DOI:10.22337/2587-9618-2022-18-1-14-23

APPLICATION OF THE THEORY OF THE MULTICOMPONENT DRY FRICTION IN SOME OF CONTROL ROBOT SYSTEMS

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Abstract: The implementations of the theory of multicomponent dry friction [1-19] for analyze the dynamics of some robotic systems, such as a butterfly robot [16-18, 20] or a humanoid robot is proposed. Since the main controlled element of these systems is a spherical, elastic composite shell, it is required to calculate the distribution of normal contact stresses inside the contact spot. The contact pressure distribution for such elements is constructed using the S. A. Ambartsumyan's equation for a transversally isotropic spherical shell. This equation is modified by introducing the averaged contact pressure and normal displacements for the shell. The construction of the resolving integral equation for the contact pressure is based on the principle of superposition and the method of Green's functions. For this, the corresponding Green's function is constructed, which is the normal displacement of the shell as a solution to the problem of the effect of concentrated pressure. Green's function as well as the contact pressure, it is sought in the form of series expansions in Legendre polynomials, taking into account additional relations for the reduced contact pressure and normal displacements. Using the Green's function, an integral equation solving the problem is constructed. As a result, the problem is reduced to determining the expansion coefficients in a series of the reduced contact pressure. Restricting ourselves to a finite number of terms in the series of expansions, using the discretization of the contact area and the properties of Legendre polynomials, the problem is reduced to solving a system of algebraic equations for the expansion coefficients for the reduced pressure. After that, from the additional relation, the coefficients of the required expansion of the contact pressure in a series in Legendre polynomials are determined. To describe the conditions of shell contact with the surface, the theory of multicomponent anisotropic dry friction is used, taking into account the combined kinematics of shell motion (simultaneous sliding, rotation and rolling). The coefficients of the dry friction model can be calculated using simple explicit formulas [1-19] based on numerical experiments.

Keywords: spherical composite shell; contact problem; theory of the multicomponent anisotropic dry friction.

ПРИМЕНЕНИЕ ТЕОРИИ МНОГОКОМПОНЕНТНОГО СУХОГО ТРЕНИЯ В НЕКОТОРЫХ УПРАВЛЯЕМЫХ РОБОТОТЕХНИЧЕСКИХ СИСТЕМАХ

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Аннотация: Предложена реализация теории многокомпонентного сухого трения [1-15] для анализа динамики некоторых роботизированных систем, таких как робот-бабочка или робот-гуманоид. Поскольку основным управляемым элементом этих систем является сферическая эластичная композитная оболочка, требуется рассчитать распределение нормальных контактных напряжений внутри пятна контакта. Распределение контактного давления для таких элементов построено с использованием уравнения С. А. Амбарцумяна для поперечно-изотропной сферической оболочки. Это уравнение модифицируется путем введения усредненного контактного давления и нормальных перемещений для оболочки. Построение разрешающего интегрального уравнения для контактного давления основано на принципе суперпозиции и методе функций Грина. Для этого строится соответствующая функция Грина, представляющая собой нормальное смещение оболочки как решение проблемы влияния концентрированного давления. Функция Грина, так же, как и контактное давление, ищется в виде разложений в ряды в полиномах Лежандра с учетом дополнительных соотношений для пониженного контактного давления и нормальных перемещений. Используя функцию Грина, строится интегральное уравнение, решающее задачу. В результате задача сводится к определению коэффициентов расширения в ряду пониженного контактного давления. Ограничиваясь конечным числом членов в ряду разложений, используя дискретизацию площади контакта и свойства полиномов Лежандра, задача сводится к решению системы алгебраических уравнений для коэффициентов расширения для пониженного давления. После этого из дополнительного соотношения определяются коэффициенты требуемого расширения контактного давления в ряду в полиномах Лежандра. Для описания условий контакта оболочки с поверхностью используется теория многокомпонентного анизотропного сухого трения, учитывающая комбинированную кинематику движения оболочки (одновременное скольжение, вращение и качение). Коэффициенты модели сухого трения могут быть рассчитаны с помощью простых явных формул [2], основанных на численных экспериментах.

Ключевые слова: сферическая композитная оболочка; контактная задача; теория многокомпонентного анизотропного сухого трения.

INTRODUCTION

The theory of the multicomponent dry friction is very effective instrument for the correctly describing of effects of the combined dry friction in many engineering systems. One of the distinguished features of this theory is possibility to carry out analytical investigation of the equations of motions. Connection between the parameters, which are defined the force state inside of contact spot, and the kinematical parameters is given by the simple analytical functions. It’s so-called the approximate model of the combined dry friction or phenomenological model. Only six numeric’s coefficients are required to calculate. These coefficients can be calculated analytically, numerically or defined from the experiments. Procedure of analytical or numerical definitions of the friction model coefficients is based on the calculation of the first moments of the distribution of the normal pressure inside of contact patch. In order to use these results to analyze the dynamics of some robotic systems, such as a butterfly robot or a humanoid robot, it is necessary to calculate the normal pressure inside the contact spot for various composite spherical shells.

In investigation described below the contact pressure distribution is constructed using the S.A. Ambartsumyan’s equation for a transversally isotropic spherical shell. This equation is modified by introducing additional relationships for the reduced contact pressure and normal displacements. The construction of the resolving integral equation for the contact pressure is based on the principle of superposition and the method of Green's functions. For this, the corresponding Green's function is constructed, which is the normal displacement of the shell as a solution to the problem of the effect of concentrated pressure. Green's function as well as the contact pressure, it is sought in the form of series expansions in Legendre polynomials, taking into account additional relations for the reduced contact pressure and normal displacements.

Using the Green's function, an integral equation solving the problem is constructed. As a result, the problem is reduced to determining the expansion coefficients in a series of the reduced contact pressure. Restricting ourselves to a finite number of terms in the series of expansions, using the discretization of the contact area and the properties of Legendre polynomials, the problem is reduced to solving a system of algebraic equations for the expansion coefficients for the reduced pressure. After that, from the additional relation, the coefficients of the required expansion of the contact pressure in a series in Legendre polynomials are determined.

CONTACT PROBLEM SOLUTION

To determine the contact pressure, we pose a static contact problem for a spherical shell of radius R, thickness h and an absolutely rigid reference plane Π [21]. We assume that the shell is made of a transversely isotropic material in such a way that the main direction of elasticity, perpendicular to the plane of isotropy, at each point of the shell coincides with the outer normal $\mathbf{n}$ to the middle surface of the shell.

Contact between shell and reference plane Π occurs along a flat circular area (contact patch) Ω some radius r belonging to the plane Π: $\Omega \in \Pi$ (fig. 1). Taking into account the small size of the contact area $(r \ll R)$ the radius of the contact spot in the zero approximation is determined from the condition of intersection of the undeformed middle surface of the shell

$$r = R\sin\beta_*,\ \beta_* = \arccos\frac{R - w_T}{R}, \qquad (1.1)$$

where w_T - displacement at the frontal point of the shell.

In this case, in the contact area, the normal displacements of the shell are determined as follows (Fig. 1)

$$w = R\left(1-\cos\beta\right) - w_T. \qquad (1.2)$$

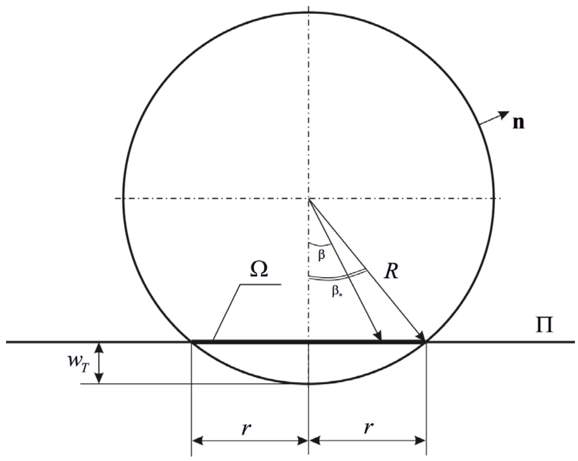

Figure 1. Contact problem

Assuming that the contact problem is axisymmetric, we use the S.A. Ambartsumyan [21] for a transversely isotropic spherical shell, connecting the normal displacements of the shell w with influencing pressure on her p

$$\left[c^2\left(\Delta+1\right)^2 + 1 - h^*\Delta\right]\left(\Delta+2\right)w =$$
$$= \frac{R^2}{Eh}\left(1-h^*\Delta\right)\left(\Delta+1-\nu\right)p, \qquad (1.3)$$
$$\Delta = \frac{1}{\sin\beta}\frac{\partial}{\partial\beta}\left(\sin\beta\frac{\partial}{\partial\beta}\right),$$

$$c^2 = \frac{h^2}{12\left(1-\nu^2\right)R^2},$$

$$h^* = \frac{Eh^2}{10\left(1-\nu^2\right)R^2G'},$$

E - Young's modulus for directions in the plane of isotropy, ν – Poisson's ratio, which characterizes the contraction in the plane of isotropy under tension in the same plane, G' - shear modulus for planes normal to the plane of isotropy.

Note that the structure of equation (1.3) does not allow us to apply expansions in series in Legendre polynomials, since the presence of the operator factor $\Delta+2$ on the left side turns it to zero at $n=1$, where n Is the number of a member of the expansion series. To overcome this difficulty, in contrast to the solution proposed in [21], we introduce auxiliary functions of averaged pressure as well as averaged deflection for the shell as follows:

$$p=\left(1+\tfrac{1}{2}\Delta\right)\tilde{p},\quad \tilde{w}=\left(1+\tfrac{1}{2}\Delta\right)w. \quad (1.4)$$

Then equation (1.3) in new functions takes the form

$$\begin{aligned}&\left[c^2(\Delta+1)^2+1-h^*\Delta\right]\tilde{w}=\\&=\frac{R^2}{Eh}\left(1-h^*\Delta\right)\left(\Delta+1-\nu\right)\tilde{p}.\end{aligned} \quad (1.5)$$

In this case, the form of equation (1.5) allows us to apply the expansion in series in Legendre polynomials to the solution. To solve the contact problem, we use the Green's function $G(\beta,\xi)$, which is a solution to the following equation

$$\begin{aligned}&\left[c^2(\Delta+1)^2+1-h^*\Delta\right]G(\beta,\xi)=\\&=\frac{R^2}{Eh}\left(1-h^*\Delta\right)\left(\Delta+1-\nu\right)\delta(\beta-\xi),\end{aligned} \quad (1.6)$$

where $\delta(\beta-\xi)$ Is the Dirac delta function.

We expand the required function $G(\beta,\xi)$ and $\delta(\beta-\xi)$ in series in Legendre polynomials

$$\begin{aligned}&G(\beta;\xi)=\sum_{n=0}^{\infty}G_n(\xi)P_n(\cos\beta),\\&\delta(\beta-\xi)=\sum_{n=0}^{\infty}\delta_n(\xi)P_n(\cos\beta),\\&\delta_n(\xi)=\frac{2n+1}{2}P_n(\cos\xi)\sin\xi.\end{aligned} \quad (1.7)$$

Substitution of (1.7) into (1.6) taking into account the relation $\Delta P_n(\cos\beta)=-mP_n(\cos\beta)$, $m=n(n+1)$ leads to the equation

$$\begin{aligned}&\left[c^2(1-m)^2+1+h^*m\right]G_n(\xi)=\\&=\frac{R^2}{Eh}\left(1+h^*m\right)\left(1-m-\nu\right)\delta_n(\xi).\end{aligned}$$

Where does it follow

$$\begin{aligned}&G_n(\xi)=A_nP_n(\cos\xi)\sin\xi,\\&A_n=R^2\frac{2n+1}{2Eh}\frac{\left(1+h^*m\right)\left(1-m-\nu\right)}{c^2(1-m)^2+1+h^*m}.\end{aligned} \quad (1.8)$$

Using the Green's function, we obtain an integral connection between normal displacements and the function $\tilde{p}$ [22-26]

$$\tilde{w}(\beta)=\int_0^{\pi}G(\beta,\xi)\,\tilde{p}(\xi)\,d\xi. \quad (1.9)$$

Let's expand $\tilde{p}(\beta)$ in a series in Legendre polynomials

$$\tilde{p}(\beta)=\sum_{k=0}^{\infty}\tilde{p}_kP_k(\cos\beta). \quad (1.10)$$

Substituting (1.10) into (1.9), we obtain

$$\begin{aligned}&\tilde{w}(\beta)=\\&=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}A_n\tilde{p}_kP_n(\cos\beta)c_{nk},\\&c_{nk}=\int_0^{\pi}P_n(\cos\xi)P_k(\cos\xi)\sin\xi d\xi\end{aligned} \quad (1.11)$$

Insofar as $c_{nk}=\frac{2}{2n+1}\delta_{kn}$, where δ_{kn} Is the Kronecker symbol, relation (1.11) takes the form

$$\tilde{w}(\beta)=\sum_{n=0}^{\infty}B_n\tilde{p}_nP_n(\cos\beta),$$
$$B_n=\frac{R^2}{Eh}\frac{(1+h^*m)(1-m-\nu)}{c^2(1-m)^2+1+h^*m}. \quad (1.12)$$

Let us construct a system of linear algebraic equations for the coefficients $\tilde{p}_n$... On the right-hand side of (1.12), we restrict ourselves to taking into account the first $N+1$ terms

$$\tilde{w}(\beta)\approx\sum_{n=0}^{N}B_n\tilde{p}_nP_n(\cos\beta). \quad (1.13)$$

Suppose that displacements $w(\beta)$ set in the contact area $\Omega=\{\beta:\beta\in[0,\beta^*]\}$, where $\beta^*=\arcsin r/R$ - define the boundary of the contact area.
Select in the contact area $N+1$ points with coordinates $\beta_k\in\Omega$, $k=0,1,2,...N$... Replacing the approximate equality (1.13) by the exact one, for each k-th point β_k we obtain an algebraic equation containing $N+1$ unknown $\tilde{p}_n$. Thus, since the number of such equations $N+1$, we get the system from $N+1$ equations for $N+1$ unknown

$$\mathbf{B}\tilde{\mathbf{p}}=\tilde{\mathbf{w}},$$
$$\mathbf{B}=(b_{kn})_{N+1\times N+1},\ \tilde{\mathbf{p}}=(\tilde{p}_n)_{N+1\times 1},$$
$$\tilde{\mathbf{w}}=(\tilde{w}_k)_{N+1\times 1}, \quad (1.14)$$
$$b_{kn}=B_nP_n(\cos\beta_k),$$
$$w_k=w(\beta_k),$$

the solution of which is the vector $\tilde{\mathbf{p}}$ expansion coefficients (1.10).
Desired contact pressure $p(\beta)$ can also be represented as a series expansion in Legendre polynomials

$$p(\beta)=\sum_{n=0}^{\infty}p_nP_n(\cos\beta). \quad (1.15)$$

Representation (1.4) implies a connection between the coefficients p_n and $\tilde{p}_n$

$$p_n=(2-m)\tilde{p}_n, \quad (1.16)$$

from which the first $N+1$ expansion coefficients in a series of the required contact pressure.
As an example, consider a contact problem for a shell with the following parameters: $R=1$ m, $h=1/20$ m, $\nu=0.3$, $E=2\cdot10^{11}$ Pa, $G'=0.7\cdot10^{10}$ Pa, $w_T=0.01R$.
Figure 2 shows the distribution of shell displacements in the contact zone $\Omega=\{\beta:\beta\in[0,\beta^*]\}$

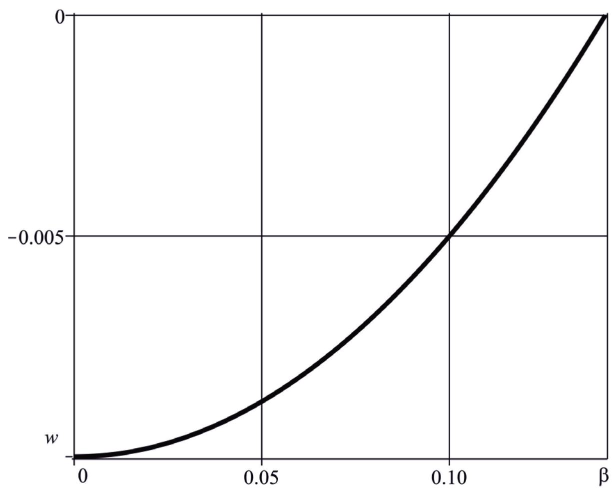

Figure 2. Distribution of displacements over the contact area

Figure 3 shows the distribution of contact pressure as a solution to problem (1.14) - (1.16). The solid curve corresponds to $N=20$, dashed line - $N=30$...
Further improvement of the solution of the contact problem based on the transient function approach [27] could require higher-order shell theories approximating the three-dimensional

stress state in irreducibility domains near contact spot boundaries (e. g. see [28-30]).

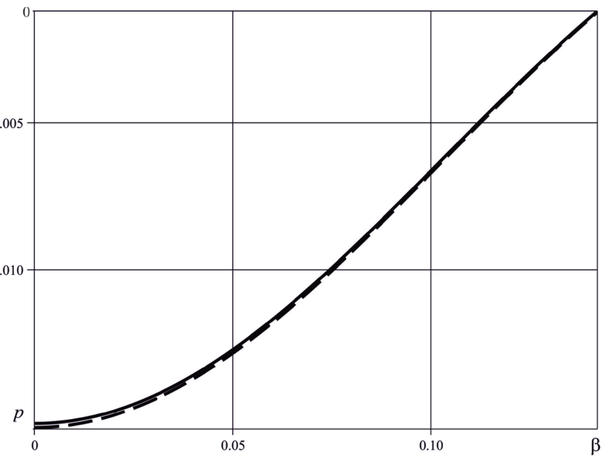

Figure 3. Contact pressure distribution

DRY FRICTION MODEL OF SHELL CONTACT WITH THE ROUGH PLANE

The dynamic interaction of a weakly deformed solid with a rough reference plane is determined by the normal reaction $\mathbf{N}$, the resulting vector of tangential forces $\mathbf{T}$, the rolling resistance moment $\mathbf{M}_\tau$ and the dry friction moment $\mathbf{M}_\nu$ [2]. These values can be determined by integrating the normal contact pressure, as well as the total tangential pressure obtained under the assumption of the validity of the differential form of the Amonton-Coulomb law for a small element of the area inside the contact spot [1-13] along the contact area S.

Taking into account the anisotropy of dry friction, the integral model of the force state inside the contact spot has the form

$$\mathbf{N} = \int_S \sigma_0 \left[\mathbf{e}_3 + \frac{\mathbf{r}_\tau \times (\mathbf{h} \cdot \mathbf{w}_\tau)}{|\mathbf{w}_\tau|} \right] dS$$

$$\mathbf{M}_\tau = \int_S \sigma_0 \mathbf{r}_\tau \times \left[\mathbf{e}_3 + \frac{\mathbf{r}_\tau \times (\mathbf{h} \cdot \mathbf{w}_\tau)}{|\mathbf{w}_\tau|} \right] dS; \qquad (2.1)$$

$$\mathbf{T} = -\int_S \sigma_0 \left[1 + \frac{\mathbf{r}_\tau \times (\mathbf{h} \cdot \mathbf{w}_\tau)}{|\mathbf{w}_\tau|} \cdot \mathbf{e}_3 \right] \frac{\mathbf{f} \cdot (\mathbf{v}_0 - R\mathbf{w}_\tau \times \mathbf{e}_3 + \mathbf{w}_\nu \times \mathbf{r}_\tau)}{|\mathbf{v}_0 + \mathbf{w}_\nu \times \mathbf{r}_\tau|} dS$$

Here $\mathbf{v}_0$ is the longitudinal absolute velocity; $\mathbf{w}_\tau$ angular rolling velocity; $\mathbf{w}_\nu$ angular spinning velocity; $R(M)$ radius of curvature of a rolling body calculated at a point M; $\mathbf{r}_\tau(M)$ radius vector of a point $M \in S$ in the contact plane; $\mathbf{e}_3$ normal unit vector of the contact plane; $\mathbf{h} = h_{\alpha\beta}\mathbf{e}^\alpha\mathbf{e}^\beta$ "rolling friction tensor" for an anisotropic elastic body:

$$\forall \mathbf{w}_\tau = \mathbf{w}_\tau(\mathbf{q}) \quad \mathbf{w}_\tau^{\mathrm{T}} \cdot \mathbf{h} \cdot \mathbf{w}_\tau > 0.$$

A detailed analysis of equations (2.1) was carried out in [2]. In particular, it was shown that in most engineering problems, it is sufficient to consider orthropic dry friction defined by the following friction tensor as it was shown in [2, 5, 10]:

$$\mathbf{f} = f \begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}, \; f \neq 0, \; \kappa \neq 0 \qquad (2.2)$$

where f and κf are the main components of the friction tensor.

It is convenient to write the proposed friction model (2.1) in a coordinate system Oxy, with the origin in the center of the contact spot, such that the corresponding basis vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ are collinear to the main directions of the friction tensor. In addition, it is natural to assume that static contact pressure has the property of axial symmetry: $\sigma_0(x,y) = \sigma_0(\pm x, \pm y)$, and rolling friction is isotropic.

An approximate analytical model of the force inside interaction the contact spot is constructed under the assumption that the rolling shell moves with longitudinal velocity $\mathbf{v}_0 = v\mathbf{e}_1'$ along the axis of the global stationary coordinate system, with angular velocity of rolling $\mathbf{w}_{\tau} = -\Omega_{r}\mathbf{e}_2$ and angular velocity of spinning ω.

It is assumed that the contact area has axial symmetry with a characteristic size of the contact spot R (for example, the diameter of the corresponding set on the plane $\{x, y\}$).

In the presence of motion, tangential stresses arise, leading to distortion of the symmetrical diagram of the distribution of normal contact stresses. Assuming that the displacement of the center of gravity of the contact spot relative to the geometric center is described by a vector $\mathbf{d}$ whose modulus was calculated in [1, 6-12], the symmetry breaking can be represented by the following formula:

$$\sigma(x, y) = \sigma_0(x, y)(1 + d_x x + d_y y)$$

where d_x and d_y are the projections of the vector $\mathbf{d}$ on the axis x and y, respectively.

The resulting friction force vector can be represented as the sum of two components: $\mathbf{T} = T_{P}\mathbf{e}_1 + T_{\perp}\mathbf{e}_2$, where T_{P} is the longitudinal abut $T_{\perp}$ is transverse components of the friction force. As it was shown in [1-12], the latter of them arises due to the relationship of friction effects.

As a result, integral representations (2.1) can be substantially simplified, as it was implemented in [1-12].

However, integral relations are too complex to apply to the analysis of the dynamics of real systems, while their approximations by analytical functions are quite accurate and simple at the same time. Using the technique described in detail in previous works [1-12], an approximate analytical model of friction describing the interaction of an elastic rolling shell with a solid surface, in the case of combined kinematics and orthotropic friction, can be presented in the following form:

$$F_{P} = \frac{F_0 v}{\sqrt{v^2 + au^2}}, \quad F_{\perp} = \frac{\mu_0 k F_0 v u^2}{\sqrt{(v^6 + b\omega^6)}}, \qquad (2.3)$$
$$M_{v} = \frac{M_0 u}{\sqrt{u^2 + mv^2}}$$

Here $u = \omega_{v} R$, F_0 is the longitudinal component of the friction force of the beginning of the motion, but M_0 is the friction torque of the beginning of the motion. The coefficients of the model (2.3) can be calculated using simple explicit formulas [2] based on numerical experiments, the results of which are presented in Fig. 2-3.

CONCLUSIONS

A formulation is proposed and a method is developed for solving the problem of the motion of a composite spherical transversely isotropic shell on a solid surface, taking into account the combined dry friction. Taking into account the assignment of the reduced contact pressure function, S.A. Ambartsumyan's modified equation was used, which allows us to apply series expansions by Legendre polynomials. Using the Green's function, the problem is reduced to solving a system of algebraic equations with respect to the coefficients of decomposition into a series of functions of the reduced contact pressure and displacements. The relationship between the true and reduced contact pressure allows us to determine the coefficients of the contact pressure decomposition series.

To describe the conditions of shell contact with the surface, the theory of multicomponent anisotropic dry friction is used, taking into account the combined kinematics of shell motion (simultaneous sliding, rotation and rolling).

The coefficients of the approximated dry friction model (2.3) can be calculated using simple explicit formulas [2] based on numerical experiments, the results of which are presented in Fig. 2-3.

ACKNOWLEDGEMENTS

A.A. Kireenkov and G.V. Fedotenkov were supported by the RFBR under the grant Nr. 20-08-01120. Investigations performed by S. I. Zhavoronok were a part of the State Task for Basic Researches Nr. 121112200124-1.

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