

USING DISCRETE-CONTINUOUS APPROACH FOR THE SOLUTION OF UNSTEADY-STATE MOISTURE TRANSFER EQUATION FOR MULTILAYER BUILDING WALLS

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Abstract: Moisture regime of enclosing structures is one of the most complicated and controversial directions in construction industry. Temporary climate impact on enclosing structures and low moisture inertia of building materials lead to the situation in which it is impossible to calculate the steady-state moisture regime. Numerical methods are usually used to assess the moisture behaviour of the enclosing structures. In the current paper, a differential equation of moisture transfer is formulated. The solution of the unsteady-state equation of moisture transfer was obtained using the discrete-continuous approach. Thus, a formula which allows scientists to calculate unsteady-state moisture transfer in multilayer walls of buildings was obtained. A two-layer building enclosing structure with aerated concrete base and mineral wool insulation was calculated.

Keywords: moisture regime, mathematical model, discrete-continuous method, moisture potential, multilayer enclosing structure.

ИСПОЛЬЗОВАНИЕ ДИСКРЕТНО-КОНТИНУАЛЬНОГО ПОДХОДА К РЕШЕНИЮ УРАВНЕНИЯ НЕСТАЦИОНАРНОГО ВЛАГОПЕРЕНОСА В МНОГОСЛОЙНЫХ СТЕНАХ ЗДАНИЙ

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Аннотация: Влажностный режим ограждающих конструкций зданий является одним из наиболее сложных и противоречивых направлений в строительстве. Постоянное изменение климатических воздействий на ограждающую конструкцию и низкая влажностная инерция строительных материалов приводят к ситуации, в которой невозможно производить расчеты стационарного влажностного режима. Для оценки влажностного состояния ограждающей конструкции используются численные методы. В настоящей работе сформулировано дифференциальное уравнение влагопереноса. Получено решение нестационарного уравнения влагопереноса с помощью дискретно-континуального подхода. В результате была получена формула, которая позволяет рассчитывать нестационарный влагоперенос в многослойных стенах зданий. Была рассчитана двухслойная ограждающая конструкция с основанием из газобетона и утеплителем из минеральной ваты.

Ключевые слова: влажностный режим, математическая модель, дискретно-континуальный метод, потенциал влажности, многослойная ограждающая конструкция.

1. INTRODUCTION

Heat and moisture transfer inside enclosing structures is a vital problem in modern construction industry [1–7].

In nowadays, there are many multilayers walls that are used in building, so it is crucial to assess heat-conductivity coefficients under various climate conditions [8, 9], durability of building materials [10–13] and influence of the moisture content inside enclosures on human health [14–18].

Calculations of moisture transfer are based on a transfer potential. For instance, it can be gradient of water vapor partial pressure [19]. Moreover, moisture transportation can be described by some moisture transfer potentials. For example, gradient of water vapor partial pressure and gradient of capillary pressure [20] or liquid content pressure [21]. The most convenient method is a moisture potential theory, which allows scientists to solve only one moisture transfer equation using the moisture potential [22]. A huge number of moisture potentials exist but in Russian Federation the moisture potential F , which is included in regulatory documents, was developed by V.G. Gagarin and V.V. Kozlov [23].

The moisture potential F can be written as a function of moisture and temperature [23]:

$$F(w, t) = E_i(t) \cdot \varphi(w) + \frac{1}{\mu_0} \int_0^w \beta(\zeta) d\zeta. \quad (1)$$

where F – moisture potential, Pa; E_i – saturated water vapor pressure, Pa; φ – relative air humidity, %; μ – vapor permeability coefficient, $\text{kg}/(\text{m} \cdot \text{s} \cdot \text{Pa})$; β – moisture conductivity coefficient, $\text{kg}/(\text{m} \cdot \text{s} \cdot \text{kg}/\text{kg})$, which depends on moisture, t – temperature, °C; w – material moisture, % by weight (1 kg/kg = 100 % by weight).

Moisture transfer differential equation based on the moisture potential F can be formulated as [23]:

$$\frac{\partial F(w, t)}{\partial \tau} = \left(\frac{1}{\mu} \beta(w) + \frac{\partial \varphi(w)}{\partial w} E_i(t) \right) \frac{\mu}{\gamma_0} \frac{\partial^2 F(w, t)}{\partial x^2}. \quad (2)$$

where γ_0 – enclosing structure dry material density, kg/m^3 , τ – time, s; x – coordinate, m.

In 2010, the new discrete-continuous approach was developed by Zolotov A.B., Akimov P.A., Sidorov V.N. and Mozgaleva M.L. This approach gives an opportunity to find an analytical solution of the unsteady-state heat transfer equation [24, 25].

The heat transfer equation can be formulated as [24, 25]:

$$\frac{\partial t}{\partial \tau} = a \cdot \frac{\partial^2 t}{\partial x^2}. \quad (3)$$

where a – thermal diffusivity coefficient, m^2/s . First-order boundary conditions for the heat transfer equation can be written as [24, 25]:

$$t_{x=0} = t_{ext}. \quad (4)$$

$$t_{x=l} = t_{in}. \quad (5)$$

where $t_{x=0}$ – temperature in $x=0$, °C; $t_{x=l}$ – temperature in $x=l$, °C; t_{ext} – temperature of outside air, °C; t_{in} – temperature of inside air, °C; l – thickness of researched enclosing structure, m.

If inside and outside temperatures do not change during time, it is possible to use discrete-continuous formula:

$$\bar{U}(\tau) = e^{A\tau} \cdot \bar{U}_0 - A^{-1} (E - e^{A\tau}) \cdot \bar{S}. \quad (6)$$

where \bar{U} – temperature distribution column vector; \bar{U}_0 – initial temperature distribution column vector; A – coefficient matrix; \bar{S} – boundary conditions column vector.

Opportunities of the formula (6) has been developed by V.N. Sidorov and S.M. Matskevich [26–28]. First-order boundary

conditions varied with time, and temperature distribution was described by the following expression at any moment of time:

$$\bar{U}(\tau) = e^{A \cdot \tau} \cdot \bar{U}_0 + \int_0^\tau e^{A(\tau-\sigma)} \cdot \bar{S}(\sigma) d\sigma. \quad (7)$$

The integral in equation (7) can be determined by method of trapezoidal.

2. THE PROBLEM

To obtain analytical solution of the unsteady-state moisture transfer equation (2) for multilayer building walls using discrete-continuous method.

3. MATERIALS AND METHODS

The formula (2) was reformulated as [29,30]:

$$\frac{\partial F(w, t)}{\partial \tau} = \kappa_{F_0} \cdot E_t(t) \frac{\partial^2 F(w, t)}{\partial x^2}. \quad (8)$$

where κ_{F_0} – average material heat-humidity characteristic coefficient, $m^2/(s \cdot Pa)$.

Thus, saturated water vapor pressure E_t depends on temperature and can be calculated by the following expression:

$$E_t(t) = 1.84 \cdot 10^{11} \cdot \exp(-5330 / (273 + t)). \quad (9)$$

In order to simplify equation (8) let us consider the steady-state heat-transfer equation with third order boundary conditions:

$$\frac{\partial^2 t}{\partial x^2} = 0. \quad (10)$$

$$-\lambda \left. \frac{\partial t}{\partial x} \right|_{i=1} = \alpha_{ext} (t_{ext} - t_1). \quad (11)$$

$$\lambda \left. \frac{\partial t}{\partial x} \right|_{i=N} = \alpha_{in} (t_{in} - t_N). \quad (12)$$

where t_1 – temperature of the enclosing structure surface which contacts with outside air, °C; α_{ext} – heat exchange coefficient of outside air and enclosing structure section, $W/(m^2 \cdot ^\circ C)$; t_N – temperature of the enclosing structure surface which contacts with inside air, Pa; α_{in} – heat exchange coefficient of inside air and enclosing structure section, $W/(m^2 \cdot ^\circ C)$.

Third-order boundary condition for moisture transfer equation can be written as:

$$-\mu \left. \frac{\partial F}{\partial x} \right|_{i=1} = \beta_{ext} (F_{ext} - F_1). \quad (13)$$

$$\mu \left. \frac{\partial F}{\partial x} \right|_{i=N} = \beta_{in} (F_{in} - F_N). \quad (14)$$

where F_{ext} – outside air moisture potential equal to partial pressure of outside air water vapor, Pa; F_{in} – inside air moisture potential equal to partial pressure of inside air water vapor, Pa; F_1 – moisture potential of the enclosing structure surface which contacts with outside air, Pa; F_N – moisture potential of the enclosing structure surface which contacts with inside air, Pa; β_{in} – moisture exchange coefficient of inside air and enclosing structure section, $kg/(m^2 \cdot s \cdot Pa)$; β_{ext} – moisture exchange coefficient of outside air and enclosing structure section, $kg/(m^2 \cdot s \cdot Pa)$.

According to the analytical expressions (9) – (14), there is a possibility to find discrete-continuous solution of the moisture-transfer equation for the multi-layer enclosing structure:

$$\begin{aligned} \bar{F} = & p \cdot ((G + K \cdot E_t \cdot A)^{-2} \cdot e^{(G+K \cdot E_t \cdot A) \cdot \tau} - \\ & - \tau \cdot (G + K \cdot E_t \cdot A)^{-1} - (G + K \cdot E_t \cdot A)^{-2} \cdot \bar{L} + \\ & + (G + K \cdot E_t \cdot A)^{-1} (e^{(G+K \cdot E_t \cdot A) \cdot \tau} - E) \cdot \bar{B} + \\ & + e^{(G+K \cdot E_t \cdot A) \cdot \tau} \cdot \bar{F}_0. \end{aligned} \quad (15)$$

where G – matrix of coefficients for materials joint; K – matrix, which takes into account the differences in the thermal and moisture

properties of the materials of the calculating enclosing structure; A – matrix of coefficients for a multilayer enclosing structure; \bar{L} – a column vector, the first element of which is equal to one, other elements are equal to 0 for a multilayer enclosing structure \bar{B} – a column vector, the first and last elements of which describe the boundary conditions on the outer and inner surfaces of the enclosing structure, other elements are equal to 0 for a multi-layer enclosing structure; E_i – matrix of the saturated water vapour pressure; p – the coefficient of the external boundary condition for a multilayer building enclosing structure, Pa/s^2 . A computer program based on formula (15) has been created. It was made by MATHLAB application, which is able to use an engineer's work.

4. RESULTS AND DISCUSSION

The new discrete-continuous formula was used for calculation of the moisture regime of the building wall with aerated concrete base and mineral wool insulation. The climate data of Moscow (Russian Federation) for temperature and moisture field was taken as initial conditions for moisture behaviour assessment.

The results of the moisture behaviour calculation in the building wall with aerated concrete base

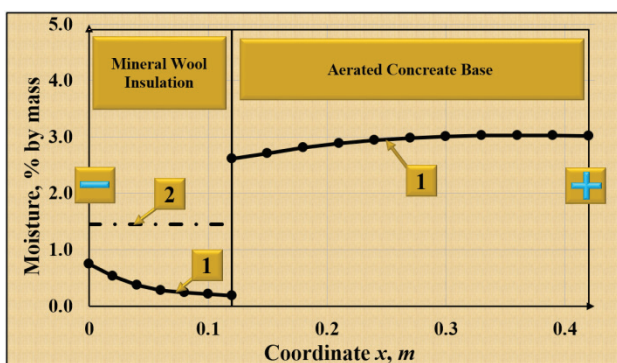


Figure 1. Result of the moisture behaviour calculation in building wall with aerated concrete base and mineral wool insulation in January (1 – a moisture behaviour inside enclosing structure; 2 – a maximum sorption zone).

and mineral wool insulation in January are given at (Figure 1).

5. CONCLUSIONS

The new efficient method was proposed for HVAC (heating, ventilation and air conditioning) engineers. This method is based on the discrete-continuous approach, which allows scientists calculate unsteady-state moisture transfer by final formula (15).

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