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CALCULATIONS OF 3D ANISOTROPIC MEMBRANE STRUCTURES UNDER VARIOUS CONDITIONS OF FIXING

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Abstract: A unified approach in solving equilibrium problems of standard cells of membrane structures made of various absolutely flexible materials, including anisotropic (orthotropic) materials is presented in this paper. The objects of study are rectangular membranes under various conditions of fixing and/or supporting. The problems were considered in a geometrically nonlinear formulation, with the deformations and the squares of the rotation angles thereunder being considered to be comparable with each other, but small compared to unity. A resolving system of differential equations in partial derivatives written in displacements is obtained therewith. These equations combining with the presented boundary conditions are numerical models of a number of fragments of real membrane structures. The closed nonlinear system of equations was integrated using the discrete braking method. Therewith both longitudinal and transverse vibrations of anisotropic masses were analyzed using a conditional dynamic model to select the initial values of the displacements. The problem of equilibrium of a square isotropic membrane with a free boundary under a uniformly distributed load is presented as an example. The resulting graphs and tables show the distribution of forces and displacements. They may be used for calculating membrane structures. The developed technique may be applied to those values of the initial parameters under which the calculations have not yet been made.

Keywords: membranes, membrane structures, geometrically nonlinear formulation, dimensionless form of resolving equations, compliant contour, free boundary, absolutely rigid fixing, peculiar points, method of undetermined reduction factors, discrete braking method, uniaxial state of stress zone, displacement equation, dynamic models, anisotropic masses.

РАСЧЕТЫ ПРОСТРАНСТВЕННЫХ АНИЗОТРОПНЫХ МЕМБРАННЫХ КОНСТРУКЦИЙ ПРИ РАЗЛИЧНЫХ УСЛОВИЯХ ЗАКРЕПЛЕНИЯ

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Аннотация: В работе в геометрически нелинейной постановке сформулирован единый подход к решению задач равновесия типовых ячеек мембранных конструкций из различных, абсолютно гибких, в том числе и анизотропных (ортотропных) материалов. Объектами исследования являются прямоугольные мембраны при различных условиях закрепления и/или подкрепления. Задачи рассматривались в геометрически нелинейной постановке, согласно которой деформации и квадраты углов поворота считаются соизмеримыми между собой, но малыми по сравнению с единицей. При этом получена разрешающая система дифференциальных уравнений в частных производных, записанная в перемещениях. Эти уравнения в комбинации с представленными краевыми условиями являются численными моделями ряда фрагментов реальных мембранных конструкций. Замкнутая нелинейная система уравнений интегрировалась с использованием метода дискретных торможений. При этом для выбора начальных значений смещений с помощью условной динамической модели анализировались продольные и поперечные колебания анизотропных масс. В качестве примера представлена задача равновесия квадратной изотропной мембраны со свободным краем под равномерно распределенной нагрузкой. Полученные при этом графики и таблицы показывают распределение усилий и смещений.

Они могут быть использованы при расчетах мембранных конструкций. Для тех значений исходных параметров, при которых вычисления еще не проводились, можно прибегнуть к разработанной методике.

Ключевые слова: мембраны, мембранные конструкции, геометрически нелинейная постановка, безразмерный вид разрешающих уравнений, податливый контур, свободный край, абсолютно жесткое закрепление, особенные точки, метод неопределенных коэффициентов приведения, метод дискретных торможений, складчатая зона, уравнения в перемещениях, динамические модели, анизотропные массы.

INTRODUCTION

Membrane structures widely applied in construction practice are implemented in materials with various properties and structure (metal sheet, polymer films, technical fabrics, etc.). When calculating, such structures are divided into standard cells being the anisotropic membranes either fixed and/or supported on four, three or two sides [6, 8, 13, 14, 16-18].

A vast amount of references [15] is devoted to the solution of equilibrium problems for rectangular membranes fixed along a contour or along two boundaries, with a detailed review thereof being considered herein [2].

This article is devoted to the calculation of rectangular orthotropic membranes with different fixing conditions.

Therewith the load was assumed to be uniformly distributed. The problem is considered in a geometrically nonlinear formulation, whereby the deformations and the squares of the rotation angles are considered to be comparable with each other, but small as compared to unity [1, 3, 4]. The following geometric relations correspond to these conditions:

$$\begin{gathered}\varepsilon_x = K_u\frac{\partial u}{\partial x} + \left(\frac{1}{2}\right)K_w^2\left(\frac{\partial w}{\partial x}\right)^2;\\ \varepsilon_y = K_v\frac{\partial v}{\partial y} + \left(\frac{1}{2}\right)K_w^2\left(\frac{\partial w}{\partial y}\right)^2;\\ \gamma_{xy} = K_u\frac{\partial u}{\partial y} + K_v\frac{\partial v}{\partial x} + K_w^2\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}.\end{gathered} \tag{1}$$

To give a more convenient dimensionless form to the resolving system and the subsequent solution the method of undetermined reduction factors [5, 6] is used here [5, 6]: $U = K_u au$, $V = K_v av, W = K_\omega aw$, where K_u, K_v, K_w – are unknown yet arbitrary constants, u, v, w – are dimensionless functions proportional to displacements in the direction of the coordinate axes (Fig. 1).

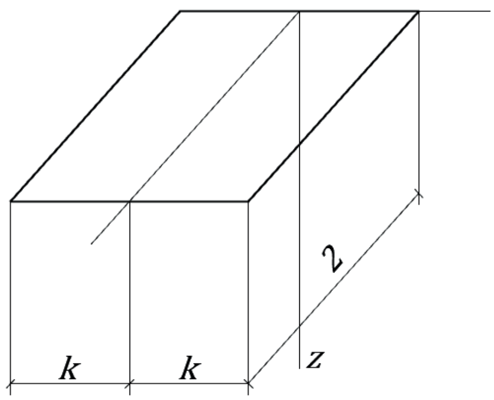

Figure 1. Original membrane scheme

The orthotropy axes of the membrane material were considered to be oriented parallel to the sides thereof, thereby enabling to simplify the physical relations to the form:

$$\begin{gathered}\varepsilon_x = \lambda n_x - \nu n_y;\\ \varepsilon_y = -\nu n_x + n_y, \gamma_{xy} = \frac{n_{xy}}{\mathrm{g}}.\end{gathered} \tag{2}$$

Here and elsewhere, a number of dimensionless parameters are used:

$$\begin{gathered}n_x = N_x/(E_yH)\,;\; n_y = N_y/(E_yH)\,;\\ \mu(\lambda - \nu^2) = 1;\; \alpha = \mu\nu + \mathrm{g};\\ \mathrm{g} = G_{xy}/E_y\,;\; k = b/a.\end{gathered} \tag{3}$$

where, N_x, N_y, N_{xy} – are forces; $2a, 2b$ и H – are the plan dimensions and the membrane thickness, respectively; $E_x, E_y, G_{xy}, \nu_{xy}, \nu_{yx}$, – are elastic characteristics of the material.

Let us assume that the structures of the considered membranes and the nature of loading thereof do not allow for uniaxial state of stress zones. Equilibrium equations in a similar formulation are:

$$\frac{\partial n_x}{\partial x}+\frac{\partial n_x}{\partial y}=0;\ \frac{\partial n_{xy}}{\partial x}+\frac{\partial n_y}{\partial y}=0;$$
$$\left[n_x\frac{\partial^2 w}{\partial x^2}+n_y\frac{\partial^2 w}{\partial y^2}+2n_{xy}\frac{\partial^2 w}{\partial x\partial y}\right]K_w^3E_yH = -qa. \quad (4)$$

A resolving system written in displacements may be obtained under (1)-(4) after a number of transformations

$$\mu\frac{\partial^2 u}{\partial x^2}+g\frac{\partial^2 u}{\partial y^2}+\alpha\frac{\partial^2 v}{\partial x\partial y}+\alpha\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x\partial y}+ +\mu\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2}+g\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial y^2}=0;$$
$$\alpha\frac{\partial^2 u}{\partial x\partial y}+\mathrm{g}\frac{\partial^2 v}{\partial x^2}+\alpha\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x\partial y}+g\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x^2}+ +\mu\lambda\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2}+\mu\lambda\frac{\partial^2 v}{\partial y^2}=0; \quad (5)$$
$$\mu\left[\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\nu\frac{\partial v}{\partial y}+\frac{\nu}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]\frac{\partial^2 w}{\partial x^2} +2g\left[\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right]\frac{\partial^2 w}{\partial x\partial y} +\mu\left[\nu\frac{\partial u}{\partial x}++\frac{\nu}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{1}{\lambda}\frac{\partial v}{dy}+\frac{1}{2\lambda}\left(\frac{\partial w}{\partial y}\right)^2\right]\frac{\partial^2 w}{\partial y^2} = -1.$$

This system was used in numerical calculations. Equations (10) combined with boundary conditions may be interpreted as a model of a number of standard cells in membrane structures found in actual designing.

Therefore, we had a system of three equations for displacements written in a specific dimensionless form that in the case of an isotropic membrane coincides with the system of resolving equations obtained by M. S. Kornishin [11] for plates, with the cylindrical rigidity thereof tending to zero.

A closed nonlinear system composed of equations (5) and the corresponding boundary conditions (Table 1) was integrated by the discrete braking method [10, 19]. Application of this method in solving 3D problems is known to deal with unacceptable costs of computer time, since the integration step must be small due to stability limitations associated with a large difference in the periods of longitudinal and transverse oscillations, being avoided by introducing anisotropic masses of simpler forms than in paper [12] to the dynamic models. Information on the hypothetical laws of variation of the required functions Q was included to these conditional concepts (Fig. 2), in addition to the weight factors p.

Table 1. Main kinds of boundary conditions

Kind of support contour	Boundary conditions at		
	$x=0$	$y=\kappa$	$x=2$
Absolutely rigid	$u_0=0$	$u_\kappa=0$	$u_2=0$
	$v_0=0$	$v_\kappa=0$	$v_2=0$
	$w_0=0$	$w_\kappa=0$	$w_2=0$
Three sides are absolutely rigid, with the one being free	$u_0=0$	$u_\kappa=0$	$n_{x2}=0$
	$v_0=0$	$v_\kappa=0$	$n_{xy2}=0$
	$w_0=0$	$w_\kappa=0$	$n_{y2}\cdot\frac{\partial^2 w}{\partial y^2}=0$
Flexible, of symmetrical cross section	$\eta_\kappa\cdot\frac{\partial^4 u}{\partial y^4}=n_{x0}+[\Sigma n_{y\kappa}-n_{xy0}]\cdot\frac{\partial^2 u}{\partial y^2}$	$\psi_\kappa\cdot\frac{\partial u}{\partial x}=\Sigma n_{x0}-n_{xy\kappa}$	$\eta_\kappa\cdot\frac{\partial^4 u}{\partial y^4}=n_{x2}+[\Sigma n_{y\kappa}-n_{xy2}]\cdot\frac{\partial^2 u}{\partial y^2}$
	$\psi_\kappa\cdot\frac{\partial v}{\partial y}=\Sigma n_{y\kappa}-n_{xy0}$	$\eta_\kappa\cdot\frac{\partial^4 v}{\partial x^4}=n_{y\kappa}+[\Sigma n_{x0}-n_{xy\kappa}]\cdot\frac{\partial^2 v}{\partial x^2}$	$\psi_\kappa\cdot\frac{\partial v}{\partial y}=\Sigma n_{y\kappa}-n_{xy2}$
	$\eta_\kappa\cdot\frac{\partial^4 w}{\partial y^4}=n_{x0}\cdot\frac{\partial w}{\partial x}+ +[\Sigma n_{y\kappa}-n_{xy0}]\cdot\frac{\partial^2 w}{\partial y^2}$	$\eta_\kappa\cdot\frac{\partial^4 w}{\partial x^4}=n_{y\kappa}\cdot\frac{\partial w}{\partial y}+ +[\Sigma n_{x0}-n_{xy\kappa}]\cdot\frac{\partial^2 w}{\partial x^2}$	$\eta_\kappa\cdot\frac{\partial^4 w}{\partial y^4}=n_{x2}\cdot\frac{\partial w}{\partial x}+ +[\Sigma n_{y\kappa}-n_{xy2}]\cdot\frac{\partial^2 w}{\partial y^2}$

The symbols $\eta_к = \frac{E_к I_к}{E_y}$ u $\psi_к = \frac{E_к A_к}{E_y}$ are used in the table to denote the relative rigidity of the contour for bending and compression. An appropriate combination of the given boundary conditions enables obtaining of a mathematical model of any kind of support for membrane structures [3, 7, 8].

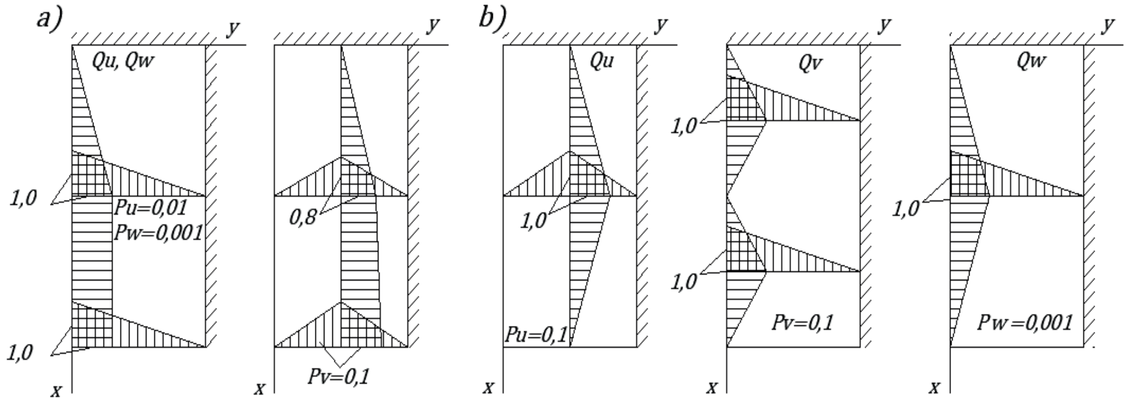

Figure 2. Characteristics of anisotropy of conditional masses of dynamic models

As a result of the numerical solution of the difference analogues of the above equations, the values of the displacement components at all points were obtained. Therewith, central differences with an error of the reminder term of the order of the second degree of strength were used on a square net; extrapolation of the desired functions beyond the contour was made by means of the Lagrange polynomial. The final solutions for displacements may be presented in dimensional form as necessary:

$$U = u\left(\frac{qa}{E_y H}\right)^{\frac{2}{3}} a;\ V = v\left(\frac{qa}{E_y H}\right)^{\frac{2}{3}} a;\quad W = w\left(\frac{qa}{E_y H}\right)^{\frac{2}{3}} a, \tag{6}$$

with one being able to proceed therefrom to dimensional forces to get a full view of the stress-strain behavior.

The problem of equilibrium of a square isotropic membrane with a free boundary under a uniform load is presented as an example. Half of the membrane only was examined because of the symmetry. Continuous characteristics with reasonable ordinates were received (Fig. 3). In a small neighborhood of the extreme right point of the free boundary, there were jumps of normal n_x, n_y and tangential n_{xy} forces to be explained by ambiguity thereof at this peculiar point. An analysis of the graphs enables conditional division of the area of the deformed membrane in the direction of the x axis into three zones: the first one (in the neighborhood of the y axis) - here all the characteristics behave as in a membrane with four absolutely rigid boundaries; the second one (central part) is a zone where the pattern of the stress-strain behavior is similar to that in the case of a cylindrical bend; the third one (in the neighborhood of the free boundary) - the most of the characteristics have extreme values.

Rectangular membranes with one free boundary were also calculated for various aspect ratios. The free boundary at k > 1.5 was found not to affect the stress state of the membrane in the neighborhood of the opposite rigidly fixed side. Therewith, the stress and strain fields have been slightly changed as compared with a similar membrane rigidly fixed on four sides.

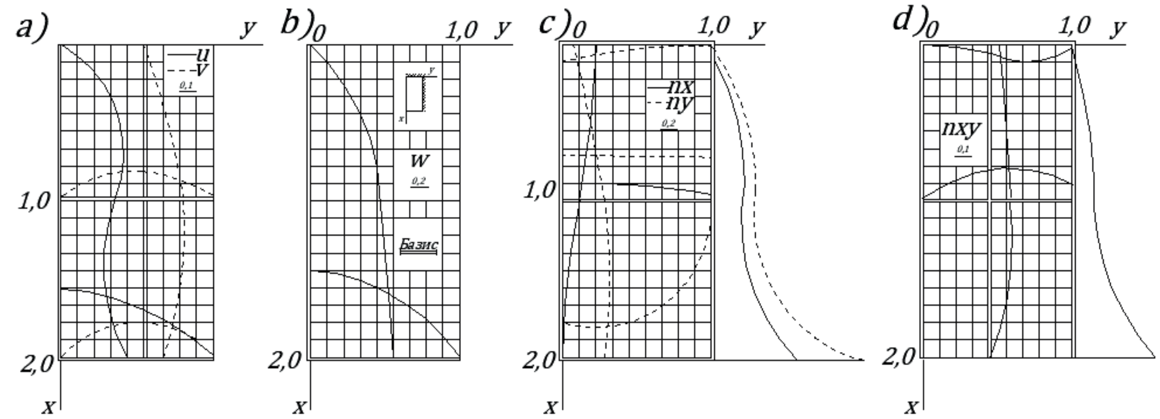

Figure 3. Characteristic functions of deformations in the plane of the membrane (a), displacements in the vertical direction (б), normal components of the stress state (в), and shear components (г)

CONCLUSION

In conclusion we note that the graphs and tables obtained show the nature of the distribution of forces and displacements in membranes for a quite wide range of changes in the main parameters of structures thereof. They may be used in calculations of membrane systems. For the same values of the parameters for which the calculations were not made you may use the developed methodology and the finished program.

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REFERENCES

1. **Bedov A., Vagapov R., Gabitov A., Salov A.** // Nonlinear problems of equilibrium for axisymmetric membranes // APEM 2021. IOP Conf. Series: Materials Science and Engineering 11674 (2021) 012009.
2. **Vagapov R.F.** // Review of papers on the theory of membrane calculation // Ufa Petroleum Institute. - Ufa, 1987. - 40 p. Dep. in VNIIIS, 1988, No. 7964.
3. **Vagapov R.F.** Equilibrium of rectangular orthotropic membranes with a free boundary // Structural Analysis. - 1990. - No.1. P.62-65.
4. **Vagapov R.F., Grigoriev A.S., Konovalov M.B., Trushina V.M.** // Problems of equilibrium of membranes and membrane structures // Proceedings of the 14th All-Union Conference on the Theory of Plates and Shells. - Tbilisi, 1987. P. 255-260.
5. **Grigoriev A.S., Trushina V.M., Shadrin V.A.** // Equilibrium of rectangular and round orthotropic membranes at large

deflections // Current problems in Mechanics and Aviation. - M., 1982. - P. 106-113.
6. **Demenev M.G.** Development of coating structures with the application of membrane panels // PhD Tech.Thesis M.: 1990. P.234.
7. **Bedov A.I., Gabitov A.I., Gaisin A.M., Salov A.S., Chernova A.R.** CAD technologies under thermal properties analysis of wall cladding of framed buildings. IOP Conference Series: Materials Science and Engineering: Volume 465, 2018.VII International Symposium Actual Problems of Computational Simulation in Civil Engineering 1–8 July 2018, Novosibirsk, Russian Federation. P.1-8.
8. **Eremeev P.G., Tusnin A.R.** Influence of the eccentric fixing of the membrane to the support contour on the redistribution of forces in the system // Structural Analysis. 1990. - No.1. P. 8-13.
9. **Eremeev P.G.** // 3D thin sheet metal structures of coatings. M: ASV, 2006. P.560.
10. **Kislooky V.N.** // Algorithm for the numerical solution of problems of statics and dynamics of nonlinear systems // Applied Mechanics. - Kyiv, 1966. - Vol. 2. - Issue. 6. P. 87-92.
11. **Kornishin M.S.** // Nonlinear problems of the theory of plates and depressed shells and methods for solution thereof. – M.: Science, 1964. P. 192
12. **Malyi V.I., Dolzhnikov I.L., Alyautdinov M.I., Kulikov V.L.** // Calculation of membrane coatings with a flexible contour // Structural Analysis. – 1981. No. 2. P. 18-22
13. Membrane constructions of buildings and structures: Reference. Under general editorship of V.I. Trofimov and P.G. Eremeeva; Central Research Institute of Building Structures named after V.A. Kucherenko. M.: Stroyizdat, 1990. P.446.
14. **Novikov Yu.N.** // Architectural membrane structures [Electronic resource]: - Access mode: http://www.flexpro.ru / text.htm
15. **Perelmuter A.V., Slivker V.I.** // Calculation models of structures and the possibility of analysis thereof. - M.: DMK Press, 2007. P.600.
16. Pneumatic building structures // Ermolov V.V. et al. - M:. Stroyizdat, 1983. P.439.
17. **Tusnin A.R.** Development of structures for the membrane coating of industrial buildings // PhD Tech. Thesis M.: 1989. P. 219.
18. **Trofimov V.I., Mikulin V.B., Pritsker A.Ya., Reusov V.A., Semyonov V.T.** // Membrane constructions of buildings and structures. Kyiv: Budivelnik, 1986. P.177.
19. **Vinichenko V., Ryazanova V.A., Gabitov A.I., Udalova Y.A., Salov A.S.** Efflorescence processes in exterior wall surface of buildings // Materials Science Forum 968 MSF, 2019. P. 115-121.

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