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DERIVATION OF THE EQUATION OF UNSTEADY-STATE MOISTURE BEHAVIOUR IN THE ENCLOSING STRUCTURES OF BUILDINGS USING A DISCRETE-CONTINUOUS APPROACH

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Abstract: Two differential equations of moisture transfer based on the theory of moisture potential have been considered. The first equation includes the record of moisture transfer mechanisms of vapor and liquid phases and their relationship. The second equation is a simplified form of the first equation which makes it possible to apply a discrete-continuous approach. The peculiar properties of the boundary conditions setting of the outside air for temperature and humidity fields have been presented. It is proved that the use of the discrete-continuous method provides high accuracy of calculations and can be used in engineering practice to assess the unsteady humidity regime of enclosing structures.

Keywords: moisture regime, discrete-continuous approach, moisture transfer equation.

ВЫВОД УРАВНЕНИЯ НЕСТАЦИОНАРНОГО ВЛАГОПЕРЕНОСА В ОГРАЖДАЮЩИХ КОНСТРУКЦИЯХ ЗДАНИЙ С ПРИМЕНЕНИЕМ ДИСКРЕТНО-КОНТИНУАЛЬНОГО ПОДХОДА

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Аннотация: Рассмотрено два дифференциальных уравнения влагопереноса, основанных на теории потенциала влажности. Первое уравнение включает учет механизмов влагопереноса парообразной и жидкой фазы и их связь между друг другом. Второе уравнение представляет собой упрощенный вид первого уравнения специально для возможности применения дискретно-континуального подхода. Представлены особенности задания граничных условий наружного воздуха для температурного и влажностного полей. Доказано, что применение дискретно-континуального метода дает высокую точность вычислений и может использоваться в инженерной практике для оценки нестационарного влажностного режима ограждающих конструкций.

Ключевые слова: влажностный режим, дискретно-континуальный подход, уравнение влагопереноса.

1. INTRODUCTION

Heat and moisture transfer in enclosing structures is one of the most complicated and pending issues in construction [1,2]. The main feature of the problem mentioned is that the processes of moisture transfer are non-stationary [3]. The assessment of the moisture state of building materials by stationary methods can lead to serious errors in the design of buildings and structures [4].

Experimental studies of heat and mass transfer problems are constantly being carried out by scientists from all over the world [5,6] in various branches of technology. As an example, we can mention the problem of drying wood. In construction, new architectural and designer solutions have a great influence on the choice of enclosing structures [7,8].

The humidity regime affects energy saving, thermal protection of buildings, as well as the durability of the enclosing structures [9,10]. The moisture regime assessment is accentuated by the fact that moisture can be in different relationship with the skeleton of the building material and different aggregate states.

The earliest notions about the calculation of building enclosure moisture state were based on the differential equation for the water vapor transfer [11]. Subsequently, it was found that water vapor is not the only moisture transfer potential. In the pores of building materials, some processes, such as capillary fluid flow, moisture transfer under the temperature and air filtration, and the mutual influence of temperature and humidity fields occur. [12]

In after years, the researchers worked with a system of differential equations describing various transfer potentials. However, back in the XX century V.N. Bogoslovsky created the function of the moisture potential that makes it possible to replace several separate transfer potentials with one single moisture potential, which greatly simplified the work with mathematical models of the moisture regime. At present, a large number of moisture or mass transfer potentials have been developed, for instance, the Bogoslovsky potential, the Lykov potential, the L. Pel potential, the H.M. Kunzel, etc [13].

There are diverse regulations governing the assessment of the moisture state of building envelopes all over the world. In Europe it is customary to assess the humidity regime using the WUFI computer program. In the Russian Federation the Set of Rules 50.13330.2012 “Thermal protection of buildings”, which also contains a section called “Protection against water logging of enclosing structures”, is applied. This regulatory document is based on the theory of moisture potential by V.G. Gagarin and V.V. Kozlov [14]. According to the Set of Rules 50.13330.2012 "Thermal protection of buildings", the problem of stationary moisture transfer under the moisture potential *F*, which uniformly takes into account the movement of vaporous and liquid moisture, is considered [15]:

$$\frac{\partial}{\partial x}(\mu \frac{\partial F(w,t)}{\partial x}) = 0. \quad (1)$$

where F – moisture potential, *Pa*; μ – vapor permeability coefficient, *kg/(m·s·Pa)*; w – material moisture , % by weight (1 *kg/kg* = 100 % by weight), t – temperature,°*C*; x – coordinate, *m*.

Nevertheless, creating formulas and methods that allow us to assess the unsteady humidity regime without applying numerical methods remains relevant.

In this regard, the most promising areas of research are discrete-continuous calculation methods, which are used in various construction problems [16,17].

The closest to moisture transfer is the heat transfer problem.

In a discrete-continuous formulation, the heat equation was studied by Zolotov A.B., Akimov P.A., Sidorov V.N., Mozgaleva M.L. and S.M. Matskevich [18-22].

2. THE PROBLEM

Derive the equation of unsteady moisture transfer using the discrete-continuous approach and evaluate its efficiency in comparison with the finite difference method.

3. MATERIALS AND METHODS

In earlier studies, a differential moisture transfer equation was derived based on the moisture potential *F* [15]:

$$\frac{\partial F(w,t)}{\partial \tau} = \kappa_F(w,t) \cdot E_t(t) \cdot \frac{\partial^2 F(w,t)}{\partial x^2}. \quad (2)$$

where E_t – saturated water vapor pressure, *Pa*; τ – time, *s*; *x* – coordinate, *m*, κ_F – material heat-humidity characteristic coefficient, $m^2/(s \cdot Pa)$.

However, it is impossible to apply a discrete-continuous method to the equation (2), since the coefficient κ_F depends on the mass moisture content of the material and temperature. This means that in discretization of the space-time domain the coefficient κ_F will have a different value at any point in space, as well as at any time.

To simplify the equation (2), it is proposed to replace the material heat-humidity characteristic coefficient κ_F with the average material heat-humidity characteristic coefficient κ_{F0}.

Thus, the equation (2) will be represented as:

$$\frac{\partial F(w,t)}{\partial \tau} = \kappa_F \cdot E_t(t) \cdot \frac{\partial^2 F(w,t)}{\partial x^2}. \quad (3)$$

Since moisture transfer processes in the building materials are by four orders of magnitude slower than heat transfer processes, it was proposed that the outdoor air temperature outside the structure will be constant for a month (Figure 1) and the humidity potential outside the building enclosure will have the form of a piecewise linear function (Figure 2).

Inside the structure, both the temperature and the humidity potentials were taken constant throughout the year.

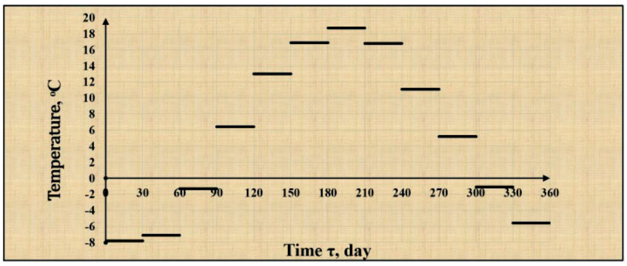

Figure 1. Boundary conditions for the temperature field outside the structure for Moscow (Russian Federation)

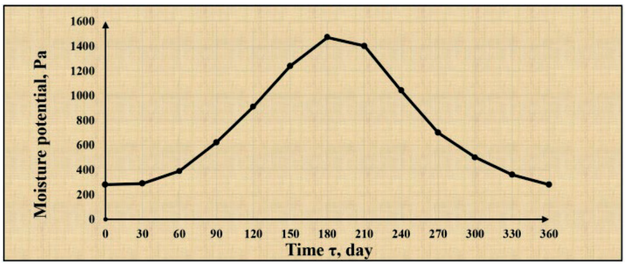

Figure 2. Boundary conditions for the field of moisture potential outside the structure for Moscow (Russian Federation)

Thus, the saturated water vapor pressure in the equation (3) will depend on the coordinate but not on the time during the calculated month.

Setting the boundary conditions of moisture exchange of the third kind for the equation (3) and applying the discrete-continuous approach, we obtain a system of equations in matrix form which represents the Cauchy problem and has an analytical solution in matrices:

$$\begin{cases} \overline{F}'_\tau = E_t \cdot A \cdot \overline{F} + \overline{S} \\ \overline{F}(0) = \overline{\chi}, 0 \le x \le l \end{cases} . \quad (4)$$

where E_t – a matrix of the saturated water vapour pressure; A – a matrix of coefficients; $\overline{F}$ – a matrix of initial distribution of moisture potential; $\overline{S}$ – a matrix of boundary conditions; $\overline{F}'_\tau$ – a matrix of derivatives to the moisture potential in different sections of the considered area.

The solution of the Cauchy problem (4) can written as the equation:

$$\overline{F} = \int_0^\tau (e^{E_t \cdot A \cdot (\tau - \sigma)} \cdot \overline{S}(\sigma) \cdot d\sigma) + e^{E_t \cdot A \cdot \tau} \cdot \overline{F_0}. \quad (5)$$

The integral in the equation (5) will depend on the function of the boundary conditions.

To take the integral, we represent the matrix of the boundary conditions as the following an expression:

$$\overline{S} = p \cdot \tau \cdot \overline{L} + \overline{B}. \quad (6)$$

where p – the coefficient of the external boundary condition for a building enclosing structure, Pa/s^2; $\overline{B}$ – a column vector, the first and last elements of which describe the boundary conditions on the outer and inner surfaces of the enclosing structure, other elements are equal to 0 for a multi-layer enclosing structure; $\overline{L}$ – a column vector, the first element of which is equal to one, other elements are equal to 0 for a building enclosing structure.

Substituting (6) into (5), we can obtain the final expression for the moisture potential using the discrete-continuous approach:

$$\overline{F} = p\cdot\left(\left(E_t\cdot A\right)^{-2}\cdot e^{E_t\cdot A\cdot\tau} - \tau\cdot\left(E_t\cdot A\right)^{-1} - \left(E_t\cdot A\right)^{-2}\right)\cdot\cdot$$

$$\cdot\overline{L} + \left(E_t\cdot A\right)^{-1}\left(e^{E_t\cdot A\cdot\tau} - E\right)\cdot\overline{B} + e^{E_t\cdot A\cdot\tau}\cdot\overline{F}_0. \qquad (7)$$

As the equation (7) is a numerical-analytical formula and it does not require numerical calculations, it can be used in engineering calculation methods.

4. RESULTS AND DISCUSSION

Replacing the coefficient $\kappa_F(w,t)$ in the equation (2) with the coefficient κ_F in the equation (3) leads to the simplification of the mathematical model of heat-moisture transfer. To prove the effectiveness of the proposed new discrete-continuous formula (7), the results of the non-stationary moisture regime calculations using the finite difference method according to an explicit difference scheme by the equation (2) and by the analytical expression (7) were compared.

A comparison of moisture distribution in the enclosing structure of aerated concrete in Moscow (the Russian Federation) made by the thickness of the enclosing structure (Figure 3) and by the time throughout the year is demonstrated (Figure 4).

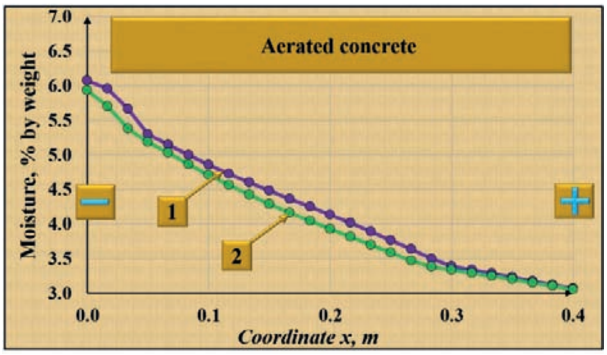

Figure 3. The results of the moisture regime calculations for a single-layer enclosing structure made of aerated concrete by its thickness (1 - solving the non-stationary moisture transfer equation by the finite difference method according to an explicit difference scheme; 2 - solving the non-stationary moisture transfer equation using a discrete-continuous approach according to the formula (7))

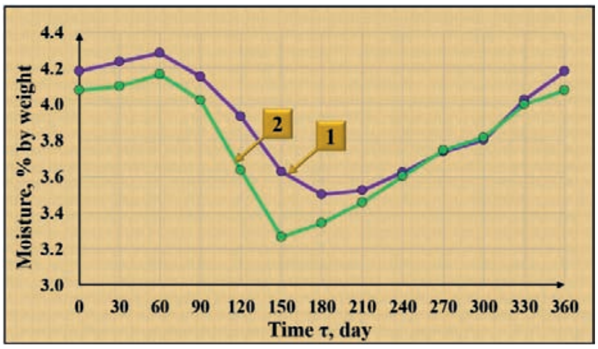

Figure 4. The results of the average moisture content calculations for a single-layer enclosing structure made of aerated concrete throughout a year (1 - solving the non-stationary moisture transfer equation by the finite difference method according to an explicit difference scheme; 2 - solving the non-stationary moisture transfer equation using the discrete-continuous approach according to formula (7))

As can be seen from the graphs, the solution of the moisture transfer equation according to the proposed method both quantitatively and qualitatively coincides with the solution of the moisture transfer equation according to the finite difference method. However, the discrete-continuous approach is carried out according to the final formula, which simplifies the calculation.

5. CONCLUSIONS

The discrete-continuous approach enables us to propose a new efficient method for assessing the unsteady humidity regime which can be used by engineers in the design of enclosing structures.

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