\eta^2 + \eta^3\right) .$$

Further let us imagine that the entire time interval τ is divided into a number of small intervals Δt during which the value δ_i is considered constant and equal to its value at the beginning of each of such intervals. The η_i и $\dot{\eta}_i$ are consider as the parameters of displacement and speed at $t_1 = 0$. The equation solution (20) is to be written in the following form:

$$\eta = (\eta_i + \delta_i)\cos\bar{p}t_1 + \frac{\dot{\eta}_i}{\bar{p}}\sin\bar{p}t_1 - \delta_i \quad (21)$$

As a result of time differentiation (21), we got:

$$\frac{\dot{\eta}}{\bar{p}} = -(\eta_i + \delta_i)\sin \bar{p}t_1 + \frac{\dot{\eta}_i}{\bar{p}}\cos \bar{p}t_1 \quad . \qquad (22)$$

(21) and (22) are to be squared and summed up. The result is:

$$(\eta + \delta_i)^2 = (\eta_i + \delta_i)^2 + \left(\frac{\eta_i}{\bar{p}}\right)^2 \quad . \qquad (23)$$

Thus, we make sure that the equation solution (20) is performed on the $(\eta, \dot{\eta}/\bar{p})$ phase plane of the other circle with the center on the η axis and δ_i coordinate.

Firstly, the value $\ddot{\eta}_0$ is calculated and then the $\dot{\eta}_1$ and η_1 approximate values are determined from [10] at the initial conditions $\eta = \eta_0$ and $\dot{\eta} = 0$ for $t = 0$:

$$\dot{\eta}_1 = \dot{\eta}_0 + \ddot{\eta}_0 \Delta t \,; \quad \eta_1 = \eta_0 + \frac{\dot{\eta}_0 + \dot{\eta}_1}{2} \,.$$

Substituting η_1 to the equation (20) instead of η, $\ddot{\eta}_1$ can be found . The most accurate $\dot{\eta}_1$ and η_1 approximations could be derived with this value from the expressions:

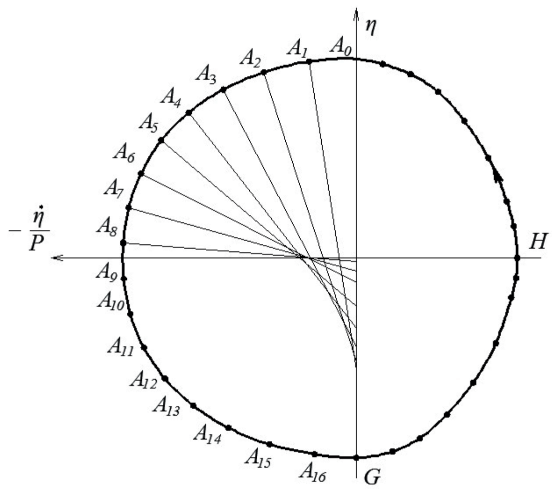
$$\dot{\eta}_1 = \dot{\eta}_0 + \frac{\ddot{\eta}_0 + \ddot{\eta}_1}{2}\Delta t \,; \quad \eta_1 = \eta_0 + \frac{\dot{\eta}_0 + \eta_1}{2} \,.$$

The values η_2, $\dot{\eta}_2$ and $\ddot{\eta}_2$ for the moment $t_2 = 2\Delta t$ is to be calculated by repeating in order this procedure which was described above.

This values for the moments $t_3 = 3\Delta t$, $t_4 = 4\Delta t$, .., $t_n = n\Delta t$ are calculated analogical.

The numerical integration results are performed on the phase plane (Fig. 5).

Time of the first fluctuation mass cycle m is equal to $\tau = 0{,}939\frac{1}{p}$ with the 2,3% error according to the phase curve shown in the graph. The restoring force grows faster than the deflection in the case when the initial displacement values are increased in this phase curve section. The mass m will take the position indicated by the letter G which corresponds to the central joint vertical displacement critical value at the fluctuation second cycle.



<u>Figure 5</u>. speed-displacement curve on the phase plane for the system fluctuation full cycle

The sign in front of the second summand expression in parentheses is reversed at the phase curve construction for the second fluctuation cycle in the motion equation (20). Due to this operation at increasing its initial displacement value the restoring force growth slows down.

The phase trajectory will have a spiral form instead of a closed curve at the damped fluctuation process. The phase speed will not be equal 0 anywhere in this case, despite it will be continuously decreased as the point approximating to the origin of coordinates.

Comparing the derived data, we come to the conclusion that the two-tier dome part structure snap-through phenomenon can be caused by the rod system fluctuations excitation with a nonlinear restoring force as a result of its central joint deviation from the equilibrium state in the vertical displacement parameter critical value surroundings.

The fact that fluctuations period is inversely related to the amplitude η_0 in the considered case has to be denoted.

Thus, it could be argued that the danger of its snap-through increases significantly in the case when the rod structure fluctuates with initial speed. Such result in the structure operation is practically inevitable in the case when its carrying capacity is completely exhausted due to the critical load close level. From this it follows that the mass disturbed motion total energy will exceed the difference between the initial potential energy and the potential energy of the system at the considered moment.

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