DOI:10.22337/2587-9618-2022-18-2-85-97

DEFINITION OF THE BEAMS FROM A NONLINEARLY DEFORMED MATERIAL BY THE RITZ-TIMOSHENKO METHODS AND FINITE DIFFERENCES TAKING INTO ACCOUNT THE DEGRADATION RIGIDITY FUNCTIONS

Vladimir P. Selyaev, Sergey Yu. Gryaznov, Delmira R. Babushkina

National Research Mordovia State University, Saransk, RUSSIA

Abstract. The article solves the problem of determining the deflections of a beam made of a nonlinearly deformable material – a polyester composite held in water using the numerical methods of Ritz-Timoshenko and finite differences. The influence of an aggressive environment on the material of the structure was taken into account by introducing the degradation function of stiffness into the calculation algorithms of the above methods. The problem of determining the time and conditions for the onset of the limiting state of the structure in the second group in accordance with the current norms and rules has been solved.

Keywords: beam, nonlinearity, deflection, Ritz-Timoshenko method, finite difference method, stiffness, degradation function, limiting state.

ОПРЕДЕЛЕНИЕ ПРОГИБОВ БАЛКИ ИЗ НЕЛИНЕЙНО ДЕФОРМИРУЕМОГО МАТЕРИАЛА МЕТОДАМИ РИТЦА-ТИМОШЕНКО И КОНЕЧНЫХ РАЗНОСТЕЙ С УЧЕТОМ ДЕГРАДАЦИОННЫХ ФУНКЦИЙ ЖЕСТКОСТИ

В.П. Селяев, С.Ю. Грязнов, Д.Р. Бабушкина

Национальный исследовательский Мордовский государственный университет, г. Саранск, РОССИЯ

Аннотация. В статье решена задача определения прогибов балки из нелинейно деформируемого материала – полиэфирного композита, выдержанного в воде, численными методами Ритца-Тимошенко и конечных разностей. Влияние агрессивной среды на материал конструкции было учтено введением деградационной функции жесткости в расчетные алгоритмы, вышеназванных методов. Решена задача по определению времени и условий наступления предельного состояния конструкции по второй группе согласно действующим нормам и правилам.

Ключевые слова: балка, нелинейность, прогиб, метод Ритца-Тимошенко, метод конечных разностей, жесткость, деградационная функция, предельное состояние.

INTRODUCTION

Construction materials and structures may be subject to the destructive effects of aggressive environments at any stage of the life cycle of the object. The probability of occurrence of some adverse events is taken into account at the design stage of structures by the introduction of conditional reserve coefficients that guarantee the impossibility of the occurrence of limit states. However, in practice, emergency situations often occur in which the structure can go into the limit state in a fairly short period of time. For example, numerous studies have proved the nonlinearity of the development of chemical degradation processes [1-8].

Consequently, in the existing mathematical models, all unreasonable coefficients should be replaced by functional dependencies that take into account the influence of complex stochastic and chronological processes [9-11]. Among the known approaches to assessing durability and reliability [12-20], the most accurate, perfect and experimentally reasonable is the method of assessing the chemical resistance of construction materials and structures using degradation functions [10, 21-24].

The purpose of the work: to determine the change in deflections of a polyester composite operating under the combined effects of mechanical loads and an aggressive environment; to determine the conditions for the onset of the limit state for group 2, using the method of degradation functions.

MATERIALS AND METHODS

It has been experimentally established [10] that the process of destruction of the material within the cross-sectional area of the element has an uneven character. The aggressive medium begins to penetrate deep into the material through weak areas, through pores, capillaries, amorphous particles. Consequently, the elastic-strength characteristics will change non-linearly along the cross-section, which can be traced on the corresponding graphs – isochrones of degradation (Fig. 1, *b*). The position of the degradation isochrones is characterized by three parameters: the coordinate of the destruction front – the depth index (*a*), the characteristic of the linearity of the degradation mechanism – (φ), and the chemical resistance coefficient of the material – (β).

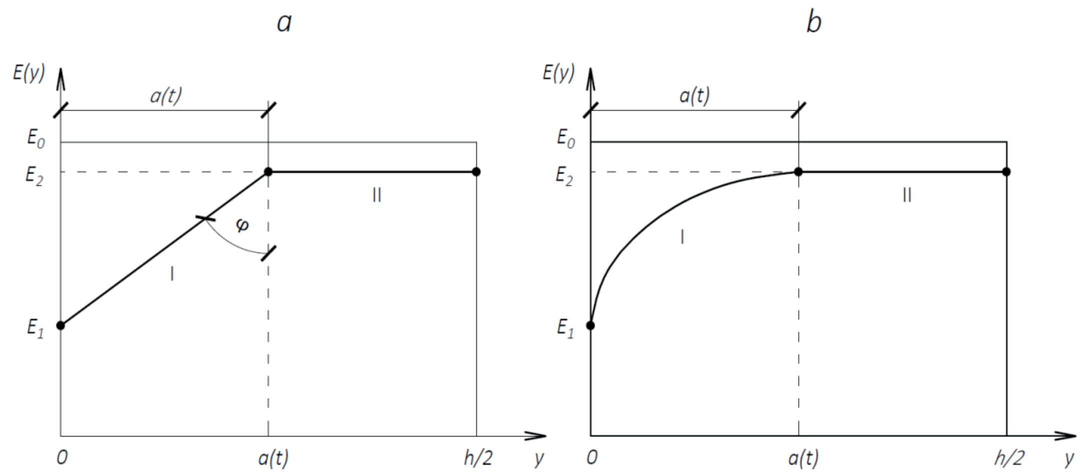

Figure 1. Basic models of deformation modulus change: a – linear, b – nonlinear

It has been experimentally established that the numerical values of the modulus of deformation and micro-hardness, determined with high accuracy by sclerometric methods, have a directly proportional relationship. Therefore, the degradation function of stiffness $F(B)$, characterize by the law of variation of the modulus of deformation over the section of the element, was determined as the basic one. It is important to note that in this work, the process of transferring an aggressive medium into the sample was studied only along the *y* axis.

Let is consider a single-span, pivotally supported beam with a length $l = 10\ m$ c with a constant cross-section $b \times h = 0{,}3 \times 0{,}4\ m$ of polyester composite (Table. 1), loaded along the entire length with a uniformly distributed load $q = 17\ kN/m$ (Fig. 2).

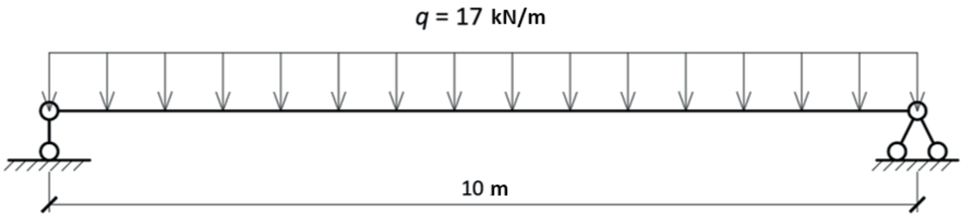

Figure 2. The design scheme of the beam

For bendable elements, the degradation stiffness function, based on a combined power approximation of the isochron of degradation, in relative coordinates has the following form:

$$F(B) = k_{\text{x.c.2}} - (k_{\text{x.c.2}} - k_{\text{x.c.1}}) \cdot \left(\frac{a}{h}\right)^3 \cdot \frac{2p}{p+3}\ , (1)$$

where is $k_{\text{x.c.1}} = \beta_1 = H_1/H_0 = E_1/E_0$ – the coefficient of chemical resistance of the material, determined by the degradation isochrones, using numerical values of the change in micro-hardness over time on the sample surface [25];

$k_{\text{x.c.2}} = \beta_2 = H_2/H_0 = E_2/E_0$ – the coefficient of chemical resistance of the material, determined at the depth a of the damaged material layer;

h – the height of the cross section of the element;

$a = k(\xi)\sqrt{Dt}$ – coordinate of the leading edge of corrosion, characterizing the depth of damage to the material (depth indicator);

$k(\xi) = 0{,}1$ – a coefficient that takes into account the instrumental accuracy of determining the coordinate$a$;

$D = 0{,}02\ cm^2/day$ – the diffusion coefficient of the aggressive medium into the material determined experimentally;

t – time of exposure to aggressive solution;

a/h – relative characteristics of the corrosion front.

In formula (1), the parameter p characterizes the type of isochron degradation, their position and shape. It can be determined experimentally from the analysis of isochron degradation, or selected from a pre-formed statistical database of compliance of materials, conditions of physico-chemical and mechanical effects, as well as the duration of the aggressive environment. Numerically, the parameter p can be equal to any positive rational number, provided the inequality is one.

For the polyester composite exposed in water, the values of elastic strength characteristics, as well as the values of degradation functions (1), were obtained at each time point under consideration t (Table 1).

To solve the problem of bending a beam from a non-linearly deformable material, an analytical function of the following form approximating the deformation diagram «σ – ε», was chosen:

$$\sigma_i = \alpha \cdot \varepsilon_i - \beta \cdot {\varepsilon_i}^5\ , \qquad (2)$$

where the constants $\alpha = E_0(t_0) \approx 3{,}16 \cdot 10^4\ MPa$, $\beta = (E_b - E_{bu})/\varepsilon_{bu}^4 \approx 5{,}12 \cdot 10^{12}\ MPa$ are determined from the condition of conformity of the approximating function (2) to the normalized indicators [26].

RESULTS AND DISCUSSION

To solve the problem of determining the deflections of the beam, 2 methods were used: the Ritz-Timoshenko method (MRI) and the finite difference method (MD). Both options were automated in Microsoft Excel 2010. To verify the software algorithms, additional calculations were performed in the Lira-CAD 2013 software package (Fig. 3).

Table 1. Elastic strength characteristics of polyester composite

Parameter	Exposure time of samples in water t, day					
	0	15	30	175	265	400
1	2	3	4	5	6	7
σ_{ue}, MPa	132.853	122.581	117.153	84.819	74.592	60.477
ε_{ue}	0.0042	0.0047	0.0051	0.005	0.005	0.0049
E_0, MPa	31631.667	26081.064	22971.176	16963.8	14918.4	12342.245
σ_{bu}, MPa	150	140.572	135.304	99.773	93.466	82.197
ε_{bu}	0.006	0.0065	0.0066	0.0076	0.0083	0.0097
E_{bu}, MPa	25000	21626.462	20500.606	13128.026	11260.964	8473.918
$k_{x.c.1}$	1	0.825	0.726	0.536	0.472	0.39
$k_{x.c.2}$	1	1	1	1	1	1
a, cm	0	5.5	7.7	18.7	23	28.3
a/h	0	0.1375	0.1925	0.4675	0.575	0.7075
$F(B)$	1	1	0.998	0.962	0.92	0.827

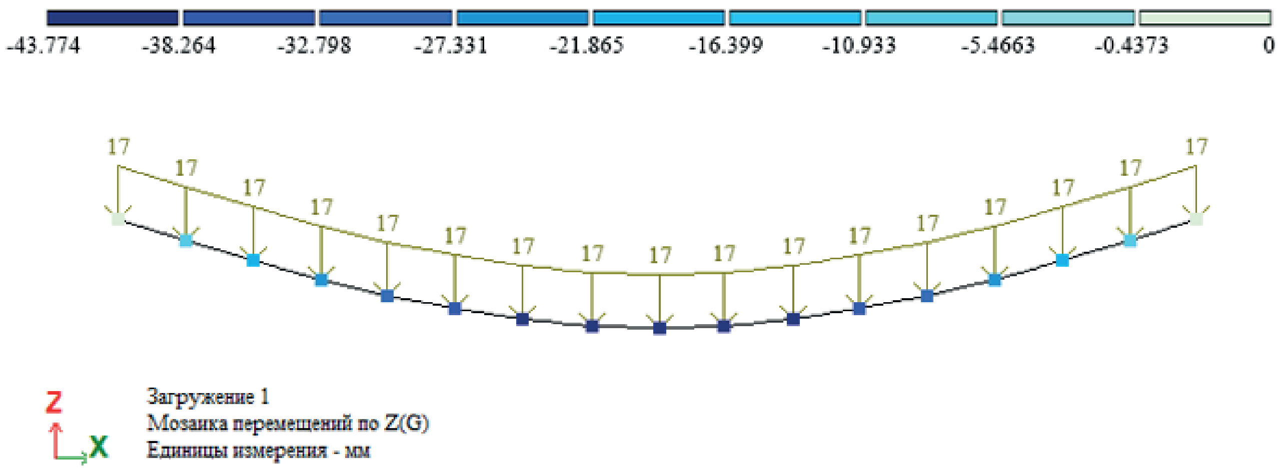

Figure 3. Linear calculation of beam deflections in the Lira-CAD 2013 PC

Table 2. Comparison of the results of linear calculations of Lira, MRI and MD at $F(B) = 1$

Cross-section coordinate x, m	Deflections w, mm				
	Lira	MRI	Δ%	MD	Δ%
0	0	0	0	0	0
0.625	8.680	8.681	0.01	8.707	0.31
1.25	16.980	16.982	0.01	17.029	0.29
1.875	24.566	24.570	0.01	24.635	0.28
2.5	31.158	31.162	0.01	31.241	0.27
3.125	36.524	36.529	0.01	36.619	0.26
3.75	40.485	40.491	0.01	40.588	0.26
4.375	42.913	42.919	0.01	43.021	0.25
5	43.730	43.737	0.01	43.840	0.25
| max |	43.730	43.737	0.01	43.840	0.31

Table 3. Comparison of the results of linear calculations of Lira, MRI and MD at $F(B) = 0{,}827$

Cross-section coordinate x, m	Deflections w, mm				
	Lira	MRI	Δ%	MD	Δ%
0	0	0	0	0	0
0.625	10.495	10.497	0.02	10.527	0.30
1.25	20.531	20.535	0.02	20.590	0.29
1.875	29.704	29.710	0.02	29.785	0.27
2.5	37.674	37.681	0.02	37.774	0.26
3.125	44.163	44.171	0.02	44.275	0.26
3.75	48.952	48.961	0.02	49.075	0.25
4.375	51.888	51.897	0.02	52.0160	0.25
5	52.876	52.886	0.02	53.007	0.25
| max |	52.876	52.886	0.02	53.007	0.30

Note to Tables 2 and 3: the beam is symmetrical relative to the middle of the span, so the results are presented in abbreviated form.

The introduction of the degradation stiffness function into the algorithm for solving the problem by the Ritz-Timoshenko method (MRI) was implemented as follows.

This method is based on the Lagrange-Dirichlet theorem on the minimum of the total potential energy of a body in equilibrium. Taking into account formula (2), the expression for the total potential energy of the beam will be written as follows:

$$V = \frac{1}{2}\alpha J_o \int_0^l \left(\frac{d^2w}{dx^2}\right)^2 dx - \frac{1}{6}\beta J_n \int_0^l \left(\frac{d^2w}{dx^2}\right)^6 dx\,, \quad (3)$$

where is $J_o = bh^3/12$ и $J_n = bh^7/448$ – the moments of inertia of the beam section (axial and higher order, respectively).

In equation (3), the multipliers αJ_o and βJ_n represent nothing else than the stiffness of the beam, linear and higher order, respectively. Multiplying them by the value $F(B)$ obtain formulas for recording the stiffness of the beam taking into account the degradation function:

$$B_o = \alpha J_o F(B)\,;\; B_n = \beta J_n F(B)\,. \quad (4)$$

The operation of an external distributed load $q(x)$ is determined by the formula:

$$A_q = \int_0^l q(x) w dx\,. \quad (5)$$

Adding (5) and (3) taking into account (4), obtain a formula for determining the total bending energy of the beam

$$Э(W) = \frac{1}{2}B_o \int_0^l (w'')^2 dx - \frac{1}{6}B_n \int_0^l (w'')^6 dx - \int_0^l q(x) w dx\,. \quad (6)$$

The deflection of a beam can be represented as a series with a finite number of terms:

$$w(x) = \sum_{n=1}^{N} K_n \varphi_n(x), \quad (n = 1, 2, \dots N), \quad (7)$$

where is K_n – the desired constants (generalize coordinates);

$\varphi_n(x)$ – approximating functions (constructed by the method of initial parameters), each of which must satisfy geometric boundary conditions.

Formula (6) from the generalize coordinates [27] will be written as:

$$Э(K) = f_1 K^2 - f_2 K^6 - f_3 K\,. \quad (8)$$

From the condition of the minimum of the total potential energy of the beam, obtain the following nonlinear algebraic equation with respect to the deflection amplitude K:

$$\frac{dЭ}{dK} = 2f_1 K - 6f_2 K^5 - f_3 = 0\,. \quad (9)$$

Here the coefficients f_1, f_2, f_3 are determined by the formulas:

$$f_1 = \frac{1}{2}B_o \int_0^l (\varphi'')^2 dx\,;\; f_2 = \frac{1}{6}B_n \int_0^l (\varphi'')^6 dx\,;\; f_3 = \int_0^l q(x)\varphi dx\,. \quad (10)$$

The final resolving equation will be written as:

$$a \cdot K^5 + b \cdot K + c = 0\,. \quad (11)$$

It follows from formula (7) that in order to determine the deflections of the beam, it is necessary to find at least one real root of equation (11), at which the identical equality of this equation is fulfilled. However, this equation is not solved in radicals, i.e. there are no formulas that would make it possible to calculate the roots by coefficients. This was first proved by the Norwegian mathematician Nils Abel. However, the roots of the 5th degree equation can be found with any predetermined accuracy using numerical methods. In this case, the actual roots were calculated with $1 \cdot 10^{-5}$ precision. Thus, the maximum deflection of the beam, determined by the above method (MRI), taking into account the degradation function $F(B) = 0{,}827$ was $w_{max} = 52{,}888\, mm$. At the same time, the root of equation (11) was $K \approx 1{,}69 \cdot 10^{-5}$.

To solve this problem by the finite difference method (MD), the following algorithm for introducing the degradation function of stiffness into the calculation is proposed.

The basic differential equation of bending of a beam made of a nonlinear elastic material has the form:

$$J_c(w)\frac{d^4w}{dx^4} + 2\frac{dJ_c(w)}{dx}\cdot\frac{d^3w}{dx^3} + \frac{d^2J_c(w)}{dx^2}\cdot\frac{d^2w}{dx^2} = q(x)\,. \qquad (12)$$

Equation (12) includes both the stiffness variable along the length of the beam and its derivatives. In this case, they can be calculated only numerically using finite-difference approximation formulas [28].

After a series of transformations, equation (12), written in finite-difference form, can be transformed relative to the deflection value w_i by the following formula:

$$w_i = \frac{q_i^* - w_{i-2}\cdot A_i - w_{i-1}\cdot B_i - w_{i+1}\cdot E_i - w_{i+2}\cdot F_i}{C_i}\,, \qquad (13)$$

where the variable coefficients depending on the stiffness will be recalculated at each new iteration stage according to the formulas:

$$A_i = \frac{J_{ci}}{\Delta x^4} - \frac{J'_{ci}}{\Delta x^3}\,;\, B_i = -4\frac{J_{ci}}{\Delta x^4} + 2\frac{J'_{ci}}{\Delta x^3} + \frac{J''_{ci}}{\Delta x^2}\,;\, C_i = -6\frac{J_{ci}}{\Delta x^4} - 2\frac{J''_{ci}}{\Delta x^2}\,;$$
$$E_i = -4\frac{J_{ci}}{\Delta x^4} - 2\frac{J'_{ci}}{\Delta x^3} + \frac{J''_{ci}}{\Delta x^2};\, F_i = \frac{J_{ci}}{\Delta x^4} + \frac{J'_{ci}}{\Delta x^3}\,. \qquad (14)$$

Consequently, the smaller the step of dividing the beam lengthwise into finite elements Δx, the smaller the error value of the finite-difference approximation. Thus, to take into account the stiffness degradation process, it is proposed to multiply formulas (14) by the value of the degradation function $F(B)$.

Then the main equation (13) will be written as:

$$w_i = \frac{q_i^* - (w_{i-2}\cdot A_i - w_{i-1}\cdot B_i - w_{i+1}\cdot E_i - w_{i+2}\cdot F_i)\cdot F(B)}{C_i\cdot F(B)}\,. \qquad (15)$$

Solving the system of finite-difference equations (15) with respect to deflections w_i in each section of the beam under consideration (we take $\Delta x = 0{,}625\, m$), we determine the deflections along the entire length, taking into account the degradation function.

The boundary conditions in the finite-difference form when the beam length is split (hinted along the edges, Fig. 4) into n = 16 parts will be determined by the formulas [28]:

$$w_a = w_b = 0\,;\, w_{a-1} + w_{a+1} = 0\,;$$
$$w_{a-2} + w_{a+2} = 0\,;\, w_{b-1} + w_{b+1} = 0\,;$$
$$w_{b-2} + w_{b+2} = 0\,. \qquad (16)$$

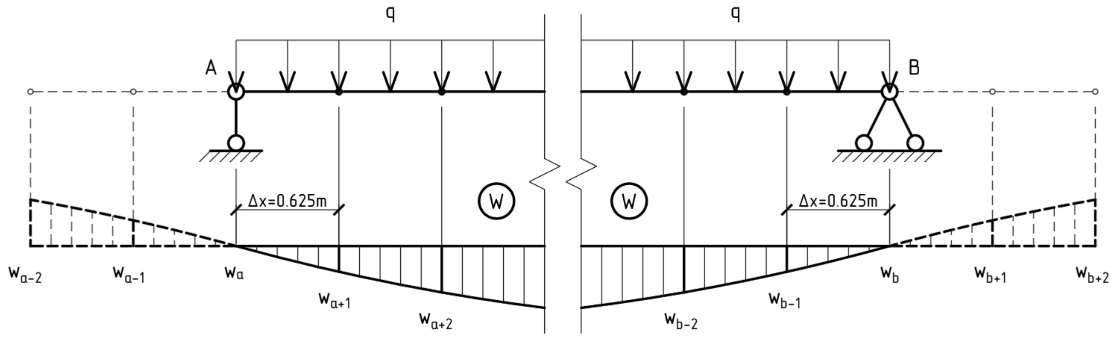

Figure 4. Accounting for boundary conditions in a finite difference scheme

The solution of a system of linear algebraic equations involves an iterative calculation process. This task was solved using the built-in iterative processor in Microsoft Excel 2010. The maximum number of iterations was 32767 with the accuracy of the calculation $1 \cdot 10^{-6}$. Thus, the maximum deflection of the beam, determined by the above method (MKR), taking into account the degradation function $F(B) = 0{,}827$ was $w_{max} = 53{,}013\ mm$.

The discrepancies in the values with the previous calculations turned out to be insignificant, however, it is important to note that the beam was loaded by 16,91% of the destructive load, as evidenced by the magnitude of the relative deformations in the middle of the span $\varepsilon = 0{,}00101$, while the limit deformations for this material $\varepsilon_{bu} = 0{,}006$. That is, the beam material under such a load works linearly elastic.

According to clause 15.1.1 of SP 20.13330.2016, when calculating building structures for the second group of limit states, the condition must be met:

$$w \leq w_u\,, \qquad (17)$$

where is w – the deflection and displacement of the structural element (or the structure as a whole), determined taking into account the factors affecting their values;

w_u – the maximum deflection or displacement established by the norms. For a beam with a span of 10 m, the maximum deflection is 0,0478 m.

In order to predict the moment of the onset of the limit state for the 2 group of limit states, as a result of the influence of an aggressive environment, taken into account with the help of the degradation function, the formula can be used:

$$w_{F(B)} = \frac{w_0}{F(B)}\,, \qquad (18)$$

where is w_0 – the initial deflection (before the start of the aggressive environment).

Therefore, solving the inverse problem, it is possible to determine the critical value of the degradation function at which the limiting state occurs:

$$F(B)_{cr} = \frac{w_0}{w_u} = \frac{0{,}0437}{0{,}0478} = 0{,}914\,. \qquad (19)$$

The graph of the change in the degradation function $F(B)$ over time (Fig. 5) can be approximated with a high degree of accuracy by higher-order polynomial dependencies, however, for this particular case, the accuracy of the approximation turns out to be high already when using the quadratic equation.

Substituting the limiting value of the degradation function (19) into the quadratic equation shown in the graph (Fig. 5) instead of the value y, obtain an expression for determining the approximate moment of time x of the onset of the limiting state:

$$y = 9{,}5712 \cdot 10^{-7} \cdot x^2 - 5{,}0601 \cdot 10^{-5} \cdot x + 8{,}6421 \cdot 10^{-2} = 0\,. \qquad (20)$$

Solving the quadratic equation (20) obtain $x \approx 275\ day$ (Fig. 5).

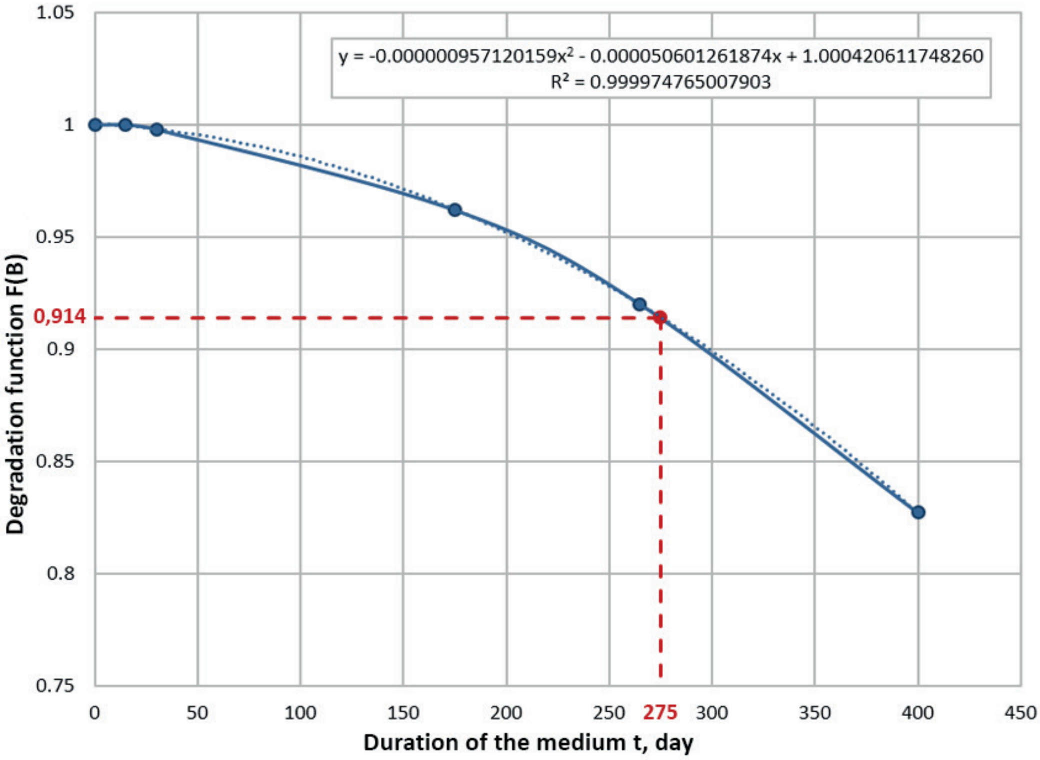

Figure 5. Dependence of the degradation function $F(B)$ on the duration of the medium t, day for the conditions according to Table 1

CONCLUSIONS

Thus, based on the above, the following conclusions can be made:

1. In the work, numerical methods of Ritz-Timoshenko and finite differences were used to determine the deflections of a polyester composite beam. The control calculation of the design in question was performed in the Lira-CAD 2013 PC, thereby confirming the correctness of the proposed automation algorithms for MRI and MD in the Microsoft Excel 2010 program.
2. In the calculation formulas of the methods under consideration, an additional mathematical dependence was introduced, which is a degradation function of the stiffness of the bent element. Thanks to this, it was possible to determine the deflections of the beam taking into account the aggressive effects of the environment, in this particular case – water.
3. It was found that the limit state for the second group, for a polyester composite beam, taking into account a given uniformly distributed load, occurs already at the linear elastic stage of the material. Thus, it can be concluded that the geometric parameters of the bent element under consideration are not optimal, therefore, the solution of the optimization problem is required.
4. In the work, the value of the limiting degradation function of stiffness was determined, at which the limit state for the structure occurs in the second group.
5. The construction of a graphical interpretation of the dependence of the degradation function of stiffness on the duration of the aggressive medium, followed by approximation by polynomial dependencies, allowed us to determine with a sufficient degree of accuracy the moment of the onset of the limiting state of the structure for the second group.

REFERENCES

1. **Selyaev V.P., Selyaev P.V., Sorokin E.V., Kechutkina E.L.** Forecasting the durability of reinforced concrete bending elements by the method of degradation functions. // Housing construction Vol. 12, 2014, pp. 8-12.
2. **Selyaev V.P., Selyaev P.V., Sorokin E.V., Kechutkina E.L.** Evaluation of the strength of structural elements made of cement composites exposed to the action of an aggressive environment. // Bulletin of the Volga State Technological University. Ser.: Materials. Constructions. Technologies Vol. 1(5), 2018, pp. 58-64.
3. **Selyaev V.P., Selyaev P.V., Sorokin E.V., Kechutkina E.L.** Assessment of the influence of chemically active aggressive media on the degradation of composites. // Durability of building materials, products and structures, 2018, pp. 170-174.
4. **Yao Y., Wang Z., Wang L.** Durability of concrete under combined mechanical load and environmental actions: A review. // Journal of Sustainable Cement-Based Materials Vol. 1(2), 2012, pp. 2-15.
5. **Vorechovska D., Somodikova M., Podrouzek J., Lehky D., Teply B.** Concrete structures under combined mechanical and environmental actions: Modelling of durability and reliability. // Computers and Concrete Vol. 1, 2017, pp. 99-110.
6. **Wittmann F.H., Zhao T., Jiang F., Wan X.** Influence of combined actions on durability and service life of reinforced concrete structures exposed to aggressive environment. // Restoration of Buildings and Monuments Vol. 2, 2012, pp. 105-112.
7. **Glasser F.P., Marchand J., Samson E.** Durability of concrete – Degradation phenomena involving detrimental chemical reactions. // Cement and Concrete Research Vol. 2, 2008, pp. 226-246.
8. **Basheer L., Kropp J., Cleland D.J.** Assessment of the durability of concrete from its permeation properties: a review. // Construction and building materials Vols. 2-3, 2001, pp. 93-103.
9. **Rzhanitsyn A.R.** Teoriya rascheta stroitel'nykh konstruktsiy na nadezhnost' [The theory of calculating building structures for reliability]. Moscow, Stroyizdat, 1978, 239 pages (in Russian).
10. **Solomatov V.I., Selyaev V.P., Sokolova Yu.A.** Khimicheskoye soprotivleniye materialov [Chemical resistance of materials]. Moscow, RAASN, 2001, 284 pages (in Russian).
11. **Karpenko N.I., Karpenko S.N., Yarmakovsky V.N., Erofeev V.T.** On modern methods of ensuring the durability of reinforced concrete structures. // Academia. Architecture and construction Vol. 1, 2015, pp. 93-102.
12. **Pujonto L.M.** Dolgovechnost' zhelezobetonnykh konstruktsiy sooruzheniy [Durability of reinforced concrete structures of structures]. Moscow, ASV, 2004, 423 pages (in Russian).
13. **Stepanova V.F., Falikman V.R.** Problems of ensuring the durability of concrete and reinforced concrete structures today. // Second Polakov Readings: Collection of scientific articles based on the materials of the international scientific and technical conference dedicated to the 105th anniversary of the birth of prof. Alexei Filippovich Polak, 2017, pp. 97-111.
14. **Stepanova V.F., Falikman V.R., Koroleva E.N.** Monitoring and analysis of regulatory documents in the field of design of reinforced concrete structures according to their life cycle. // Construction Materials Vol. 7, 2018, pp. 14-19.
15. **Bolotin V.V.** Metody teorii veroyatnostey i teorii nadezhnosti v raschetakh sooruzheniy [Methods of the theory of probability and the theory of reliability in the calculations of structures]. Moscow, Stroyizdat, 1982, 351 pages (in Russian).
16. **Selyaev V.P., Selyaev P.V., Petrov I.S.** Probabilistic methods for assessing the durability of reinforced concrete bending ele-

ments. // Academia. Architecture and construction Vol. 3, 2009, pp. 87-90.
17. **Solomatov V.I., Selyaev V.P.** Chemical resistance of concrete. // Concrete and reinforced concrete Vol. 8, 1984, pp. 16-17.
18. **Petrov V.V., Ovchinnikov I.G., Shikhov Yu.M.** Raschet elementov konstruktsiy, vzaimodeystvuyushchikh s agressivnoy sredoy [Calculation of structural elements interacting with an aggressive environment]. Saratov, Izdatel'stvo Saratovskogo universiteta, 1987, 288 pages (in Russian).
19. **Tuutti K.** Korroziya stali v betone. Shvedskiy institut issledovaniy tsementa i betona [Corrosion of steel in concrete]. Stockholm, Shvedskiy institut issledovaniy tsementa i betona, 1982, 469 pages (in Russian).
20. **Bazant Z.** Physical model for steel corrosion in concrete sea structures theory. // Journal of the Structural Division Vol. 105 (ST6), 1979, pp. 1137-1153.
21. **Zhuravleva V.N., Selyaev V.P., Solomatov V.I.** Calculation of flexible polymer concrete elements interacting with aggressive media. // Technology and mechanization of waterproofing works of industrial, civil and energy structures, 1983, pp. 78-80.
22. **Zhuravleva V.N., Selyaev V.P., Solomatov V.I.** Experimental method for determining degradation functions for polymer concrete. // Improving the durability of concrete transport structures, 1980, pp. 86-95.
23. **Selyaev V.P.** Osnovy teorii rascheta kompozitsionnykh konstruktsiy s uchetom deystviya agres-sivnykh sred: Avtoreferat dissertatsii doktora tekhnicheskikh nauk [Fundamentals of the theory of calculation of composite structures taking into account the action of aggressive area: Abstract of the dissertation of Doctor of Technical Sciences]. Moscow, 1984, 36 pages (in Russian).
24. **Zhuravleva V.N., Selyaev V.P., Solomatov V.I.** Degradation models for structural polymer concrete. // Improving the durability of concrete of transport structures, 1982, pp. 27-31.
25. **Selyaev V.P., Selyaev P.V.** Fiziko-khimicheskiye osnovy mekhaniki razrusheniya tsementnykh kompozitov: monografiya [Physicochemical foundations of fracture mechanics of cement composites: monograph]. Saransk, Izdatel'stvo Mordovskogo universiteta, 2018, 220 pages (in Russian).
26. **Selyaev V.P., Selyaev P.V., Gryaznov S.Yu., Babushkina D.R.** Verification of dependencies approximating the deformation diagrams of cement and polymer concretes by the method of standardized indicators. // Construction and reconstruction Vol. 1(93), 2021, pp. 125-133.
27. **Selyaev V.P., Utkina V.N., Gryaznov S.Yu., Babushkina D.R.** Determination of deflections of a beam from a nonlinear elastic material by the Ritz-Timoshenko method when approximating deformation diagrams by combined power dependences. // Expert: theory and practice Vol. 2(11), 2021, pp. 42-50.
28. **Petrov V.V., Krivoshein I.V.** Metody rascheta konstruktsiy iz nelineyno-deformiruyemogo materiala [Methods for calculating structures made of nonlinearly deformable material]. Moscow, ASV, 2008, 208 pages (in Russian).

СПИСОК ЛИТЕРАТУРЫ

1. **Селяев В.П., Селяев П.В., Сорокин Е.В., Кечуткина Е.Л.** Прогнозирование долговечности железобетонных изгибаемых элементов методом деградационных функций. // Жилищное строительство, 2014, №12, с. 8-12.
2. **Селяев В.П., Селяев П.В., Сорокин Е.В., Кечуткина Е.Л.** Оценка прочности элементов конструкций из цементных композитов, подверженных действию агрессивной среды. // Вестник Поволжского государственного технологического университета. Сер.: Материалы. Кон-

струкции. Технологии, 2018, № 1(5), с. 58-64.
3. **Селяев В.П., Селяев П.В., Сорокин Е.В., Кечуткина Е.Л.** Оценка влияния химически активных агрессивных сред на процесс деградации композитов. // Долговечность строительных материалов, изделий и конструкций, 2018, с. 170-174.
4. **Yao Y., Wang Z., Wang L.** Durability of concrete under combined mechanical load and environmental actions: A review. // Journal of Sustainable Cement-Based Materials, 2012, № 1(2), pp. 2-15.
5. **Vorechovska D., Somodikova M., Podrouzek J., Lehky D., Teply B.** Concrete structures under combined mechanical and environmental actions: Modelling of durability and reliability. // Computers and Concrete, 2017, № 1, pp. 99-110.
6. **Wittmann F.H., Zhao T., Jiang F., Wan X.** Influence of combined actions on durability and service life of reinforced concrete structures exposed to aggressive environment. // Restoration of Buildings and Monuments, 2012, № 2, pp. 105-112.
7. **Glasser F.P., Marchand J., Samson E.** Durability of concrete – Degradation phenomena involving detrimental chemical reactions. // Cement and Concrete Research, 2008, № 2, pp. 226-246.
8. **Basheer L., Kropp J., Cleland D.J.** Assessment of the durability of concrete from its permeation properties: a review. // Construction and building materials, 2001, № 2-3, pp. 93-103.
9. **Ржаницын А.Р.** Теория расчета строительных конструкций на надежность. – М.: Стройиздат, 1978. – 239 с.
10. **Соломатов В.И., Селяев В.П., Соколова Ю.А.** Химическое сопротивление материалов, 2-е изд., перераб. и дополн. – М.: РААСН, 2001. – 284 с.
11. **Карпенко Н.И., Карпенко С.Н., Ярмаковский В.Н., Ерофеев В.Т.** О современных методах обеспечения долговечности железобетонных конструкций. // Academia. Архитектура и строительство, 2015, № 1, с. 93-102.
12. **Пухонто Л.М.** Долговечность железобетонных конструкций сооружений. – М.: АСВ, 2004. – 423 с.
13. **Степанова В.Ф., Фаликман В.Р.** Проблемы обеспечения долговечности бетонных и железобетонных конструкций сегодня. // Вторые Полаковские чтения: Сб. науч. статей по материалам международной науч.-тех. конференции, посвященной 105-летию со дня рождения проф. Алексея Филипповича Полака, 2017, с. 97-111.
14. **Степанова В.Ф., Фаликман В.Р., Королева Е.Н.** Мониторинг и анализ нормативных документов в области проектирования железобетонных конструкций по их жизненному циклу. // Строительные материалы, 2018, № 7, с. 14-19.
15. **Болотин В.В.** Методы теории вероятностей и теории надежности в расчетах сооружений. – М.: Стройиздат, 1982. – 351 с.
16. **Селяев В.П., Селяев П.В., Петров И.С.** Вероятностные методы оценки долговечности железобетонных изгибаемых элементов. // Academia. Архитектура и строительство, 2009, № 3, с. 87-90.
17. **Соломатов В.И., Селяев В.П.** Химическое сопротивление бетонов. // Бетон и железобетон, 1984, № 8, с. 16-17.
18. **Петров В.В. Овчинников И.Г., Шихов Ю.М.** Расчет элементов конструкций, взаимодействующих с агрессивной средой. – Саратов: Сарат. ун-т, 1987. – 288 с.
19. **Tuutti K.** Corrosion of stel in concrete. Swedish Cement and Concrete Research Inst. – Stockholm, 1982. – 469 p.
20. **Bazant Z.** Physical model for steel corrosion in concrete sea structures theory. // Journal of the Structural Division, 1979, № 105 (ST6), pp. 1137-1153.
21. **Журавлева В.Н., Селяев В.П., Соломатов В.И.** Расчет изгибаемых полимербетонных элементов, взаимодействующих с агрессивными средами. // Технология и механизация гидроизоляцион-

ных работ промышленных, гражданских и энергетических сооружений, 1983, с. 78-80.
22. **Журавлева В.Н., Селяев В.П., Соломатов В.И.** Экспериментальный метод определения деградационных функций для полимербетонов. // Повышение долговечности бетона транспортных сооружений, 1980, с. 86-95.
23. **Селяев В.П.** Основы теории расчета композиционных конструкций с учетом действия агрессивных сред: Автореф. дисс. д-ра техн. наук. – М., 1984. – 36 с.
24. **Журавлева В.Н., Селяев В.П., Соломатов В.И.** Модели деградации конструкционных полимербетонов. // Повышение долговечности бетонов транспортных сооружений, 1982, с. 27-31.
25. **Селяев В.П., Селяев П.В.** Физико-химические основы механики разрушения цементных композитов: монография. – Саранск: Изд-во Мордов. ун-та, 2018. – 220 с.
26. **Селяев В.П., Селяев П.В., Грязнов С.Ю., Бабушкина Д.Р.** Верификация зависимостей, аппроксимирующих диаграммы деформирования бетонов цементного и полимерного методом нормируемых показателей. // Строительство и реконструкция, 2021, № 1(93), с. 125-133.
27. **Селяев В.П., Уткина В.Н., Грязнов С.Ю., Бабушкина Д.Р.** Определение прогибов балки из нелинейно-упругого материала методом Ритца-Тимошенко при аппроксимации диаграмм деформирования комбинированными степенными зависимостями. // Эксперт: теория и практика, 2021, № 2(11), с. 42-50.
28. **Петров В.В., Кривошеин И.В.** Методы расчета конструкций из нелинейно-деформируемого материала. – М.: АСВ, 2008. – 208 с.

Vladimir P. Selyaev, Academician of the Russian Academy of Architectural and Construction Sciences, Professor, Doctor of Technical Sciences, National Research Ogarev Mordovian State University, Head of the Department of Building Structures; 430006, Russia, Saransk, Bolshevistskaya St., 68; phone: +7 (8342) 47-71-56, e-mail: ntorm80@mail.ru.

Sergey Yu. Gryaznov, Postgraduate student of the Department of Building Structures, National Research Ogarev Mordovian State University; 430006, Russia, Saransk, Bolshevistskaya St., 68; e-mail: sergey.gryaznov.97@mail.ru.

Delmira R. Babushkina, Postgraduate student of the Department of Building Structures, National Research Ogarev Mordovian State University; 430006, Russia, Saransk, Bolshevistskaya St., 68; e-mail: delmira2009@yandex.ru.

Селяев Владимир Павлович, Академик РААСН, профессор, доктор технических наук, Национальный исследовательский Мордовский государственный университет им. Н.П. Огарёва, заведующий кафедрой строительных конструкций; 430006, Россия, г. Саранск, ул. Большевистская, д. 68; тел.: +7 (8342) 47-71-56, e-mail: ntorm80@mail.ru.

Грязнов Сергей Юрьевич, аспирант кафедры строительных конструкций, Национальный исследовательский Мордовский государственный университет им. Н.П. Огарёва; 430006, Россия, г. Саранск, ул. Большевистская, д. 68; e-mail: sergey.gryaznov.97@mail.ru.

Бабушкина Дельмира Рафиковна, аспирант кафедры строительных конструкций, Национальный исследовательский Мордовский государственный университет им. Н.П. Огарёва; 430006, Россия, г. Саранск, ул. Большевистская, д. 68; e-mail: delmira2009@yandex.ru.