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DETERMINATION OF STRAIN-STRESS PARAMETERS OF A MULTI-STOREY REINFORCED CONCRETE BUILDING ON AN ELASTIC FOUNDATION WITH ALLOWANCE FOR DIFFERENT RESISTANTANCE OF MATERIALS AND CRACKING

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Abstract: In this paper, we consider the construction of a finite-element model for determining the stress-strain state of a multi-storey building made of monolithic reinforced concrete on an elastic foundation. This takes into account the dependence of the mechanical characteristics of concrete on the form of the stressed state, the development of plastic deformation in the reinforcement, cracking. Confirmed that the account of the complicated properties of the material is necessary for obtaining correct estimates of the stress-strain state of reinforced concrete structures under conditions of progressive cracking.

Keywords: finite element method, reinforced concrete, multi-storey building, multimodulus behavior, cracking, hybrid finite element, elastic foundation

ОПРЕДЕЛЕНИЕ ПАРАМЕТРОВ НАПРЯЖЕННО-ДЕФОРМИРОВАННОГО СОСТОЯНИЯ МНОГОЭТАЖНОГО ЖЕЛЕЗОБЕТОННОГО ЗДАНИЯ НА УПРУГОМ ОСНОВАНИИ С УЧЕТОМ РАЗНОСОПРОТИВЛЯЕМОСТИ И ТРЕЩИНООБРАЗОВАНИЯ

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Аннотация: В данной статье, рассмотрено построение конечно-элементной модели для определения напряженно-деформированного состояния многоэтажного здания из монолитного железобетона на упругом основании. При этом учитывалась зависимость механических характеристик бетона от вида напряженного состояния, развитие пластических деформации в арматуре, трещинообразование. Подтверждено, что учёт усложнённых свойств материала необходим для получения корректных оценок напряженно-деформированного состояния железобетонных конструкций в условиях прогрессирующего трещинообразования.

Ключевые слова: метод конечных элементов, железобетон, многоэтажное здание, разносопротивляемость, трещинообразование, гибридный конечный элемент, упругое основание

INTRODUCTION

The intensive development of technology and the science of materials in the last decade, as well as the ever increasing demands on the economy and reliability of building structures, make serious demands on the development of construction mechanics. With a detailed study of the deformation of some materials widely used in engineering practice, such as concrete, it was found that their behavior is significantly different from the usual representations. The deformation and strength characteristics of such materials show sensitivity to the form of the stress state realized at the point, and under operational loads the dependences between stresses and deformations are essentially nonlinear. To determine the stress-strain state of nonlinear multimodulus materials, a number of defining relations were proposed [1-6]. However, almost all of these models have significant drawbacks, limiting their application for the calculation of structures in a complex stress state [2]. Therefore, in this paper, we use a version of the equations of state of isotropic, multimodulus materials, based on the method of normalized stress spaces [1].

It is important to bear in mind the fact that in order to take into account the entire complex of effects associated with modeling the behavior of nonlinear multimodulus materials, fracture of material in the form of cracking, plastic deformations in the armature, it is necessary to improve the corresponding calculation base, since existing software packages, as well as known mathematical models do not always satisfy the requirements for carrying out calculations with the necessary accuracy [1, 2, 7-11]. Therefore, in this study, a variant of a hybrid finite element (FE) is proposed, taking into account the physically nonlinear behavior of the material and its destruction in the form of cracks [7]. The creation of new mathematical models for describing the mechanical behavior of reinforced concrete structures with the most complete account of complicated properties, as well as the improvement of the corresponding design models, is undoubtedly an actual task of the construction industry and mechanics of a deformable solid [9].

As shown in [2, 9, 10], hybrid modifications of hybrid FE with three degrees of freedom at the node are quite effective for the calculation of reinforced concrete structures [12]. The direct application of R.Kuk's finite elements to the calculation of reinforced concrete spatial structures showed that they do not take into account longitudinal forces and displacements in the middle surface, and also do not allow to determine the generalized forces vector in the center of the FE quite simply and accurately [2, 8]. Therefore, a modification of the hybrid FE with five degrees of freedom in the node and a stiffness matrix obtained directly for an arbitrary plane triangular element was developed. On the basis of the chosen defining relations, the model of a hybrid flexural triangular finite element with 5 degrees of freedom in a node, taking into account the longitudinal forces and deformations of the transverse shear, allowing simple and effective study of stress-strain state of the construction of arbitrary geometry is considered. The procedures associated with obtaining the stiffness matrix of the hybrid finite element are described in detail in [2, 12, 13].

1. FINITE ELEMENT FORMULATION

To describe the connection between deformations and displacements, the following relations are used in the framework of S.P. Timoshenko's hypotheses:

$$\begin{gathered} e_{11}=u_{1,1}+x_3\psi_{2,1};\ e_{22}=u_{2,2}-x_3\psi_{1,2}; \\ e_{33}=0;\gamma_{12}=u_{1,2}+u_{2,1}+x_3\left(\psi_{2,2}-\psi_{1,1}\right); \\ \gamma_{13}=\psi_2+w_{,1};\ \gamma_{23}=-\psi_1+w_{,2}, \end{gathered} \tag{1}$$

where u_k - horizontal displacements; x_3 - coordinate in thickness; ψ_k - angles of rotation of the middle surface; γ_k - deformation of transverse shear; $w_{,k}$ - the derivative of the deflection.

Equilibrium equations are written in the traditional form:

$$\begin{gathered} N_{11,1}+N_{12,2}=0;N_{12,1}+N_{22,2}=0; \\ M_{11,1}+M_{12,2}=Q_1;M_{12,1}+M_{22,2}=Q_2; \\ Q_{1,1}+Q_{2,2}=q. \end{gathered} \tag{2}$$

The relationship between strains and stresses, which follows from the potential W_1 [1] looks like following:

$$\{e\}=[A]\{\sigma\}, \tag{3}$$

where

$$\{\sigma\}=\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \tau_{12}\\ \tau_{13}\\ \tau_{23}\end{Bmatrix};\ \{e\}=\begin{Bmatrix}e_{11}\\ e_{22}\\ \gamma_{12}\\ \gamma_{13}\\ \gamma_{23}\end{Bmatrix};$$

$$[A]=\begin{bmatrix}A_{11} & A_{12} & A_{16} & A_{14} & A_{15}\\ & A_{22} & A_{26} & A_{24} & A_{25}\\ & & A_{66} & A_{64} & A_{65}\\ & sim & & A_{44} & A_{45}\\ & & & & A_{55}\end{bmatrix}.$$

Here A_{11}, A_{12}, A_{16}, A_{14}, A_{15}, A_{22}, A_{26}, A_{24}, A_{26}, A_{66}, A_{64}, A_{65}, A_{44}; A_{45}, A_{55} are the components of a symmetric matrix $[A]$ including the functions and potential constants W_1, denoted by R_k [1].

The relationship between stresses and strains can be represented in the form [2]:

$$\{\sigma\}=[B]\{e\}, \tag{4}$$

where $[B]=[A]^{-1}$.

Then the forces in the section of the element defined as follows [2]:

$$N_{ij}=\int_{-h/2}^{h/2}\sigma_{ij}dx_3;\ M_{ij}=\int_{-h/2}^{h/2}\sigma_{ij}x_3dx_3;$$
$$Q_i=\int_{-h/2}^{h/2}\tau_{i3}dx_3,\ (i,j=1,2) \tag{5}$$

The connection between the vector of generalized forces and the vector of generalized deformations of the middle surface takes the form:

$$\{M\}=[D]\{\varepsilon\},$$

where

$$M=\begin{Bmatrix}M_{11}\\ M_{22}\\ M_{12}\\ Q_1\\ Q_2\\ N_{11}\\ N_{22}\\ N_{12}\end{Bmatrix};\ \{\varepsilon\}=\begin{Bmatrix}\psi_{2,1}\\ -\psi_{1,2}\\ \psi_{2,2}-\psi_{1,1}\\ \gamma_{13}\\ \gamma_{23}\\ u_{1,1}\\ u_{2,2}\\ u_{1,2}+u_{2,1}\end{Bmatrix};$$

$$[D]=\begin{bmatrix}D_{11} & D_{12} & D_{16} & K_{14} & K_{15} & K_{11} & K_{12} & K_{16}\\ & D_{22} & D_{26} & K_{24} & K_{25} & K_{12} & K_{22} & K_{26}\\ & & D_{66} & K_{64} & K_{65} & K_{16} & K_{26} & K_{66}\\ & & & C_{44} & C_{45} & C_{14} & C_{24} & C_{46}\\ & & & & C_{55} & C_{15} & C_{25} & C_{65}\\ & Sim & & & & C_{11} & C_{12} & C_{16}\\ & & & & & & C_{22} & C_{26}\\ & & & & & & & C_{66}\end{bmatrix}; \tag{6}$$

$$C_{km}=\int_{-h/2}^{h/2}B_{km}dx_3;\ K_{km}=\int_{-h/2}^{h/2}B_{km}x_3dx_3;$$
$$D_{km}=\int_{-h/2}^{h/2}B_{km}x_3^2dx_3$$

are the integral stiffness parameters obtained as a result of numerical integration over the thickness of the element and depending on the stress state.

It is obvious that the mathematical model for determining the stress-strain state of reinforced slabs of which the building consists should take into account the specific features of the interaction of the complex environment "concrete-reinforcement" at various stages, be quite foreseeable and practically realizable. This model can not be completely free from additional technical hypotheses, in particular, the following is considered fair [8, 9]: 1) the loading is simple, the deformation is active, creep deformations of concrete are not considered; 2) the dimensions of the slabs of the structure in plan are large in comparison with the average distance between reinforcing bars, the reinforcement is modeled by a smeared layer, taking into account the coefficient of layer reinforcement; 3) in view of the structural heterogeneity in thickness, the plates are broken up into a series of fictitious layers: a) non-reinforced (concrete) layers without cracks; b) reinforced (reinforced) layers without cracks; c) non-reinforced (concrete) layers with cracks; d) reinforced (reinforced) layers with cracks; e) reinforced (reinforced) layers with intersecting cracks. 4) The stresses within the reinforced layers of the element are defined as the sum of the stresses in the concrete and reinforcement, and the condition for compatibility of concrete and reinforcement assumes the equality of deformations of these two materials; 5) the middle surface of the plate is represented by a network of hybrid finite elements, taking into account the thickness partitioning into a number of fictitious layers; 6) the stiffness characteristics calculated for the center of the fictitious layer of this finite element extend to the entire layer; 7) the criterion for the strength of concrete in each fictitious layer is adopted according to P.P. Balandin's condition [8]; 8) cracks in the region of the cracked fictitious layer within the finite element are considered to be continuous and parallel to each other. The effect of stretched concrete is taken into account by the coefficient of V.I. Murashev and the characteristic of concrete damage [8, 15]; 9) in the presence of cracks, concrete within the fictitious layer is modeled by a transversely isotropic body with an isotropic plane parallel to the plane of cracks.

2. FICTITIOUS LAYERS

Non-reinforced (concrete) layers without cracks. The relationship between strains and stresses is:

$$\{e\} = [A]\{\sigma\}, \qquad (7)$$

where $[A]$ the symmetrical square matrix is 5 × 5 (neglecting voltages in the calculation σ_{33}).

$$A_{11} = \{2\left(R_1 + 2R_2\right)/3 + R_3\xi\left(3 - 2\xi^2\right)/3 + R_4 \times$$
$$\times[\xi\left(2-\eta^2\right) + 4\left(\sigma_{11} - 2\sigma_{22}\right)/9S_0] + R_5[\eta \times$$
$$\times \cos 3\varphi\left(1+\xi^2\right) + 2\sqrt{2}\xi - 2\cos 3\varphi - \sqrt{2}\frac{\sigma_{22}}{S_0}]\}/3;$$
$$A_{12} = \{2\left(R_1 - R_2\right)/3 + \left(R_3 + R_4/3\right)\xi + R_5[\cos 3\varphi \times$$
$$\times\left(1-\xi\right) - \sqrt{2}\xi]\}/3;$$
$$A_{16} = \left(2R_4/3 + \sqrt{2}R_5\right)\tau_{12}/3S_0; \; A_{26} = A_{16};$$
$$A_{14} = \left(2R_4/3 + \sqrt{2}R_5\right)\tau_{13}/3S_0;$$
$$A_{15} = 2\left(R_4/3 - \sqrt{2}R_5\right)\tau_{23}/3S_0;$$
$$A_{22} = \{2\left(R_1 + 2R_2\right)/3 + R_3[\xi\left(3 - 2\xi^2\right)/3 + R_4[\xi \times$$
$$\times\left(2-\eta^2\right) + 4\left(\sigma_{22} - 2\sigma_{11}\right)/9S_0] + R_5[\eta\cos 3\varphi \times$$
$$\times\left(1+\xi^2\right) + 2\sqrt{2}\xi - 2\cos 3\varphi - \sqrt{2}\sigma_{11}/S_0]\}/3;$$
$$A_{24} = 2\left(R_4/3 - \sqrt{2}R_5\right)\tau_{13}/3S_0;'$$
$$A_{25} = \left(2R_4/3 + \sqrt{2}R_5\right)\tau_{23}/3S_0;$$
$$A_{66} = 2\{2R_2 - R_3\xi^3 + R_4[\xi\left(2-\eta^2\right) - \left(\sigma_{11} + \sigma_{22}\right)/$$
$$/3S_0] + R_5[\sqrt{2}\eta\left(\sigma_{11} - \sigma_{22}\right)/2 - \eta^3\cos 3\varphi]\}/3;$$
$$A_{64} = \sqrt{2}R_5\tau_{23}/S_0; \quad A_{65} = \sqrt{2}R_5\tau_{13}/S_0;$$
$$A_{44} = 2\{2R_2 - R_3\eta^3 + R_4[\xi\left(2-\eta^2\right) - \left(\sigma_{11} + \sigma_{22}\right)/$$
$$/3S_0] + R_5[\sqrt{2}\eta\left(\sigma_{11} - 2\sigma_{22}\right)/2 - \eta^3\cos 3\varphi]\}/3;$$
$$A_{45} = \sqrt{2}R_5\tau_{12}/S_0;$$
$$A_{55} = 2\{2R_2 - R_3\eta^3 + R_4[\xi\left(2-\eta^2\right) - \left(\sigma_{11} + \sigma_{22}\right)/$$
$$/3S_0] + R_5[\sqrt{2}\eta\left(\sigma_{22} - 2\sigma_{11}\right)/2 - \eta^3\cos 3\varphi]\}/3.$$

Here: $\xi = \sigma/S_0$, $\eta = \tau/S_0$ - normalized normal and shear stresses on the octahedral platform; $S_0 = \sqrt{\sigma^2 + \tau^2}$ - module of the total voltage vector on the octahedral platform; $\sigma = \delta_{ij}\sigma_{ij}/3$ and $\tau = \sqrt{S_{ij}S_{ij}/3}$ - normal and tangential stresses; $Cos 3\varphi = \sqrt{2}\det(S_{ij})/\tau^3$; φ - phase of stresses; $S_{ij} = \sigma_{ij} - \delta_{ij}\sigma$.

The matrix of "elasticity" $[B]$ for each of the unreinforced layers of the FE is expressed through the matrix of compliance:

$$[B]=[A]^{-1}. \quad (8)$$

Reinforced layers without cracks. Taking into account the accepted hypotheses, the stresses in the reinforced concrete layer are taken as the sum of the stresses in concrete and reinforcement, from which the matrix of "elasticity" for reinforced layers without cracks follows:

$$[B]=[A]^{-1}+[B_S], \quad (9)$$

where

$$[B_S]=\begin{bmatrix} \frac{\mu_{11}E_s}{1-\nu_s^2\mu_{11}} & \frac{\mu_{11}E_s\nu_s}{1-\nu_s^2\mu_{11}} & 0 & 0 & 0 \\ \frac{\mu_{22}E_s\nu_s}{1-\nu_s^2\mu_{22}} & \frac{\mu_{22}E_s}{1-\nu_s^2\mu_{22}} & 0 & 0 & 0 \\ 0 & 0 & B_{S3,3} & 0 & 0 \\ 0 & 0 & 0 & \frac{\mu_{22}E_s}{1+\nu_s\mu_{22}} & 0 \\ 0 & 0 & 0 & 0 & \frac{\mu_{11}E_s}{1+\nu_s\mu_{11}} \end{bmatrix};$$

$$B_{s3,3}=\frac{\mu_{11}E_s}{1+\nu_s\mu_{11}}+\frac{\mu_{22}E_s}{1+\nu_s\mu_{22}};$$

E_S - modulus of elasticity of reinforcement, ν_s - Poisson's ratio of reinforcement;

$$\mu_{11}=A_{Si}/S_{i11}h_S,\ \mu_{22}=A_{si}/S_{i22}h_S$$

- the reinforcement factors in the direction of the axes X_1 and X_2 the initial coordinate system, respectively; A_{si} - cross-sectional area of the reinforcing bar; S_{i11}, S_{i22} - the pitch of the rods parallel to the axes X_1 and X_2; h_S - total thickness of reinforced layers. Note that the matrix components $[A]^{-1}$ in the expression (9) are determined by the formulas in which instead of the total stresses σ_{ij} the stresses in the concrete σ_{Bij} should be used.

Not reinforced (concrete) layers with a crack. According to the hypothesis No. 7 cracks will be formed if the following condition is fulfilled:

$$\begin{aligned}&\sigma_{11}^2+\sigma_{22}^2+3\cdot\left(\tau_{12}^2+\tau_{23}^2+\tau_{13}^2\right)-\left(\sigma_{11}\sigma_{22}\right)-\\&-\left(R_{bt}-R_b\right)\left(\sigma_{11}+\sigma_{22}\right)-R_{bt}R_b>0\end{aligned}, \quad (10)$$

where $\sigma_{11},\sigma_{22},\tau_{12},\tau_{13},\tau_{23}$ - stresses in concrete at the time of crack formation, calculated for the center of the fictitious layer. Here R_{bt},R_b - the ultimate strength of concrete for axial tension and compression, respectively.

When inequality (10) is satisfied, a crack is formed in the concrete layer along the areas orthogonal to the direction of the greatest of the principal tensile stresses, calculated from the formula for a plane stress state:

$$\sigma_{1t}=[\sigma_{11}+\sigma_{22}+\sqrt{(\sigma_{11}-\sigma_{22})^2+4\tau_{12}^2}]/2.$$

The direction of the development of cracks is determined by the magnitude of the angle between the normal to the crack and the axis x_1:

$$\chi_1=arctg[(\sigma_{1t}-\sigma_{11})/\tau_{12}].$$

We note that when parallel cracks arise in the region of the layer of a given finite element, the initially isotropic concrete acquires the properties of orthotropy. In this regard, the acceptability of potential defining relationships is lost, oriented to a nonlinear dilatable, mutually resisting isotropic material. We assume that the validity of the selected potential relationships is only valid for directions along the cracks, where the integrity of the concrete is not violated. In the indicated direction, the physically nonlinear properties of concrete will be approximated by the secant modulus of elasticity E_B and the secant coefficient of transverse deformations ν_B determined from equation

$$e_{22}^{*} = A_{12}^{*}\sigma_{11}^{*} + A_{22}^{*}\sigma_{22}^{*} = \left(\sigma_{22}^{*} - \nu_{B}\sigma_{11}^{*}\right)/E_{B},$$

i.e.

$$E_{B} = 1/A_{22}^{*};\ \nu_{B} = -A_{12}^{*}/A_{22}^{*},$$

where A_{12}^{*}, A_{22}^{*} are the components of the compliance matrix, calculated from the formulas for concrete layers without cracks; σ_{ij}^{*} - stresses in concrete, calculated in an orthogonal coordinate system $X_1^{*}OX_2^{*}$, rotated relative to the initial system X_1OX_2 by an angle χ_1.
Taking into account the foregoing, the relationship between strains and stresses in a rotated coordinate system can be represented as:\

$$\{e^{*}\} = [A^{*}]\{\sigma_{B}^{*}\},$$

where

$$\{e^{*}\} = \begin{Bmatrix} e_{11}^{*} \\ e_{22}^{*} \\ \gamma_{12}^{*} \\ \gamma_{13}^{*} \\ \gamma_{23}^{*} \end{Bmatrix};\{\sigma_{B}^{*}\} = \begin{Bmatrix} \sigma_{B11}^{*} \\ \sigma_{B22}^{*} \\ \tau_{B12}^{*} \\ \tau_{B13}^{*} \\ \tau_{B23}^{*} \end{Bmatrix};$$

$$[A^{*}] = \begin{bmatrix} A_{11}^{*} & A_{12}^{*} & 0 & 0 & 0 \\ & A_{22}^{*} & 0 & 0 & 0 \\ & & A_{66}^{*} & 0 & 0 \\ & Sim & & A_{44}^{*} & 0 \\ & & & & A_{55}^{*} \end{bmatrix};$$

or for the case with a crack in the direction of the axis X_1^{*}:

$$[B^{*}] = [A^{*}]^{-1} = \begin{bmatrix} B_{11}^{*} & B_{12}^{*} & 0 & 0 & 0 \\ & B_{22}^{*} & 0 & 0 & 0 \\ & & B_{66}^{*} & 0 & 0 \\ & Sim & & B_{44}^{*} & 0 \\ & & & & B_{55}^{*} \end{bmatrix};$$

where

$$B_{11}^{*} = 0;\ B_{12}^{*} = 0;\ B_{22}^{*} = E_{B};B_{44}^{*} = B_{66}^{*} = 0;$$
$$B_{55}^{*} = E_{B}/\left[2(1+\nu)\right].$$

Then, having carried out the transformation of coordinates from the system $X_1^{*}OX_2^{*}$ to the initial one, we obtain a stiffness matrix for the cracked concrete $[B^{c}]$:

$$[A^{c}]^{-1} = [B^{c}] = \begin{bmatrix} B_{11}^{c} & B_{12}^{c} & B_{16}^{c} & 0 & 0 \\ & B_{22}^{c} & B_{26}^{c} & 0 & 0 \\ & & B_{66}^{c} & 0 & 0 \\ & \text{sim} & & B_{44}^{c} & 0 \\ & & & & B_{55}^{c} \end{bmatrix}.$$

where

$$B_{11}^{c} = B_{22}^{*}Sin^{4}\chi_1;\ B_{22}^{c} = B_{22}^{*}Cos^{4}\chi_1;$$
$$B_{12}^{c} = B_{22}^{*}Sin^{2}\chi_1 Cos^{2}\chi_1;$$
$$B_{16}^{c} = B_{22}^{*}Cos\chi_1 Sin^{3}\chi_1;\ B_{26}^{c} = B_{22}^{*}Cos^{3}\chi_1 Sin\chi_1;$$
$$B_{66}^{c} = 4B_{22}^{*}Sin^{2}\chi_1 Cos^{2}\chi_1;\ B_{44}^{c} = B_{55}^{*}Sin^{2}\chi_1;$$
$$B_{55}^{c} = B_{55}^{*}Cos^{2}\chi_1.$$

Reinforced (reinforced) layers with a crack. The appearance of cracks is determined by the triggering of the Balandin condition within the fictitious FE layer:

$$\sigma_{B11}^{2} + \sigma_{B22}^{2} + 3\cdot\left(\tau_{12}^{2} + \tau_{23}^{2} + \tau_{13}^{2}\right) -$$
$$-\left(\sigma_{B11}\sigma_{B22}\right) - \left(R_{bt} - R_{b}\right)\left(\sigma_{B11} + \sigma_{B22}\right) -, \quad (11)$$
$$-R_{bt}R_{b} > 0$$

where σ_{Bij} - stresses in reinforced concrete.
The direction of the development of cracks is determined by the magnitude of the angle χ_1 between the normal to the crack and the axis X_1 of the original system:

$$\chi_1 = arctg\left[\frac{(\sigma_{1t} - \sigma_{B11})}{\tau_{12}}\right],$$

where σ_{1t} - the largest of the main tensile stresses in the carrier layer (concrete). Further, taking into account the arguments given above, we have:

$$e_{22}^* = A_{12}^*\sigma_{B11}^* + A_{22}^*\sigma_{B22}^* = \\ = \left(\sigma_{B22}^* - \nu_B\sigma_{B11}^*\right)/E_B \qquad (12)$$

ie,

$$E_B = 1/A_{22}^*;\ \nu_B = -A_{12}^*/A_{22}^*,$$

where A_{12}^*, A_{22}^* are the components of the compliance matrix, calculated from the formulas in which σ_{11} and σ_{22} it is necessary to replace by the stresses calculated in the orthogonal coordinate system $X_1^*OX_2^*$ rotated relative to the initial system X_1OX_2 by an angle χ_1; σ_{B11}^*, σ_{B22}^* - respectively, the stresses in concrete in this coordinate system.
Then the dependencies between strains and stresses in the rotated coordinate system will take the form:

$$\{e^*\} = [A^*]\{\sigma_B^*\}, \qquad (13)$$

where

$$\{e^*\} = \begin{Bmatrix} e_{11}^* \\ e_{22}^* \\ \gamma_{12}^* \\ \gamma_{13}^* \\ \gamma_{23}^* \end{Bmatrix};\ \{\sigma_B^*\} = \begin{Bmatrix} \sigma_{B11}^* \\ \sigma_{B22}^* \\ \tau_{B12}^* \\ \tau_{B13}^* \\ \tau_{B23}^* \end{Bmatrix};$$

$$[A^*] = \begin{bmatrix} A_{11}^* & A_{12}^* & 0 & 0 & 0 \\ & A_{22}^* & 0 & 0 & 0 \\ & & A_{66}^* & 0 & 0 \\ & Sim & & A_{44}^* & 0 \\ & & & & A_{55}^* \end{bmatrix};$$

$$A_{11}^* = 1/(E_B\omega);\ A_{12}^* = -\nu_B/E_B;\ A_{22}^* = 1/E_B;$$
$$A_{44}^* = A_{66}^* = 2(1+\nu_B)/(E_B\omega);$$
$$A_{55}^* = 2(1+\nu)/E_B,$$

where in the direction of the axis X_1^* the secant modulus of elasticity (modulus of deformation of concrete) is determined by the value $E_B\omega$ (ω - the function by which the degree of concrete damage is taken into account $0 < \omega \le 1$). Then in the initial coordinate system, the compliance matrix for the cracked concrete takes the form $[A^C]$:

$$[A^c] = \begin{bmatrix} A_{11}^c & A_{12}^c & A_{16}^c & 0 & 0 \\ & A_{22}^c & A_{26}^c & 0 & 0 \\ & & A_{66}^c & 0 & 0 \\ & \text{sim} & & A_{44}^c & 0 \\ & & & & A_{55}^c \end{bmatrix}. \qquad (14)$$

The matrix of "elasticity" for the reinforcement of a cracked reinforced layer in the original orthogonal coordinate system X_1OX_2 has the form:

$$[B_S^C] = \begin{bmatrix} B_{S1,1}^C & B_{S1,2}^C & 0 & 0 & 0 \\ B_{S2,1}^C & B_{S2,2}^C & 0 & 0 & 0 \\ 0 & 0 & B_{S3,3}^C & 0 & 0 \\ 0 & 0 & 0 & B_{S4,4}^C & 0 \\ 0 & 0 & 0 & 0 & B_{S5,5}^C \end{bmatrix}, \qquad (15)$$

where

$$B_{S1,1}^C = \frac{\mu_{11}E_{s11}\chi_{11}}{(1-\nu_{s11}^2\mu_{11})\lambda_{11}},\ B_{S1,2}^C = \frac{\mu_{11}E_{s11}\nu_{s11}\chi_{11}}{(1-\nu_{s11}^2\mu_{11})\lambda_{11}},$$
$$B_{S2,1}^C = \frac{\mu_{22}E_{s22}\nu_{s22}\chi_{22}}{(1-\nu_{s22}^2\mu_{22})\lambda_{22}},\ B_{S2,2}^C = \frac{\mu_{22}E_{s22}\chi_{22}}{(1-\nu_{s22}^2\mu_{22})\lambda_{22}}$$
$$B_{S\,3,3}^C = \frac{E_{s11}\chi_{11}^k}{(1+\nu_{s11}\mu_{11})\lambda_{11}\text{ctg}^2\chi_1} + \\ + \frac{E_{s22}\chi_{22}^k}{(1+\nu_{s22}\mu_{22})\lambda_{22}\text{tg}^2\chi_1};$$

$$B^C_{S\,4,4}=\frac{E_{s11}\chi^k_{11}}{\left(1+\nu_{s11}\mu_{11}\right)\lambda_{11}\mathrm{ctg}^2\chi_1};$$
$$B^C_{S\,5,5}=\frac{E_{s22}\chi^k_{22}}{\left(1+\nu_{s22}\mu_{22}\right)\lambda_{22}\mathrm{tg}^2\chi_1},$$

where E_{skk} - the secant modulus of deformation of the reinforcement located along the axes X_1 and X_2, ν_{skk} - the secant coefficient of transverse deformation of the reinforcement (*k* = 1,2).

$$\chi^k_{11}=1+\frac{\mu_{11}\left(1-\lambda_{11}\psi_s\right)E_{s11}\cos^2\chi_1}{E_B},$$
$$\chi^k_{22}=1+\frac{\mu_{22}\left(1-\lambda_{22}\psi_s\right)E_{s22}\sin^2\chi_1}{E_B},$$
$$\lambda_{11}=\frac{\eta_\tau\mu_{11}}{\eta_\tau\mu_{11}+\mu_{22}\mathrm{ctg}^2\chi_1},\ \lambda_{22}=\frac{\eta_\tau\mu_{22}}{\eta_\tau\mu_{22}+\mu_{11}\mathrm{tg}^2\chi_1},$$

η_τ – coefficient taking into account the increased flexibility of reinforcing bars to tangential movements in concrete near a crack $\eta_\tau \approx 16$ [19].

In order to take into account the development of plastic deformations in the reinforcement, we will calculate their values by the formulas:

$$E_{Skk}=\begin{cases}E_S & npu\ \ \sigma_{Skk}<\sigma_p\mu_{kk}\\ \sigma_p/e_{kk} & npu\ \ \sigma_{Skk}\geq\sigma_p\mu_{kk}\end{cases},k=1,2,$$

where σ_p is the yield stress of the reinforcing material.

The matrix of "elasticity" of the reinforced layer represented in the form

$$\left[B\right]=\left[B^C_B\right]+\left[B^C_S\right], \tag{16}$$

where

$$\left[B^C_B\right]=\left[A^C\right]^{-1}.$$

To specify the model of a cracked reinforced layer, a damage function ω is defined, defined with the help of the parameter of V. I. Murashev ψ_S, which takes into account the work of stretched concrete in the areas between the cracks:

$$\psi_S=E_{sn}/\left(E_B\omega+E_{sn}\right), \tag{17}$$

where E_{sn} is the elastic modulus of the reinforcement in the direction along the normal to the crack,

$$E_{sn}=E_{S11}\mu_{11}\cos^4\chi_1+E_{S22}\mu_{22}\sin^4\chi_1. \tag{18}$$

Solving jointly equations (17) and (18) with respect to the damage function, we obtain an expression of the form:

$$\omega=\left(E_{S11}\mu_{11}\cos^4\chi_1+E_{S22}\mu_{22}\sin^4\chi_1\right)\times \\ \times\left(1/\psi_s-1\right)/E_B. \tag{19}$$

The parameter ψ_S is calculated using the empirical formula recommended in the works of G.A. Geniev, V.N. Kissyuk. and Tyupin G.A. [15]:

$$\psi_S=1-0{,}7R_{bt}/\sigma^*_{11}, \tag{20}$$

where it is assumed that

$$\sigma^*_{B11}=0{,}7R_{bt},$$

σ^*_{11}, σ^*_{B11} - the normal stresses in reinforced concrete and concrete on sites coinciding with the crack.

Expression for σ^*_{B11} taking into account the rules of transformation of coordinates for stresses σ_{Bij} we obtain a nonlinear equation with respect to ω:

$$\left(B^C_{B11}e_{11}+B^C_{B12}e_{22}+B^C_{B16}\gamma_{12}\right)\cos^2\chi_1+ \\ +\left(B^C_{B12}e_{12}+B^C_{B22}e_{22}+B^C_{26}\gamma_{12}\right)\sin^2\chi_1+ \\ +\left(B^C_{B16}e_{11}+B^C_{B26}e_{26}+B^C_{B66}\gamma_{12}\right)\sin 2\chi_1=0{,}7R_{bt}. \tag{21}$$

The solution of this equation constructed in the framework of the method of successive approximations. Thus, the matrix $\left[A^C\right]$ and components of the "elasticity" matrix $\left[B\right]$ are determined from the calculated damage function and the parameter of V.I. Murashev.

Reinforced layers with intersecting cracks. Taking into account the accepted model of the reinforced layer with cracks and the hypotheses introduced earlier, the matrix of "elasticity" within the fictitious layer is obtained in the form:

$$\left[B\right]=\left[B_S^C\right], \tag{22}$$

where the matrix $\left[B_S^C\right]$ is determined according to condition (15).

3. MODEL TESTING

To demonstrate the features developed by the authors of the model, the problem solved on determining the stress-strain state of a building on an elastic foundation. The building contains 12 floors with the same layout (Figure 1), the exterior of the building in the form of a 3D model is shown in Figure 2.

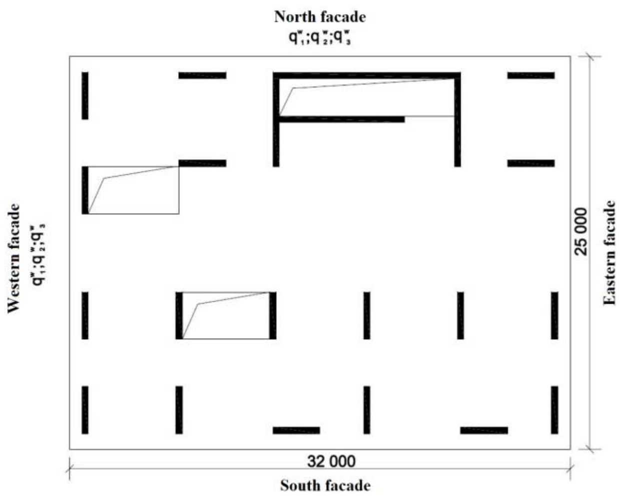

Figure 1. Plane plan of the building scheme.

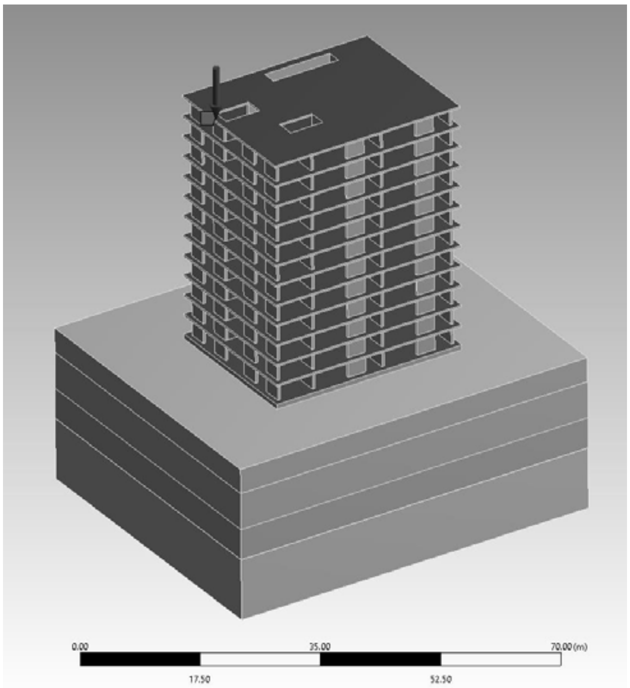

Figure 2. General view of the building (model in ANSYS Workbench 2019 R1).

The following assumptions were made: 1) the design model of the building consists only of horizontal and vertical load-bearing elements of a monolithic reinforced concrete skeleton, respectively - interfloor ceilings and pylons; 2) the thickness of all the inter-floor ceilings is the same and equal to 300 mm (the thickness of the foundation slab is 800 mm); 3) the thickness of all pylons assumed to be the same and equal to 400 mm; 4) all the interface nodes of the elements of the supporting skeleton between themselves are taken rigid; 5) the deformation of the horizontal and vertical elements of the supporting skeleton takes into account the destruction processes, 6) the elastic base is modeled in accordance with [16-18], based on the defining relations for anisotropic differently resisting materials [3]. As the main material of the building was used concrete with a compressive strength $R^{-}=28,4$ MPa [1, 2]. Foundation slab reinforcement adopted by A400 rods with a diameter of 16 mm in the form of a grid symmetrically disposed in the cross-section of the plate (protective layer 50 mm, yield strength

Table 1. Characteristics of soil layers.

Layer number	Mechanical characteristics (all deformation and shear moduli are given in MPa, Poisson's ratios in fractions of a unit)								
1	E_1^+	E_1^-	E_2^+	E_2^-	E_3^+	E_3^-	G_{12}	G_{13}	G_{23}
	31,05	27,95	29,74	24,16	26,35	19,12	10,72	9,39	10,1
	ν_{12}^+	ν_{12}^-	ν_{13}^+	ν_{13}^-	ν_{23}^+	ν_{23}^-	–	–	–
	0,38	0,32	0,37	0,32	0,37	0,32	–	–	–
2	E_1^+	E_1^-	E_2^+	E_2^-	E_3^+	E_3^-	G_{12}	G_{13}	G_{23}
	40,55	32,42	42,01	29,42	25,35	20,24	14,33	10,6	11,14
	ν_{12}^+	ν_{12}^-	ν_{13}^+	ν_{13}^-	ν_{23}^+	ν_{23}^-	–	–	–
	0,33	0,25	0,33	0,25	0,33	0,25	–	–	–
3	E_1^+	E_1^-	E_2^+	E_2^-	E_3^+	E_3^-	G_{12}	G_{13}	G_{23}
	80,00	64,15	70,9	55,3	60,0	35,0	27,36	23,81	25,64
	ν_{12}^+	ν_{12}^-	ν_{13}^+	ν_{13}^-	ν_{23}^+	ν_{23}^-	–	–	–
	0,27	0,2	0,27	0,2	0,22	0,17	–	–	–
4	E_1^+	E_1^-	E_2^+	E_2^-	E_3^+	E_3^-	G_{12}	G_{13}	G_{23}
	10,05	7,5	12,25	9,12	10,35	8,28	3,83	3,92	3,55
	ν_{12}^+	ν_{12}^-	ν_{13}^+	ν_{13}^-	ν_{23}^+	ν_{23}^-	–	–	–
	0,27	0,23	0,27	0,23	0,25	0,25	–	–	–

$\sigma_p = 400$ MPa), reinforcement of the slabs is adopted by A400 rods with a diameter of 14 mm in the form of a grid symmetrically disposed in the cross-section of the plate (protective layer 35 mm, yield strength $\sigma_p = 400$ MPa), pylon reinforcement adopted by A400 rods with a diameter of 12 mm in the form of two nets symmetrically located in the cross-section of the pylon (protective layer 30 mm, yield strength $\sigma_p = 400$ MPa). The characteristics of the soil base layers given in Table 1.

In our study, we took into account the vertical, evenly distributed load on the slabs (on all floor slabs and the foundation slab), as well as the horizontal wind load. Loads are given in Table 2.

Figures 5-6 show the results of calculating the vertical deflections (m) in the floors of the building, at characteristic points - the floors of the 12th and 6th floors (maximum deflections are shown). Deflections are presented in comparison with a similar calculation performed in ANSYS Workbench \ ANSYS APDL.

Table 2. Loads.

Load	Load step		Maximum load	
Units:	kPa	kN/m	kPa	kN/m
Cover - q_{rf}	1,5	-	30	-
Floor slab – q_{fl}	1,5	-	30	-
Wind - q_{w1}	0,5	0,152	10	3,04
Wind 2 - q_{w2}	0,7	0,212	14	4,24
Wind 3 - q_{w3}	0,45	0,136	8	2,42

Three calculation options presented: 1) the model proposed by the authors of the work based on Treschev A.A. theory for concrete model; 2) calculation in ANSYS 2019 taking into account the nonlinear strain-stress diagram for concrete $R^- = 28,4$ MPa [8]; 3) calculation in ANSYS 2019 taking into account the linear strain-stress diagram for $R^- = 28,4$ MPa concrete.

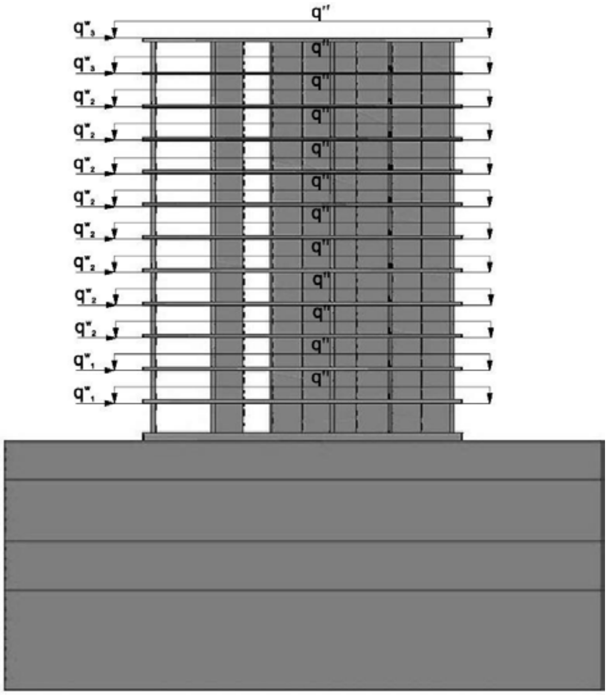

Figure 3. South façade.

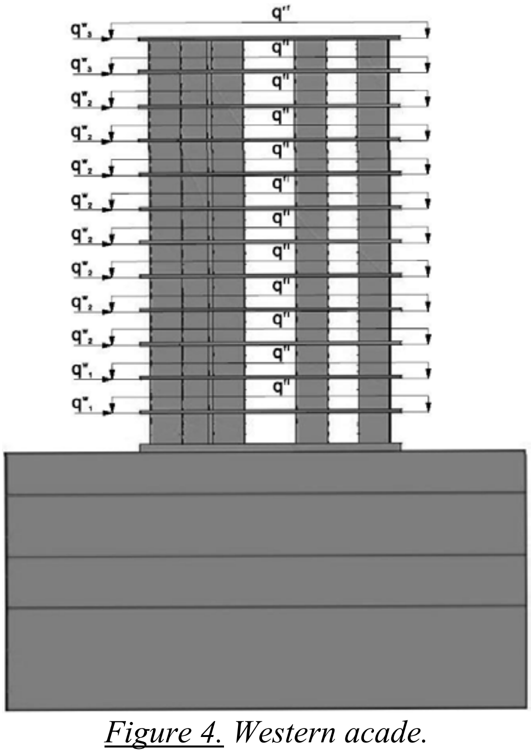

Figure 4. Western acade.

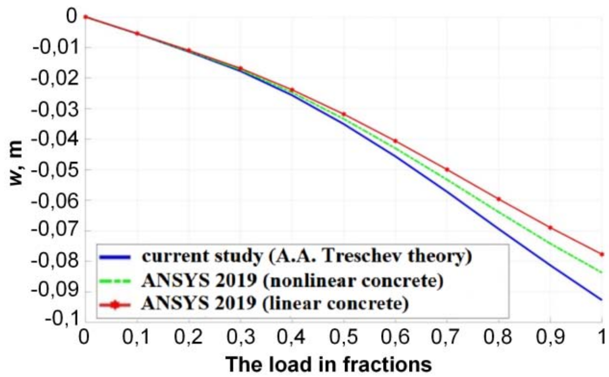

Figure 5. Maximum vertical movement at a point (12th floor).

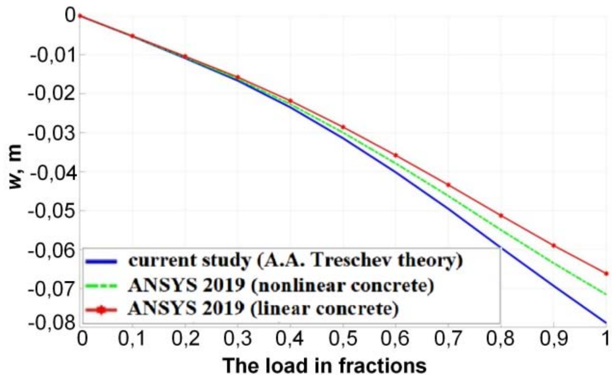

Figure 6. Maximum vertical movement at a point (6th floor).

4. CONCLUSION

Based on the results obtained, the following conclusions can be drawn:

1. The model of different resistance, adopted in accordance with the work of [2, 3, 8], showed good results in terms of determining displacements, the results differ from those obtained as a result of nonlinear calculation in the ANSYS program, at the extremum point by 24 %.
2. A numerical experiment to solve the problem of determining the stress-strain state of a building allows us objectively state that this model has several advantages over existing ones implemented in popular CAD systems. The model allows one to take into account the material's different resistance, cracking, plastic deformations in the reinforcement, the final element constructed is obtained in the form convenient for its software implementation, which was demonstrated in this work, it is also possible to take into account the work

of the elastic state for calculating combined structures.

Failure to take into account the phenomenon of different resistance, as well as the effects associated with crack formation in reinforced concrete structures, leads to significant errors in the calculation of the main characteristics of the stress-strain state of structures. To obtain reliable results of engineering calculations and to prevent the occurrence of emergency conditions of structural elements and structures, it is necessary to take into account the influence of the complicated properties of materials in full.

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